Continuum Random Phase Approximation

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The description of the energy spectrum above the nucleon emission threshold requires a proper treatment of the continuum part of the single particle configuration space. The correct expression of the creation operator of an excited state $|\nu>$, $Q^\dagger_\nu$, in the RPA theory is given by:

$$Q^\dagger_\nu = \sum_{ph} \sum_{\epsilon_p > \epsilon_F}^{\infty} \left(X^\nu_{ph}(\epsilon_p) a^\dagger_p a_h - Y^\nu_{ph}(\epsilon_p) a^\dagger_h a_p \right) \quad (1)$$

that considers a sum on the discrete energy state of bound particle and an integration up to infinity on the continuum energy. The infinite limit of Eq. (1) makes the RPA equations numerically intractable in their traditional formulation. The most used approach is to use a truncated, and often also discrete, configuration space. An alternative solution is found by rewriting the RPA equations in coordinate space representation using two new variables: the channel functions $f_{[p]h}(r)$ and $g_{[p]h}(r)$ defined as:

$$f_{[p]h}(r) = \sum_{\epsilon_p > \epsilon_F}^{\infty} X_{ph}(\epsilon_p) R_p(r, \epsilon_p) \quad (2)$$

$$g_{[p]h}(r) = \sum_{\epsilon_p > \epsilon_F}^{\infty} Y_{ph}(\epsilon_p) R_p(r, \epsilon_p) \quad (3)$$

where $[p]$ indicates all the quantum numbers characterizing the single particle state except the energy ϵ_p. With these two variables the RPA equations are transformed into a set of integro-differential equations. A possible way of solving the continuum RPA equations is to discretize the r space on a given mesh and to use finite difference numerical methods [2]. This method is applicable when zero range interactions are used. Since we want to use finite range interactions and deal with the exchange matrix elements (differently from what has been done in Refs. [3–5]), we use a technique consisting in expanding the channel functions on a complete basis [6], [7]. We choose a set of functions which already obey the correct boundary conditions: the Sturmian functions defined as in expression (4) if $\epsilon_p > 0$ and (5) if $\epsilon_p < 0$ [8]:

$$\Phi^\mu_p(r \to \infty) \to \lambda \frac{H^-_p(\epsilon_p, r)}{r} \quad (4)$$

$$\Phi^\mu_p(r \to \infty) \to \chi \frac{1}{r} exp \left[-r \left(\frac{2m|\epsilon_p|}{\hbar^2} \right)^{\frac{1}{2}} \right] \quad (5)$$

where with H^- we indicate the Hankel functions. As example we show in the upper panel of Fig. 1 the real and imaginary part of a set of Sturmian functions for protons of positive energy of 30 MeV in ^{16}O with orbital angular momentum $l = 1$ and total angular momentum $j = 3/2$. In the lower panel we show the Sturmian functions for protons of negative energy of -30 MeV with orbital angular momentum and $l = 0$ and total angular momentum $j = 1/2$ and $l = 1$ and $j = 3/2$.

The number of nodes of the Sturmian functions is linked to the index μ. We observe that when the index increases by one unity an additional node appears in the wave function. The Sturmian functions are constructed to have the same asymptotic behaviour independently of the number of nodes: Hankel function behaviour for Sturmian with positive energy and decreasing exponential behaviour otherwise.

We expand the channel functions $f^{[p_0]h_0}_{[p]h}$ and $g^{[p_0]h_0}_{[p]h}$ on the basis of the orthogonalized Sturmian functions ($\widetilde{\Phi}$) according to

$$\begin{aligned} f^{[p_0]h_0}_{[p]h}(r) &= R_{p_0}(r)\delta_{[p][p_0]}\delta_{hh_0} + \sum_\mu c^{+\mu}_{ph} \widetilde{\Phi}^{+\mu}_p(r) \\ g^{[p_0]h_0}_{[p]h}(r) &= \sum_\mu c^{-\mu}_{ph} \widetilde{\Phi}^{-\mu}_p(r) \end{aligned} \quad (6)$$

where the symbols $+$ and $-$ indicate that the sturmian functions are evaluated for $\epsilon_p = \epsilon_h + \omega$ or $\epsilon_p = \epsilon_h - \omega$ respectively where ω is the excitation energy and R is the free radial wave functions. In this way we obtain a system of linear equations whose unknowns are the expansion coefficients.

We show some results of this approach obtained with the interactions defined in [9]. We start our discussion of the 1^- excitation in the ^{16}O nucleus. We checked that the spurious isoscalar excitation related to the center of mass motion, is not present in the excitation spectrum. In the upper panel of Fig. 2 we compare the results obtained with the data of Ref. [10].

We first remark that the four responses are rather similar: the number and the position of the

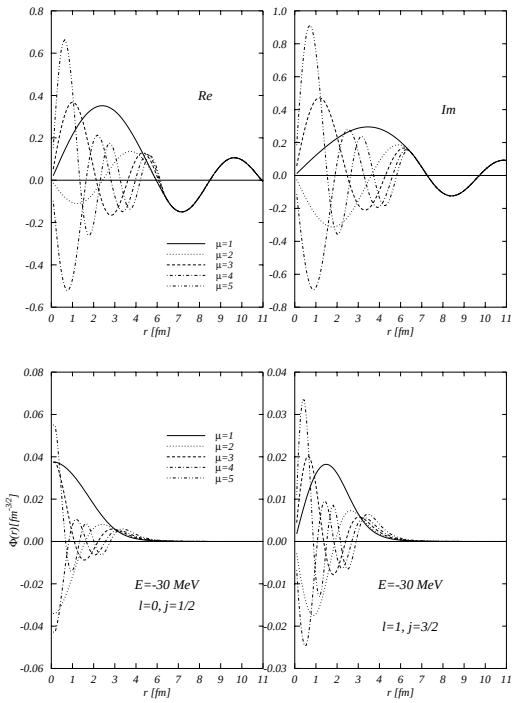

Figure 1. Sturmian functions relative to an energy of 30 MeV (upper panel) and −30 MeV (lower panel). These Sturmian functions have been calculated for a proton with $l = 1$ and $j = 3/2$ in ^{16}O in the upper panel and for a proton with $l = 0$ and $j = 1/2$ and $l = 1$ and $j = 3/2$ in the lower panel.

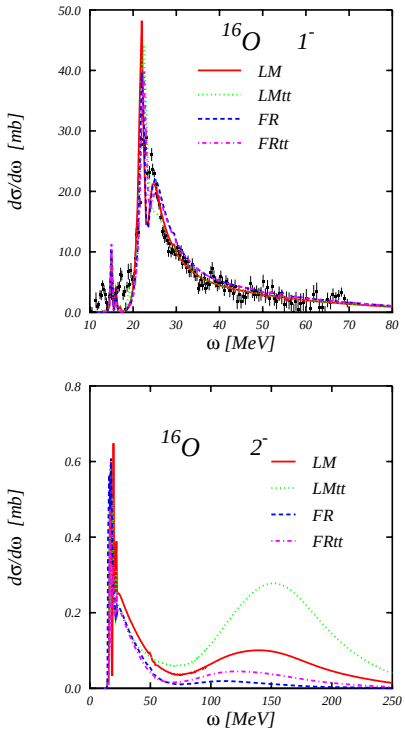

Figure 2. Photoabsorption cross sections for the 1^- excitation compared with the data of Ref. [10] and for 2^- excitation.

maxima are essentially the same for the four calculations. The comparison with the data shows that the position of the peak is well reproduced. We should remember that the parameters of the interaction have been adjusted to reproduce properties of the low-lying spectrum, by using a discrete, and limited, single particle configuration space. The position of the peak of the resonance is a genuine prediction of our calculations. Clearly the magnitude of the cross section is too high and the width is narrower than the experimental one. These are well known features of the Continuum RPA calculations [3,11,2,5]. The inclusion of excitations more complex than the 1p-1h considered by the RPA is probably curing these problems [12]. The situation is quite different for the 2^- excitation (lower panel of Fig. 2): now the responses show a remarkable sensitivity to the finite range of the force, and also to the presence of the tensor channel.

Even if the results we have shown are limited to the ^{16}O nucleus and to the contribution to the total photoabsorption cross section, we think that the potentiality of our approach is demonstrated.

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