Recent advances in AdS/CFT correspondence

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An entirely new way to explore fundamental interactions of matter is offered by the correspondence between quantum theories of gauge fields and string theories. The best understood example of a gauge theory that possesses a dual string theory is the maximally supersymmetric $\mathcal{N} = 4$ super Yang-Mills (SYM) theory, which certainly provides a unique way to quantitatively explore, via its dual string model in an AdS space-time, the behavior of strongly coupled gauge fields and to understand their relation to gravity. A fundamental insight of the last years is that planar $\mathcal{N} = 4$ SYM and its string dual, free Type IIB superstrings on the $AdS_5 \times S^5$ background, might be described by an integrable model. Notable efforts have been made in order to translate the conjectured integrability into the solvability of the theory and there is now strong evidence that the first ever exact solution of a four-dimensional interacting quantum gauge theory is within our reach. Concrete tools (Thermodynamic Bethe Ansatz, or equivalently Y-system) have been proposed to solve the full planar spectrum of both theories, and it is also becoming clear that many other observables, such as scattering amplitudes and correlation functions, can be determined in this way. This approach will undoubtedly provide a basis for studying a large class of gauge theories relevant to Nature, and it should improve at least qualitatively our understanding of general features of QCD. It is a general fact that, while $\mathcal{N} = 4$ SYM and QCD are in many details different, a compared analysis of their properties has already been crucial for a deeper understanding of both of them. Integrability itself appeared for the first time in four-dimensional gauge field theories in a QCD context, in the high-energy Regge behavior of scattering amplitudes and in the scale dependence of composite operators. A notable common issue between $\mathcal{N} = 4$ SYM and QCD is their infrared structure, and it is believed that QCD would benefit a lot from an ultimate all-loop solution of its superconformal version, since this would provide a representation for the dominant part of the perturbative gluon dynamics. Furthermore, several quantities show a universal behaviour throughout the class of four-dimensional gauge theories, e.g. tree-level gluon scattering, highest transcendentality part etc. The latter, so-called maximum transcendentality principle, is the observation that the anomalous dimensions of leading twist operators of the theory (in $\mathcal{N} = 4$ SYM, as well as in QCD, this is twist-two) may be extracted directly from the corresponding anomalous dimensions of QCD by keeping terms of highest transcendentality. Even deeper connections exist between this line of research and the theory of strong interactions, the more celebrated one being the case of the cusp anomaly. For this important observable of all four-dimensional gauge theories which governs the scaling behavior of many gauge-invariant quantities (logarithmic growth of anomalous dimensions for twist operators, Sudakov asymptotics of form factors, etc.) as well as the energy of the relevant dual string configurations (e.g. strings spinning inside the AdS space with large angular momentum), integrability techniques have provided in the case of $\mathcal{N} = 4$ SYM an integral equation. This has been solved at weak and at strong coupling, and its agreement both with gauge perturbative results from gluon scattering amplitudes and superstrings is one of the most striking tests of the AdS/CFT correspondence. Another fascinating and as yet not fully understood link between QCD, $\mathcal{N} = 4$ SYM and string theory is centered on the so-called reciprocity, and consists in a surprising pattern that emerges in studying all the available anomalous dimensions of twist-two operators in QCD, their analogue in $\mathcal{N} = 4$ SYM together with the energies of their dual string configurations. The reciprocity condition is a constraint on the large spin behavior of a transform of the anomalous dimension, which should run in even negative powers of the Casimir of the collinear group $SL(2, R)$.

This constraint, arising in the QCD context, has been presented in as a special (space-time symmetric) reformulation of the parton distribution function evolution equations, while in it has been approached from the point of view of the large spin expansion and generalized to operators of arbitrary twist. Reciprocity has been already checked in all the available twist-two anomalous dimensions in QCD , various multi-loop calculations of weakly coupled $\mathcal{N} = 4$ SYM gauge theory as well as in the dual string theory framework. Its predictive power for the spectrum of the theories has been already successfully employed to work out, via integrability techniques, multiloop analytic analytic formulas various anomalous dimensions. On the other side of the correspondence, strings in the $AdS_5 \times S^5$ background (described by a Green-Schwarz action whose bosonic part is a coset sigma model and for which fermionic degrees of freedom are included via a fermionic Wess-Zumino term with local κ-symmetry) are classically integrable, in that they possess an infinite number of conserved non-local quantities. The classical integrable structure of the theory can also be seen via an algebraic curve approach, in which the investigation of the monodromy of the Lax connection for the $AdS_5 \times S^5$ superstring action leads to the derivation of a spectral curve for any solution of the classical string equations of motion. The study of relevant classical solutions such as the folded string of or the minimal surface ending on two antiparallel lines in AdS has been crucial to identify the correct string dual description for the gauge twist operators, or for the $\mathcal{N} = 4$ SYM generalized version of the quark-antiquark potential. However, already at one-loop in sigma model perturbation theory, the evaluation of quantum corrections to the energies of classical string solutions in $AdS_5 \times S^5$ is in general a hard mathematical problem. The task is simplified considering scaling limits of some semiclassical parameters, in which the solutions become linear in the world-sheet coordinates thus making constant the coefficients in the fluctuation Lagrangian. Similar powerful simplifications apply for higher loop calculations. In this framework, recent findings reveal that fluctuations over classical, integrable solutions are governed by special, integrable, differential operators. These observations might lead to other ways of detecting the quantum integrability of strings in curved backgrounds, which is up to now an unproven assumption. Another important description of the integrable structures for the planar AdS/CFT system arises when treating it as a two-dimensional particle model. On the gauge theory side, this is due to the correspondence between the $\mathcal{N} = 4$ SYM theory and the one-dimensional spin chain, with fundamental excitations (magnons) identified as impurities in the chain whose scattering is described by an internal S-matrix constrained by symmetries. From the string theory point of view, one also considers a world-sheet S-matrix, describing the scattering of the excitations in the two-dimensional quantum field theory description of closed strings propagating in AdS backgrounds. For this description to be consistent, one has to introduce a suitable decompactification limit (for which the worldsheet does not have anymore the topology of a cylinder and becomes an infinite plane in which asymptotic states can be defined) as well as relax any constraint on the particle momenta. The scattering of these massive excitations has been described at tree level, showing that S-matrix exhibits the same structure appearing in the gauge theory and, in support of integrability, has tree-level factorization. At higher loop orders, results have been only possible in the simplified setting of the so-called near-flat limit. As mentioned above, an area that has also seen significant recent progress and presents related integrable structures is the calculation of spacetime scattering amplitudes in supersymmetric gauge theories. In particular, the connection with Wilson loops have led to a deeper understanding of the gauge theory. Many of these results have been possible because of the development of powerful new tools, among which generalized unitarity methods for evaluating on-shell quantities in Yang-Mills theories. However, in the dual string framework it is still missing a satisfying argument supporting, beyond the classical level, the proposed connection between scattering amplitudes and Wilson loop.

The detailed activity of our group can be found in the references.

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