About the breakup of aggregates in turbulent flows

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In many different applications (see e.g. [1]), it happens that small solid aggregates such as clusters or flocs are suspended in turbulent flows. By a vigorous stirring of the vessel where the liquid/solid solution is contained, it is possible to breakup such aggregates. Hydrodynamic stresses due to the turbulent nature of the flow act to prevent the formation of very large aggregates and contribute to disperse them. However, turbulence is also crucial for the reverse process, i.e. transformation of small particles of colloidal size into aggregates of few micrometres to millimetres. So turbulence is used to improve the performance of a liquid/solid separator.

From a theoretical point of view, breakup is assumed to be a first-order kinetic process, i.e. the breakup rate is proportional to the mass concentration of the solid phase. This can be recast in a simple equation for the time evolution of the distribution $Q(x,t)$ of clusters of mass x:

$$\partial_t Q(x,t) = -f(x,t)Q(x,t) + \int_x^\infty f(y,t)g(x,y)Q(y,t)dy\,, \qquad (1)$$

where $f(x)$ is the breakup rate function. In this equation, the first term accounts for loss of clusters of mass x due to breakup, while the second term accounts for the production of fragments of mass $(x, x+dx)$ formed by the the breakup of clusters of larger mass $y \geq x$.
Over the years, many efforts have been devoted to modelling the breakup rate function $f(x)$ and the fragment mass distribution $g(x,y)$, in a turbulent solution. As a starting point for our study, we focused on the modelling of the breakup rate $f(x)$, based on the work of K.A. Kusters [2] and V.I. Loginov [3]. In particular, the latter considers that the breakup of a single small fragment in a stationary turbulent flow is a very fast process (almost instantaneous), fully determined by the fluctuations of the kinetic energy dissipation ϵ in its vicinity. For the sake of clarity, we recall that

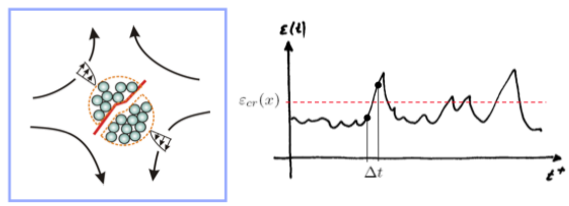

Figure 1. A cartoon of the breakup process associated to the existence of a critical value of the turbulent kinetic energy dissipation ϵ_{cr}.

kinetic energy dissipation in a turbulent flow is defined as $\epsilon = \nu/2\sum_{i,j}(\partial_i u_j + \partial_j u_i)^2$, where ν is the fluid kinematic viscosity and $\partial_j u_i$ are the fluid velocity gradients.
The idea is that for any cluster of mass x there exists a critical value of the energy dissipation $\epsilon_{cr} = \epsilon_{cr}(x)$, so that the aggregate breaks up as soon as it experiences such critical value along its trajectory (see Loginov 1985, for more details).

Preliminary results have been obtained for the measure of the breakup rate of very small aggregates, behaving as tracer objects (with no inertia with respect to the fluid), in a fully developed turbulent flow [4]. For this, we have used data obtained from high resolution Direct Numerical Simulations of a statistically homogeneous and isotropic turbulent flow, seeded with millions of tracer particles, at Taylor scale based Reynolds number $Re_\lambda = 400$ (with a numerical resolution of 2048^3 grid points) [5].

Results suggest that the Loginov model well captures the basic features of the breakup phenomenon: a large mass breaks very easily, but it is also true that it is not very probable to have a large mass in the system. Indeed a large mass in the system is observed only under the condition

that it has spent all the time before the breakup event in regions of low energy dissipation. So for large masses, the breakup frequency cannot be very high: in the limit of very large masses $x \to \infty$, the breakup rate has to go to a finite limit.
On the other hand, breakup of smaller and smaller masses should be less and less frequent. So in the limit of fragments of very small mass, the breakup rate has to vanish.

In Figure 2, it is shown the curve for breakup rate function $f(\epsilon_{cr}(x))$ as measured from the Loginov model. This is compared to a simple closure model that we derived. The agreement of the two curves is good for large values of the critical energy dissipations -corresponding to aggregates of small mass; while it does not properly work in the limit of small critical energy dissipations -corresponding to large masses. Further work is needed.

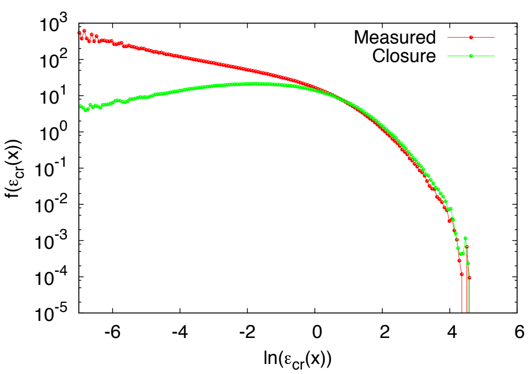

Figure 2. Plot of the breakup rate function $f(\epsilon_{cr}(x))$ obtained from the Loginov model, compared to the approximate form that we propose. Data are obtained from DNS of homogeneous and isotropic turbulence at $Re_\lambda = 400$, seeded with tracer particles.

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REFERENCES

1. M. Soos et al., J. Colloid Interface Sci. **319**, 577–589 (2008).
2. K.A. Kusters, "The influence of turbulence on aggregation of small particles in agitated vessels", PhD Thesis, Technische Universiteit Enindhoven, 1991.
3. V.I. Loginov, *Dynamics of the process of breakup of a liquid in a turbulent stream*, J. App. Mech. Tech. Phys. **26**, Number 4, 509-515, 1985 DOI: 10.1007/BF01101633. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 6673, JulyAugust, 1985.
4. Uriel Frisch, "Turbulence: the legacy of A.N. Kolmogorov", Cambridge University Press, Boston (1995).
5. J. Bec, L. Biferale, M. Cencini, A.S. Lanotte, & F. Toschi, *Intermittency in the velocity distribution of heavy particles in turbulence*, Journ. Fluid Mech. **646**, 527536 (2010).