Thermostatistics in the neighborhood of the π–mode solution for the Fermi–Pasta–Ulam β system: from weak to strong chaos

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Since its discovery, the celebrated Fermi–Pasta–Ulam (FPU) system [1] has represented a paradigmatic model for the analysis of energy equipartition, stochastic resonances [2] and thermalization in nonlinear systems (for a recent account, see [3]). In order to explain its rich phenomenology, several approaches have been proposed. Zabusky and Kruskal [4], by analyzing the string dynamics in the continuum limit, discovered solitary waves and started the modern theory of nonlinear integrable systems. Another approach, due to Izrailev and Chirikov [5] and many others, was addressed to the numerical determination of stochasticity thresholds, that marked the transition from recurrences to thermalization and equilibrium.

In the last two decades it has been shown that many complex systems possess weakly chaotic regimes, such as those exhibiting long–range particle interactions, strong correlations, scale invariance, properties of multifractality, etc. New physical phenomena are expected at the edge of chaos.

The approach first proposed in [6] aims to a generalization of the standard statistical mechanics. As is well known, the Boltzmann–Gibbs thermostatistics offers the natural theoretical framework to describe nonintegrable and fully chaotic dynamics. This yields eventually to ergodicity and mixing in phase space. A natural question is how to describe situations when the system exhibits a weakly chaotic behavior, the ergodic hypothesis typically is not verified and the statistical mechanics of Boltzmann and Gibbs (BG) fails to provide a correct theoretical framework. The classical picture is usually restored in the strongly chaotic regime. The approach, nowadays called *nonextensive statistical mechanics* [7], has been proposed in order to handle these more general situations, and in particular deals with the case of power–law divergencies of the sensitivity to the initial conditions. At the heart of the theory there is a generalization of the Boltzmann–Gibbs entropy, the S_q entropy, that depends on a real parameter, the entropic index q.

The literature on this topic has been increasing dramatically in the recent years (for a regularly updated bibliography, see [8]).

Motivated by this current research, in [9] we have analyzed the statistical behaviour of the FPU β system [1] when a π–mode solution is initially excited. We have describe both numerically and analytically, as a function of the energy density, the transition of the system from a *stable* to a *strongly chaotic regime* by following the time evolution of a suitable observable associated to the *exact π–mode solution*. This observable physically corresponds to a geometric *symmetry* of the system that is lost when the system is perturbed. The analysis of this observable offers a very accurate tool for the study of the evolution of the system.

An interesting result of our investigation is that in the weakly chaotic regime the model appears to be described by the Tsallis (q–Gaussian) statistics.

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