Sinusoidal and Gaussian Kicking of a Dissipative Oscillator Map

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It is know that a very interesting case of maps is that of a conservative harmonic oscillator perturbed by sequence of periodically applied δ-like pulses (kicks) at times $t = T, 2T, 3T, \ldots$. This system has been proposed as a model for studying quantum chaos in trapped ions [1,2], ionization of atoms by a train of pulses [3] or electronic transport in semiconductor super-lattices [4]. Usually, the system is resolved by integrating the equations of motion over the period T [5,6]. When the perturbation is sinusoidal and the dissipation is zero a well-know discrete map is obtained, called *web-map* [7]. We follows a different approach. We start from the equation, which describes the motion of a linear oscillator subjected to an external position-dependent force $\ddot{x} + \omega_0^2 x + 2\gamma\dot{x} = F(x)$. By discretizing the time $t_n = n\Delta t$, and introducing suitable variables and parameters, we obtains the map

$$\begin{aligned} z_{n+1} &= [bw_n + k\varphi(z_n)]\sin(\alpha) + z_n\cos(\alpha) \\ w_{n+1} &= [bw_n + k\varphi(z_n)]\cos(\alpha) - z_n\sin(\alpha), \end{aligned}$$

where $\varphi(z)$ models the perturbation, b the dissipation, and $\alpha = 2\pi/q$ (q = integer) sets the resonance condition. In fact, discretization introduces an additional half degree of freedom and produces an effect equivalent to the action of a periodic sequence of δ-pulses. Besides, differently from what happens for the standard treatment, the map can be continued to $b = 0$, where becomes one-dimensional.

For weak k, dissipation destroys the stochastic net of phase plane ($b = 1$) and periodic attractors P_N of period $N = nq$, $(n = 1, 2, \ldots)$ appear and disappear by saddle node bifurcation and period doubling.

Increasing k toward the instability region of origin, chaotic bands and strange attractors appear. We studied (i) sinusoidal $\varphi(z) = \sin(z)$ and (ii) Gaussian $\varphi(z) = ze^{-z^2}$ pulses, which give complementary properties to the system: forcing symmetry and resonance symmetry dominance, respectively. With (i), coexistence between chaotic and periodic attractors (P_1 attractors, together their fast period doubling cascade) is evidenced in particular intervals of the forcing parameter k. With (ii), q-symmetrical periodic points are found also for very large k, and chaotic states bifurcate by crisis in periodic windows.

Therefore, the system shows various and interesting properties, which can be investigated in the parameters space.

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