A weakly random Universe?

V.G.Gurzadyan[1] A.E.Allahverdyan[1] T.Ghahramanyan[1] A.L.Kashin[1] H.G.Khachatryan[1] A.A.Kocharyan[12] S.Mirzoyan[1 3 4] E.Poghosian[1] D.Vetrugno[5 6] G.Yegorian[1]

[1]Yerevan Physics Institute and Yerevan State University, Yerevan, Armenia

[2]School of Mathematical Sciences, Monash University, Clayton, Australia

[3]Dipartimento di Fisica "E.R. Caianiello", Universita' degli Studi di Salerno, Fisciano, Italy

[4]Institute of Theoretical Physics, University of Zurich, Zurich, Switzerland

[5]Dipartimento di Fisica, Università del Salento, Via per Arnesano, I-73100 Lecce, Italy

[6]Istituto Nazionale di Fisica Nucleare, Sezione di Lecce, Via per Arnesano, I-73100 Lecce, Italy

1. Introduction

The high-accuracy Planckian spectrum, the quadrupole anisotropy, and the acoustic peaks of the power spectrum [1–3] of the cosmic microwave background (CMB) appear to be carriers of valuable information on the early Universe. Among other peculiarities of the CMB signal is that it is described well by a Gaussian distribution of initial fluctuations. This is remarkable at least for two reasons. Gaussian fluctuations follow from the simplest single scalar field version of inflation. The Gaussian is also a limiting distribution, in accordance with the central limit theorem, as the sum of many independent random variables of finite variance. CMB fitting the Gaussian certainly does not reveal its randomness, as it might arise from both random and regular distributions.

Is it possible to reveal the fractions of random and regular signals in the CMB?

One can deal with this problem using the Kolmogorov distribution and the stochasticity parameter as a rigorous measure of randomness [4,5]. Arnold applied this technique to arithmetical and geometrical progressions and to number sequences [5]), and it has also been applied to CMB temperature datasets [7]. Now, based on the 7-year CMB data obtained by the Wilkinson Microwave Anisotropy Probe (WMAP) [8] we have derived a general result, i.e., that the random signal yields about 0.2 fraction in the CMB total signal.

We start by briefly outlining the approach. We consider a sequence $\{X_1, X_2, \dots, X_n\}$ of a random variable X ordered in an increasing manner $X_1 \le X_2 \le \dots \le X_n$. Cumulative distribution function of X is defined as the probability of the event $X \le x$

$$F(x) = P\{X \le x\},$$

and an empirical distribution function $F_n(x)$ as

$$\begin{aligned} F_n(x) = 0\ , \quad & for\, x < X_1\ ; \\ k/n\ , \quad & for\, X_k \le x < X_{k+1},\quad k = 1, 2, \dots, n-1\ ; \\ 1\ , \quad & for\, X_n \le x\ . \end{aligned}$$

Then the Kolmogorov's stochasticity parameter λ_n, which is also a random quantity, is defined as the normalized deviation of those two distribution functions

$$\lambda_n = \sqrt{n}\ \sup_x |F_n(x) - F(x)|\ . \quad (1)$$

Here, the multiplier $\sqrt{n}$ appears to account for the deviation expected for n independent observations of a genuinely random variable.

The remarkable result is Kolmogorov's proof [4] ("the astonishing Kolmogorov's theorem" as Arnold [6] called it), concerning the asymptotic behavior of the distribution of λ for any continuous F

$$\lim_{n\to\infty} P\{\lambda_n \le \lambda\} = \Phi(\lambda)\ ,$$

where the limiting distribution for any real $\lambda > 0$ is

$$\Phi(\lambda) = \sum_{k=-\infty}^{+\infty} (-1)^k\ e^{-2k^2\lambda^2},\ \Phi(0) = 0, \quad (2)$$

the convergence is uniform, and Φ, the Kolmogorov's distribution, is independent of the distribution F of the initial random variable X. The interval of probable values of λ yields $0.3 \le \lambda_n \le 2.4$ [4]. This universality of Kolmogorov's distribution marks it as a measure of a stochasticity degree of datasets [5]).

This objective measure enables one to consider the composition of signals of various randomness; e.g., the behavior of λ_n and Φ was studied at numerical simulations of sequences [9,10].

In this report, we undertake a novel analysis of the cosmological CMB signal revealed by the

Φ distribution. We obtain the Kolmogorov distribution $\Phi(\lambda)$ for different regions of sky and WMAP's 7-year W-band data [8] in HEALPix representation [11].

Considering CMB as a composition of random and regular signals, we solve the inverse problem of recovering of their mutual fractions from the temperature sky maps. Deriving the empirical Kolmogorovs function in the Wilkinson Microwave Anisotropy Probes maps, we obtain the fraction of the random signal to be about 20 per cent; i.e., the cosmological sky is a weakly random one.

For further details, we address the reader to the paper [12].

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