Electromagnetic nuclear charge operator to one-loop order of ChPT

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One of the main advantages of the chiral effective field theory (χEFT) approach to nuclear structure is its capability of a unified description of interactions and currents: the latter can be viewed at the same time as the Noether currents of the global symmetries of strong interactions, and as external (weakly coupled) sources. It is then possible to describe electroweak processes in few-nucleon systems in a fully consistent way, with potentials and currents derived in the same framework.

We have already developed in Ref. [1] a NN potential and one- and two-body electromagnetic currents up to one loop in the perturbative chiral series, corresponding to the next-to-next-to-next-to-leading order (N3LO). The calculation was performed within the framework of time-ordered perturbation theory, in order to clearly disentangle two-nucleon reducible and irreducible contributions: the former have to be discarded since they are generated by the iteration of the dynamical (Lippmann-Schwinger) equation. In this respect, particular care has to be taken in the treatment of recoil corrections to the reducible diagrams, since they produce subtle partial cancellations of the irreducible ones. Loop diagrams are handled in dimensional regularization and the renormalization program is consistently applied.

The above work has been extended in Ref. [2] to derive systematically the two-nucleon electromagnetic charge operator to the same accuracy. This is not a trivial step, since it demands to address issues related to the non-static contributions to the one-pion-exchange and two-pion-exchange potentials: the origin of such a complication is the fact that the electromagnetic charge operator starts to contribute at one order less than the corresponding current operator, in the chiral counting. We have therefore been lead to explore more precisely the connection between the amplitudes calculated in χEFT and the strong and electromagnetic potentials, which are derived from it and are used in quantum-mechanical formulations, based on the Lippmann-Schwinger or Schroedinger equations. Consider the perturbative expansion for the two-nucleon scattering amplitude $\langle f \mid T \mid i \rangle$,

$$\langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\,\eta} H_1 \right)^{n-1} \mid i \rangle \ , \quad (1)$$

where $\mid i \rangle$ and $\mid f \rangle$ represent the initial and final two-nucleon states of energy $E_i = E_f$, H_0 is the Hamiltonian describing free pions and nucleons, and H_1 is the Hamiltonian describing interactions among these particles. The evaluation of this amplitude is carried out in practice by inserting complete sets of H_0 eigenstates between successive terms of H_1. Power counting is then used to organize the expansion in powers of $p/\Lambda_\chi \ll 1$, where $\Lambda_\chi \simeq 1$ GeV is the typical hadronic mass scale. Notice that the expansion above contains both irreducible and reducible contributions: the latter stem from purely nucleonic intermediate states, and are enhanced in the chiral counting since the corresponding energy denominators involve only nucleon kinetic energies $\sim O(p^2)$. Unsuppressed energy denominators are further expanded as

$$\frac{1}{E_i - E_I - \omega_\pi} = -\frac{1}{\omega_\pi} \left[1 + \frac{E_i - E_I}{\omega_\pi} + \dots \right] \ , \quad (2)$$

where E_I denotes the kinetic energy of the intermediate two-nucleon state, and ω_π, generically, the pion energies. The ratio $(E_i - E_I)/\omega_\pi$ is $\sim O(p)$. As a result, the two-nucleon amplitude admits the following expansion

$$T = T^{(0)} + T^{(1)} + T^{(2)} + \dots \ , \quad (3)$$

where $T^{(n)} \sim O(p^n)$. The needed two-nucleon potential v has to be defined such that, when iterated in the Lippmann-Schwinger equation,

$$v + v\, G_0\, v + v\, G_0\, v\, G_0\, v + \dots \ , \quad (4)$$

where G_0 is the free two-nucleon propagator, it leads to the T-matrix in Eq. (3), order by order in the power counting. Assuming that v has an expansion

$$v = v^{(0)} + v^{(1)} + v^{(2)} + \dots \,, \tag{5}$$

where the yet to be determined $v^{(n)}$ is $\sim O(p^n)$, we obtain

$$v^{(0)} = T^{(0)} \,, \tag{6}$$

$$v^{(1)} = T^{(1)} - \left[v^{(0)}\, G_0\, v^{(0)}\right] \,, \tag{7}$$

$$v^{(2)} = T^{(2)} - \left[v^{(0)}\, G_0\, v^{(0)}\, G_0\, v^{(0)}\right] - \left[v^{(1)}\, G_0\, v^{(0)} + v^{(0)}\, G_0\, v^{(1)}\right] \,, \tag{8}$$

$$v^{(3)} = T^{(3)} - \left[v^{(0)}\, G_0\, v^{(0)}\, G_0\, v^{(0)}\, G_0\, v^{(0)}\right] - \left[v^{(1)}\, G_0\, v^{(0)}\, G_0\, v^{(0)} + \text{permutations}\right] - \left[v^{(2)}\, G_0\, v^{(0)} + v^{(0)}\, G_0\, v^{(2)}\right] \,, \tag{9}$$

where $v^{(n)}$ is the "recoil-corrected" two-nucleon potential. As it appears, the very definition of the potential at a given order involves the lower orders potentials *off the energy shell*. This introduces an ambiguity, concerning $v^{(2)}$ in our case, which however has no observable consequences, since the different possible choices leads to unitarily equivalent potentials. This was observed already in the 70s by Friar at the level of the one-pion-exchange. We have therefore proved that this holds also at the two-pion-exchange level in the framework of the chiral expansion.

Analogously, the electromagnetic transition operator can be expanded as

$$T_\gamma = T_\gamma^{(-3)} + T_\gamma^{(-2)} + T_\gamma^{(-1)} + \dots \,, \tag{10}$$

where $T_\gamma^{(n)}$ is of order $e\,p^n$ (e is the electric charge). The nuclear charge, ρ, and current, $\mathbf{j}$, operators follow from $v_\gamma = A^0\,\rho - \mathbf{A}\cdot\mathbf{j}$, where $A^\mu = (A^0, \mathbf{A})$ is the electromagnetic vector field, and it is assumed that v_γ has a similar expansion as T_γ. The $v_\gamma^{(n)}$ are then fixed from the requirement that, when iterated in the Lippmann-Schwinger equation, v_γ matches T_γ order by order in the chiral expansion. The chiral expansion for the charge operator ρ starts with $\rho^{(-3)}$, and thus we need a N4LO computation to achieve an accuracy of order *ep*, as done for the current. The relevant diagram are shown in Figg. 1 and 2 We have explicitly verified by direct computation that different off-shell extensions lead to different charge operators, which however are unitarily equivalent, related through the same unitary transformation as the potentials. Thus, provided a consistent set is adopted, predictions for physical observables, such as the few-nucleon charge form factors, will remain unaffected by the non-uniqueness associated with off-the-energy-shell effects. It is important to stress that in the present work we have

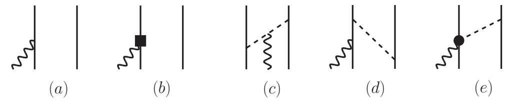

Figure 1. Diagrams illustrating one- and two-body charge operators entering at LO ($e\,p^{-3}$), panel (a), N2LO ($e\,p^{-1}$), panels (b), (c) and (d), and N3LO ($e\,p^0$), panels (e). There are no NLO contributions. Nucleons, pions, and photons are denoted by solid, dashed, and wavy lines, respectively. The square in panel (b) represents the $(p/m_N)^2$, or $(v/c)^2$, relativistic correction to the LO one-body charge operator, whereas the solid circle in panel (e) is associated with a $\gamma\pi N$ charge coupling of order $e\,Q$. Only one among the possible time orderings is shown in panels (c), (d), and (e).

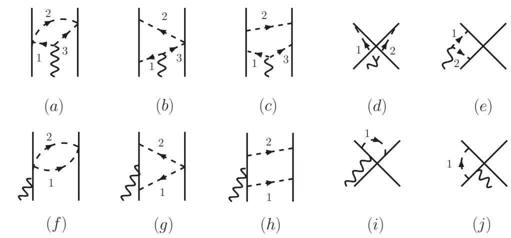

Figure 2. Diagrams illustrating one–loop charge operators entering at N^{4}LO ($e\,Q$), notation is as in Fig. 1. Only one among the possible time orderings is shown for each contribution.

only examined those off-the-energy-shell effects relating to pion retardation which arise, in TOPT amplitudes, from energy denominators containing pion (in addition to nucleon kinetic) energies. There are, of course, additional non-static corrections originating from the non-relativistic reduction of interaction vertices (generated by fully relativistic Lagrangians). It would be interesting to explore the constraints, in the present χEFT setting, that relativistic covariance and power counting impose on these non-static terms of the potentials and electromagnetic charge and current operators.

REFERENCES

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