Particle Theory at Chicago in the Late Sixties

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Abstract

As a contribution requested by the editors of a Memorial Volume for Peter G.O. Freund (1936-2018) we recall the lively particle theory group at the Enrico Fermi Institute of the University of Chicago in the late sixties, of which Peter was a memorable member. We also discuss a period some twenty years later when our and Peter's research overlapped on the topic of p-adic strings.

In memory of Peter G.O. Freund

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1   Introduction

It is a pleasure and an honour to write in memory of Professor Peter G.O. Freund (1936-2018) who spent most of his career, from 1965 to 2018, at the University of Chicago. As can be read elsewhere in this book he had been born and initially educated in Romania then acquired his PhD at the University of Vienna which explains his European sophistication. He had a passion for theoretical physics and inspired many young people, including myself, in their quest to extend human knowledge.

2   Enrico Fermi Institute

At the beginning of September 1968, I arrived at the University of Chicago group in the Enrico Fermi Institute for a two-year postdoctoral position having just finished a D.Phil in Oxford. I had a happy and productive time within the following group of particle theorists: *Faculty:* Peter Freund, Yoichiro Nambu, Reinhard Oehme, J.J. Sakurai. *Postdocs:* Lay Nam Chang, Paul Frampton, Bodo Hamprecht, Ray Rivers, Joe Scanio.

Every weekday, most of the group would attend a meeting for tea where new developments in particle theory were discussed. On one day a week there was typically a seminar by an external speaker. On Thursday afternoons, there was mandatory attendance (mentioned in the offer letter) at an Enrico Fermi seminar where a speaker was chosen from the attendees without prior warning.

After I arrived we discussed the recently-discovered Veneziano model, especially with Nambu at first then the year later with Freund. The progress by Nambu in string theory was remarkable. By the end of 1968, after some early help by me with the 4-point function he had factorised the N-point function and derived its exponential degeneracy. The Veneziano model was discovered too late for my 1968 Oxford thesis on finite-energy sum rules but was used in a similar follow-up Oxford D.Phil in 1969 by Michael Kosterlitz with the same supervisor, J.C. Taylor. Kosterlitz later went on to be awarded a Nobel prize in 2016 for very different research in condensed matter theory.

Before discussing my joint work with Peter, just to memorialise the daily tea meetings, let me recall the four faculty, all by now unfortunately deceased.

Oehme (1928-2010) was a formal expert on quantum field theory and did not usually participate in socialising but was accessible in his office for intense discussions. Sakurai (1928-1982) was a true phenomenologist who favoured the idea that photon interactions with hadrons are dominated by vector mesons, including in a paper we published jointly, later at UCLA [1]. Nambu (1921-2015) had a legendary reputation from several major accomplishments, including the discovery of spontaneous symmetry breaking in field theory for which he won a Nobel Prize in 2008. I discussed the Veneziano model with him [2] and we published one paper together [3].

Peter Freund was the youngest and most gregarious of the faculty in the group. Every afternoon he had some new idea to discuss with anybody willing to listen. His enthusiasm was contagious and he produced many publications. It was said he could conceive of an idea, do the relevant calculations and write up a paper all in one day. Certainly he had the energy of a dynamo but the flexibility to drop any idea which was demonstrably false. I did not collaborate with him during my first year because I was discussing the bosonic string with Nambu almost every weekday, as well as writing other papers with co-postdocs.

It seemed inevitable that in my second year we did coauthor two interesting papers. The Veneziano model could be interpreted as a possible description of the duality in scattering of two mesons each composed of a $(q\bar{q})$ pair of quark-antiquark. The question addressed in [4,5] was how to extend the description to baryons (qqq) and exotic hadrons such as $qq\bar{q}\bar{q}$, $qqqq\bar{q}$, etc. It was argued that the additional quarks are focused to a point on the string worldsheet. Explicit formulas were suggested to describe these situations. In [5], an imaginative construction based on a *Star-of-David* rule was hypothesised.

These two papers were written at an early time (1970) when the string picture was not fully developed but I do remember how much fun we had in writing and publishing them and they well illustrated the creative imagination of Freund who generated ideas while the rest of us did calculations.

According to my list, I coauthored work either at Chicago or later with all the other four postdocs in the group [6–10] which illustrate how Peter Freund successfully encouraged such meetings of the minds.

3 P-adic strings

Starting from 1968, string theory experienced an up-and-down history. After the enthusiasm and support for dual models of strong interactions from 1968 to 1974 interest fell off in favour of QCD as the correct strong interaction theory. But as an attempt to unify all elementary particle forces including gravity, superstrings made a huge revival during 1984-87 when anomaly-cancellation and Calabi-Yau spaces appeared to make a major breakthrough feasible. By 1987, a dramatic contact to the real world, for example derivation of one or more of the many parameters in the standard model, looked less likely because string theory, especially its numerous possible compactifications, itself contained as many or more parameters.

In 1987, by coincidence, Peter and I simultaneously noticed a surprising paper [11] coming from Russia which made a curious new observation about the Veneziano model. By expressing the Euler gamma functions therein in terms of Riemann zeta functions, different p-adic strings appeared, one for each prime number p. This impressed also Witten and my postdoc Okada and the Chicago and UNC groups began a friendly competition to understand p-adic strings better. One motivation was surely the appearance of algebraic number theory and the excitement generated by the idea that prime numbers could be useful in theoretical physics.

Freund and Witten [12] showed that the adelic infinite product over the p-adic 4-point functions gave the original Veneziano formula.

With the superscript (p) denoting a prime number and the convention that $p = \infty$ denotes the real number field so that $Q_p \to \mathcal{R}$ as $p \to \infty$ the Veneziano model is

$$A_0^{(\infty)}(s,t,u) = \int_0^1 dx |x|^{-\alpha(s)-1} |1-x|^{-\alpha(t)-1} = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \qquad (1)$$

where $\alpha(s) = 1 + \frac{1}{2}s$, $s + t + u = -8$ and therefore $\alpha(s) + \alpha(t) + \alpha(u) = -1$.

Adding the terms related by crossing symmetry

$$A^{(\infty)} = A_0^{(\infty)}(s,t,u) + A_0^{(\infty)}(t,u,s) + A_0^{(\infty)}(u,s,t) \qquad (2)$$

and define $B^{(\infty)}$ by

$$A^{(\infty)}(s,t,u) = g_\infty^2 B^{(\infty)}(-\alpha(s), -\alpha(t)) \qquad (3)$$

with

$$B^{(\infty)}(-\alpha(s), -\alpha(t)) = \int_{-\infty}^{+\infty} dx |x|^{\alpha(s)-1} |1-x|^{-\alpha(t)-1} \tag{4}$$

in which the integration range is the real field $\mathcal{R} = Q_\infty$ rather than just $(0,1)$.

The p-adic string amplitude is defined by replacing $\mathcal{R}$ with the p-adic number field Q_p.

$$B^{(p)}(-\alpha(s), -\alpha(t)) = \int_{Q_p} dx |x|_p^{-\alpha(s)-1} |1-x|_p^{\alpha(t)-1}$$

$$= \Pi_{x=s,t,u} \left(\frac{1 - p^{-\alpha(x)-1}}{1 - p^{+\alpha(x)}} \right) \tag{5}$$

Recall the definition of the Riemann zeta function:

$$\zeta(z) = \Pi_p \left(\frac{1}{1 - p^{-z}} \right) \equiv \Sigma_{r=1}^{\infty} \left(\frac{1}{r^z} \right) \tag{6}$$

in order to rewrite

$$\Pi_p B^{(p)}(-\alpha(s), -\alpha(t)) = \Pi_{x=s,t,u} \frac{\zeta(-\alpha(x))}{\zeta(1 + \alpha(x))} \tag{7}$$

Using the relationship

$$\Gamma(z)\zeta(z) = (2\pi)^{s-1} \sin(\frac{\pi z}{2}) \Gamma(1-z)\zeta(1-z) \tag{8}$$

leads to the Freund-Witten adelic infinite-product formula

$$\Pi_p B^{(p)}(-\alpha(s), -\alpha(t)) = \left[B^{(\infty)}(-\alpha(s), -\alpha(t)) \right]^{-1} \tag{9}$$

Eq.(9) was their main result. The right-hand-side is (the inverse of) the Veneziano model which is now seen to be equal to an infinite product of p-adic amplitudes. This provided the initial excitement which led to a flurry of activity in 1988 because it suggested that the p-adic string in Eq.(5) might be more basic than the bosonic string.

The next step was to generalise to the N-point functions [13] for $N \geq 5$,

Consider therefore the p-adic 5-point function

$$A_5^{(p)} - \int_{Q_p} dxdy |x|_p^{-\alpha_{35}-1}|1-x|_p^{-\alpha_{23}-1}|y|_p^{-\alpha_{45}-1}|1-y|_p^{-\alpha_{24}-1}|x-y|_p^{-\alpha_{34}-1} \quad (10)$$

where $\alpha_{ij} = 1 + \frac{1}{2}(k_i + k_j)^2$.

This double integral gives the additive form

$$A_5^{(p)} = \Sigma_{(ij)(kl)} \left(\frac{(1-p^{-1})(1-p^{-1})}{(1-p^{\alpha_{ij}})(1-p^{\alpha_{kl}})} \right) \cdot$$
$$- (2-p^{-1})\Sigma_{(ij)} \left(\frac{(1-p^{-1})}{(1-p^{\alpha_{ij}})} \right) + (2-p^{-1})(3-p^{-1}) \quad (11)$$

where the summations are over compatible (by duality) poles with 15 and 10 terms respectively.

For N=6, we find

$$A_6^{(p)} = \Sigma\Pi_{i=1}^{i=3} \frac{(1-p^{-1})}{(1-p^{\alpha_i})} - (2-p^{-1})\Sigma\Pi_{i=1}^{i=2} \frac{(1-p^{-1})}{(1-p^{\alpha_i})}$$
$$+ (2-p^{-1})(3-p^{-1})\Sigma_{ij} \frac{(1-p^{-1})}{(1-p^{\alpha_{ij}})}$$
$$+ (2-p^{-1})^2 \Sigma_{ijk} \frac{(1-p^{-1})}{(1-p^{\alpha_{ijk}})}$$
$$- (2-p^{-1})(3-p^{-1})(4-p^{-1}) \quad (12)$$

where $\alpha_{ijk} = 1 + (k_i + k_j + k_k)^2$. The 1st and 2nd sums both have 105 terms; the 3rd and 4th have 15 and 10 terms respectively.

When we examine factorisation of $N = 5$, we find

$$A_5^p \xrightarrow{\alpha_{12} \to 0} \left(\frac{(1-p^{-1})}{(1-p^{\alpha_{12}})} \right) A_4^p.$$
$$\sim - \left(\frac{1-p^{-1}}{\ln p} \right) \left(\frac{1}{\alpha_{12}} \right) A_4^p \quad (13)$$

while, for $N = 6$, at a 2-particle pole

$$A_6^p \overset{\alpha_{12} \to 0}{\longrightarrow} \left(\frac{(1 - p^{-1})}{(1 - p^{\alpha_{12}})} \right) A_5^p.$$

$$\sim \quad -\left(\frac{1 - p^{-1}}{\ln p} \right) \left(\frac{1}{\alpha_{12}} \right) A_5^p \tag{14}$$

and at a 3-particle pole

$$A_6^p \overset{\alpha_{123} \to 0}{\longrightarrow} A_4^p \left(\frac{(1 - p^{-1})}{(1 - p^{\alpha_{123}})} \right) A_4^p.$$

$$\sim \quad A_4^p \left[-\left(\frac{1 - p^{-1}}{\ln p} \right) \right] \left(\frac{1}{\alpha_{123}} \right) A_4^p \tag{15}$$

From formulas (13), (14) and (15), the prescription for the p-adic propagator is

$$\frac{(1 - p^{-1})}{(1 - p^{\alpha(k^2)})} \tag{16}$$

and the p-adic degree m vertex V_m is

$$V_m = (-1)^{m+1} \Pi_{n=2}^{m-2} (n - p^{-1}) \tag{17}$$

for $m \geq 4$ and $V_3 = 0$.

In [13], we were able to derive these Feynman rules more quickly than the Chicago group [15] by starting from the N-point function of [16] which is an Eulerian integral of the first kind, as is Eq.(1), while the Chicago group began with the N-point function of [17].

For fixed p, the p-adic string can be described by a non-local scalar field theory [14] with lagrangian

$$L_0 = \frac{1}{2} \Phi \left(\frac{1 + p^{1 + \partial \partial / 2}}{1 - p^{-1}} \right) \Phi$$

$$- \frac{1}{(p^{-1} + 1)p^{-1}(p^{-1} - 1)} \left[(1 + \Phi)^{1 + p^{-1}} - 1 - (1 + p^{-1})\Phi \right]$$

$$- \frac{(1 + p^{-1})p^{-1}}{2} \Phi^2 \right] \tag{18}$$

which gives the Feynman rules discussed above.

After 1988, because no new insight was being gained the interest in p-adic strings subsided, although it has been used in studying the vacuum of the bosonic string [18] and may yet reappear to shed light on string dynamics.

4 Discussion

Peter Freund was a special personality in the EFI-Chicago group There was one German, Oehme, and two Japanese, Nambu and Sakurai, all intellectually humble. As an intellectual, Freund was unusually extrovert and gregarious and always had new ideas which he was eager to discuss.

Sakurai and Nambu were tied by their common language and I recall one Japanese visitor reverting to their language in mid-seminar and nobody dared to point it out. Nambu was already recognised as of extraordinary creativity. Peter contacted me in the 1990s about Nambu being recognised in Stockholm and I made suggestions. Eventually Nambu was so rewarded, decades later then he deserved, in 2008.

Peter Freund was extremely respectful of Nambu when both contributed in our daily tea meetings, with Freund loquacious and Nambu relatively silent. Yet when Nambu did say something it was usually decisive because he was almost always correct.

Peter Freund made fundamental contributions about phenomenological duality and compactifications of extra spatial dimensions. The two papers I coauthored with him [4,5] are interesting and his daily brilliance left a memorable intellectual legacy with all who had the good fortune to meet and discuss with him.

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