# Quantum Field Theory and its Anomalies for Topological Matter

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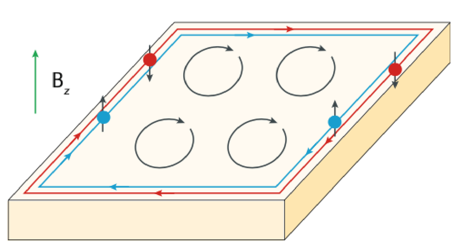
**Presented at QCD@work, Lecce, June 27-30, 2022**

## Abstract

Topology enters quantum field theory (QFT) in multiple forms: one of the most important, in non-Abelian gauge theories, is the identification of the vacuum $\theta$ angle in QCD. A very relevant aspect of this connection is through the phenomenon of chiral and conformal qft anomalies. It has been realized that a class of materials, including topological insulators and Weyl semimetals, they also exhibit the phenomenon of anomalies, which are responsible for numerous exotic phenomena. For example, the presence of edge currents, resilient to perturbations and dispersion from impurities, has been associated with qft anomalies and their topological properties. Another example comes from the response functions of these materials, which can be performed using correlation functions of the stress energy tensors in General Relativity. In this case the conformal anomaly plays an important role. In this work we briefly illustrate some salient features of this correspondence, and the effective action that describes the long-range interactions which can account for these topological effects. It can be matched with numerical studies of these materials at lattice scale.

## 1. The Quantum Hall Effect

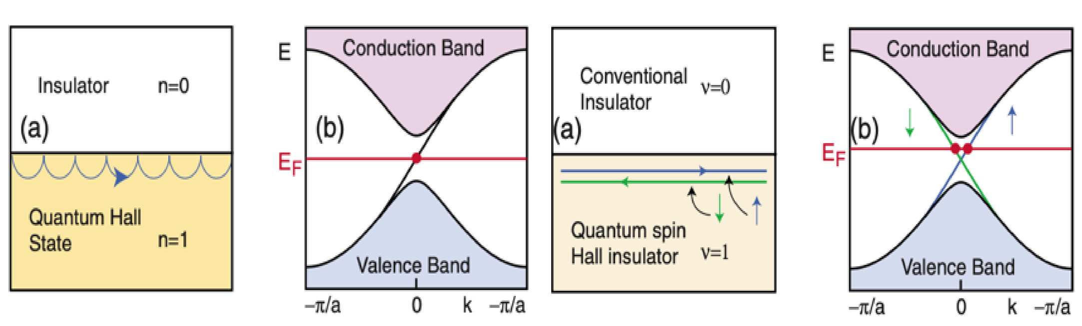
The first example of topological behavior in matter was first observed in 1980, when Klaus von Klitzing discovered a **quantized conductivity** in the Quantum Hall Effect (QHE). The QHE is the first example of a topological insulator (TI), because the bulk of the material acts as an insulator, while electrons at the boundary of the material move in a **surface current**. Therefore, the boundary is a conductor. The topological origin of such behaviour requires some comments. In fact, the quantized conductivity $\sigma_{xy} = -\frac{e^2}{2\pi\hbar}C_N$ depends on $C_N$, the Chern number, which is a topological invariant and is connected to a gauge field $A_\mu$, called the **Berry gauge potential**, and also to a field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, whose volume integral is exactly $C_N$. These currents of non-zero Chern number are **topologically protected**, since they are resilient under scattering by impurities and disorder present in the material.



(K. von Klitzing et al., 40 years of the quantum Hall effect., Nat. Rev. Phys., 2, 397-401, (2020).)

## 2. Other TI's and Bands Structure

The bands of such materials are characterized by some **edge states** located between the gapped valence band and the conductance band. We have several kind of TI's, characterized by some specific topological invariants. The QHE, nowadays recognized as the first example of TI, is characterized by a non-zero Chern number, $C_N = 1$. However, there are some materials that have Chern number $C_N = 0$, but exhibit a topological behavior. In these examples we have **two different topological surface states** associated to two separate spin-up/down configurations, despite having $\vec{B}_{ext} = 0$. We call these materials $Z_2$ TI's.




(M. Z. Hasan and C. L. Kane, Rev. of Mod. Phys., 82, (2010).)

## 3. Topological Field Theory (TFT) Description of the QHE

A Lagrangian description of the QHE is realized thanks to the Chern-Simons action. One can start from a discrete and local Hamiltonian (for example a tight binding Hamiltonian) $H[c^+_{i,\alpha}, c_{i\alpha}, A_\mu]$, describing the motion of the electrons in a periodic lattice in the presence of an external electromagnetic field. Resorting to a path integral formulation of the dynamics, we can **integrate over the fermions** and treat the EM field as a background $e^{-S_{eff}[A_\mu]} = \int Dc\dagger_{i\alpha} Dc_{i\alpha} e^{-S[c\dagger_{i\alpha}, c_{i\alpha}, A_\mu]}$. We can perform an expansion of the effective action in powers of $A_\mu$, obtaining the Chern-Simons action at the lowest order in a $A_\mu$

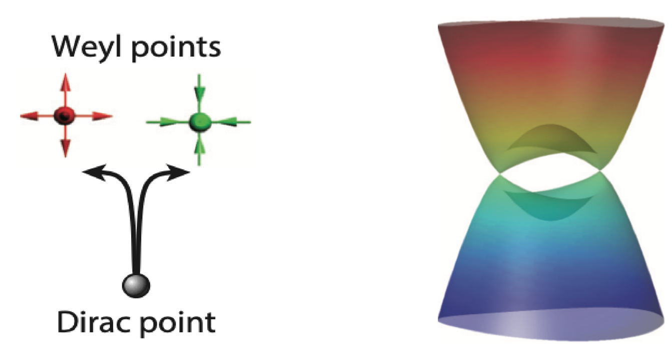
$$S_{CS} = i\frac{\sigma_H}{2}\int d^3x \epsilon_{\mu\nu\rho} A^\mu \partial^\nu A^\rho,$$

where $\sigma_H$ is a **quantized coefficient** that preserves the gauge invariance under $e^{i\theta(t)}$ of $e^{-S_{eff}}$, with $\theta(t)$ the gauge parameter. This qft description captures the long range behaviour of the material. Moreover, in (3+1)D, quite similar to (2+1)D, one may derive another expression of the topological action

$$S_\theta = \frac{i}{32\pi}\int d^4x \theta(x) \epsilon^{\mu\nu\sigma\tau} F_{\mu\nu} F_{\sigma\tau}.$$

## 4. Weyl Semimetals

In a Dirac semimetals the valence and conductance bands **touch each other in a single point**, called a Dirac point. At this specific points the energy/momentum dispersion relation is linear, similarly to the relativistic case. However this Dirac points could split into a pair of so-called Weyl points in the Weyl semimetals. In correspondence of these pairs of Weyl points the dispersion relation is exactly that of a Weyl Hamiltonian $H^{\pm}_{Weyl} = \pm\vec{p}\cdot\vec{\sigma}$. The Weyl points can be seen as **monopoles for the Berry's flux**, so they are robust and topologically protected. Also for these materials we can analyze the topological response. In fact, relative to the Weyl nodes, it happens that the **particle number is not conserved**. This means that the time variation of the densities $n_{L/R}$ is of the form $\frac{\partial}{\partial t}(n_{L/R}) = \pm\frac{e^2}{h^2}\vec{E}\cdot\vec{B}$. The presence of massless Weyl fermions is strongly and closely connected with the presence of the chiral anomaly.




(Y. Binghai and C. Felser, Topological Materials: Weyl Semimetals, Ann. Rev. of Cond. Mat. Phys., 8, 1, 337-345, (2017).)

## 5. Gravity and Topological Materials: Theoretical and Numerical Studies

Gravity can also play a fundamental role in Condensed Matter. The link is provided by the Tolman-Ehrenfest theorem on the equilibrium temperature of a gravitational system, which is position-dependent, combined with Luttinger's formula $\frac{1}{T}\nabla T = -\frac{1}{c^2}\nabla\Phi$. The basic idea is that the effect of a temperature gradient $\nabla T$ that drives a system out of equilibrium can be compensated, at linear order, by a non-uniform gravitational potential $\Phi$. Therefore, in a stationary gravitational field holds that $T(x) = T_0/\sqrt{g_{00}}$, where $g_{00}$ is the "00" component of the metric $g_{\mu\nu}$. In the case of massless QED, **renormalization induces a conformal anomaly at quantum level**

$$\langle T^\mu_\mu \rangle = b\, C^2 + b' E + c_M F_{\mu\nu}F^{\mu\nu},$$

where $T^{\mu\nu}$ is the stress energy tensor of the material and $\langle , \rangle$ is the quantum average over the fermions in the material. $C^2$ and $E$ are the square of the Weyl tensor and $E$ is the topological Euler-Poincarè density. Our investigation combines both a **theoretical and a phenomenological analysis of the response functions** associated to thermal and mechanical stresses on these materials. In particular, **the anomalous part of the response to $\langle T^\mu_\mu \rangle$, generated by a thermal/mechanical stress, can be extracted from the non-local action**

$$S_{anom}[g, A] = \frac{1}{8}\int d^4x\sqrt{g(x)}\int d^4y\sqrt{g(y)}E(x)G_4(x,y)[2bC^2(y) + b'E(y) + c_M F_{\mu\nu}F^{\mu\nu}(y)],$$

where $G_4$ is the Green function of a quartic conformally covariant operator. Its local form, obtained by the inclusion of extra scalar degrees of freedom, is central to our analysis, which is interfaced with numerical investigations of the properties of such materials at lattice level. Our study involves two different scales: the lattice scale, investigated by many-body methods, and the long-range one, that can be addressed using TFT.

## 6. References

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