Quantum Field Theory and its Anomalies for Topological Matter

1. Quantum Field Theory and its Anomalies

Topology enters quantum field theory (QFT) in multiple forms: one of the most important, in non-Abelian gauge theories, is the identification of the vacuum angle in QCD. A very relevant aspect of this connection is through the phenomenon of chiral and conformal qft anomalies. It has been realized that a class of materials, including topological insulators and Weyl semimetals, they also exhibit the phenomenon of anomalies, which are responsible for numerous exotic phenomena. For example, the presence of edge currents, resilient to perturbations and dispersion from impurities, has been associated with qft anomalies and their topological properties. Another example comes from the response functions of these materials, which can be performed using correlation functions of the stress energy tensors in General Relativity. In this case the conformal anomaly plays an important role. In this work we briefly illustrate some salient features of this correspondence, and the effective action that describes the long-range interactions which can account for these topological effects. It can be matched with numerical studies of these materials at lattice scale.

2. The Quantum Hall Effect

The first example of topological behavior in matter was first observed in 1980, when Klaus von Klitzing discovered a quantized conductivity in the Quantum Hall Effect (QHE). The QHE is the first example of a topological insulator (TI), because the bulk of the material acts as an insulator, while electrons at the boundary of the material move in a surface current. Therefore, the boundary is a conductor.

3. Topological Field Theory (TFT) Description of the QHE

A Lagrangian description of the QHE is realized thanks to the Chern-Simons action. One can start from a discrete and local Hamiltonian (for example a tight binding Hamiltonian) $H_{\text{discrete}}$, describing the motion of the electrons in a periodic lattice in the presence of an external electromagnetic field. Resorting to a path integral formulation of the dynamics, we can integrate over the fermions and treat the EM field as a background $e^{-S_{\text{fermions}}}$. We can perform an expansion of the effective action in powers of $A_\mu$, obtaining the Chern-Simons action at the lowest order in $A_\mu$

$$S_{\text{CS}} = \frac{\sigma_3}{4} \int d^2 x \epsilon_{\mu \nu \lambda} \partial \mu A_\nu \partial \lambda A^\mu,$$

where $\sigma_3$ is a quantized coefficient that preserves the gauge invariance under $e^{i\theta(t)}$ of $e^{-S_{\text{CS}}}$, with $\theta(t)$ the gauge parameter. This qft description captures the long range behaviour of the material. Moreover, in $4+1$D, quite similar to $(2+1)$D, one may derive another expression of the topological action

$$S_0 = \frac{i}{32\pi} \int d^4 \theta(x) e^{i\theta(y)} F_{\mu \nu} F^{\mu \nu}.$$

4. Weyl Semimetals

In a Dirac semimetals the valence and conductance bands touch each other in a single point, called a Dirac point. At this specific points the energy/momentum dispersion relation is linear, similarly to the relativistic case. However this Dirac points could split into a pair of so-called Weyl points in the Weyl semimetals. In correspondence of these pairs of Weyl points the dispersion relation is exactly that of a Weyl Hamiltonian $H_{\text{Weyl}} = \pm \vec{p} \cdot \vec{\sigma}$. The Weyl points can be seen as monopoles for the Berry’s flux, so they are robust and not easily destroyed. Also for these materials we can analyze the topological response. In particular, relative to the Weyl nodes, it happens that the particle number is not conserved. This means that the time variation of the densities $n_{\uparrow}/n_{\downarrow}$ is of the form $\frac{d}{dt}(n_{\uparrow}/n_{\downarrow}) = \pm \frac{1}{2} \vec{E} \cdot \vec{B}$. The presence of massless Weyl fermions is strongly and closely connected with the presence of the chiral anomaly.

5. Gravity and Topological Materials: Theoretical and Numerical Studies

Gravity can also play a fundamental role in Condensed Matter. The link is provided by the Tolman-Ehrenfest theorem on the equilibrium temperature of a gravitational system, which is position-dependent, combined with Luttinger’s formula $\mathbf{\nabla T} = \frac{1}{2} \mathbf{\nabla} \Phi$. The basic idea is that the effect of a temperature gradient $\mathbf{\nabla} T$ drives a system out of equilibrium can be compensated, at linear order, by a non-uniform gravitational potential $\Phi$. Therefore, in a stationary gravitational field holds that $T(\varphi) = T_0/\sqrt{\Omega_g}$, where $g_0$ is the $0\text{th}$ component of the metric $g_{\mu \nu}$. In the case of massless QED, renormalization induces a conformal anomaly at quantum level

$$(T^\mu_\nu) = b \nabla^\mu E + c_{\text{SM}} F_{\mu \nu} F^{\mu \nu},$$

where $T^\mu_\nu$ is the stress energy tensor of the material and $b$ is the quantum average over the fermions in the material. $c_{\text{SM}}$ and $E$ are the square of the Weyl tensor and $E$ is the topological Exler-Poincaré density. Our investigation combines both a theoretical and a phenomenological analysis of the response functions associated to thermal and mechanical stresses on these materials. In particular, the anomalous part of the response to $(T^\mu_\nu)$, generated by a thermal/mechanical stress, can be extracted from the non-local action

$$S_{\text{nonlocal}}[g, A] = \frac{1}{8} \int d^4 x \sqrt{-g(x)} \int d^4 y \sqrt{-g(y)} G_4(x, y) [2 b c_{\text{SM}}(y) + b E(y) + c_{\text{SM}} F_{\mu \nu} F^{\mu \nu}(y)],$$

where $G_4$ is the Green function of a quartic conformally covariant operator. Its local form, obtained by the inclusion of extra scalar degrees of freedom, is central to our analysis, which is interfaced with numerical investigations of the properties of such materials at lattice level. Our study involves two different scales: the lattice scale, investigated by many-body methods, and the long-range one, that can be addressed using TFT.

6. References