Neutrino and Photon Lensing by Black Holes and Radiative Lens Equations

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The Leverhulme Trust

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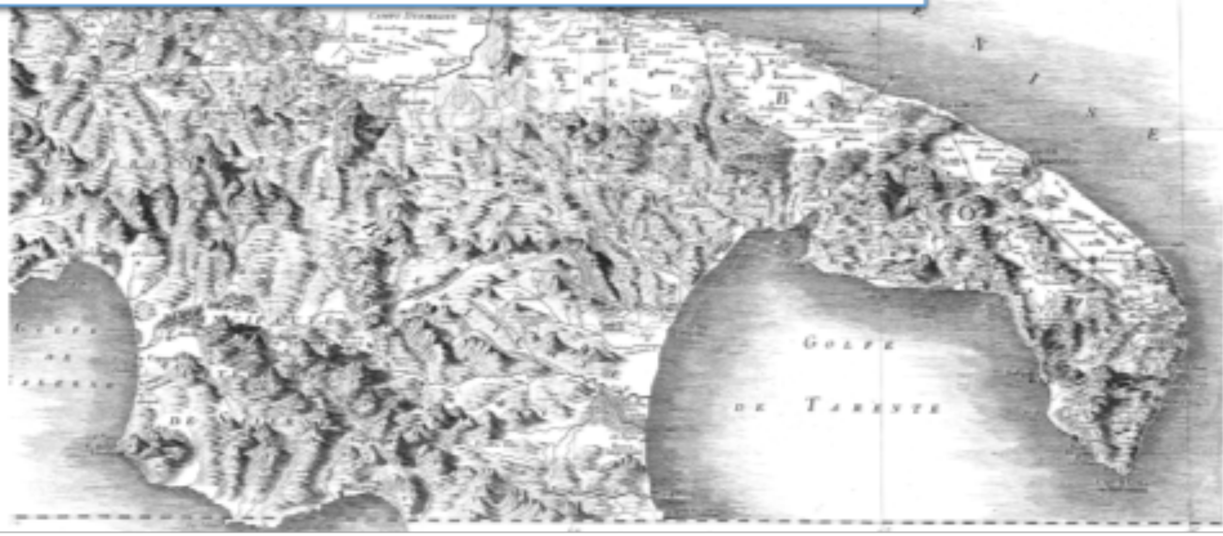


Work in collaboration with

L. Delle Rose, M. Serino,
A Costantini
M. Maglio, M. dell'Atti

Extending a perturbative analysis **with**
L. Delle Rose, M. Serino

and

L. Delle Rose, E. Gabrielli and L Trentadue

2011

Gravity and the Neutral Currents: Effective Interactions from the Trace Anomaly

Luigi Delle Rose, Mirko Serino. C.C.
arXiv:1102.4558 [hep-ph].
10.1103/PhysRevD.83.125028.
Phys.Rev. D83 (2011) 125028.

1-loop analysis of the interaction between the Standard Model and Gravity in the Neutral Currents sector

The TVV vertex is renormalizable if the Higgs is conformally coupled and can be studied in the broken electroweak phase

1) **Fermion Scattering in a Gravitational Background: Electroweak Corrections and Flavour Transitions**
Luigi Delle Rose, Emidio Gabrielli, Luca Trentadue, C.C.
arXiv:1312.7657 [hep-ph].
10.1007/JHEP03(2014)136.
JHEP 1403 (2014) 136.

2013-2014

2) **Mass Corrections to Flavor-Changing Fermion-Graviton Vertices in the Standard Model**
Luigi Delle Rose, Emidio Gabrielli, Luca Trentadue, C.C.
arXiv:1303.1305 [hep-th].
10.1103/PhysRevD.88.085008.
Phys.Rev. D88 (2013) 085008.

L. Delle Rose
E. Gabrielli
L. Trentadue

3) **One loop Standard Model corrections to flavor diagonal fermion-graviton vertices**
Luigi Delle Rose, Emidio Gabrielli, Luca Trentadue, C.C.
arXiv:1212.5029 [hep-ph].
10.1103/PhysRevD.87.054020.
Phys.Rev. D87 (2013) 5, 054020.

APPLICATIONS TO GRAVITATIONAL LENSING OF SINGLE PARTICLES

2015

Neutrino and Photon Lensing by Black Holes: Radiative Lens Equations and Post-Newtonian Contributions
Antonio Costantini, Marta Dell'Atti, Luigi Delle Rose, C.C.
arXiv:1504.01322 [hep-ph].
10.1007/JHEP07(2015)160.
JHEP 1507 (2015) 160.

Electroweak Corrections to Photon Scattering, Polarization and Lensing in a Gravitational Background and the Near Horizon Limit
Luigi Delle Rose, Matteo Maria Maglio, Mirko Serino, C.C.
arXiv:1411.2804 [hep-ph].
10.1007/JHEP01(2015)091.
JHEP 1501 (2015) 091.

Develop a formalism which can help us understand the structure of the radiative corrections in weak gravity,

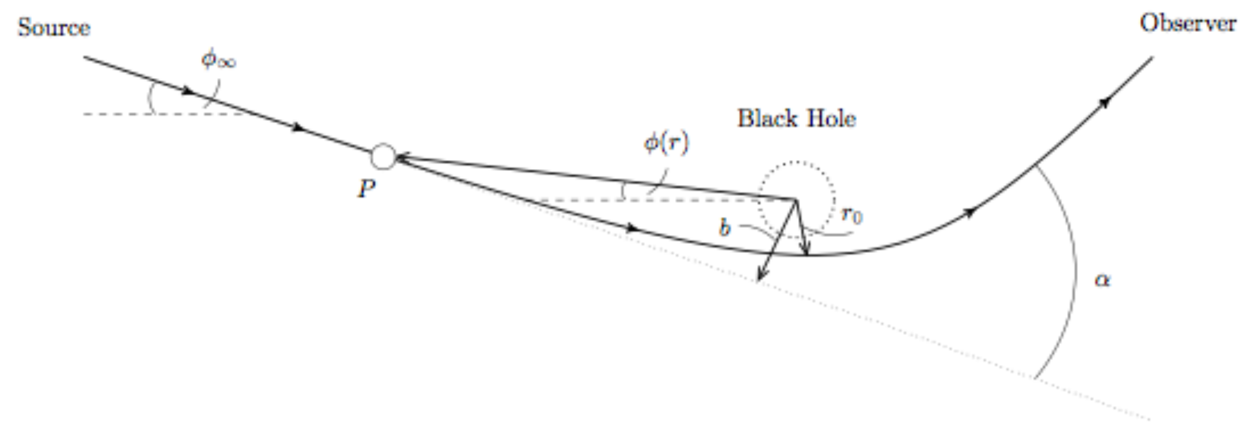
Explore the implications of the conformal anomaly

Quantify the impact of these corrections in the deflection of neutrinos and photons

Lensing effects can be studied both for weak and strong lensing

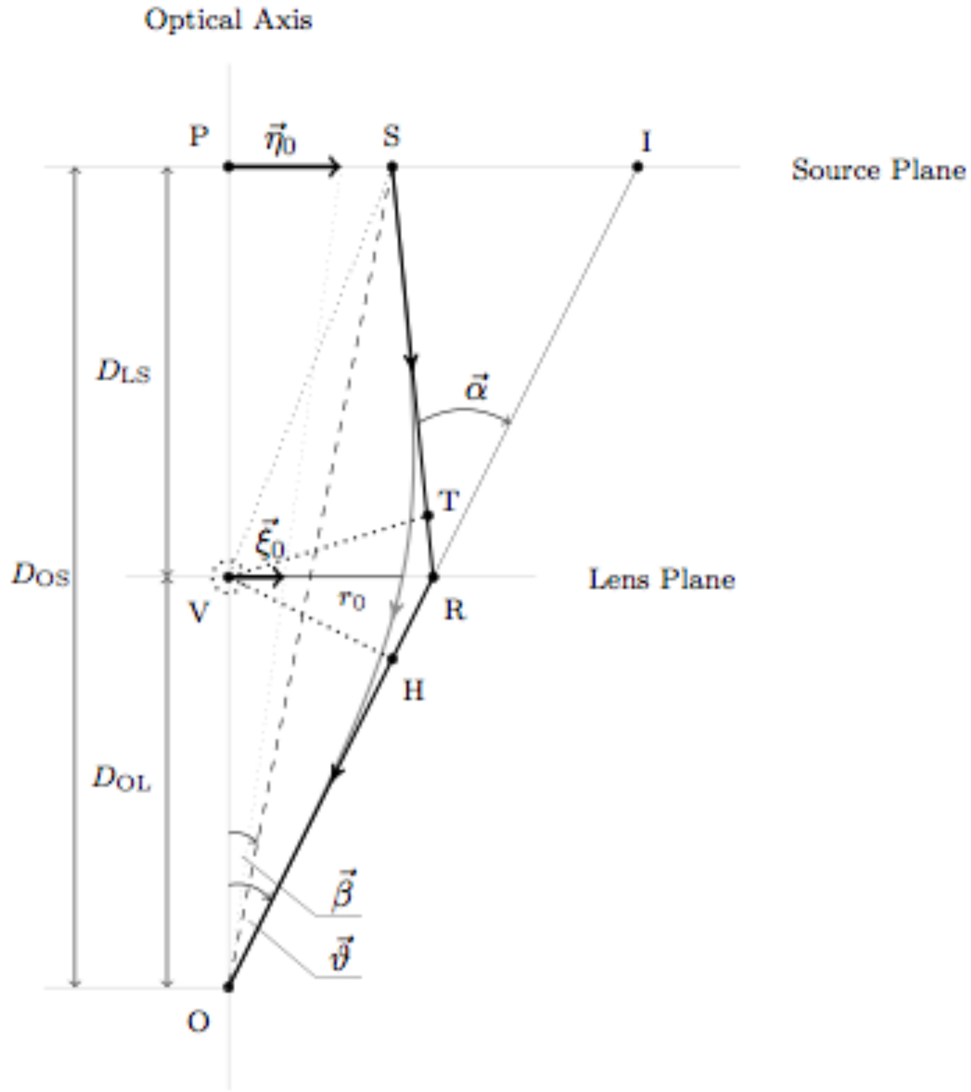
Weak lensing: Deflections are about 1-2 arcseconds

Strong lensing : 30 arcseconds or more

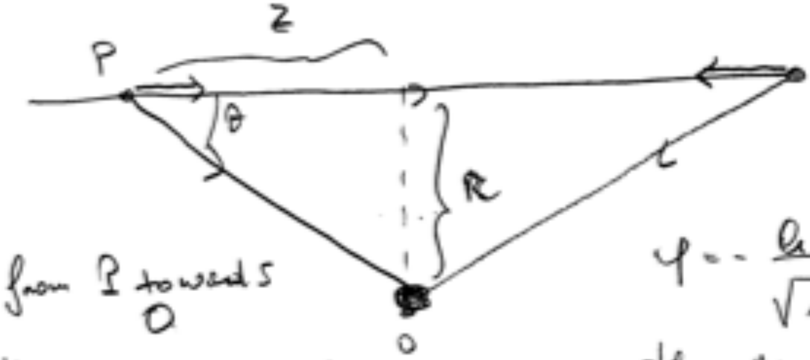


Source
ϕ_∞
Black Hole
$\phi(r)$
P
r_0
b
α
Observer

The lens geometry



Scattering off a fixed point mass.



longitudinal components give zero net contribution

Consider also from P towards O

$$a = \frac{GM}{r^2+z^2}$$

$$\varphi = -\frac{GM}{\sqrt{r^2+z^2}}$$

$$\frac{d\varphi}{dr} = a_\downarrow = \frac{GMr}{(r^2+z^2)^{3/2}}$$

$$a_\perp = \frac{GM}{(r^2+z^2)} \cdot \sin\theta \quad , \text{ use } \sqrt{r^2+z^2}\,\sin\theta = r$$

$$\Leftarrow \quad \frac{r}{\sqrt{r^2+z^2}} \simeq \theta$$

$$a_\perp = \frac{GM\,r}{(r^2+z^2)^{3/2}}$$

The change in the radial velocity, with $z = ct$, $dt = \frac{dz}{c}$

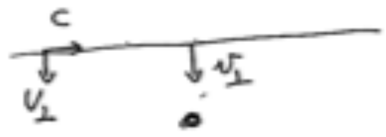
$$v_\perp = \int a_\perp dt = \int \frac{GMr}{(r^2+z^2)^{3/2}} \frac{dz}{c}$$

$$= \boxed{\frac{2}{c}\int_0^\infty \frac{GMr\,dz}{(r^2+z^2)^{3/2}}}$$

GLL

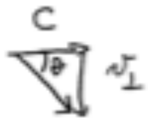
integrate $$\int_0^{\infty} \frac{dz}{(r^2+z^2)^{3/2}} = \frac{1}{r^2}$$

then $$v_\perp = \frac{2}{cr^2} GMr = \boxed{\frac{2GM}{cr}}$$



to obtain the bending angle

$$\boxed{\theta \simeq \frac{v_\perp}{c} = \frac{2GM}{c^2 r}}$$



$\frac{v_\perp}{c} = \tan\theta \approx \theta$

in GR $\Rightarrow$ $$\boxed{\frac{2GM}{c^2 r} = \frac{4GM}{c^2 r}}$$ factor of 2

The factor of 2

$$g_{\mu\nu} = \begin{pmatrix} 1 + \frac{2\Phi}{c^2} & 0 & 0 & 0 \\ 0 & -(1 - \frac{2\Phi}{c^2}) & 0 & 0 \\ 0 & 0 & -(1 - \frac{2\Phi}{c^2}) & \\ 0 & 0 & 0 & -(1 - \frac{2\Phi}{c^2}) \end{pmatrix}$$

Now light propagates at zero eigentime, $\mathrm{d}s = 0$, from which we gain

$$\left(1 + \frac{2\Phi}{c^2}\right) c^2 \mathrm{d}t^2 = \left(1 - \frac{2\Phi}{c^2}\right) (\mathrm{d}\vec{x})^2 \, .$$

$$n = c/c' = \frac{1}{1 + \frac{2\Phi}{c^2}} \approx 1 - \frac{2\Phi}{c^2} \, .$$

c' is lower than in vacuum.

The light speed in the gravitational field is thus

$$c' = \frac{|\mathrm{d}\vec{x}|}{\mathrm{d}t} = c\sqrt{\frac{1+\frac{2\Phi}{c^2}}{1-\frac{2\Phi}{c^2}}} \approx c\left(1+\frac{2\Phi}{c^2}\right) ,$$

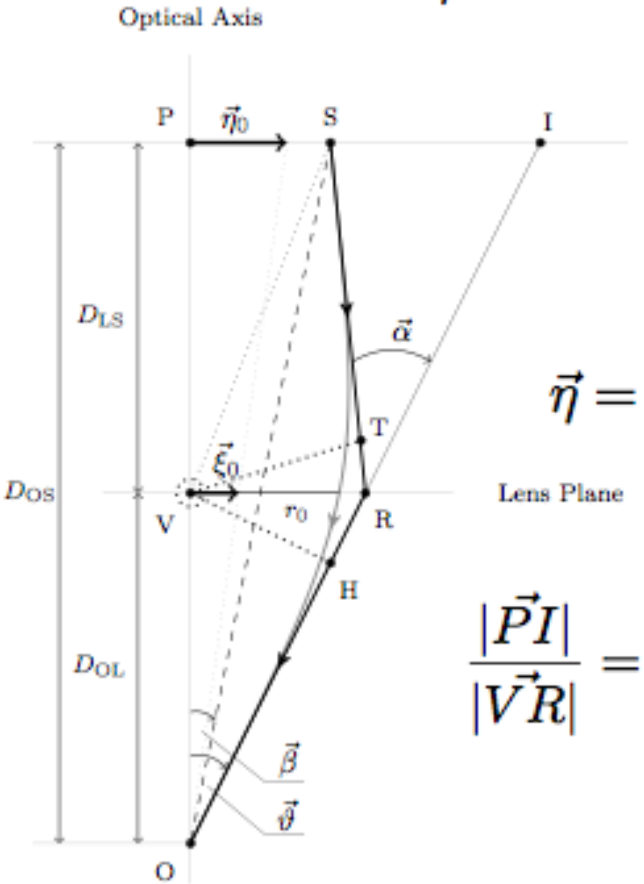
where we have used that $\Phi/c^2 \ll 1$ by assumption. The index of refraction is thus

$$\int_A^B n[\vec{x}(l)]\mathrm{d}l ,$$

$$\delta \int_A^B n[\vec{x}(l)]\mathrm{d}l = 0 .$$

The lens geometry

$$\vec{\eta} \equiv \vec{PS} = \vec{\beta} D_{OS} \qquad \vec{SI} = \hat{\vec{\alpha}} D_{LS} \qquad \vec{PI} = \vec{\theta}_I D_{OS}.$$

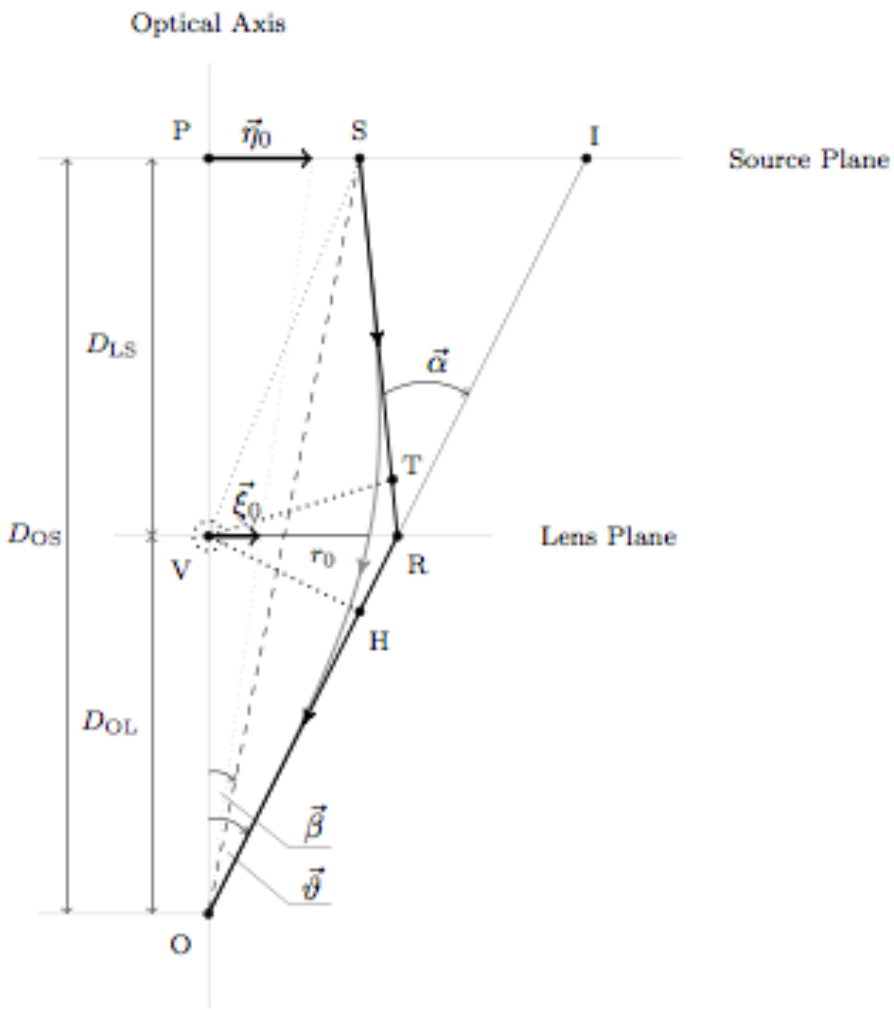


$$\vec{PI} = \vec{PS} + \vec{SI} \qquad \text{i.e.} \qquad \vec{\beta} = \vec{\theta}_I - \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}}.$$

$$\vec{\eta} = \eta_0 \, \vec{y} \qquad \vec{\xi} \equiv \vec{VR} = \xi_0 \vec{x} \qquad \frac{\eta_0}{\xi_0} = \frac{D_{OS}}{D_{LS}}.$$

$$\frac{|\vec{PI}|}{|\vec{VR}|} = \frac{D_{OS}}{D_{OL}},$$

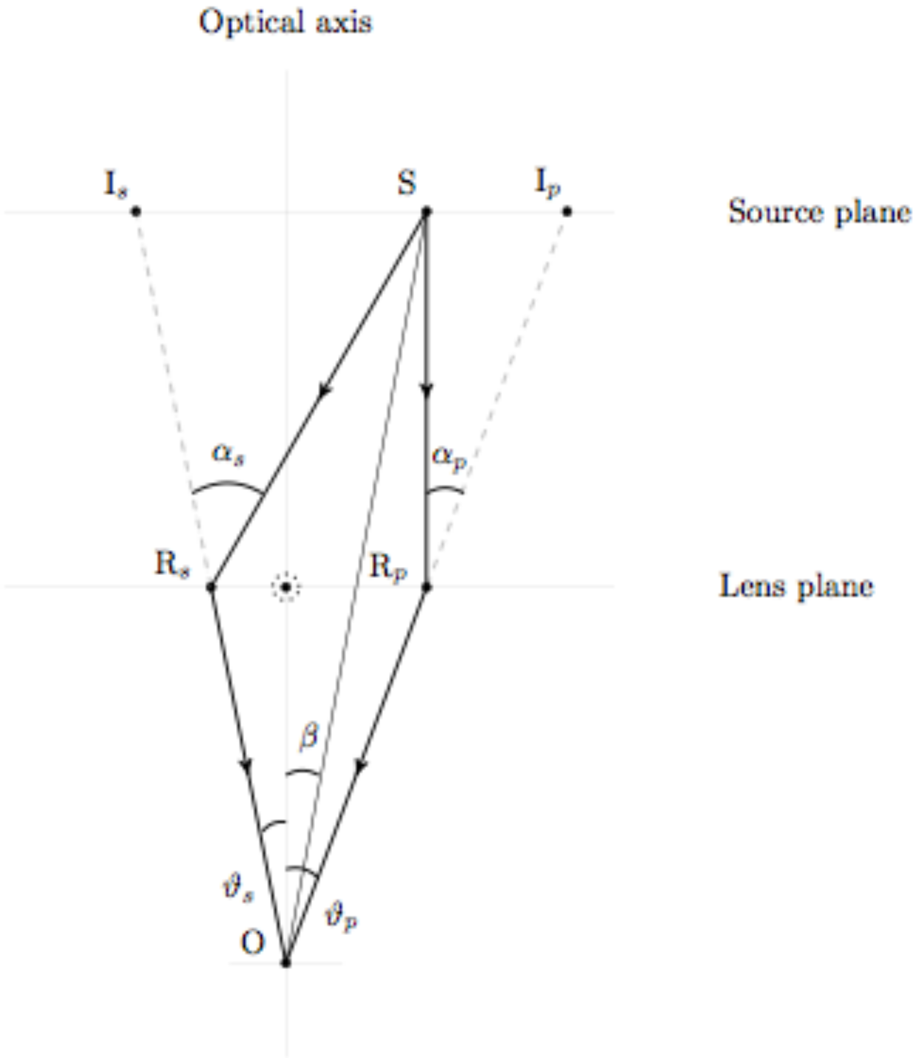
$$\vec{y} = \vec{x} - \hat{\vec{\alpha}} \frac{D_{LS} \, D_{OL}}{D_{OS} \, \xi_0} \equiv \vec{x} - \vec{\alpha}$$



$$\beta = \theta_I - \alpha \frac{D_{LS}}{D_{OS}},$$

$$\beta = \theta_I - \frac{\theta_E^2}{\theta_I} \qquad \theta_E^2 = \frac{D_{LS}}{D_{OS}} \frac{4GM}{D_{OL}},$$

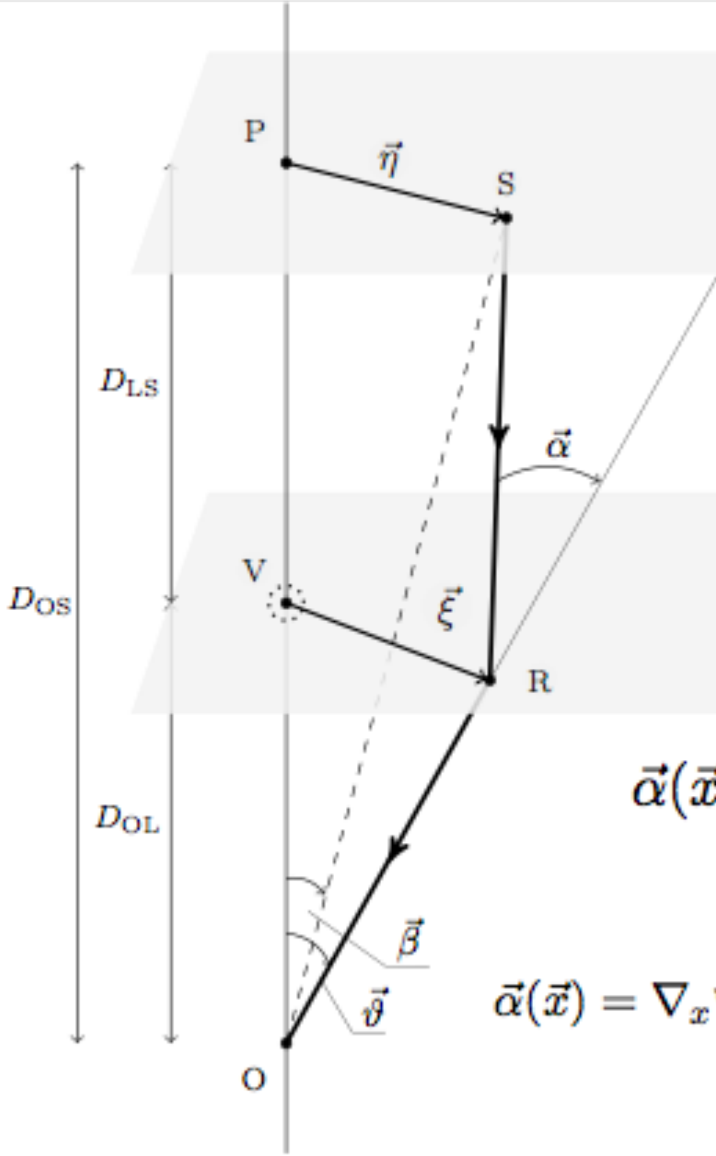
$$\alpha = 4GM/b$$



$$\theta_{I\pm} = \frac{\beta}{2} \pm \frac{1}{2}\left(\beta^2 + 4\theta_E^2\right)^{1/2}.$$

$$\beta = \theta_I - \frac{\theta_E^2}{\theta_I} - \sum_{n\geq 2} \frac{\theta_E^{(n)}}{\theta_I^n},$$

$$\theta_E^{(n)} \equiv r_s^n a_n \frac{D_{LS}}{D_{OS} D_{OL}^n}.$$



$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z)\, dz.$$

$$\hat{\vec{\alpha}}(\vec{\xi}) = 4\, G\, M \int d^2\xi' \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}\, \Sigma(\vec{\xi}')$$

after the rescaling $\vec{\xi} = \xi_0 \vec{x}$,

$$\hat{\vec{\alpha}}(\vec{x}) = 4\, G\, M \int d^2x' \xi_0 \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^2}\, \Sigma(\xi_0 \vec{x}')\,.$$

$$\vec{\alpha}(\vec{x}) = 4\, G\, M\, \frac{D_{LS}\, D_{OL}}{D_{OS}}\, \nabla_x \int d^2x' \log|\vec{x} - \vec{x}'| \Sigma(\xi_0 \vec{x})$$

$$\vec{\alpha}(\vec{x}) = \nabla_x \boldsymbol{\Psi}(\vec{x}) \qquad \boldsymbol{\Psi}(\vec{x}) \equiv \frac{1}{\pi} \int d^2x' \log|\vec{x} - \vec{x}'|\, \frac{\Sigma(\xi_0 \vec{x})}{\Sigma_{cr}}$$

$1/b^n$ **corrections to lensing for discrete and continuous mass distributions in GR**

$$\hat{\vec{\alpha}}(\vec{\xi}) = \int d^2\xi' (\vec{\xi} - \vec{\xi'}) \left(4GM \frac{\Sigma(\vec{\xi'})}{|\vec{\xi} - \vec{\xi'}|^2} + \frac{15\pi}{4} (GM)^2 \frac{\Sigma^2(\vec{\xi'})}{|\vec{\xi} - \vec{\xi'}|^3} + \frac{128}{3} (GM)^3 \frac{\Sigma^3(\vec{\xi'})}{|\vec{\xi} - \vec{\xi'}|^4} \right)$$

$$\mathbf{\Psi}(\vec{x}) = \frac{1}{\pi \, \Sigma_{cr}} \int d^2x' \left(\log |\vec{x} - \vec{x}'| \, \Sigma(\xi_0 \vec{x}) - \frac{15\pi}{16} \frac{1}{|\vec{x} - \vec{x}'|} \frac{G\,M}{\xi_0} \Sigma^2(\xi_0 \vec{x}) - \frac{128}{24} \frac{1}{|\vec{x} - \vec{x}'|^2} \frac{G^2M^2}{\xi_0^2} \Sigma^3(\xi_0 \vec{x}) \right).$$

Costantini, Delle Rose, Dell'Atti, C.C.

CLASSICAL LENSING

$$g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0,$$

with λ an affine parameter of the geodesic. The equations of motion can be separated in the form

$$\left(1 - \frac{2M}{r}\right)\frac{dt}{d\lambda} = E \qquad r^2\frac{d\phi}{d\lambda} = J \qquad \frac{d\theta}{d\lambda} = 0,$$

By setting $u \equiv \dfrac{J}{E}$ with u denoting the impact parameter ($u \equiv b_h$).

the geodesic equation becomes

$$\left(1 - \frac{2M}{r}\right)\frac{1}{r^2} + \frac{1}{J^2}\left(\frac{dr}{d\lambda}\right)^2 - \frac{1}{u^2} = 0,$$

$$\theta_d(r_0) = \int_{r_0}^{\infty} dr \frac{2}{r^2} \left[\frac{1}{u^2} - \frac{1}{r^2} \left(1 - \frac{2M}{r} \right) \right]^{-1/2} - \pi.$$

r_0 is the point of closest radial approach between the source and the beam extremum condition $dr/d\lambda = 0$

$$u = r_0 \left(1 - \frac{2M}{r_0} \right)^{-1/2}$$

$$\theta_d(r_0) = \int_{r_0}^{\infty} dr \frac{2}{r^2} \left[\frac{1}{r_0^2} \left(1 - \frac{2M}{r_0} \right) - \frac{1}{r^2} \left(1 - \frac{2M}{r} \right) \right]^{-1/2} - \pi.$$

with $x_0 \equiv r_0/(2M)$ gives

$$b_h \equiv u = x_0 \left(1 - \frac{1}{x_0}\right)^{-1/2},$$

$$\theta_d(x_0) = 2\int_{x_0}^{\infty} \frac{dx}{x\sqrt{\left(\frac{x}{x_0}\right)^2 \left(1 - \frac{1}{x_0}\right) - \left(1 - \frac{1}{x}\right)}} - \pi.$$

$$x_0 = \frac{\sqrt[3]{\frac{2}{3}}b_h^2}{\sqrt[3]{\sqrt{3}\sqrt{27b_h^4 - 4b_h^6} - 9b_h^2}} + \frac{\sqrt[3]{\sqrt{3}\sqrt{27b_h^4 - 4b_h^6} - 9b_h^2}}{\sqrt[3]{2}3^{2/3}}.$$

Delle Rose, Maglio, Serino C.C.

a singularity located at $b_h = 3/2\sqrt{3} \equiv b_h^0$ (i.e. $x_0 = 3/2$),

$$\theta_d(x_0) = -\pi - 4\,\mathbf{F}\left(\phi(x_0), \lambda(x_0)\right)\Sigma(x_0)$$

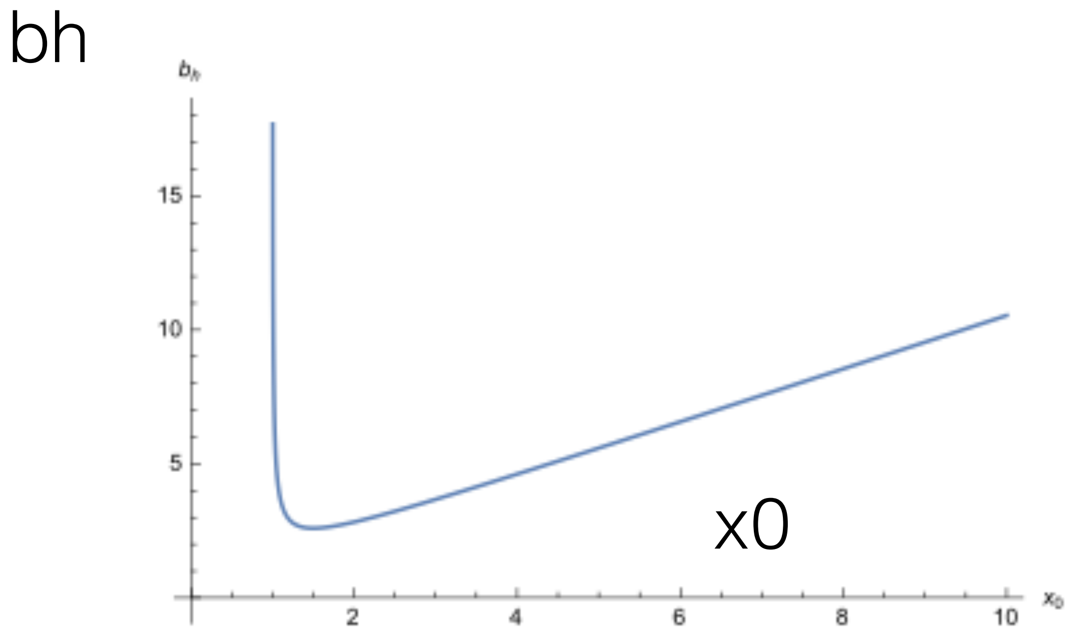
The two factors are imaginary,
Their product is real for x0 >3/2

$$\mathbf{F}(\phi, \lambda) = \int_0^{\phi} \frac{d\theta}{\sqrt{1-\lambda^2 \sin^2\theta}}$$

elliptic integral

$$\begin{aligned}
\phi_0(x_0) &= \mathrm{Arcsin}(\tau(x_0))\,, \\
\tau(x_0) &= \sqrt{\frac{-3+x_0-\sqrt{-3+2x_0+x_0^2}}{2(-3+2x_0)}}, \\
\Sigma(x_0) &= \sqrt{\frac{x_0\left(-\sqrt{x_0{}^2+2x_0-3}+3x_0-3\right)}{(3-2x_0)\left(\sqrt{x_0{}^2+2x_0-3}-x_0+1\right)}},
\end{aligned}$$

$$\lambda(x_0) = \frac{3-x_0-\sqrt{-3+2x_0+x_0^2}}{3-x_0+\sqrt{-3+2x_0+x_0^2}}.$$



Plot of b_h versus x_0, showing the singularity at the position of the photon sphere for $x_0 = 3/2$.

$$\theta_d(x_0) = 4\sqrt{\frac{2\,x_0}{Y}}\left[\mathbf{F}\left(\frac{\pi}{2},\kappa\right) - \mathbf{F}\left(\mathrm{Arcsin}\left(\sqrt{2}\sqrt{\frac{2\,x_0-2}{6\,x_0+Y-6}}\right),\kappa\right)\right] - \pi$$

$$Y = \sqrt{4(x_0-1)(x_0+3)}\,, \qquad \kappa = \frac{-2\,x_0+Y+6}{2Y}.$$

Alternative formula

The nature of the singularity around $x_0 = 3/2$ can be easily worked out by setting $x_0 = 3/2+\epsilon$

$$\begin{aligned}\theta_d(3/2+\epsilon) &\sim -4\mathbf{F}\left(\mathrm{Arcsin}\left(\frac{1}{\sqrt{3}}\right),1\right) - \pi + \log(324) - 2\log\epsilon \\ &= 0.00523507 - 2\log\epsilon\end{aligned}$$

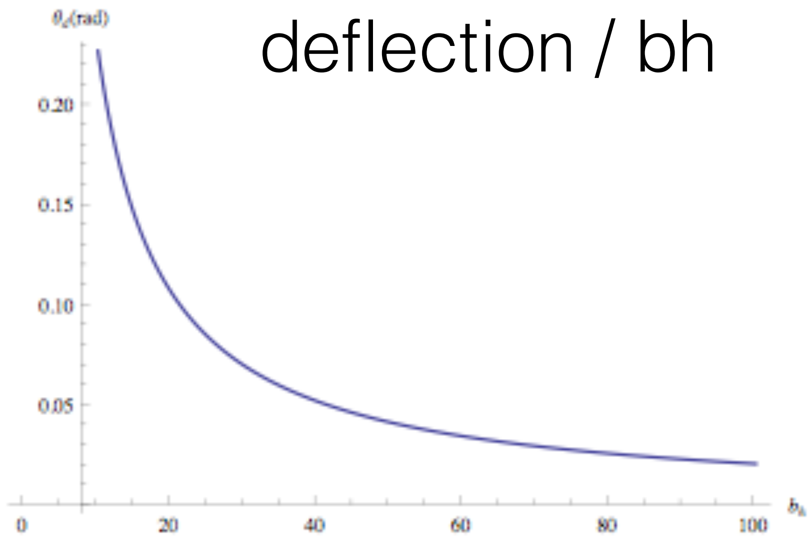
which proves to be logarithmically divergent as the beam approaches the photon sphere ($\epsilon \to 0$).

Weak field limit

The weak field expansion,valid for $x_0 \gg 3/2$, obtained from the elliptic solution, takes the form

$$\theta_d(x_0) = \frac{2}{x_0} + \left(-1 + \frac{15}{16}\pi\right)\frac{1}{x_0^2} + O(1/x_0^3).$$

deflection / bh



: The deflection angle as a function of the impact parameter in the classical GR solution.

We are looking for a method to incorporate quantum effects in a classical Lens equation

(Costantini, Delle Rose, dell'Atti. CC., JHEP 2015)

RADIATIVE LENS EQUATION

Suggestion:
Compute the quantum corrections to the deflection of a particle, for scatterings characterised by a certain impact parameter (b)

$$b_h(\alpha) \qquad b_h \equiv \frac{b}{2GM}$$

Insert $\alpha(b_h)$ into the lens equations

RADIATIVE CORRECTIONS TO THE PHOTON-GRAVITON VERTEX

R. DELBOURGO and P. PHOCAS-COSMETATOS
Physics Department, Imperial College, London SW7 2BZ, UK

Received 1 August 1972

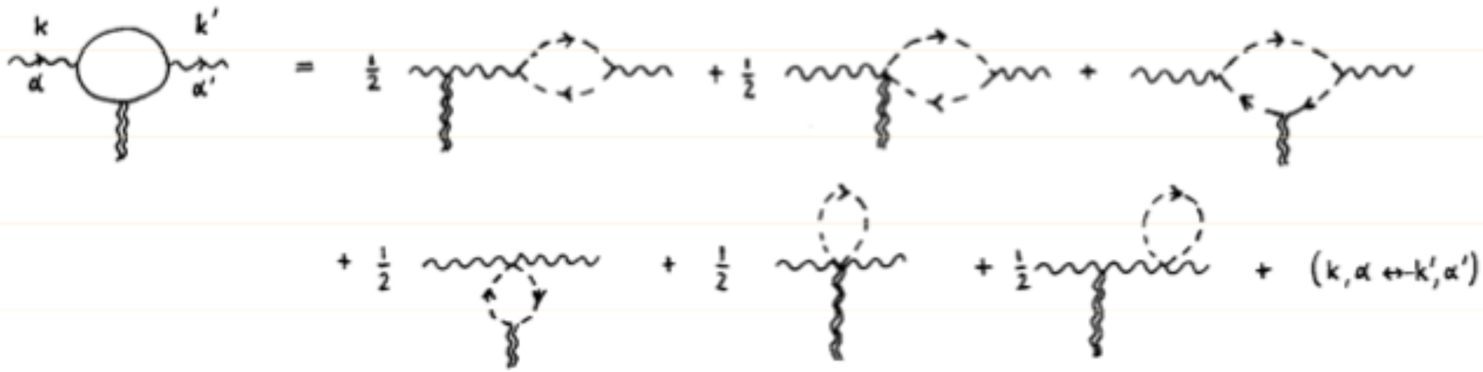
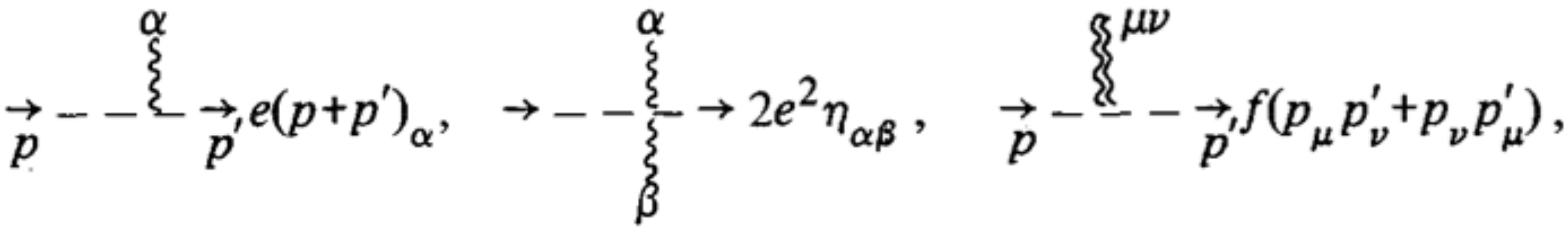


Fig. 1. Pion loop contributions to the γγg vertex.

$$-\frac{1}{4}g^{\mu\nu}\left[g^{\kappa\lambda}F_{\mu\kappa}F_{\nu\lambda}+\frac{\alpha}{720m^2}(-1)^{2J}(2J+1)F_{\kappa\lambda}\overset{\leftrightarrow}{\partial}_\mu\overset{\leftrightarrow}{\partial}_\nu F_{\kappa\lambda}\right],$$



$$\left|\frac{T^{++}}{8\pi M_\odot}\right| \approx \frac{4GM}{\theta^2}\left(1-\frac{\theta^2}{24}-\frac{4\alpha\omega^2\theta^2}{90\pi m^2}\right), \qquad \theta_\odot = \frac{4GM}{R_\odot}\left[1-\frac{1}{24}\left(1+\frac{16\alpha\omega^2}{15\pi m^2}\right)\left(\frac{4GM}{R_\odot}\right)^2 \log\left(\frac{R_\odot}{4GM}\right)+\ldots\right].$$

$$\mathrm{d}\sigma/\mathrm{d}\Omega = |T/8\pi M_\odot|^2 = -\tfrac{1}{2}\,\mathrm{d}b^2/\mathrm{d}(\cos\theta)$$

ANNALS OF PHYSICS **98**, 225–236 (1976)

Quantum Electrodynamical Corrections to Graviton-Matter Vertices

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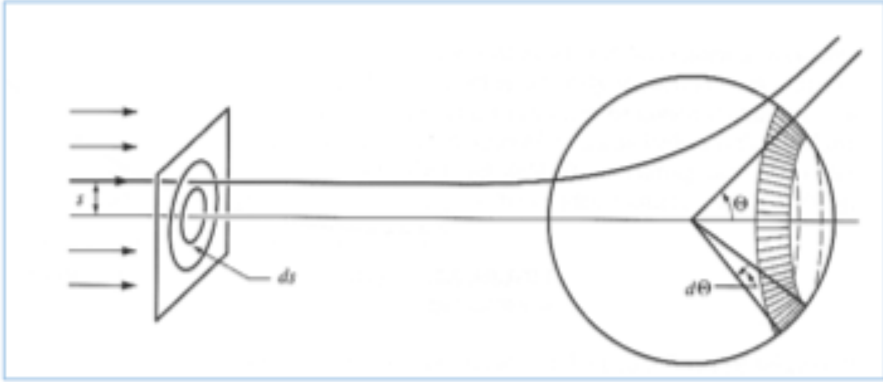
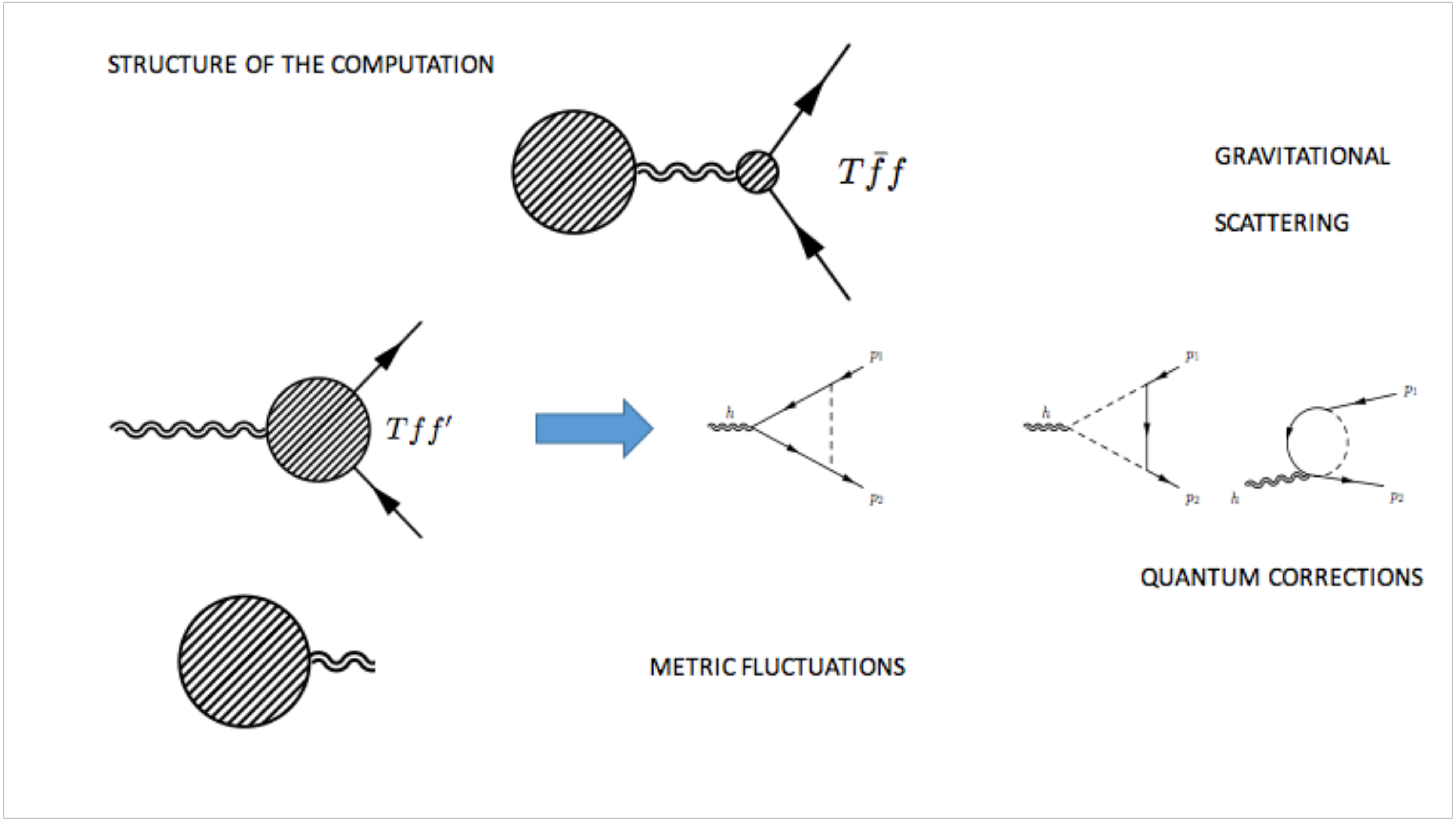
Received December 15, 1975



$$2\pi I b db = 2\pi \sigma(\theta) I \sin\theta d\theta$$

$$\frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{d\sigma}{d\Omega}.$$

The expression above defines a differential equation for the impact parameter whose solution relates b to θ, the classical angle of deflection. This allows a comparison between the two approaches, giving a deflection which is in agreement with Einsten's prediction in the case of weak lensing.


STRUCTURE OF THE COMPUTATION
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QUANTUM CORRECTIONS
METRIC FLUCTUATIONS

The study of these cross sections can be used to derive generalized **corrections to Einstein's formula for the angular deflection,** which becomes **energy dependent**.

The approach can be applied to any quantum correction to the propagation of fields in a gravitational background, also with dynamical gravity.

The Standard Model Lagrangian in a gravitational background: the fermion sector

$$\mathcal{S} = \mathcal{S}_{SM} + \mathcal{S}_G + \mathcal{S}_I$$

$$\begin{aligned} \mathcal{S}_G &= -\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\,R \\ \mathcal{S}_I &= \chi\int d^4x\sqrt{-g}\,R\,H^\dagger H \end{aligned}$$

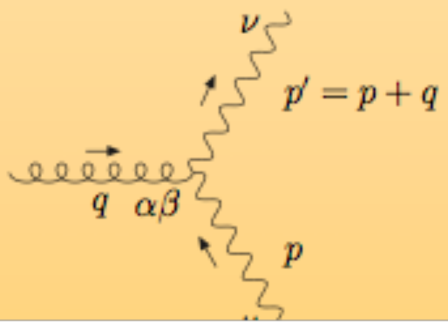
$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \kappa h_{\mu\nu} + O(\kappa^2) \\ g^{\mu\nu} &= \eta^{\mu\nu} - \kappa h^{\mu\nu} + O(\kappa^2) \\ \sqrt{-g} &= 1 + \frac{\kappa}{2}h + O(\kappa^2), \end{aligned}$$

$$\mathcal{S}_{int} = -\frac{\kappa}{2}\int d^4x\, T_{\mu\nu}h^{\mu\nu}\,,$$

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta\left(S_{SM} + S_I\right)}{\delta g^{\mu\nu}}\bigg|_{g=\eta}.$$

$$
\begin{aligned}
\mathcal{L}_M &= -\frac{\sqrt{-g}}{4} g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu} \\
&= \underbrace{-\frac{1}{4}\eta^{\alpha\beta}\eta^{\mu\nu} F_{\alpha\mu}F_{\beta\nu}}_{\mathcal{L}_0} - \frac{\kappa}{2} h^{\mu\nu}\left[-\eta^{\alpha\beta} F_{\alpha\mu}F_{\beta\nu} - \eta_{\mu\nu}\mathcal{L}_0\right] \\
&\quad + \frac{\kappa^2}{4}\left[\frac{1}{2}(h^2 - 2h^{\mu\nu}h_{\mu\nu})\mathcal{L}_0 + F_{\alpha\beta}F_{\mu\nu}(hh^{\alpha\mu}\eta^{\beta\nu} - 2h^{\alpha\lambda}h^{\mu}_{\lambda}\eta^{\beta\nu} - h^{\alpha\mu}h^{\beta\nu})\right] + \dots
\end{aligned}
$$

$$
\begin{aligned}
V^{\alpha\beta,\mu\nu}(p',p) &= -\frac{i\kappa}{2}[(\eta^{\alpha\beta}\eta^{\mu\nu} - \eta^{\alpha\mu}\eta^{\beta\nu} - \eta^{\alpha\nu}\eta^{\beta\mu})p'\cdot p - \eta^{\alpha\beta}p'^{\mu}p^{\nu} + \eta^{\mu\beta}p'^{\alpha}p^{\nu} \\
&\quad -\eta^{\mu\nu}p'^{\alpha}p^{\beta} + \eta^{\alpha\nu}p'^{\mu}p^{\beta} + \eta^{\beta\nu}p'^{\mu}p^{\alpha} - \eta^{\mu\nu}p'^{\beta}p^{\alpha} + \eta^{\alpha\mu}p'^{\beta}p^{\nu}]
\end{aligned}
$$



Photon/Graviton

FERMIONS IN A CURVED SPACETIME

$$\mathcal{L}_F = \sqrt{-g}\left(\frac{i}{2}\bar{\psi}\gamma^\mu(\mathcal{D}_\mu\psi) - \frac{i}{2}(\mathcal{D}_\mu\bar{\psi})\gamma^\mu\psi - m\,\bar{\psi}\psi\right),$$

$\mathcal{D}_\mu = \partial_\mu + A_\mu + \Omega_\mu$, with A_μ denoting the gauge field.

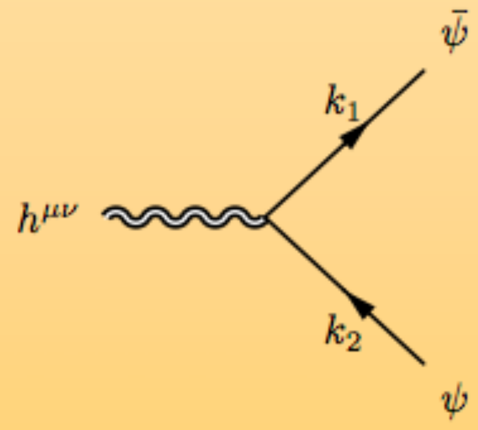
$$\Omega_\mu = \frac{1}{2}\sigma^{ab}V_a^\nu V_{b\nu;\mu}$$

$$\Omega_\mu = \frac{1}{4}\sigma^{mn}\left[V_m^\nu\left(\partial_\mu V_{n\nu} - \partial_\nu V_{n\mu}\right) + \frac{1}{2}V_m^\rho V_n^\sigma\left(\partial_\sigma V_{l\rho} - \partial_\rho V_{l\sigma}\right)V_\mu^l - (m \leftrightarrow n)\right].$$

$$V_\mu^m = \delta_\mu^m + \frac{\kappa}{2}h_\mu^m + O(\kappa^2).$$

$$\mathcal{L}_F = \mathcal{L}_0 - \frac{\kappa}{2} h_{\mu\nu} T^{(0)\mu\nu}$$

$$\mathcal{L}_0 = \frac{i}{2}\left(\bar\psi \overrightarrow{\not\partial} \psi - \bar\psi \overleftarrow{\not\partial} \psi\right) - m\bar\psi\psi$$



$$T^{(0)}_{\mu\nu} = \frac{i}{4}\left((\bar\psi\gamma_\mu\partial_\nu\psi - \partial_\nu\bar\psi\gamma_\mu\psi) + (\mu \leftrightarrow \nu)\right) - \eta_{\mu\nu}\mathcal{L}_0$$

$$V^{(0)\mu\nu} = \frac{i}{4}\left(\gamma^\mu(p_1+p_2)^\nu + \gamma^\nu(p_1+p_2)^\mu - 2\eta^{\mu\nu}(\not p_1 + \not p_2 - 2m)\right),$$

$$\hat{T}^{\mu\nu} = \bar{u}(p_2) V^{\mu\nu} u(p_1),$$

$$\langle p_2 | T^{\mu\nu}(x) | p_1 \rangle = \bar{\psi}_f(p_2) V^{\mu\nu} \psi_i(p_1) e^{iq\cdot x},$$

$$\psi_i(p_1) = \mathcal{N}_i u(p_1), \qquad \mathcal{N}_i = \sqrt{\frac{m_1}{E_1 V_l}}, \qquad \bar{u}(p_1) u(p_1) = 1,$$

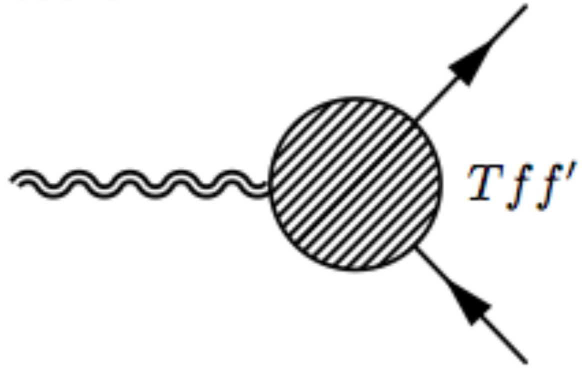
$$iS_{if} = -\frac{\kappa}{2} \int_{\mathcal{V}} d^4x \langle p_2 | h_{\mu\nu}(x) T^{\mu\nu}(x) | p_1 \rangle,$$

$$\langle p_2 | h_{\mu\nu}(x) T^{\mu\nu}(x) | p_1 \rangle \;=\; h_{\mu\nu}(x) \bar{\psi}(p_2) V^{\mu\nu} \psi(p_1) e^{iq\cdot x}.$$

COMPUTING THE CROSS SECTION

$$\begin{aligned} i\mathcal{S}_{fi} &= -\frac{\kappa}{2} h_{\mu\nu}(q)\bar{\psi}(p_2)V^{\mu\nu}\psi(p_1) \\ &= -\frac{\kappa}{2} h_{\mu\nu}(q)\mathcal{N}_i\mathcal{N}_f\hat{T}^{\mu\nu} \end{aligned}$$

GRAVITATIONAL SOURCE



$$\Box\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = -\kappa T^{ext}_{\mu\nu}$$

the EMT of the external localized source, defining $S_{\mu\nu}$, given by

$$T^{ext}_{\mu\nu} = \frac{P_\mu P_\nu}{P_0}\delta^3(\vec{x}) \, .$$

$$G_R(x,y) = \frac{1}{4\pi}\frac{\delta(x_0 - |\vec{x} - \vec{y}| - y_0)}{|\vec{x} - \vec{y}|}$$

Retarded propagator

$$\Box G_R(x,y) = \delta^4(x-y).$$

$$h^{ext}_{\mu\nu}(x) = \kappa \int d^4y G_R(x,y) S_{\mu\nu}(y),$$

$$h^L_{\mu\nu}(x) \;\; = \;\; \frac{2GM}{\kappa|\vec{x}|}\bar{S}_{\mu\nu}.$$

$$h_{\mu\nu}(q_0, \vec{q}) \;\; = \;\; 2\pi\delta(q_0) \times \left(\frac{\kappa M}{2\vec{q}^2}\right)\bar{S}_{\mu\nu},$$

$$
\begin{aligned}
i\mathcal{S}_{if}^{(0)} &= \int d^4x h_{\mu\nu}(x)\bar{\psi}_f(x)T^{(0)\mu\nu}\psi_i(x) \\
&= \left(-\frac{\kappa}{2}\right) \times \mathcal{N}_i\mathcal{N}_f \times \left(ih_{\mu\nu}(\vec{q})\bar{u}(p_2)O_V^{(0)\mu\nu}u(p_1)\right) \times 2\pi\delta(q_0).
\end{aligned}
$$

$$
\begin{aligned}
\langle|i\mathcal{S}_{if}^{(0)}|^2\rangle &= \left(-\frac{\kappa}{2}\right)^2 (\mathcal{N}_i\mathcal{N}_f)^2 \times (2\pi\delta(q_0)\mathcal{T}) \times \left(\frac{\kappa M}{2\vec{q}^2}\right)^2 \times \frac{1}{2}\mathcal{Y}_0, \\
\mathcal{Y}_0 &= \frac{1}{4m^2}Tr\left[(\not{p}_2+m)O_V^{(0)\mu\nu}(\not{p}_1+m)O_V^{(0)\alpha\beta}\right]\bar{S}_{\mu\nu}\bar{S}_{\alpha\beta} \\
&= E^2\mathrm{Tr}\left[\frac{\not{p}_2+m}{2m}\left(2\gamma^0-\frac{m}{E}\right)\frac{\not{p}_1+m}{2m}\left(2\gamma^0-\frac{m}{E}\right)\right] \\
&= 8\frac{\vec{p_1}^4}{m^2}F^{(0)}(x,\theta)
\end{aligned}
$$

with $x = m^2/\vec{p_1}^2$ and $\vec{p_1}$ the 3-momentum of the incoming fermion.

$$
F^{(0)}(x,\theta) = \cos^2\frac{\theta}{2} + \frac{x}{4} + \frac{x^2}{4} + \frac{3}{4}x\cos^2\frac{\theta}{2},
$$

$$dW = \frac{|i\mathcal{S}_{if}|^2}{j_i} dn_f$$

$$j_i = \vec{p_1}/(E_i V)$$

$$dn_f = \frac{V}{(2\pi)^3} d^3\vec{p}_f = \frac{V}{(2\pi)^3} |\vec{p_2}| E_2 dE_2 d\Omega,$$

$$\begin{aligned} \frac{d\sigma}{d\Omega} &\equiv \frac{d\sigma}{d\Omega}|_L \\ &= \left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2 F^{(0)}(x,\theta). \end{aligned}$$

Now use

$$\frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{d\sigma}{d\Omega}$$

The semiclassical relation at Born level

$$b_h^2(\alpha) = b_h^2(\bar{\theta}) + 2\int_\alpha^{\bar{\theta}} d\theta' \sin\theta' \frac{d\tilde{\sigma}}{d\Omega'},$$

The integration constant can be fixed by the condition that $b(\theta_d)$ approaches the classical GR solution as b_h goes to infinity.

equivalently

$$\lim_{\theta_d \to \pi} b^2(\theta_d) = 0\,,$$

$$\frac{db^2}{d\theta} = -2\left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2 F^{(0)}(x,\theta)\sin\theta$$

$$b^2(\theta) = (GM)^2\left(\frac{(2+x)^2}{\sin^2(\frac{\theta}{2})} + 2(4+3x)\log\left(\sin(\frac{\theta}{2})\right)\right).$$

In the small θ limit we get the relations

$$b \sim GM\left(\frac{4}{\theta} + \frac{2x}{\theta} + (1+\frac{x}{4})\theta\log\theta\right) + \mathcal{O}(x^2\theta\log\theta) + \mathcal{O}(\theta),$$

$$\theta \equiv \theta_d \sim 4\frac{GM}{b}.$$

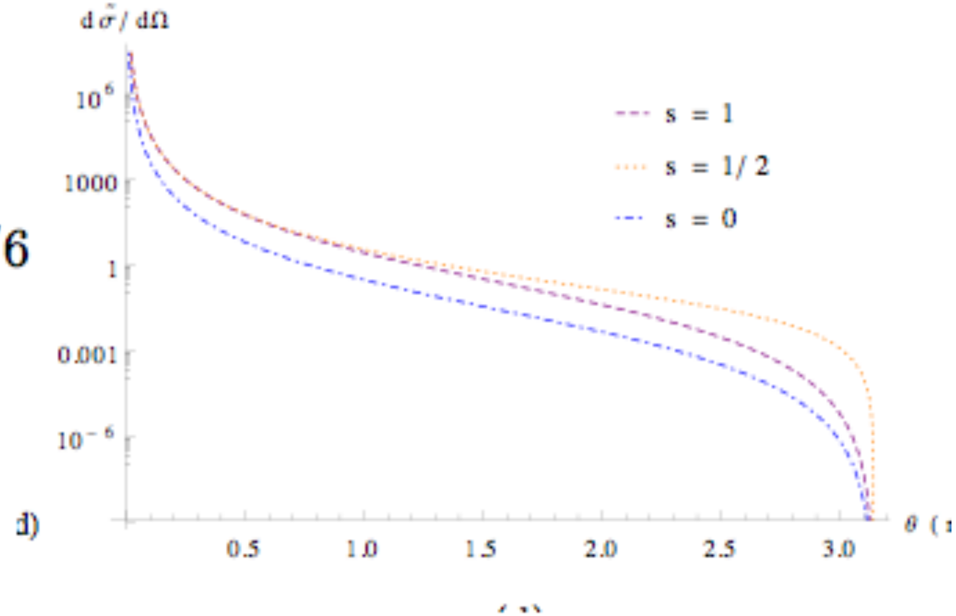
Angular deflection, in agreement with Einstein's formula

SUMMARY OF BORN LEVEL CROSS SECTIONS

$$\left.\frac{d\sigma}{d\Omega}\right|_f^{(0)} = \left(\frac{GM}{\sin^2(\theta/2)}\right)^2 \left(\cos^2\vartheta/2 + \frac{1}{4}\frac{m^2}{|\vec{p}_1|^2} + \frac{1}{4}\frac{m^4}{|\vec{p}_1|^4} + \frac{3}{4}\frac{m^2}{|\vec{p}_1|^2}\cos^2\vartheta/2\right).$$

$$\left.\frac{d\sigma}{d\Omega}\right|_\nu^{(0)} = \left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2},$$

$$\left.\frac{d\sigma}{d\Omega}\right|_s^{(0)} = \begin{cases} (GM)^2\csc^4(\theta/2) & \chi = 0 \\ \left(\frac{GM}{3}\right)^2\cot^4(\theta/2) & \chi = 1/6 \end{cases}$$



PHOTON LENSING (Born)

$$-i\,\frac{\kappa}{2}\,\hat{T}^{(0)\,\mu\nu} = V^{\mu\nu\alpha\beta}(p_1,p_2)A^i_\alpha(p_1)\,A^f_\beta(p_2)\,,$$

$$V^{\mu\nu\alpha\beta}(p_1,p_2) = -i\frac{\kappa}{2}\Big\{-p_1\cdot p_2\,C^{\mu\nu\alpha\beta} + D^{\mu\nu\alpha\beta}(p_1,p_2)\Big\}$$

$$C^{\mu\nu\alpha\beta} = \eta^{\mu\alpha}\,\eta^{\nu\beta} + \eta^{\mu\beta}\,\eta^{\nu\alpha} - \eta^{\mu\nu}\,\eta^{\alpha\beta}\,,$$

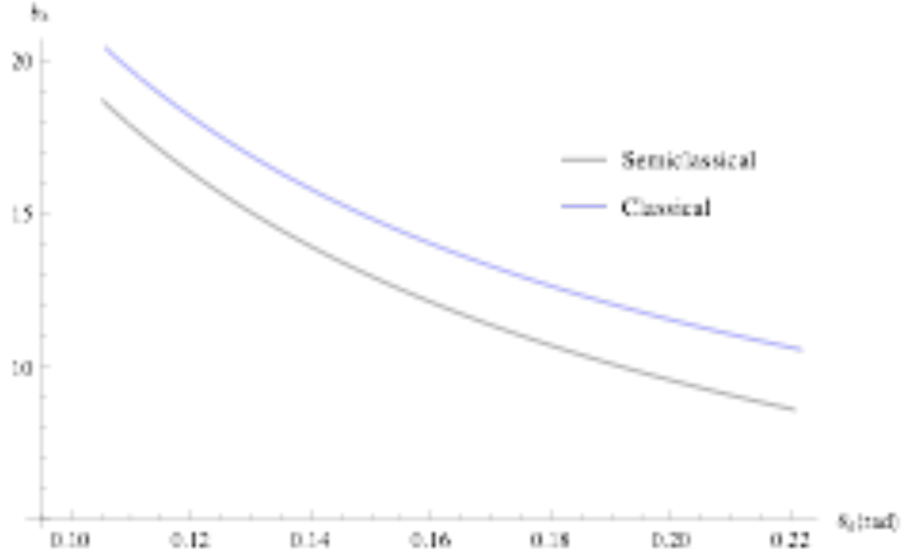
$$D^{\mu\nu\alpha\beta}(p_1,p_2) = -\eta^{\mu\nu}\,p_1^\beta\,p_2^\alpha + \Big[\eta^{\mu\beta}p_1^\nu p_2^\alpha + \eta^{\mu\alpha}\,p_1^\beta\,p_2^\nu - \eta^{\alpha\beta}\,p_1^\mu\,p_2^\nu + (\mu\leftrightarrow\nu)\Big].$$

$$\frac{d\sigma}{d\Omega_0} = (G\,M)^2\cot^4\left(\frac{\theta}{2}\right)$$

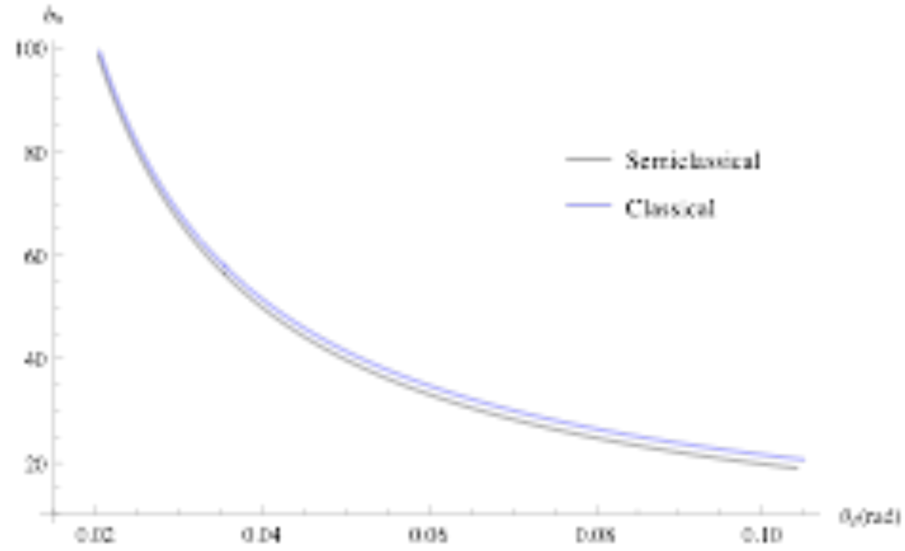
$$\frac{db_0^2}{d\theta} = -2\,(G\,M)^2\cot^4\left(\frac{\theta}{2}\right)\sin\theta\,,$$

$$b_0^2(\theta_d) = 4\,G^2M^2\,\left(\csc^2\left(\frac{\theta_d}{2}\right) + 4\,\log\sin\left(\frac{\theta_d}{2}\right) - \sin^2\frac{\theta_d}{2}\right)\,.$$

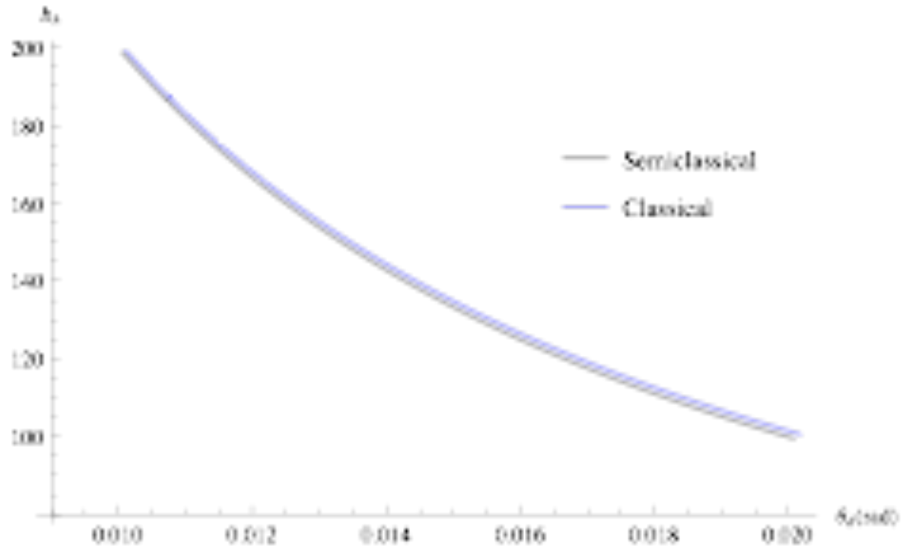
$$b_0 \sim GM\,\left(\frac{4}{\theta_d} + \frac{\theta}{6}\left(1 + 12\,\log\frac{\theta_d}{2}\right)\right)\,,$$


20
15
10
0.10
0.12
0.14
0.16
0.18
0.20
0.22
Semiclassical
Classical

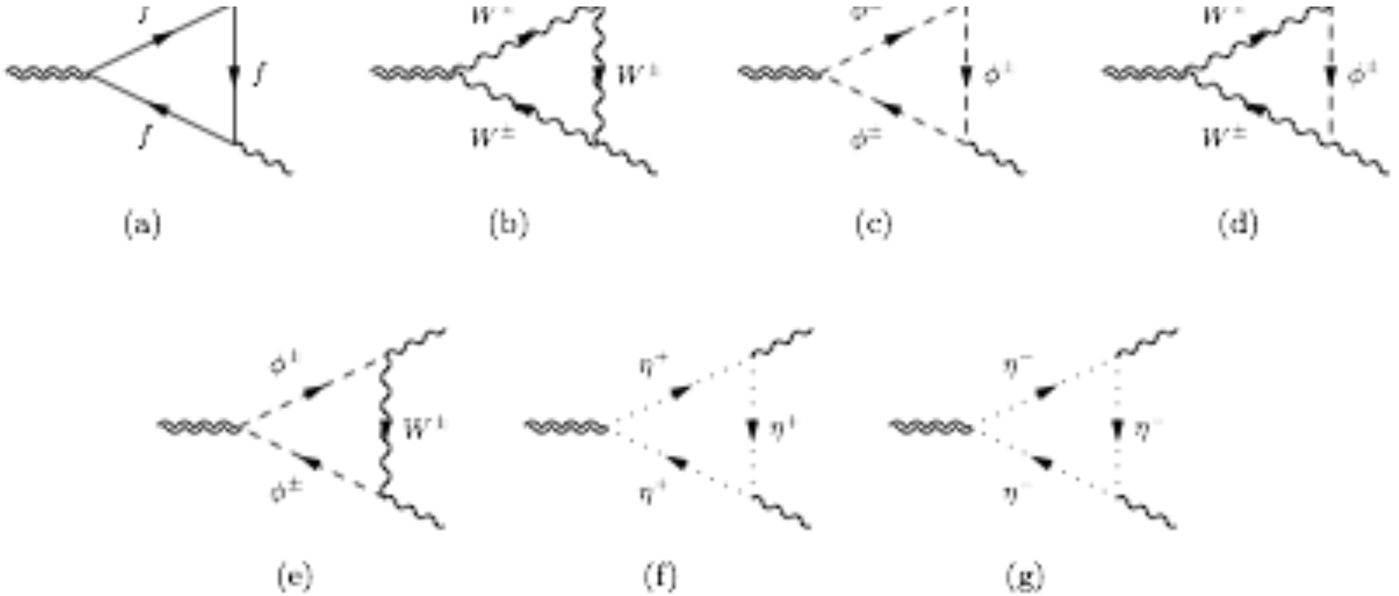
(a)

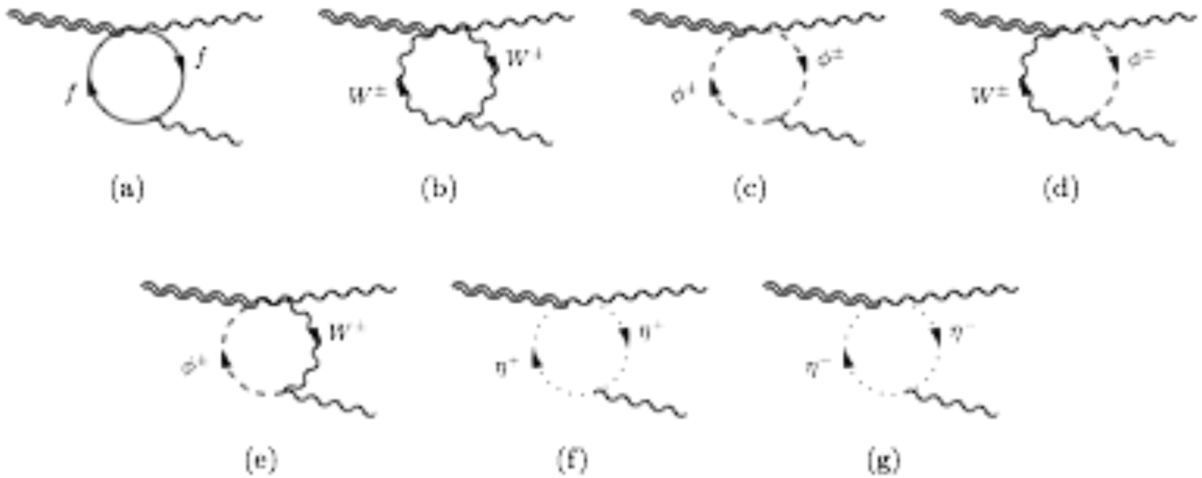

100
80
60
40
20
0.02
0.04
0.06
0.08
0.10
Semiclassical
Classical

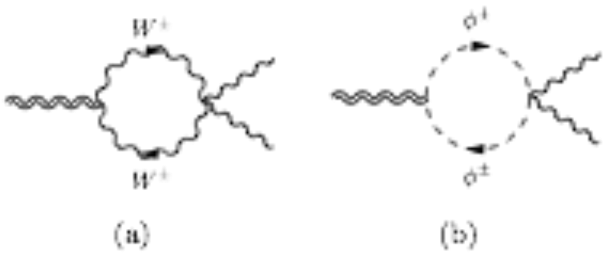
(b)

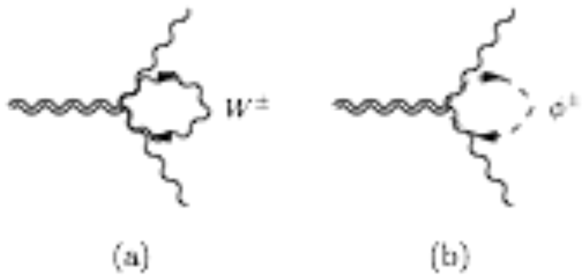

200
180
160
140
120
100
0.010
0.012
0.014
0.016
0.018
0.020
Semiclassical
Classical

(c)


(a)
(b)
(c)
(d)
(e)
(f)
(g)


(a)
(b)
(c)
(d)
(e)
(f)
(g)


(a)
(b)


(a)
(b)

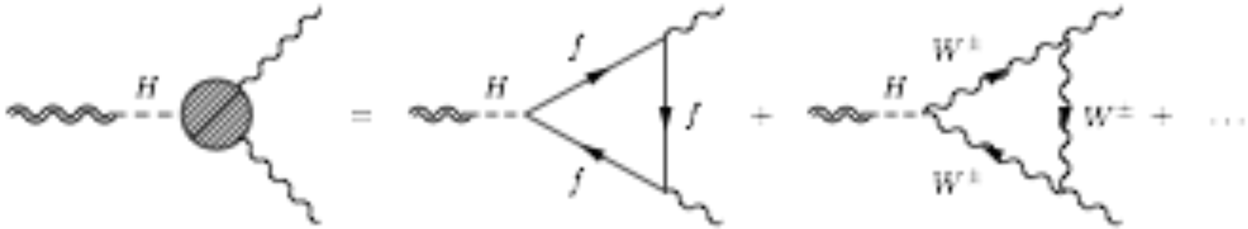
$$\Sigma^{\mu\nu\alpha\beta}(p_1,p_2) = \Sigma_F^{\mu\nu\alpha\beta}(p_1,p_2) + \Sigma_B^{\mu\nu\alpha\beta}(p_1,p_2) + \Sigma_I^{\mu\nu\alpha\beta}(p_1,p_2).$$

$$\begin{aligned}
\Sigma_F^{\mu\nu\alpha\beta}(p_1,p_2) &= \sum_{i=1}^{3} \Phi_{i\,F}(t,0,0,m_f^2)\,\phi_i^{\mu\nu\alpha\beta}(p_1,p_2)\,,\\
\Sigma_B^{\mu\nu\alpha\beta}(p_1,p_2) &= \sum_{i=1}^{3} \Phi_{i\,B}(t,0,0,M_W^2)\,\phi_i^{\mu\nu\alpha\beta}(p_1,p_2)\,,\\
\Sigma_I^{\mu\nu\alpha\beta}(p_1,p_2) &= \Phi_{1\,I}(t,0,0,M_W^2)\,\phi_1^{\mu\nu\alpha\beta}(p_1,p_2) + \Phi_{4\,I}(t,0,0,M_W^2)\,\phi_4^{\mu\nu\alpha\beta}(p_1,p_2)\,,
\end{aligned}$$

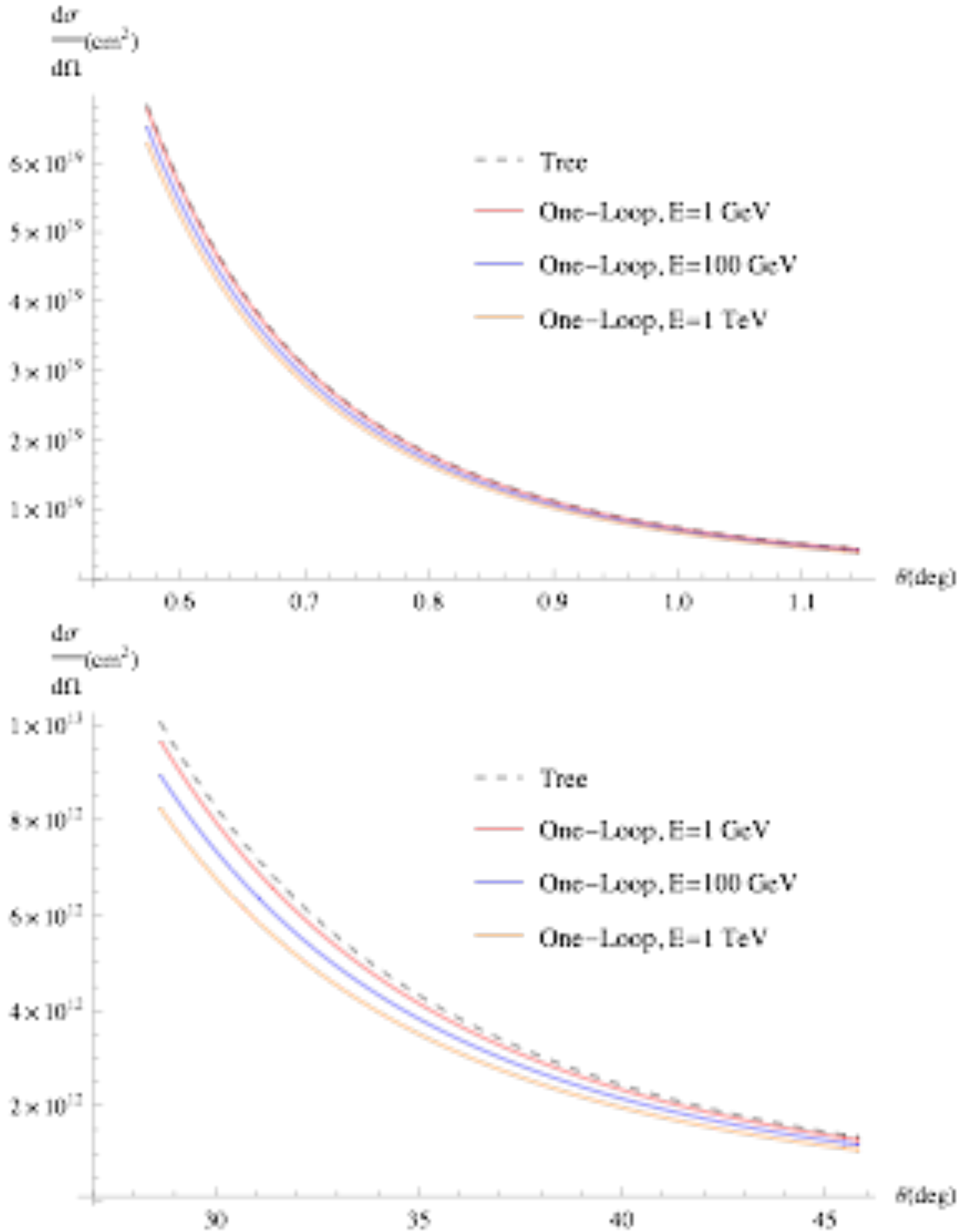
$$\begin{aligned}
\phi_1^{\mu\nu\alpha\beta}(p_1,p_2) &= (t\,\eta^{\mu\nu} - k^\mu k^\nu)\,u^{\alpha\beta}(p_1,p_2)\,,\\
\phi_2^{\mu\nu\alpha\beta}(p_1,p_2) &= -2\,u^{\alpha\beta}(p_1,p_2)\left[t\,\eta^{\mu\nu} + 2(p_1^\mu\,p_1^\nu + p_2^\mu\,p_2^\nu) + 4\,(p_1^\mu\,p_2^\nu + p_2^\mu\,p_1^\nu)\right]\,,\\
\phi_3^{\mu\nu\alpha\beta}(p_1,p_2) &= -(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu)\eta^{\alpha\beta} + \frac{t}{2}\left(\eta^{\alpha\nu}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\nu}\right) - \eta^{\mu\nu}\,u^{\alpha\beta}(p_1,p_2)\\
&\quad + \left(\eta^{\beta\nu}p_1^\mu + \eta^{\beta\mu}p_1^\nu\right)p_2^\alpha + (\eta^{\alpha\nu}p_2^\mu + \eta^{\alpha\mu}p_2^\nu)p_1^\beta\,,\\
\phi_4^{\mu\nu\alpha\beta}(p,q) &= (t\,\eta^{\mu\nu} - k^\mu k^\nu)\,\eta^{\alpha\beta}\,,
\end{aligned}$$

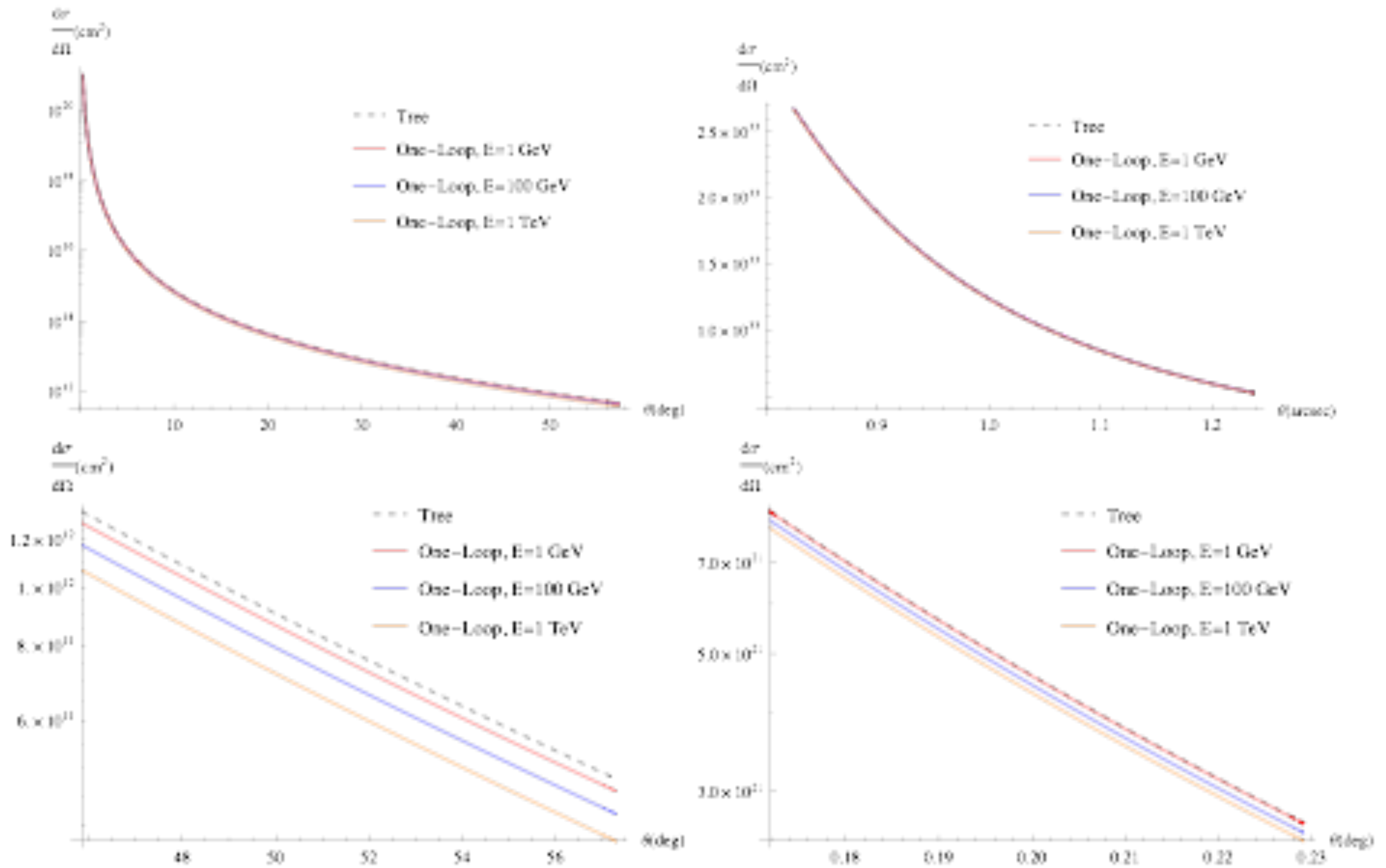
$$u^{\alpha\beta}(p_1,p_2) = p_2^\alpha\,p_1^\beta - (p_1\cdot p_2)\,\eta^{\alpha\beta}\,.$$

term of improvement

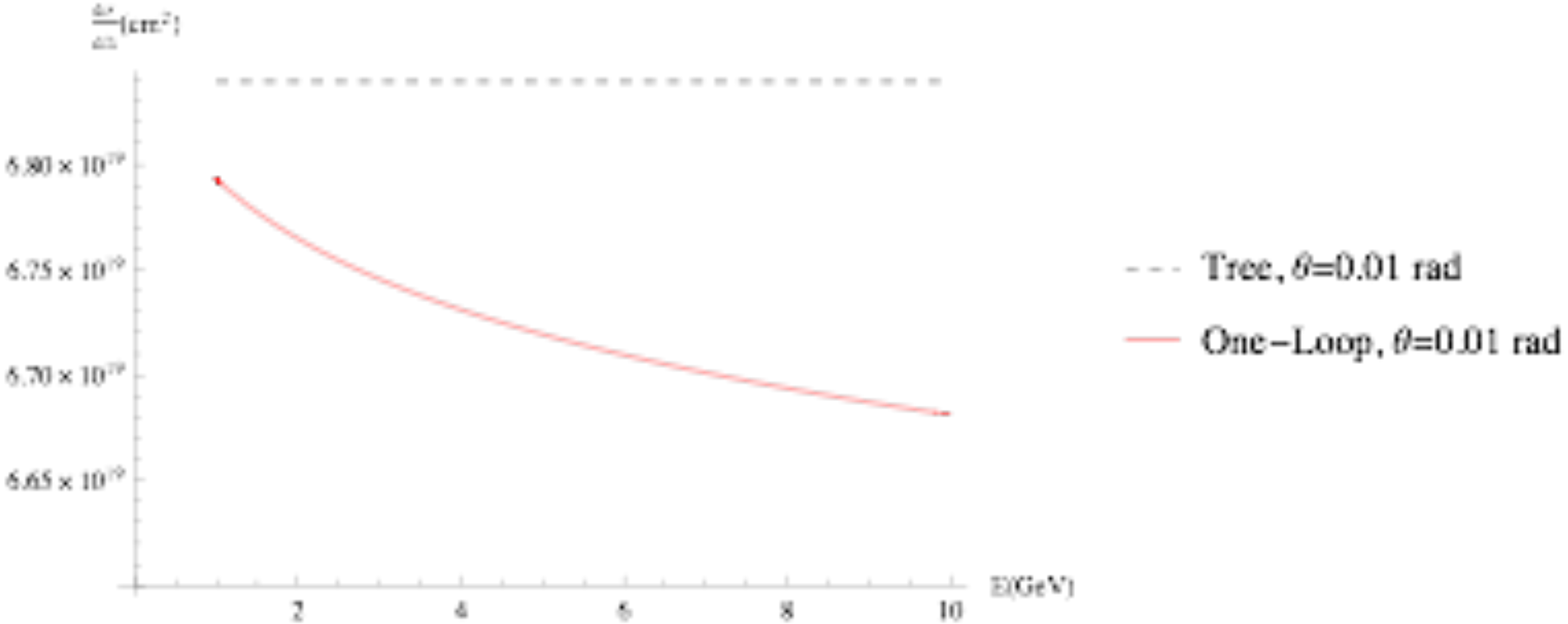


$$
\begin{aligned}
\Gamma^{\mu\nu\alpha\beta}_{(1)}(p_1,p_2) &= -i\,\frac{\kappa}{2}\left(\Sigma^{\mu\nu\alpha\beta}(p_1,p_2)+\Delta^{\mu\nu\alpha\beta}(p_1,p_2)+\delta Z_{AA}\,\phi_3^{\mu\nu\alpha\beta}(p_1,p_2)\right)\\
&\equiv -i\,\frac{\kappa}{2}\sum_{i=1}^{3}\phi_i^{\mu\nu\alpha\beta}(p_1,p_2)\,\overline{\Phi}_i\,,
\end{aligned}
$$

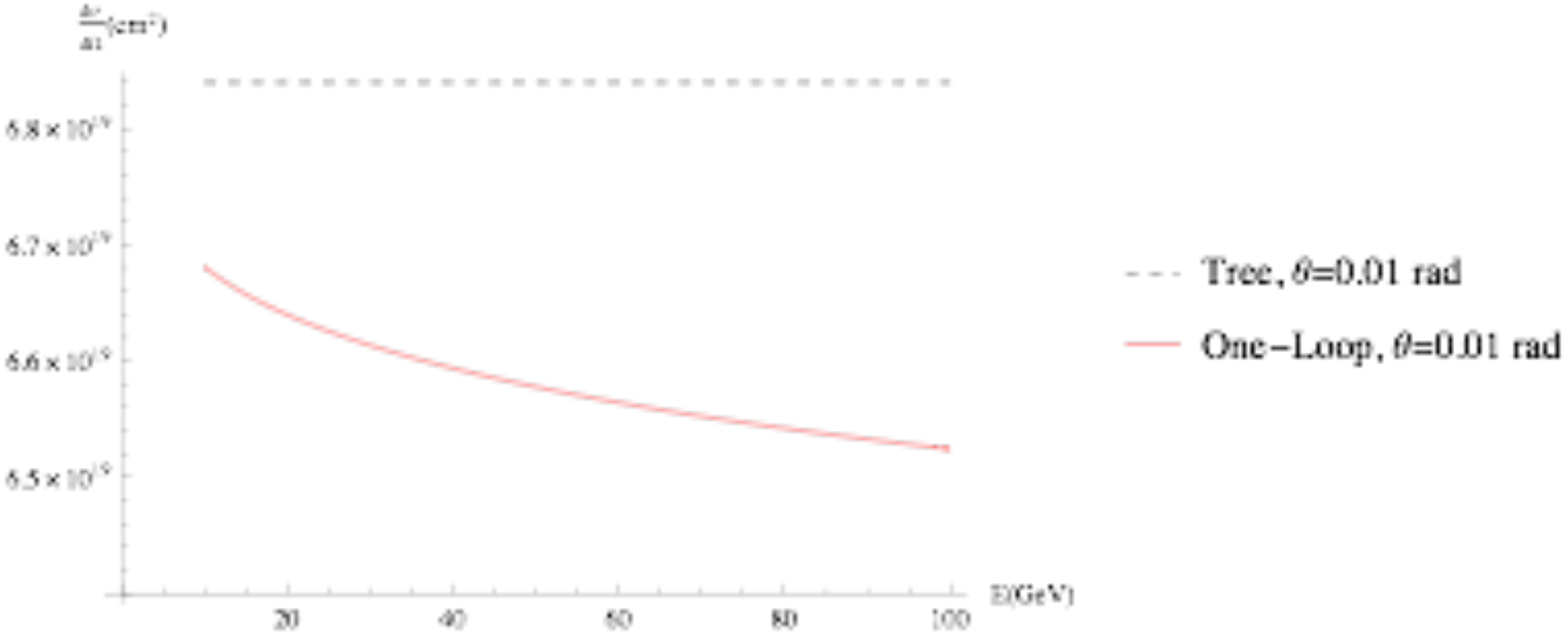

dσ/dΩ (cm²)
Tree
One−Loop, E=1 GeV
One−Loop, E=100 GeV
One−Loop, E=1 TeV
θ(deg)
0.6
0.7
0.8
0.9
1.0
1.1
dσ/dΩ (cm²)
Tree
One−Loop, E=1 GeV
One−Loop, E=100 GeV
One−Loop, E=1 TeV
θ(deg)
30
35
40
45



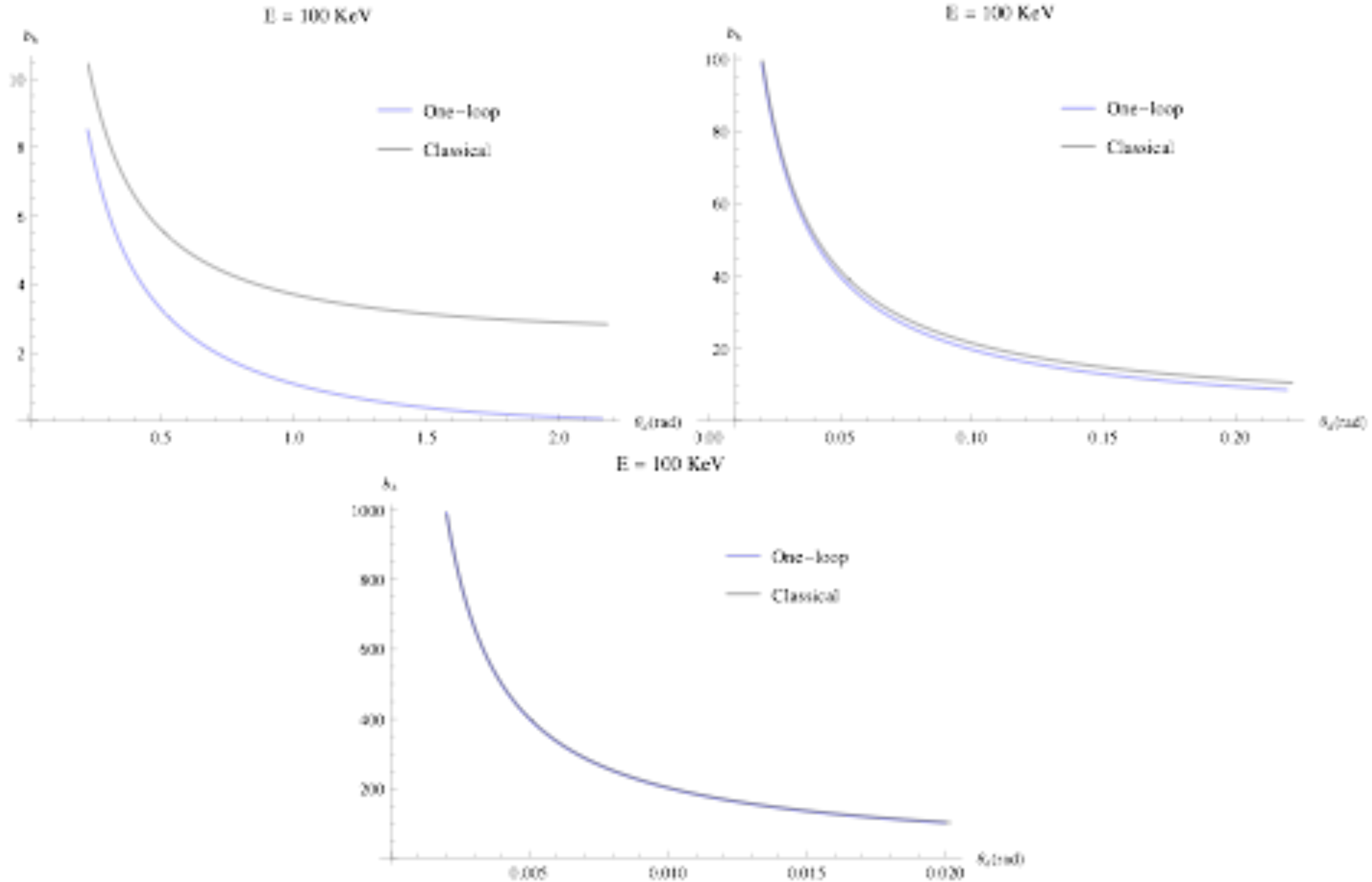
Tree
One−Loop, E=1 GeV
One−Loop, E=100 GeV
One−Loop, E=1 TeV
θ(deg)
10
20
30
40
50
θ(arcsec)
0.9
1.0
1.1
1.2
48
50
52
54
56
0.18
0.19
0.20
0.21
0.22
0.23

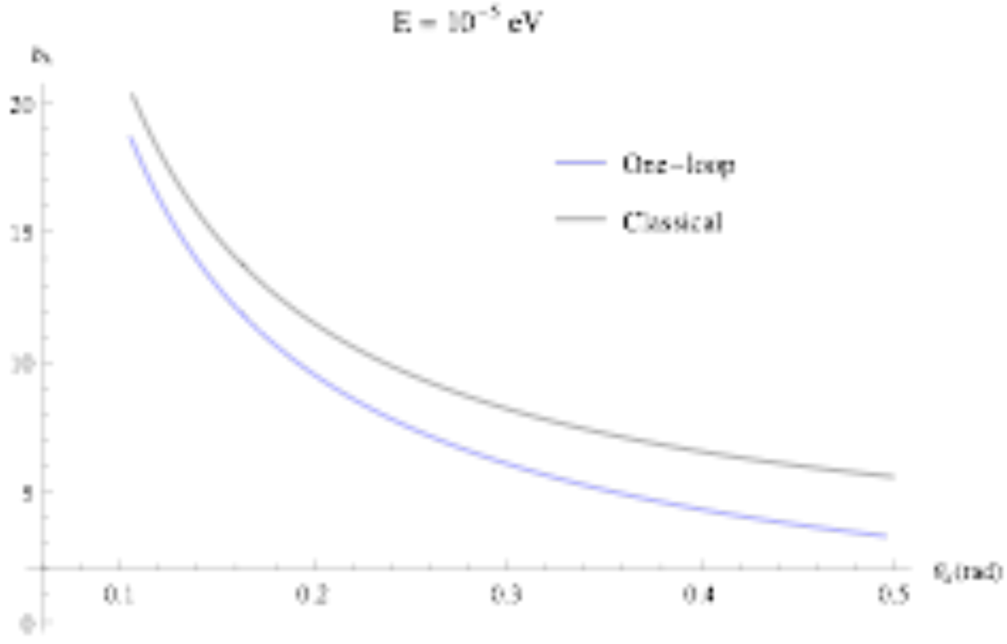


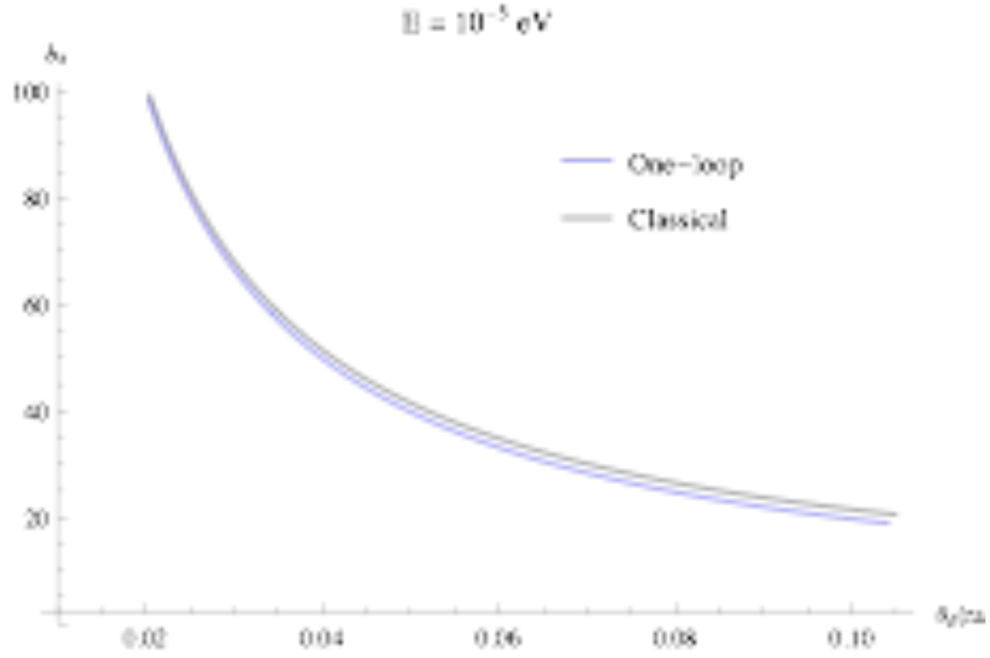
Tree, θ=0.01 rad
One−Loop, θ=0.01 rad
E(GeV)
2
4
6
8
10



Tree, θ=0.01 rad
One−Loop, θ=0.01 rad
E(GeV)
20
40
60
80
100


E = 100 KeV
One−loop
Classical
θ_c(rad)
E = 100 KeV
One−loop
Classical
θ_c(rad)
E = 100 KeV
One−loop
Classical
θ_c(rad)





CMB photons

$$b_{h,0}^2(\theta_d) \to (1+A^2)\left(\csc^2\left(\frac{\theta_d}{2}\right) + 4\log\sin\left(\frac{\theta_d}{2}\right) - \sin^2\frac{\theta_d}{2}\right).$$

$$b \sim GM(1+\frac{A^2}{2})\left(\frac{4}{\theta_d} + \frac{\theta_d}{6}\left(1 + 12\log\frac{\theta_d}{2}\right)\right).$$

$$A \equiv \alpha\frac{Q_f^2}{6\pi} = 3.87\times 10^{-4} Q_f^2,$$

Effect of the conformal anomaly

$$\theta_d = \frac{4GM}{b}(1+\frac{A^2}{2}) + \ldots$$

CONCLUSIONS

The inclusions of quantum effects in the context of scattering in the presence of a black hole takes to a violation of the equivalence principle.

It is possible to device a formalism which allows to generalize the concept of classical propagation of a photon or a neutrino. The deflection is energy dependent.

For mini black holes it allows to investigate femtolensing processes.