

Gauging Stuckelberg Axions: the Axi-Higgs

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Abstract

Variants of the usual Peccei-Quinn axion theory for the solution of the strong CP problem allows to generate more general axion-like terms in an effective Lagrangean beyond the Standard Model. One of these extensions involves Stuckelberg axions and (gauged) anomalous abelian symmetries. Similar interactions are generated by other methods, for instance by a decoupling of chiral fermions from the low energy spectrum in an anomaly-free theory. A third possibility is encoded in a scale invariant theory, where an axion, a dilaton and a dilatino are the anomaly multiplet of an N=1 Superconformal theory, in a nonlinear realization.



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General Results

Effective actions of Stuckelberg-type: $SU(3) \times SU(2) \times U(1)_Y \times U(1)'$

Generalising a PQ global symmetry to a local U(1) symmetry

(Stuckelberg axion models). Predict a **fundamental** axion (**gauged axion**) (the axi-Higgs) of a generic mass.

The mass is related to a misalignment potential which is generic.

It can cover the TeV region. Obviously, the misalignment has to be strong
For an axion at the Terascale.

Two models: MSLOM (Irges, Kiritsis, C.C.)

USSM-A (Lazarides, Irges, Mariano, C.C.) (Stuckelberg supermultiplet)

These models are built using a Wess-Zumino Lagrangean with an asymptotic and elementary axion

Decoupling of a heavy fermion and a gauged (anomaly free U(1) symmetry can also also be described by this class of models

ALTERNATIVE PATHS

AXIONS, DILATONS AS COMPOSITE

Conformal/superconformal anomaly

Dilaton interactions and the anomalous breaking of scale invariance of the Standard Model

Delle Rose, Quintavalle, Serino, C.C.
JHEP 1306 (2013) 077

Superconformal sum rules and the spectral density flow of the composite dilaton (ADD) multiplet in N=1 theories

Delle Rose, Costantini, Serino, C.C.
JHEP 1406 (2014) 136

Work to appear soon: Bandyopadhyay, Irges, Guzzi, Delle Rose, C.C.
“Heavy Axions and Dilatons”

A superconformal theory can generate these states due to the alignment of the anomaly multiplet.

Also in this case we need a dynamical breaking of supersymmetry in order to generate these states. The approach require a

Nonlinear realization of the superconformal symmetry

Axions emerge as a candidate solution of the strong CP problem

**The well known solution of the strong CP problem is due to
R. Peccei and H. Quinn (PQ)**

It is based on the introduction of an extra $U(1)$ global symmetry of the SM broken by an anomaly.



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$$E_8 \rightarrow M_8 \rightarrow E_6 \times SU(3)$$

$$E_6 \rightarrow M_6 \rightarrow SO(10) \times U(1)$$

$$SO(10) \rightarrow M_{10} \rightarrow SU(5) \times U(1)$$

$$SO(10) \rightarrow M'_{10} \rightarrow SO(6) \times SO(4)$$

$$SO(6) \sim SU(4) \quad SO(4) \sim SU(2)_L \times SU(2)_R$$

$$SU(4) \rightarrow M_4 \rightarrow SU(3)_c \times U(1)_{B-L}$$

$$SU(5) \rightarrow M_5 \rightarrow SU(3) \times SU(2) \times U(1)$$

Various effective models

$$E_6 \rightarrow SM \times U(1)$$

$$E_6 \rightarrow M_6 \rightarrow M_{10} \rightarrow M_5 \rightarrow SM \times U(1)$$

$$E_6 \rightarrow M_{10} \rightarrow M'_{10} \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$



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Generalization of the PQ proposal

Irges, Kiritsis, CC, 2005
U. of Crete, U. of Salento



Anomalous $U(1)$ extension of the Standard Model
(N. Irges, S. Morelli, C.C.)

Phenomenology: **M. Guzzi** (Manchester U.), R. Armillis, C.C.

Susy extensions: Irges (Athens TU), **A. Mariano** (Salento U.), C.C.

Cosmology: **G. Lazarides** (Thessaloniki U.), A. Mariano (Salento), C.C.

The role played by anomalies and anomaly actions in QFT can be hardly underestimated.

Anomalies describe the radiative breaking of a certain classical symmetry and *theorists have tried to use anomaly actions as a way to show the effect of the anomaly (example: chiral dynamics and the pion, AVV anomaly) but also have tried to cancel anomalies when these symmetries are gauged*

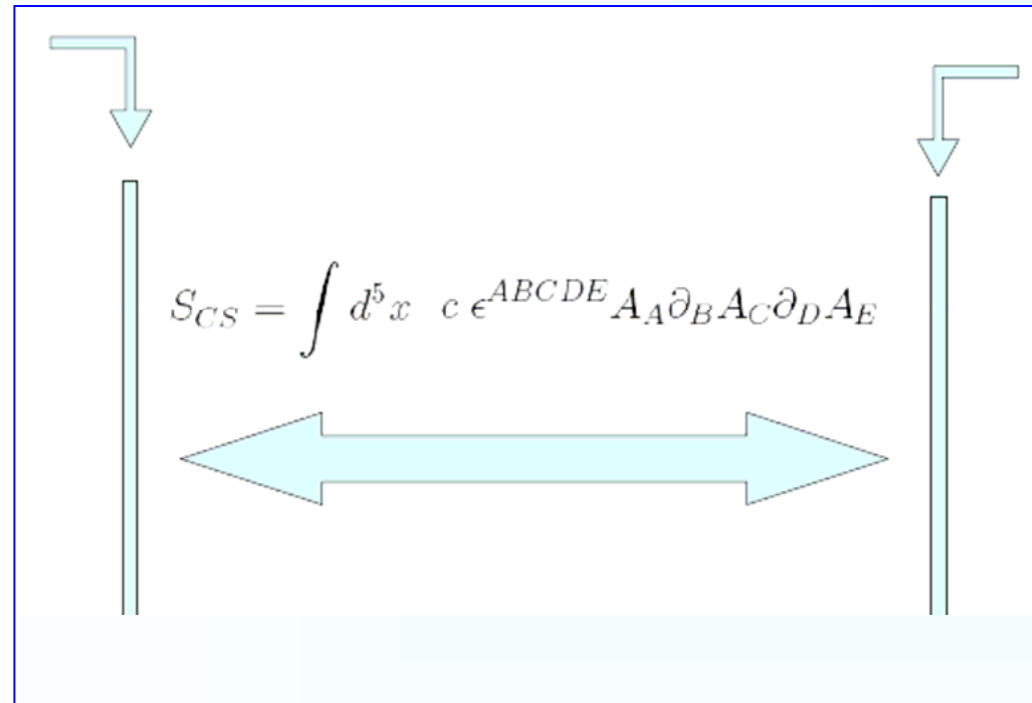
Anomaly cancellation (for a gauge symmetry):

1. **by charge assignment** in gauge theory (Standard Model):
in the exact (unbroken) phase of the theory, choose the representation in such a way that anomalous chiral interactions cancel
2. **by the introduction of extra sectors** (axions, dilatons) in the form of local actions (Wess Zumino actions)
3. More complex mechanisms such as “**anomaly inflows**”



Anomaly inflow on branes

$$\int_I d^4x \bar{\psi}_L i \not{D}_L \psi_L$$



$$\int_{II} d^4x \bar{\psi}_R i \not{D}_R \psi_R$$

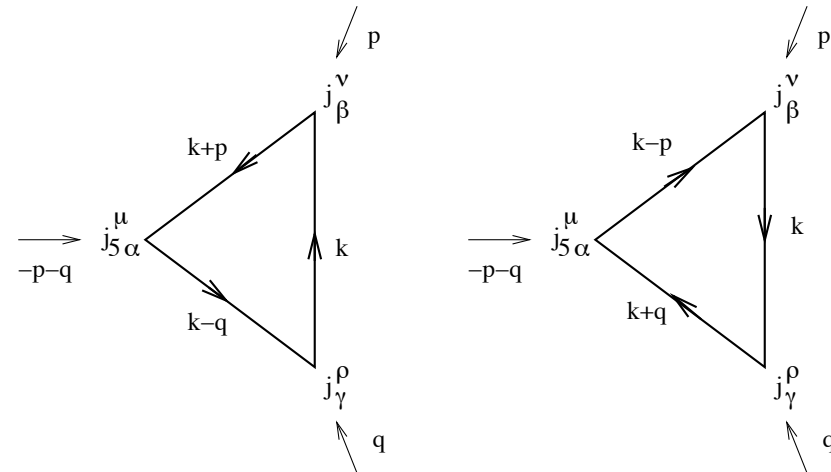
Hill, Phys. Rev. D74 (2006)

$$A_A(x_\mu, y) \rightarrow A_A(x_\mu, y) + \partial_A \theta(x_\mu, y)$$

$$S_{CS} \rightarrow S_{CS} + \frac{c}{4} \int_{II} d^4x \theta(R) \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(R) - \frac{c}{4} \int_I d^4x \theta(0) \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(0)$$

(Callan and Harvey)

	$SU(3)_c$	$SU(2)_w$	$U(1)_Y$	$U(1)'$
Q_L	3	2	1/6	z_Q
u_R	3	1	2/3	z_u
d_R	3	1	-1/3	$2z_Q - z_u$
L	1	2	-1/2	$-3z_Q$
e_R	1	1	-1	$-2z_Q - z_u$
H	1	2	1/2	z_H
$\nu_{R,k}$	1	1	0	z_k
χ	1	1	0	z_χ

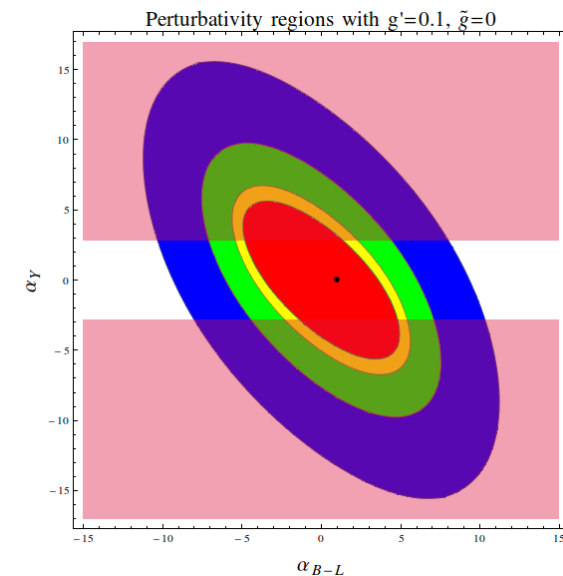


Charge assignment of fermions and scalars in the $U(1)'$ SM extension.

Constraints on Abelian Extensions of the Standard Model from Two-Loop Vacuum Stability and $U(1)_{B-L}$

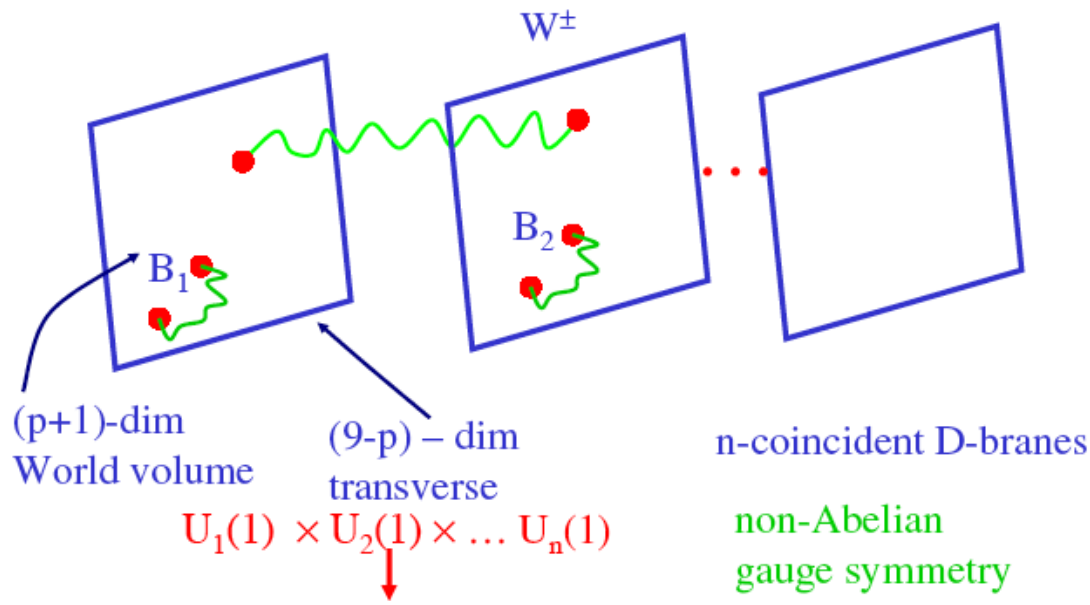


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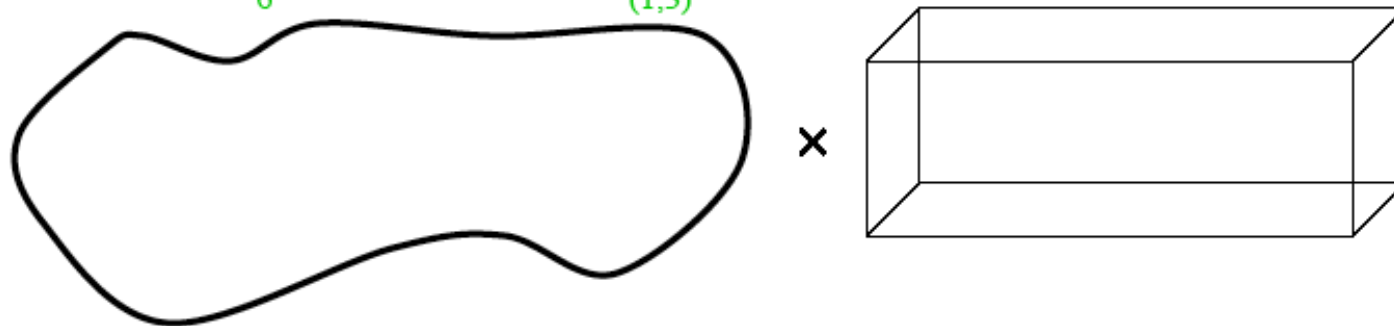
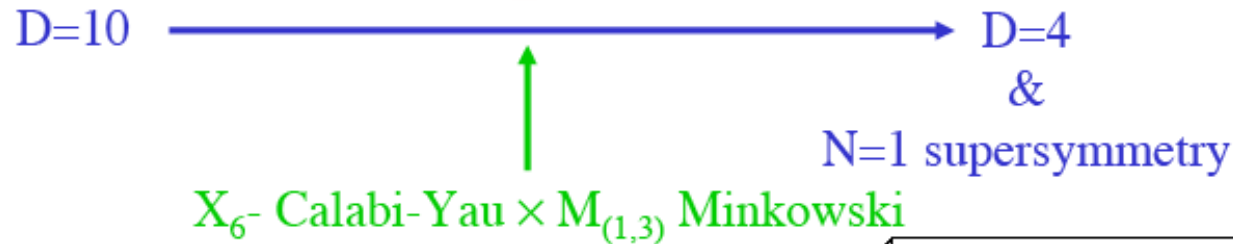


Delle Rose, Marzo, C.C.

D p-branes

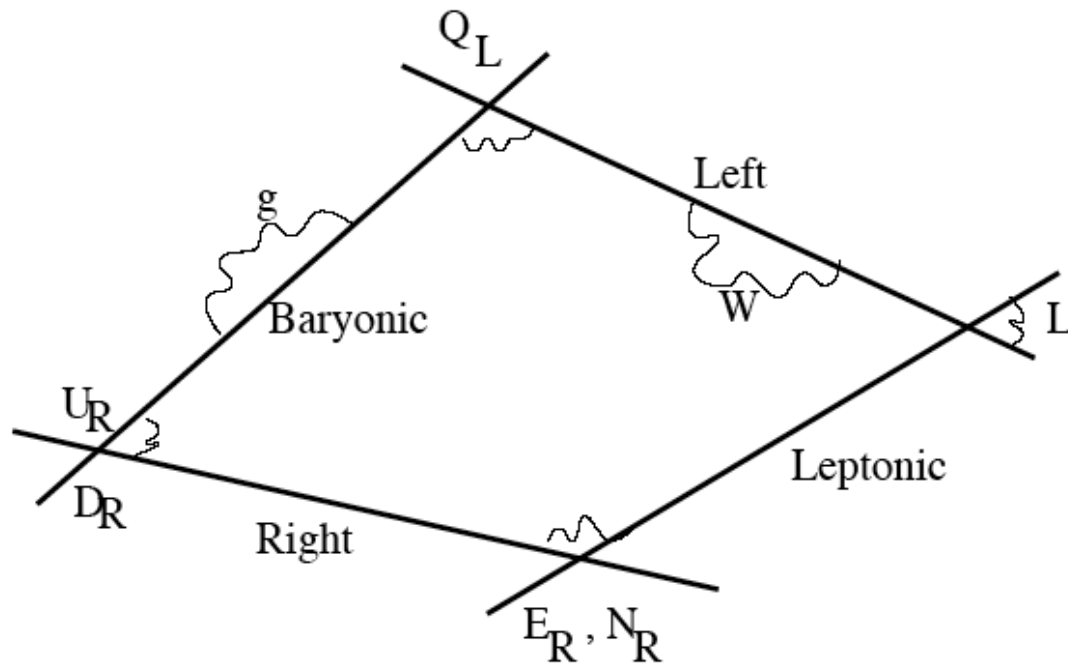


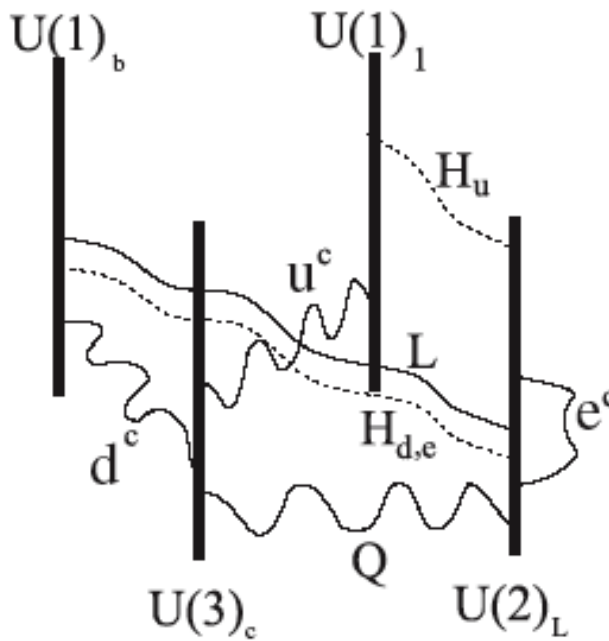
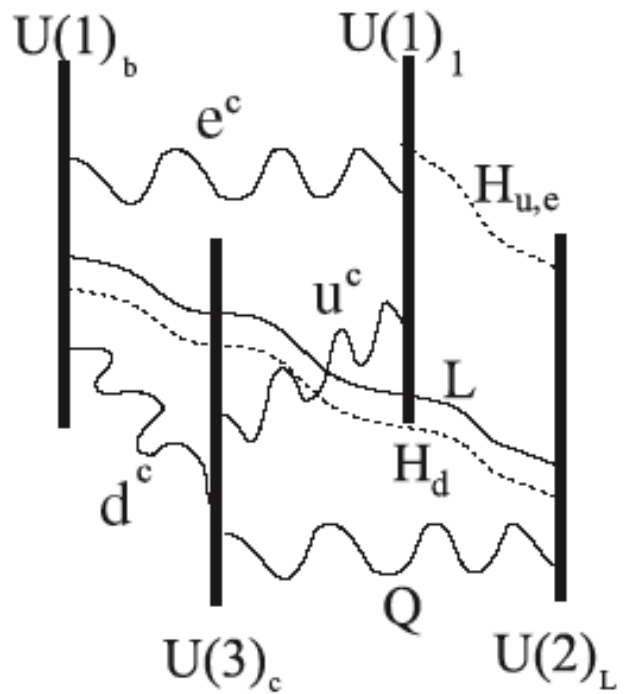
Compactification



Label	Multiplicity	Gauge Group	Name
stack a	$N_a = 3$	$SU(3) \times U(1)_a$	Baryonic brane
stack b	$N_b = 2$	$SU(2) \times U(1)_b$	Left brane
stack c	$N_c = 1$	$U(1)_c$	Right brane
stack d	$N_d = 1$	$U(1)_d$	Leptonic brane

Table 1: Brane content yielding the SM spectrum.





$$Y = -\frac{1}{3} Q_c - \frac{1}{2} Q_L + Q_1$$

$Q(\mathbf{3}, \mathbf{2}, +1, -1, 0, 0)$

$u^c(\bar{\mathbf{3}}, \mathbf{1}, -1, 0, -1, 0)$

$d^c(\bar{\mathbf{3}}, \mathbf{1}, -1, 0, 0, -1)$

$L(\mathbf{1}, \mathbf{2}, 0, +1, 0, -1)$

$e^c(\mathbf{1}, \mathbf{1}, 0, 0, +1, +1)$

$H_u(\mathbf{1}, \mathbf{2}, 0, +1, +1, 0)$

$H_d(\mathbf{1}, \mathbf{2}, 0, -1, 0, -1)$

Irges, Kiritsis, C.C.

“On the effective theory of low-scale
Orientifold vacua”

The study the effective field theory of
a class of models containing a gauge structure of the form

$$SM \times U(1) \times U(1) \times U(1)$$

$$SU(3) \times SU(2) \times U(1)_Y \times U(1) \dots$$

from which the hypercharge is assigned to be anomaly free

These models are the object of an intense scrutiny by
many groups working on intersecting branes in the past.

Antoniadis, Kiritsis, Rizos, Tomaras

Antoniadis, Leontaris, Rizos

Ibanez, Marchesano, Rabadan,

Ghilencea, Ibanez, Irges, Quevedo

See. E. Kiritsis' review on Phys. Rep.

What happens if you to have an anomalous
U(1) at low energy? What is its signature?

Gauged Stuckelberg axions: field theory realization of the Green-Schwarz mechanism of string theory

The gauging procedure requires an anomalous abelian symmetry (an anomalous $U(1)$) and a periodic potential in order to make the axion physical.

But first we are going to review the PQ axion



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Axions and the Strong CP Problem

Axions have appeared in physics in an attempt to solve the strong CP problem of QCD.

Why is the $\theta G \tilde{G}$ term so small?
Consider an $SU(2)$ gauge theory

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc} A_\mu^b A_\nu^c$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \quad G_{\mu\nu} = G_{\mu\nu}^a T^a$$

$$A_\mu \rightarrow UA_\mu U^{-1} + U\partial_\mu U^{-1}$$

$$G_{\mu\nu} \rightarrow UG_{\mu\nu} U^{-1}$$

We look for minima of the Euclidean action

$$S = -\frac{1}{2g^2} \int d^4x \text{Tr} G_{\mu\nu} G_{\mu\nu}$$

In a nonabelian theory a vanishing field strength is possible with

$$A_\mu = U \partial_\mu U^{-1}$$

(pure gauge). Solutions of this condition are instanton configurations, characterised by a topological number.

$$-16\pi^2 Q(x) = \text{Tr}[G_{\mu\nu} \tilde{G}_{\mu\nu}] = \text{Tr}[\epsilon_{\mu\nu\alpha\beta} [2\partial_\mu (A_\nu \partial_\alpha A_\beta + \frac{2}{3} A_\nu A_\alpha A_\beta)],$$

$$\tilde{G} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}, \quad Q(x) = \partial_\mu J_\mu, \quad J_\mu = -\frac{1}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} A_\nu (\partial_\alpha A_\beta + \frac{2}{3} A_\alpha A_\beta)$$

For an $SU(3)$ gauge theory such as QCD, similarly, the Lagrangean then allows a total derivative term $\theta G \tilde{G}$ which is a boundary term, but cannot be neglected. For instantons

$$G = \tilde{G}, \quad \int d^4x G \tilde{G}(x) = 32\pi^2 n,$$

Therefore \rightarrow *There is a dimension-4 operator that we can write down in the Standard Model (SM)*

$$\theta_0 G \tilde{G}$$

(violates Parity and Time reversal, CP is broken)

It is a total derivative term and as such it does not contribute *in perturbation theory*

Adding a total derivative term gives a zero momentum vertex in perturbation theory, but it contributes non-perturbatively

How?

If we consider an instanton (Euclidean) configuration, then the contribution to the path integral is

$$\sim e^{-S_0} = e^{-\frac{1}{4g^2} \int d^4x FF} = e^{-\frac{8\pi^2}{g^2}}$$

- ▶ These configurations, at small coupling, give a negligible contribution
- ▶ They are solutions of the classical eq. of motion of QCD, which is scale invariant at classical level
However, the solution of the equation $G = \tilde{G}$ involves an integration constant, the size of the instanton.
- ▶ The solution breaks scale invariance, because of the integration constant, which remains arbitrary.
It tells us where the energy of the configuration is localized.
At tree level g is constant, but at 1-loop it runs. Scale invariance is broken by renormalization.

In the functional integral we need to sum over all these configurations.

Small instantons (R)

- ▶ → large scale $\lambda \sim 1/R$
- ▶ → small coupling $g(\lambda) \ll 1$
- ▶ → large suppression in $e^{-\frac{8\pi^2}{g^2(\lambda)}}$. The contribution is perturbative, since g is small, but it is negligible.

The instanton contribution to the QCD action is dominated by large instantons ($g(\lambda)$ large). Unfortunately the contribution is non-perturbative.

- ▶ The running is controlled by the size of the instanton,
 $g = g(\lambda)$

In the functional integral we need to sum over all these configurations.

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The instanton contribution to the QCD action is dominated by large instantons ($g(\lambda)$ large). Unfortunately the contribution is non-perturbative.

- ▶ The saddle point approximation is not valid any more since the action is $O(1)$.

The partition function can be written in the form

$$\sim e^{-8\pi^2/g^2(\lambda) - i\theta_0}$$

and summing over instantons/anti instantons

$$\sum_{I\bar{I}} \sim e^{-8\pi^2/g^2(\lambda)} \cos \theta_0$$

θ_0 is not directly observable. One expects the energy density to depend on θ_0 . Notice, however, that QCD has a $U(1)_A$ anomaly, due to fermions. There is an axial symmetry

$$q \rightarrow qe^{i\gamma_5\alpha}$$

and the integration measure is not invariant

$$DqD\bar{q} \rightarrow DqD\bar{q} e^{-\frac{i}{16\pi^2}\alpha \int F\tilde{F}d^4x}$$

Therefore θ_0 is not physical because it can be shifted by a field redefinition

$$\theta_0 \rightarrow \theta_0 + 2\alpha$$

But also the quark mass term gets a phase under the chiral transformation

$$\bar{q}_L M q_R + h.c. \rightarrow \bar{q}_L M q_R e^{2i\alpha} + h.c.$$

therefore

$$\arg M \rightarrow \arg M + 2\alpha$$

and

$$\theta \equiv \theta_0 - \arg M$$

is invariant under field redefinitions. *If we have fermions in complex representations of the gauge group, θ_0 is affected by field redefinitions and is not physical, but θ is physical. This can be generalized to n_f fermions.*

$$\theta_0 \rightarrow \theta_0 + 2n_f\alpha, \quad \text{Argdet}M \rightarrow \text{Argdet}M + 2n_f\alpha$$

$$\theta \equiv \theta_0 - \text{Argdet}M$$

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is physical.

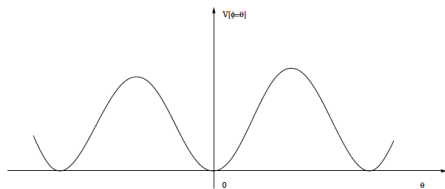
Experimentally θ is very small. We can set this value to zero assuming a cancellation between

- ▶ θ_0 (related to gluon dynamics)
- ▶ $Arg Det M$ (related to the electroweak sector, Yukawas and Higgs)

We can easily derive some properties of the vacuum energy as a function of θ .

$$e^{-VE(\theta)} = \left| \int D\Phi e^{-S[\Phi] - \frac{i}{32\pi^2} \theta \int F\tilde{F} d^4x} \right|$$

$$\leq \int D\Phi \left| e^{-S[\Phi] - \frac{i}{32\pi^2} \theta \int F\tilde{F} d^4x} \right| = e^{-VE(\theta=0)}$$



$$E(\theta) \geq E(0)$$

It is also even in θ : $E(\theta) = E(-\theta)$. Periodic of period 2π .

We can eliminate the θ_0 term and bring it completely into the fermion Mass matrix.

$$q_L \rightarrow q_L e^{+i\theta_0/2} \quad q_R \rightarrow q_R e^{-i\theta_0/2}$$

Then

$$M \rightarrow e^{-i\theta_0/2} M e^{-i\theta_0/2}$$

It can be generalized to

$$q_L^f \rightarrow q_L e^{+iQ_f\theta_0/2} \quad q_R^f \rightarrow q_R e^{-iQ_f\theta_0/2}$$

as far as

$$\text{Tr}Q_f = 1$$

(global phase is θ_0).

QCD with light quarks has a chiral symmetry (u,d)

$$U(2)_L \times U(2)_R = SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

broken by quark condensates and anomalies to

$$SU(2)_V \times U(1)_V$$

with $U(1)_V$ =baryon number. Three NG-models π^\pm, π^0 of the broken chiral symmetry. We try to fix the low energy effective action using the left-over global symmetries

$$\mathcal{Z}[J] = \int D\Phi e^{iS_{QCD}(\Phi)+J\Phi} = \int D\pi e^{iS(\pi,J)}$$

$$\begin{bmatrix} \pi^0 & \sqrt{2}\pi^+ \\ -\sqrt{2}\pi^- & -\pi^0 \end{bmatrix}$$

$$U = e^{i\pi \cdot T / f_\pi}$$

$$\mathcal{L} = \frac{f_\pi^2}{4} \left(\text{Tr} \left[\partial_\mu U^\dagger \partial^\mu U \right] + 2B_0 \text{Tr} \left[MU^\dagger + M^\dagger U \right] \right)$$

$$E(\theta, \pi) = -\frac{f_\pi^2}{4} 2B_0 2\text{ReTr} \left(\begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix} e^{i\theta/2} \text{Exp} \frac{i}{f_\pi} \begin{bmatrix} \pi^0 & 0 \\ 0 & -\pi^0 \end{bmatrix} \right)$$

$$= -m_\pi^2 f_\pi^2 \sqrt{\cos^2 \frac{\theta}{2} + \left(\frac{m_u - m_d}{m_u + m_d} \right)^2 \sin^2 \frac{\theta}{2} \cos\left(\frac{\pi^0}{f_\pi} - \phi(\theta)\right)}$$

where

$$\sin(\phi) = \frac{m_d - m_u}{m_d + m_u} \sin^2 \frac{\theta}{2}$$

A minimum is obtained for (vev) $\pi^0 = f_\pi \phi(\theta)$ (with $m_\pi^2 = B_0(m_u + m_d)$) Then

$$E(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{\theta}{2}}$$

when the mass of any of the quarks goes to zero, the θ -dependence disappears.

For $\theta = 0$ $E(0) = -m_\pi^2 f_\pi^2$

Possible solutions. Can we use any existing SM symmetry?

After we turn on the Yukawa's only B and L are left as global symmetries of the SM.

In the SM we have an anomalous symmetry B , baryon number and L , lepton number (B-L is anomaly free).

But B is not anomalous respect to $SU(3)_c$, whence it cannot produce a $F_g \tilde{F}_g$ (gluon).

We then require an extra $U(1)_{PQ}$ global symmetry.

There is another solution: if $Y_u = 0$ then we could rotate:

$$u_R \rightarrow e^{i\alpha} u_R$$

This symmetry would be anomalous under $SU(3)_c$ and we could erase the $\theta F_g \tilde{F}_g$ term.

Notice that in the electroweak case we could also consider a "weak CP" problem $\sim \theta_W F_W \tilde{F}_W$

In fact B is anomalous under $SU(2)_L$, electroweak quark doublets therefore could be redefined under $U(1)_{baryon}$, canceling the corresponding weak-CP violating term.

A second type of protection from θ_W contributions come from the fact that the theory is in a Higgs phase. The contribution is

$e^{\frac{-8\pi^2}{g_w(W)^2}}$ which are screened due to the masses of the W 's and Z .

KSVZ axion (Kim, Shifman, Vainshtein, Zakharov)

A pseudoscalar $a(x)$ that shifts under a global $U(1)_{PQ}$ symmetry (NG mode) $a(x) \rightarrow a(x) + \alpha f_a$ can do the job.

Use the Lagrangean

$$\frac{1}{2} \partial_\mu a \partial^\mu a + \frac{a(x)}{32\pi^2} F \tilde{F} + i \bar{Q} \gamma^\mu \partial_\mu Q + \lambda \phi \bar{Q}_L Q_R$$

ϕ has a typical mexican-hat potential, with $\langle \phi \rangle = v_{PQ}$. Then

$$\phi(x) = \frac{v_{PQ} + \rho}{\sqrt{2}} e^{i \frac{a(x)}{v_{PQ}}} \text{ and}$$

$$\frac{\lambda}{\sqrt{2}} \phi \bar{Q}_L Q_R \sim \lambda v_{PQ} e^{i \frac{a}{v_{PQ}}} \bar{Q}_L Q_R$$

We perform now a chiral field redefinition

$$Q \rightarrow Q' = e^{-i \frac{a}{2v_{PQ}} \gamma^5} Q \quad \frac{\lambda}{\sqrt{2}} v_{PQ} \bar{Q}'_L Q'_R$$

. We will generate a term $\delta S = \frac{a}{32\pi^2 v_{PQ}} F \tilde{F}$, since the field redefinition is anomalous under $U(1)_{PQ}$.

Now we can integrate out Q and ρ . We are left with an interaction

$$\frac{a(x)N}{32\pi^2 v_{PQ}} F \tilde{F} = \frac{a(x)}{32\pi^2 f_a} F \tilde{F}$$

for N quarks Q , with $\frac{v_{PQ}}{N} = f_a$.

DFSZ axion (PQ), (WW). This is generated using only scalars.

$$H_u, H_d, \phi$$

Up to dimension-4 involves three mexican-hat types of potentials for H_u, H_d and ϕ , and an extra contribution

V' which depends on

$$|H_u|^2, |H_d|^2, |\phi|^2, |H_u H_d^\dagger|^2, |H_u \cdot H_d|^2, H_u \cdot H_d \phi^2.$$

Collecting the phases, one can identify the NG mode of the $U(1)_{PQ}$ using the condition that it has to be orthogonal to the hypercharge

There are 3 phases. One of them will identify the Goldstone mode.

Orthogonality respect to the Goldstone of the Z boson is found by

looking at the bilinear mixing $M_Z Z_\mu \partial^\mu G_Z$

$$H_u = \frac{v_u}{\sqrt{2}} e^{i \frac{q_u a(x)}{v_{PQ}}} \quad H_d = \frac{v_d}{\sqrt{2}} e^{i \frac{q_d a(x)}{v_{PQ}}} \quad \phi = \frac{v_\phi}{\sqrt{2}} e^{i \frac{q_\phi a(x)}{v_{PQ}}}$$

$$q_\phi = -1 \quad q_u = 2 \frac{v_d^2}{v^2} \quad q_d = 2 \frac{v_d^2}{v^2}$$

$$v^2 = v_u^2 + v_d^2$$

absence of mixing with G_Z : $q_u^2 v_u^2 - q_d^2 v_d^2 = 0$. v is the electroweak vev (246 GeV).

By requiring that $a(x)$ is canonically normalized:

$v_{PQ} = v_\phi^2 + v^2 \sin 2\beta$, with $\sin \beta = \frac{v_u}{v}$ and $\cos \beta = \frac{v_d}{v}$. Notice that $a(x)$ is associated mostly to ϕ .

From the Yukawa couplings one gets

$$-Y_u \bar{q}_L H_u q_R - Y_d \bar{q}_L H_d d_R$$

$$-Y_u \bar{u}_L \frac{v_u}{\sqrt{2}} e^{2ia \sin^2 \beta \frac{a}{v_{PQ}}} u_R - Y_d \bar{d}_L \frac{v_d}{\sqrt{2}} e^{2ia \cos^2 \beta \frac{a}{v_{PQ}}} d_R$$

Doing a chiral redefinition

$$\delta \mathcal{L} = \frac{6}{32\pi^2 v_{PQ}} a F \tilde{F} \quad \bar{q}_L \gamma^\mu D_\mu q_L \rightarrow \frac{c}{v_{PQ}} \partial_\mu a \bar{q} \gamma^\mu \gamma_5 q$$

$$\mathcal{L} = \mathcal{L}_{QCD}(\theta = 0) + \frac{1}{f_a} \partial_\mu J^\mu + \left(\frac{a}{f_a} - \theta \right) \frac{1}{32\pi^2} F \tilde{F} + \frac{1}{2} \partial_\mu a \partial^\mu a$$

we can clearly redefine $a(x)$ in order to absorb θ .

Since f_a is very large, then we can treat $a(x)$ as an external source.

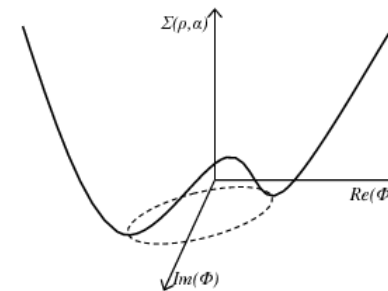
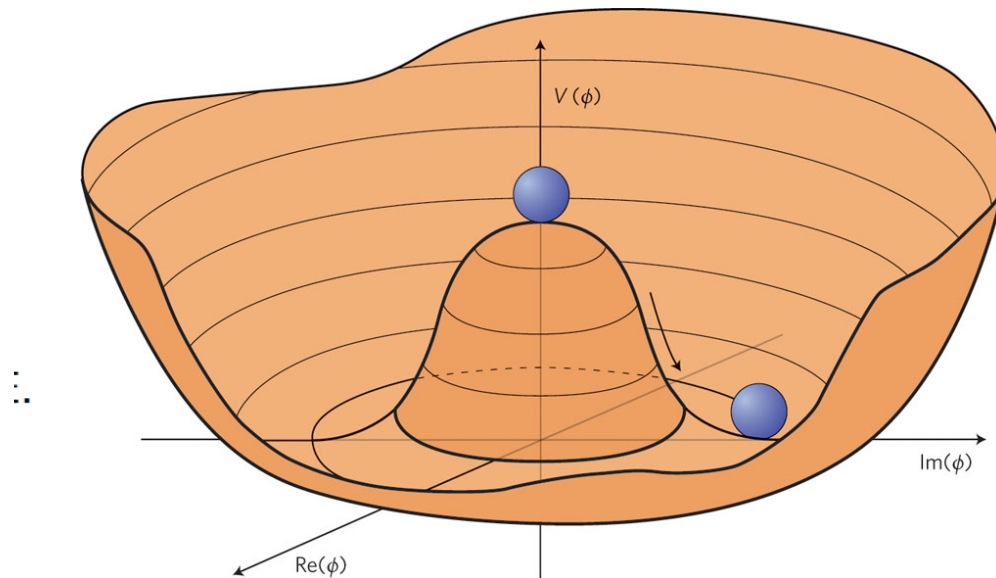
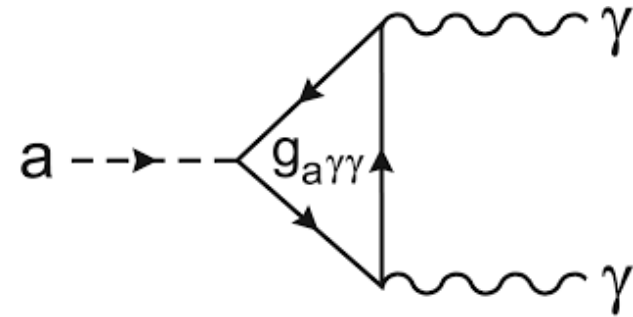
To determine its potential, we can then take $V(\theta)$ with $\theta \rightarrow a/f_a$

$$V\left(\frac{a}{f_a}\right) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2f_a}\right)}$$

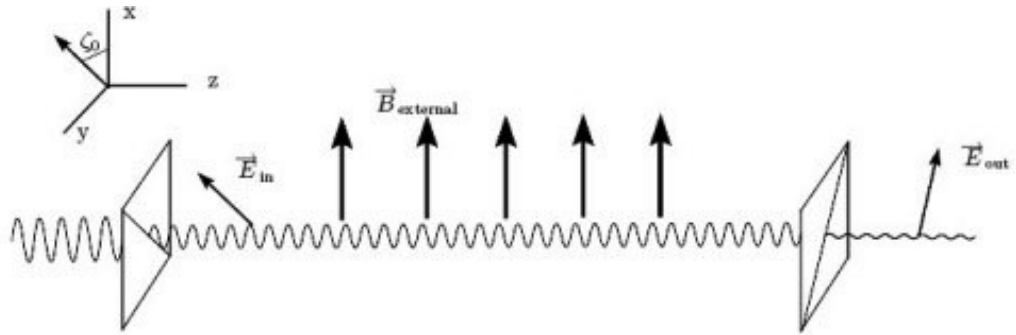
from which one can extract the axion mass

$$m_a^2 = \frac{m_\pi^2}{f_a^2} f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$

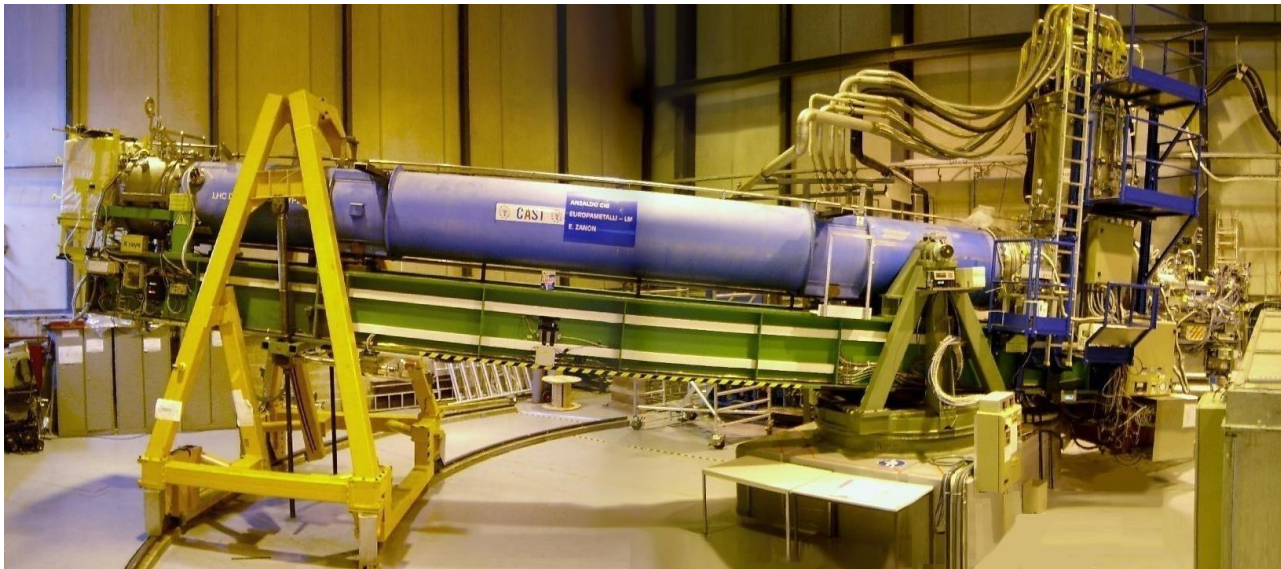
The breaking of the PQ symmetry takes place at a large scale f_a , but the wiggling of the PQ potential occurs much later, at the QCD phase transition



For a PQ axion a : $m = C/f_a$, while the $a\text{FF}$ interaction is also suppressed by $a/f_a \text{FF}$ with $f_a = 10^9 \text{ GeV}$



PVLAS (INFN)



CAST (Cern)

Optical activity

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi + \frac{1}{4}\tilde{g}\varphi F_{\mu\nu}\tilde{F}^{\mu\nu},$$

$$\left\{ \begin{array}{l} \nabla \cdot \mathbf{B} = 0, \\ \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0, \\ \square\varphi = -\tilde{g}\mathbf{E} \cdot \mathbf{B}, \\ \nabla \cdot \mathbf{E} = \tilde{g}\nabla\varphi \cdot \mathbf{B}, \\ \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = -\tilde{g}\mathbf{B}\frac{\partial\varphi}{\partial t} + \tilde{g}\mathbf{E} \times \nabla\varphi, \end{array} \right. \quad \text{PVLAS-type}$$

$$\square(\mathbf{E} - \frac{1}{2}\tilde{g}\varphi\mathbf{B}) = -\frac{1}{2}\tilde{g}\varphi\square\mathbf{B},$$

$$\square(\mathbf{B} + \frac{1}{2}\tilde{g}\varphi\mathbf{E}) = \frac{1}{2}\tilde{g}\varphi\square\mathbf{E}.$$

L. Carcagni', C.C.

$$\mathbf{D} \equiv \mathbf{E} - \frac{1}{2}\tilde{g}\varphi\mathbf{B},$$

$$\mathbf{H} \equiv \mathbf{B} + \frac{1}{2}\tilde{g}\varphi\mathbf{E}.$$

$$\Delta\mathbf{E} \equiv \mathbf{E}(L) - \mathbf{E}(0) = \frac{1}{2}\tilde{g}\Delta\varphi\mathbf{H}(0).$$

Optical activity

Gauging axionic symmetries

The chain of anomalous $U(1)$ symmetries requires

- One Stuckelberg term for each anomalous symmetry
- The $U(1)$'s are in a massive (Stuckelberg phase)
- One linear combination of them generates the anomaly free hypercharge

Possibility of describing axion-like particles.

Such types of particles have been conjectured in several phenomenological analysis.

The mass of the particle and its interactions with the photons are independent quantities.

Our suggestion: use **anomalous abelian (gauge) symmetries**

This brings us to a mechanism of cancelation of the gauge anomalies of “GreenSchwarz” type

Compared to a Peccei-Quinn axion, the new axion is gauged

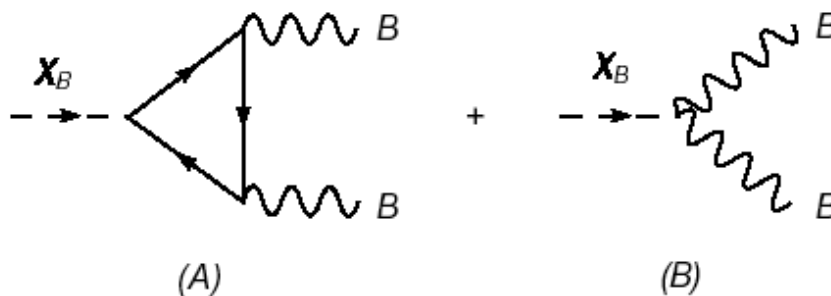
For a PQ axion a : $m = C/f_a$, while the aFF interaction is also suppressed by : $a/f_a FF$ with $f_a = 10^9 \text{ GeV}$

In the case of these models, the mass of the axion and its gauge interactions are unrelated

the mass is generated by the combination of the Higgs and the Stuckelberg mechanisms combined

The interaction is controlled by the Stuckelberg mass (M_1)

The axion shares the properties of a CP odd scalar



Asymptotic axions for Wess Zumino actions and gauge invariance

$$\mathcal{L} = -\frac{1}{4}F_B^2 + i\bar{\psi}\gamma^\mu(\partial_\mu + ig_B\gamma_5 B_\mu)\psi$$

$$\mathcal{L} = -\frac{1}{4}F_B^2 - \frac{1}{4}F_A^2 + i\bar{\psi}\gamma^\mu(\partial_\mu + ig_A A_\mu + ig_B\gamma^5 B_\mu)\psi$$

Using a Stuckelberg axion and the inclusion of local counterterms

$$B_\mu \rightarrow B_\mu - \partial_\mu\theta$$

$$b \rightarrow b + M\theta$$

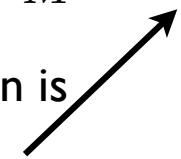
$$a_{BBB}\frac{b}{M}F_B \wedge F_B + a_{BAA}\frac{b}{M}F_A \wedge F_A$$

$$\frac{1}{2}(\partial_\mu b + MB_\mu)^2$$

One then considers the effective action

$$\mathcal{L} = -\frac{1}{4}F_B^2 + \frac{1}{2}\left(B_\mu + \frac{1}{M}\partial_\mu b\right)^2 + i\bar{\psi}\gamma^\mu(\partial_\mu + ig_B\gamma_5)\psi + a_n\frac{b}{M}F_B \wedge F_B$$

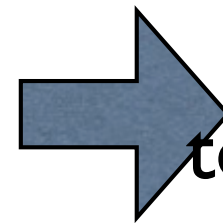
where the anomaly generated at one loop level by the fermion is removed by the Wess-Zumino counterterm



$$a_n\frac{b}{M}F_B \wedge F_B$$

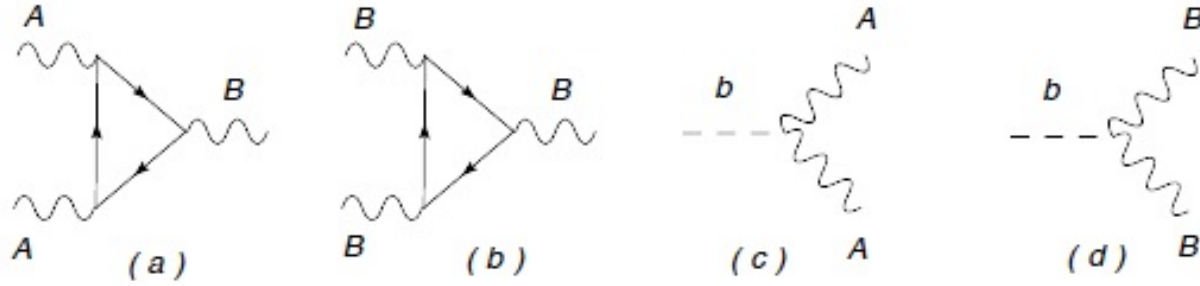
Somehow, this mechanism is viewed, from the point of view of QFT, as the mechanism of “Anomaly Cancellation”

But anomalies are not cancelled by local counterterms. One should notice that the mechanism of “anomaly cancellation”, in this case, is based on introducing an extra field degree of freedom ($b(x)$)

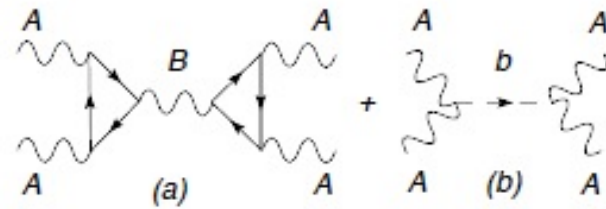


One could go to a gauge where $b(x)=0$.

In what sense, then we cancel the anomaly?



One loop vertices and counterterms in the R_ξ gauge for the $A - B$ model for the WZ case.



A typical Bouchiat-Iliopoulos-Meyer amplitude and the axion counterterm to restore gauge invariance in the R_ξ gauge in the WZ effective action.

$$\mathcal{L}_{WZ} = \frac{C_{AA}}{2!M_1} b F_A \wedge F_A + \frac{C_{BB}}{2!M_1} b F_B \wedge F_B,$$

Variants: Higgs-axion mixing

There are some variants of this Lagrangian which may help us clarify this issue

$$\mathcal{L}_0 = |(\partial_\mu + ig_B q_B B_\mu)\phi|^2 - \frac{1}{4}F_A^2 - \frac{1}{4}F_B^2 + \frac{1}{2}(\partial_\mu b + M_1 B_\mu)^2 - \lambda(|\phi|^2 - \frac{v^2}{2})^2 + \bar{\psi}i\gamma^\mu(\partial_\mu + ieA_\mu + ig_B\gamma^5 B_\mu)\psi - \lambda_1\bar{\psi}_L\phi\psi_R - \lambda_1\bar{\psi}_R\phi^*\psi_L$$

In this case we consider a model with 2 U(1)'s. The two gauge fields are A and B. The fermion has axial vector couplings to B and is vector coupled to A.

We have BBB and BAA anomalies. Vector field B is massive, A is massless

B mass generated via a combination of the Stuckelberg + Higgs mechanisms.

ϕ is the Higgs field

$$|(\partial_\mu + ig_B q_B B_\mu)\phi|^2 + \frac{1}{2}(\partial_\mu b + M_1 B_\mu)^2 - \lambda(|\phi|^2 - \frac{v^2}{2})^2$$

$$\mathcal{L}_b = \frac{C_{AA}}{M} b F_A \wedge F_A + \frac{C_{BB}}{M} b F_B \wedge F_B.$$

$$\delta_B (\mathcal{L}_b + \mathcal{L}_{an}) = 0$$

Higgs-Axion Mixing in U(1) Models: massless axi-Higgs

$$\phi = \frac{1}{\sqrt{2}}(v + \phi_1 + i\phi_2),$$

$$\mathcal{L}_q = \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \frac{1}{2}(\partial_\mu b)^2 + \frac{1}{2}(M_1^2 + (q_B g_B v)^2) B_\mu B^\mu - \frac{1}{2}m_1^2\phi_1^2 + B_\mu \partial^\mu (M_1 b + v g_B q_B \phi_2),$$

B field massive by the Higgs and Stuckelberg mechanism

Goldstone mode is a combination of Stuckelberg field and CP odd part of the Higgs

$$\mathcal{L}_q = \frac{1}{2}(\partial_\mu \chi_B)^2 + \frac{1}{2}(\partial_\mu G_B)^2 + \frac{1}{2}(\partial_\mu h_1)^2 + \frac{1}{2}M_B^2 B_\mu B^\mu - \frac{1}{2}m_1^2 h_1^2 + M_B B^\mu \partial_\mu G_B \quad \begin{aligned} m_1 &= v\sqrt{2\lambda}, \\ M_B &= \sqrt{M_1^2 + (q_B g_B v)^2}. \end{aligned}$$

The mass of the B gauge boson is a combination of the Higgs and the Stuckelberg mechanism

I physical axion (axi-Higgs) χ_B
 I Higgs h_1
 I massive gauge boson B_μ

$$\theta_B = \arccos(M_1/M_B)$$

$$\chi_B = \frac{1}{M_B}(-M_1 \phi_2 + q_B g_B v b), \quad (\phi_2, b) \rightarrow (\chi_B, G_B) \quad U = \begin{pmatrix} -\cos \theta_B & \sin \theta_B \\ \sin \theta_B & \cos \theta_B \end{pmatrix}$$

$$G_B = \frac{1}{M_B}(q_B g_B v \phi_2 + M_1 b),$$

$$b = \alpha_1 \chi_B + \alpha_2 G_B = \frac{q_B g_B v}{M_B} \chi_B + \frac{M_1}{M_B} G_B,$$

The Stuckelberg has a gauge invariant physical component, χ_B

A massive axi-Higgs (periodic potential)

ordinary Higgs potential $V = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$

$$V' = b_1 \left(\phi e^{-iq_B g_B \frac{b}{M_1}} \right) + \lambda_1 \left(\phi e^{-iq_B g_B \frac{b}{M_1}} \right)^2 + 2\lambda_2 (\phi^* \phi) \left(\phi e^{-iq_B g_B \frac{b}{M_1}} \right) + \text{c.c.}$$

extra potential allowed by the symmetry

$$c_\chi = 4 \left(\frac{b_1}{v^3} + \frac{4\lambda_1}{v^2} + \frac{2\lambda_2}{v} \right).$$

$$m_\chi^2 = -\frac{1}{2} c_\chi v^2 \frac{M_B^2}{M_1^2}.$$

massive axi-Higgs

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}tr G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}tr W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}F_{\mu\nu}^l F^{\mu\nu,l} \\
& - |(\partial_\mu + i\frac{g_2}{2}\tau^a W_\mu^a + iq_l^{(H_u)} g_l A_\mu^l)H_u|^2 - |(\partial_\mu + i\frac{g_2}{2}\tau^a W_\mu^a + iq_l^{(H_d)} g_l A_\mu^l)H_d|^2 \\
& + Q_{Li}^\dagger \sigma^\mu \mathcal{D}_\mu Q_{Li} + u_{Ri}^\dagger \bar{\sigma}^\mu \mathcal{D}_\mu u_{Ri} + d_{Ri}^\dagger \bar{\sigma}^\mu \mathcal{D}_\mu d_{Ri} \\
& + L_{Li}^\dagger \sigma^\mu \mathcal{D}_\mu L_{Li} + e_{Ri}^\dagger \bar{\sigma}^\mu \mathcal{D}_\mu e_{Ri} + \nu_{Ri}^\dagger \bar{\sigma}^\mu \mathcal{D}_\mu \nu_{Ri} \\
& + \gamma_{ij}^u H_u^T \tau^2 (Q_{Li}^t \sigma^2 u_{Rj}) + \gamma_{ij}^d H_d^t (Q_{Li}^t \sigma^2 d_{Rj}) + c.c. \\
& + \gamma_{ij}^e H_u^\dagger (L_{Li}^t \sigma^2 e_{Rj}) + \gamma_{ij}^\nu H_d^T \tau^2 (L_{Li}^t \sigma^2 \nu_{Rj}) + c.c. \\
& - \frac{1}{2} \sum (\partial_\mu a^I + g_l \mathcal{M}_l^I A_\mu^l)^2 + E_{lmn} \epsilon^{\mu\nu\rho\sigma} A_\mu^l A_\nu^m F_{\rho\sigma}^n
\end{aligned}$$

Generic extension

$$+ \sum_I (D_I a^I tr \{G \wedge G\} + F_I a^I tr \{W \wedge W\} + C_{Imn} a^I F^m \wedge F^n)$$

$$+ V(H_u, H_d, a^I).$$

The gauge symmetry under which this Lagrangian is invariant is

$$SU(3)_c \times SU(2)_W \times G_1, \quad G_1 = \prod_{l=1}^4 U(1)_l.$$

Gauge kinetic

Stuckeberg mass terms

Chern Simons abelian interactions

$$SU(3) \times SU(2) \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d.$$

$$a_I, \quad I = 1, 2, \dots, n \quad \text{Stuckelberg axions}$$

$$F_I$$

$$H_u \quad H_d$$

$$E_{lmn} \epsilon^{\mu\nu\rho\sigma} A_\mu^l A_\nu^m F_{\rho\sigma}^n$$

Abelian CS terms

Higgs sector

$$|\mathcal{D}_\mu H_u|^2 + |\mathcal{D}_\mu H_d|^2 + \frac{1}{2} \sum_I (\partial a'_I + M_I A^I)^2$$

$$\mathcal{D}_\mu H_u = \left(\partial_\mu + \frac{i}{\sqrt{2}} g_2 (T^+ W^+ + T^- W^-) + \frac{i}{2} g_2 \tau_3 W_{3\mu} + \frac{i}{2} g_Y A_\mu^Y + \frac{i}{2} \sum_I q_u^I g_I A_\mu^I \right) H_u$$

$$\mathcal{D}_\mu H_d = \left(\partial_\mu + \frac{i}{\sqrt{2}} g_2 (T^+ W^+ + T^- W^-) + i \frac{g_2}{2} \tau_3 W_{3\mu} + \frac{i}{2} g_Y A_\mu^Y + \frac{i}{2} \sum_I q_d^I g_I A_\mu^I \right) H_d$$

Typical mass terms for the gauge bosons are generated both from the Higgs and the Stuckleberg contributions

$$\begin{aligned} & \frac{1}{2} \sum_I M_I^2 (A_\mu^I)^2 + \frac{1}{4} (-g_2 W_{3\mu} + g_Y A_\mu^Y + \sum_I q_u^I g_I A_\mu^I)^2 v_u^2 \\ & + \frac{1}{4} (-g_2 W_{3\mu} + g_Y A_\mu^Y + \sum_I q_d^I g_I A_\mu^I)^2 v_d^2, \end{aligned}$$

There will be bilinear mixings in the broken (electroweak) phase

$$Z^\mu \partial_\mu \left\{ f_u C^u + f_d C^d + \sum_I g_I M_I O_{Z_I}^A a'_I \right\} + \sum_J Z_J'^\mu \partial_\mu \left\{ f_{u,J} C^u + f_{d,J} C^d + \sum_I g_I M_I O_{Z_J I}^A a'_I \right\},$$

We can extract the NG modes by a rotation, identifying 1 single physical axion

$$\begin{pmatrix} \text{Im} H_u^0 \\ \text{Im} H_d^0 \\ \cdot \\ a'_I \\ \cdot \end{pmatrix} = O^\chi \begin{pmatrix} \chi \\ G_1^0 \\ G_2^0 \\ \cdot \\ \cdot \end{pmatrix}$$

The scalar potential has an ordinary 2-Higgs doublet part and an extra contribution

$$V_{PQ} = \sum_{a=u,d} \left(\mu_a^2 H_a^\dagger H_a + \lambda_{aa} (H_a^\dagger H_a)^2 \right) - 2\lambda_{ud} (H_u^\dagger H_u) (H_d^\dagger H_d) + 2\lambda'_{ud} |H_u^T \tau_2 H_d|^2$$

$$\begin{aligned} V_{PQ} = & b (H_u^\dagger H_d e^{-i \sum_I (q_u^I - q_d^I) \frac{a'_I}{M_I}}) + \lambda_1 (H_u^\dagger H_d e^{-i \sum_I (q_u^I - q_d^I) \frac{a'_I}{M_I}})^2 \\ & + \lambda_2 (H_u^\dagger H_u) (H_u^\dagger H_d e^{-i \sum_I (q_u^I - q_d^I) \frac{a'_I}{M_I}}) + \lambda_3 (H_d^\dagger H_d) (H_u^\dagger H_d e^{-i \sum_I (q_u^I - q_d^I) \frac{a'_I}{M_I}}) + c.c. \end{aligned}$$

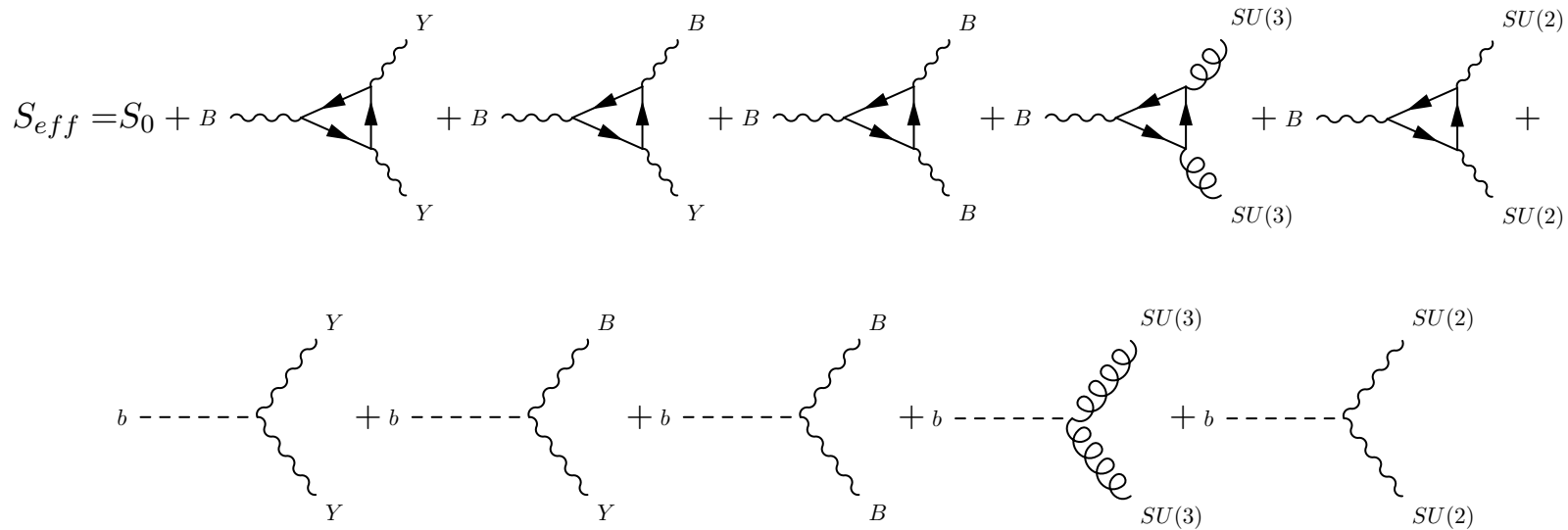
The Standard Model with 1 extra anomalous U(1) and an axion

f	Q	u_R	d_R	L	e_R
q^B	q_Q^B	$q_{u_R}^B$	$q_{d_R}^B$	q_L^B	$q_{e_R}^B$

f	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_B$
Q	3	2	1/6	q_Q^B
u_R	3	1	2/3	$q_Q^B + q_u^B$
d_R	3	1	-1/3	$q_Q^B - q_d^B$
L	1	2	-1/2	q_L^B
e_R	1	1	-1	$q_L^B - q_d^B$
H_u	1	2	1/2	q_u^B
H_d	1	2	1/2	q_d^B

The effective action has the structure given by

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_{Yuk} + \mathcal{S}_{an} + \mathcal{S}_{WZ} + \mathcal{S}_{CS}$$



Axionic contributions

$$\begin{aligned} \mathcal{S}_{WZ} = & C_{BB} \langle b F_B \wedge F_B \rangle + C_{YY} \langle b F_Y \wedge F_Y \rangle + C_{YB} \langle b F_Y \wedge F_B \rangle \\ & + F \langle b \text{Tr}[F^W \wedge F^W] \rangle + D \langle b \text{Tr}[F^G \wedge F^G] \rangle, \end{aligned}$$

Abelian/non-abelian Chern Simons terms

$$\begin{aligned} \mathcal{S}_{CS} = & +d_1 \langle BY \wedge F_Y \rangle + d_2 \langle YB \wedge F_B \rangle \\ & + c_1 \langle \epsilon^{\mu\nu\rho\sigma} B_\mu C_{\nu\rho\sigma}^{SU(2)} \rangle + c_2 \langle \epsilon^{\mu\nu\rho\sigma} B_\mu C_{\nu\rho\sigma}^{SU(3)} \rangle. \end{aligned}$$

$$\begin{aligned} C_{\mu\nu\rho}^{SU(2)} &= \frac{1}{6} \left[W_\mu^i \left(F_{i,\nu\rho}^W + \frac{1}{3} g_2 \varepsilon^{ijk} W_\nu^j W_\rho^k \right) + \text{cyclic} \right], \\ C_{\mu\nu\rho}^{SU(3)} &= \frac{1}{6} \left[G_\mu^a \left(F_{a,\nu\rho}^G + \frac{1}{3} g_3 f^{abc} G_\nu^b G_\rho^c \right) + \text{cyclic} \right]. \end{aligned}$$

With a single anomalous U(1) these terms care not essential.

$$V = V_{PQ}(H_u, H_d) + V_{\cancel{P}Q}(H_u, H_d, b).$$

$$V_{PQ} = \mu_u^2 H_u^\dagger H_u + \mu_d^2 H_d^\dagger H_d + \lambda_{uu}(H_u^\dagger H_u)^2 + \lambda_{dd}(H_d^\dagger H_d)^2 - 2\lambda_{ud}(H_u^\dagger H_u)(H_d^\dagger H_d) + 2\lambda'_{ud}|H_u^T \tau_2 H_d|^2$$

$$V_{\cancel{P}Q} = \lambda_0(H_u^\dagger H_d e^{-ig_B(q_u - q_d)\frac{b}{2M}}) + \lambda_1(H_u^\dagger H_d e^{-ig_B(q_u - q_d)\frac{b}{2M}})^2 + \lambda_2(H_u^\dagger H_u)(H_u^\dagger H_d e^{-ig_B(q_u - q_d)\frac{b}{2M}}) + \lambda_3(H_d^\dagger H_d)(H_u^\dagger H_d e^{-ig_B(q_u - q_d)\frac{b}{2M}}) + \text{h.c.},$$

$$H_u = \begin{pmatrix} H_u^+ \\ v_u + H_u^0 \end{pmatrix} \quad H_d = \begin{pmatrix} H_d^+ \\ v_d + H_d^0 \end{pmatrix}.$$

This potential is characterized by two null eigenvalues corresponding to two neutral Goldstone modes (G_0^1, G_0^2) and an eigenvalue corresponding to a massive state with an axion component (χ). In the $(\text{Im}H_d^0, \text{Im}H_u^0, b)$ CP-odd basis we get the following normalized eigenstates

$$\begin{aligned} G_0^1 &= \frac{1}{\sqrt{v_u^2 + v_d^2}}(v_d, v_u, 0) \\ G_0^2 &= \frac{1}{\sqrt{g_B^2(q_d - q_u)^2 v_d^2 v_u^2 + 2M^2(v_d^2 + v_u^2)}} \left(-\frac{g_B(q_d - q_u)v_d v_u^2}{\sqrt{v_u^2 + v_d^2}}, \frac{g_B(q_d - q_u)v_d^2 v_u}{\sqrt{v_d^2 + v_u^2}}, \sqrt{2}M\sqrt{v_u^2 + v_d^2} \right) \\ \chi &= \frac{1}{\sqrt{g_B^2(q_d - q_u)^2 v_d^2 v_u^2 + 2M^2(v_d^2 + v_u^2)}} \left(\sqrt{2}M v_u, -\sqrt{2}M v_d, g_B(q_d - q_u)v_d v_u \right) \end{aligned} \quad (14) \quad \begin{pmatrix} G_0^1 \\ G_0^2 \\ \chi \end{pmatrix} = O^\chi \begin{pmatrix} \text{Im}H_d^0 \\ \text{Im}H_u^0 \\ b \end{pmatrix},$$

$$O^\chi = \begin{pmatrix} \frac{v_d}{v} & \frac{v_u}{v} & 0 \\ -\frac{g_B(q_d-q_u)v_d v_u^2}{v\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2 + 2M^2 v^2}} & \frac{g_B(q_d-q_u)v_d^2 v_u}{v\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2 + 2M^2 v^2}} & \frac{\sqrt{2}Mv}{\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2 + 2M^2 v^2}} \\ \frac{\sqrt{2}Mv_u}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2 + 2M^2 v^2}} & -\frac{\sqrt{2}Mv_d}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2 + 2M^2 v^2}} & \frac{g_B(q_d-q_u)v_d v_u}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2 + 2M^2 v^2}} \end{pmatrix}$$

where $v = \sqrt{v_u^2 + v_d^2}$.

χ inherits $\dot{W}Z$ interaction since b can be related to the physical axion χ and to the Goldstone modes via this matrix

$$b = O_{13}^\chi G_0^1 + O_{23}^\chi G_0^2 + O_{33}^\chi \chi,$$

Stuckelberg axion

Physical axi-Higgs (gauged axion)

$$\chi = O_{31}^\chi \text{Im}H_d + O_{32}^\chi \text{Im}H_u + O_{33}^\chi b.$$

The phase-dependent potential has a well-defined periodicity. To identify the corresponding phase in the Higgs-neutral CP-odd sector we introduce a polar parametrization of the neutral components in the broken electroweak phase

$$H_u^0 = \frac{1}{\sqrt{2}} \left(\sqrt{2}v_u + \rho_u^0(x) \right) e^{i\frac{F_u^0(x)}{\sqrt{2}v_u}} \quad H_d^0 = \frac{1}{\sqrt{2}} \left(\sqrt{2}v_d + \rho_d^0(x) \right) e^{i\frac{F_d^0(x)}{\sqrt{2}v_d}}, \quad (22)$$

where we have introduced the two phases F_u and F_d of the two neutral Higgs fields. The potential is periodic with respect to the linear combination of fields

$$\theta(x) \equiv \frac{g_B(q_d - q_u)}{2M} b(x) - \frac{1}{\sqrt{2}v_u} F_u^0(x) + \frac{1}{\sqrt{2}v_d} F_d^0(x), \quad (23)$$

and using the matrix O^χ to rotate on the physical basis, the phase describing the periodicity of the potential turns out to be proportional to the physical axion, modulo a dimensionful constant (σ_χ)

$$\theta(x) \equiv \frac{\chi(x)}{\sigma_\chi}, \quad (24)$$

$$\sigma_\chi \equiv \frac{2v_u v_d M}{\sqrt{g_B^2 (q_d - q_u)^2 v_d^2 v_u^2 + 2M^2 (v_d^2 + v_u^2)}}.$$

Replaces f_a of Peccei Quinn

Notice that χ (or, equivalently, θ) is gauge invariant as one can check quite directly. In fact a $U(1)_B$

The PQ axion feels the QCD vacuum via the interaction

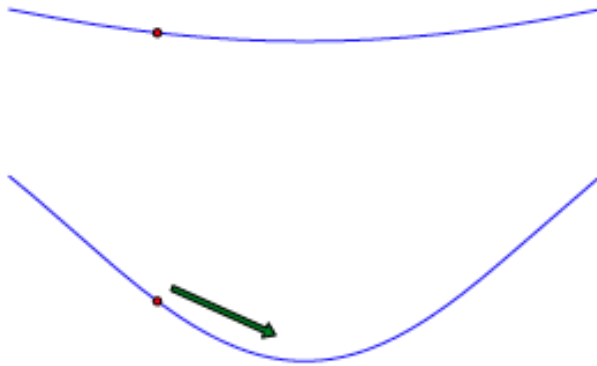
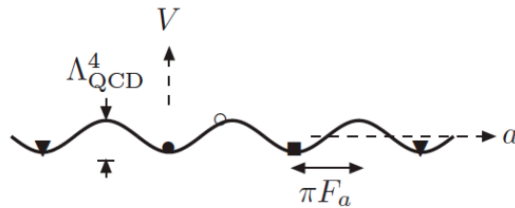
$$\frac{a}{f_a} G\tilde{G}$$

The angle of misalignment is

$$\theta = \frac{a(x)}{f_a}$$

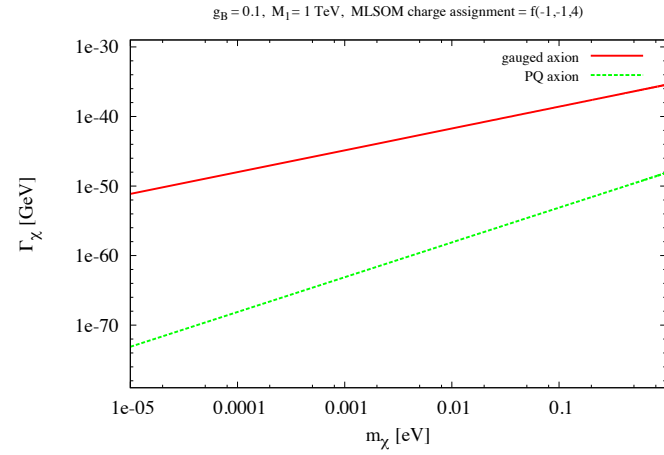
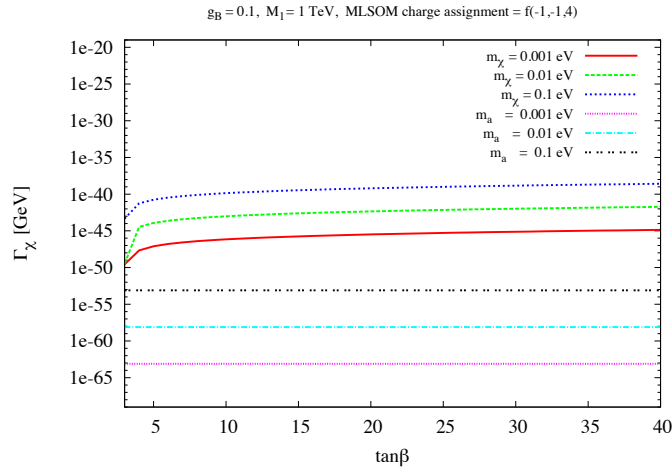
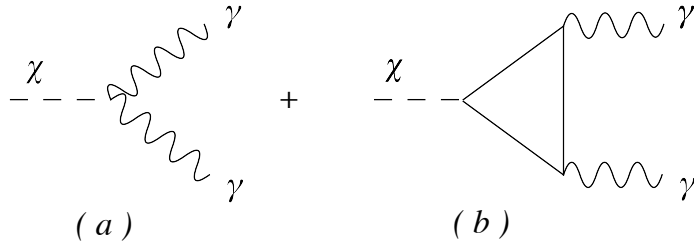
The mass is sizeable

$$10^{-3} - 10^{-4} eV$$



PQ axion. Vacuum misalignment at the QCD phase transition

If an axion has charges both under SU(3) and SU(2) we could consider the possibility of sequential misalignments. The dominant misalignment clearly comes from the largest potential



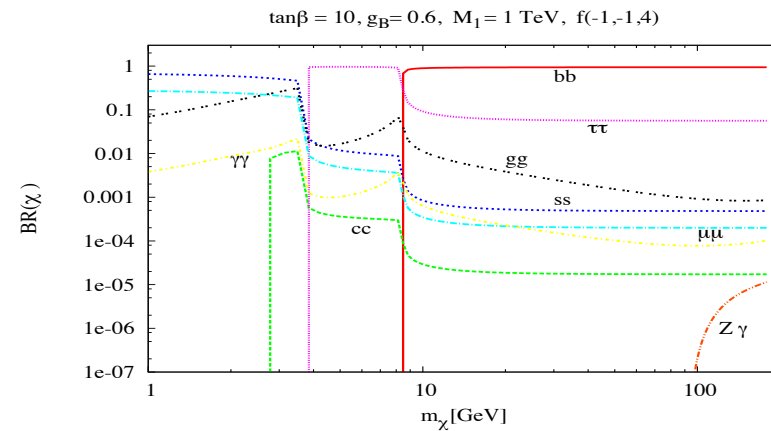
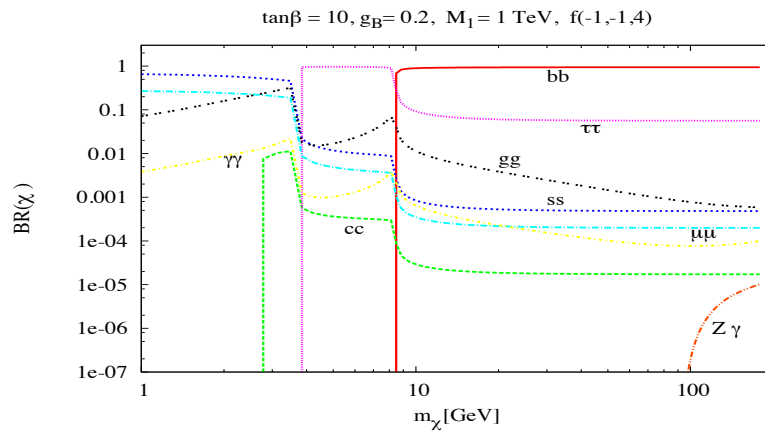
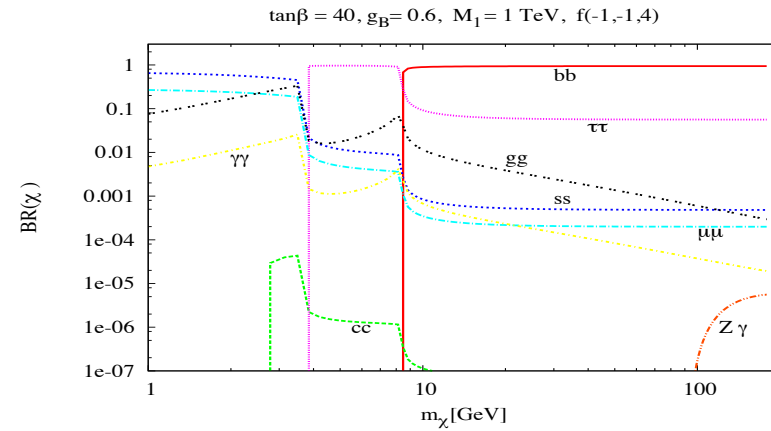
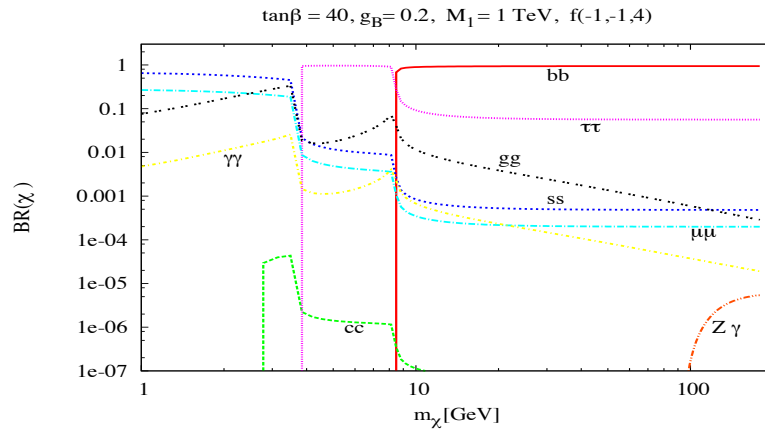
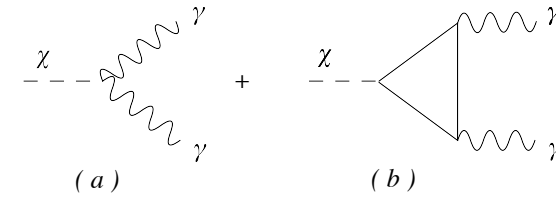
Total decay rate of the axi-Higgs for several mass values. Here, for the PQ axion, we have chosen

$$f_a = 10^{10} \text{ GeV}.$$

$$m_\chi^2 = -\frac{1}{2} c_\chi v^2 \left[1 + \left(\frac{q_u^B - q_d^B}{M_1} \frac{v \sin 2\beta}{2} \right)^2 \right] = -\frac{1}{2} c_\chi v^2 \left[1 + \frac{(q_u^B - q_d^B)^2}{M_1^2} \frac{v_u^2 v_d^2}{v^2} \right],$$

G. Lazarides, A.Mariano, C.C.

Since the mass is an independent parameter, you can also
 Consider the axi-Higgs to be in the GeV range.



Study of the branching ratios of the axi-Higgs. We analyze the dependence on the free parameters $g_B, \tan\beta$.

Conclusions

Gauged Axions are an interesting avenue for physics BSM
They can be framed in a completely supersymmetric scenario

In a local formulation these theories predict a new (gauged)

Axion, an anomalous extra Z prime. In the supersymmetric case
two forms of dark matter.

The issue of the UV completion of anomalous theories
(FROM A FIELD THEORY FRAMEWORK) remains still open.
Similar features appear in gravity, in the trace anomaly, for instance.

We are starting to discover the physical implications of anomalies
using more dynamics than geometry.
How to imbed these formulations in more sophisticated theories such as
gauged supergravities remain open.

Soon or later, these formulations have to be described
By effective actions either of MLSOM-type or of the USSM-A

BACK UP

Anomalous extra Z prime

$$\hat{D}_\mu = \left[\partial_\mu - ig_2 (W_\mu^1 T^1 + W_\mu^2 T^2 + W_\mu^3 T^3) - i\frac{g_Y}{2} \hat{Y} B_Y^\mu - i\frac{g_z}{2} \hat{z} B_z^\mu \right]$$

$$\tan \theta_W = g_Y / g_2.$$

$$\varepsilon = \frac{\delta M_{ZZ'}^2}{M_{Z'}^2 - M_Z^2}$$

$$M_Z^2 = \frac{g_2^2}{4 \cos^2 \theta_W} (v_{H_1}^2 + v_{H_2}^2) [1 + O(\varepsilon^2)]$$

$$M_{Z'}^2 = \frac{g_z^2}{4} (z_{H_1}^2 v_{H_1}^2 + z_{H_2}^2 v_{H_2}^2 + z_\phi^2 v_\phi^2) [1 + O(\varepsilon^2)]$$

$$\delta M_{ZZ'}^2 = -\frac{g_2 g_z}{4 \cos \theta_W} (z_{H_1}^2 v_{H_1}^2 + z_{H_2}^2 v_{H_2}^2).$$

$$M_Z^2 = \frac{1}{4} \left(2M_1^2 + g^2 v^2 + N_{BB} - \sqrt{(2M_1^2 - g^2 v^2 + N_{BB})^2 + 4g^2 x_B^2} \right)$$

$$\simeq \frac{g^2 v^2}{2} - \frac{1}{M_1^2} \frac{g^2 x_B^2}{4} + \frac{1}{M_1^4} \frac{g^2 x_B^2}{8} (N_{BB} - g^2 v^2),$$

$$M_{Z'}^2 = \frac{1}{4} \left(2M_1^2 + g^2 v^2 + N_{BB} + \sqrt{(2M_1^2 - g^2 v^2 + N_{BB})^2 + 4g^2 x_B^2} \right)$$

$$\simeq M_1^2 + \frac{N_{BB}}{2}.$$

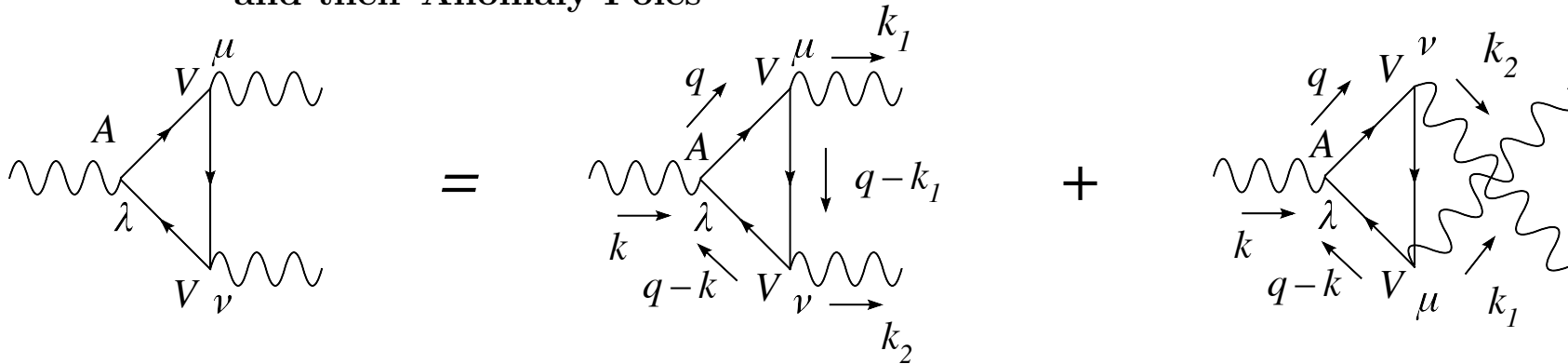
$$N_{BB} = (q_u^{B2} v_u^2 + q_d^{B2} v_d^2) g_B^2, \quad x_B = (q_u^B v_u^2 + q_d^B v_d^2) g_B.$$

$$\begin{pmatrix} A_\gamma \\ Z \\ Z' \end{pmatrix} = O^A \begin{pmatrix} W_3 \\ A^Y \\ B \end{pmatrix}$$

$$O^A \simeq \begin{pmatrix} \frac{g_Y}{g} & \frac{g_2}{g} & 0 \\ \frac{g_2}{g} + O(\epsilon_1^2) & -\frac{g_Y}{g} + O(\epsilon_1^2) & \frac{g}{2} \epsilon_1 \\ -\frac{g_2}{2} \epsilon_1 & \frac{g_Y}{2} \epsilon_1 & 1 + O(\epsilon_1^2) \end{pmatrix}$$

Armillis, Delle Rose, Guzzi, C.C.

Anomalous $U(1)$ Models in Four and Five Dimensions
and their Anomaly Poles



$$\Delta_0^{\lambda\mu\nu} = A_1(k_1, k_2)\varepsilon[k_1, \mu, \nu, \lambda] + A_2(k_1, k_2)\varepsilon[k_2, \mu, \nu, \lambda] + A_3(k_1, k_2)\varepsilon[k_1, k_2, \mu, \lambda]k_1^\nu$$

$$+ A_4(k_1, k_2)\varepsilon[k_1, k_2, \mu, \lambda]k_2^\nu + A_5(k_1, k_2)\varepsilon[k_1, k_2, \nu, \lambda]k_1^\mu + A_6(k_1, k_2)\varepsilon[k_1, k_2, \nu, \lambda]k_2^\mu.$$

$$A_1(k_1, k_2) = k_1 \cdot k_2 A_3(k_1, k_2) + k_2^2 A_4(k_1, k_2),$$

$$A_2(k_1, k_2) = k_1^2 A_5(k_1, k_2) + k_1 \cdot k_2 A_6(k_1, k_2),$$

$$A_5(k_1, k_2) = -A_4(k_2, k_1)$$

$$A_6(k_1, k_2) = -A_3(k_2, k_1).$$

Rosenberg, 1963

$$A_1(s, s_1, s_2) = -\frac{i}{4\pi^2} + \frac{i}{8\pi^2\sigma} \left\{ \Phi(s_1, s_2) \frac{s_1 s_2 (s_2 - s_1)}{s} + s_1 (s_2 - s_{12}) \log \left[\frac{s_1}{s} \right] - s_2 (s_1 - s_{12}) \log \left[\frac{s_2}{s} \right] \right\},$$

Nothing specific emerges from this computation

$$\Phi(x, y) = \frac{1}{\lambda} \left\{ 2[Li_2(-\rho x) + Li_2(-\rho y)] + \ln \frac{y}{x} \ln \frac{1 + \rho y}{1 + \rho x} + \ln(\rho x) \ln(\rho y) + \frac{\pi^2}{3} \right\},$$

where $s = k^2$, $s_1 = k_1^2$, $s_2 = k_2^2$, $s_{12} = k_1 \cdot k_2$ with $\sigma = s_{12}^2 - s_1 s_2$

$$\begin{aligned} \lambda(x, y) &= \sqrt{\Delta}, & \Delta &= (1 - x - y)^2 - 4xy, \\ \rho(x, y) &= 2(1 - x - y + \lambda)^{-1}, & x &= \frac{s_1}{s}, & y &= \frac{s_2}{s}. \end{aligned}$$

The vertex in the longitudinal/transverse (L/T) formulation and comparisons

$$W^{\lambda\mu\nu} = \frac{1}{8\pi^2} [W^{L\lambda\mu\nu} - W^{T\lambda\mu\nu}],$$

$$\text{(with } w_L = -4i/s\text{)}$$

$$\begin{aligned} W^T_{\lambda\mu\nu}(k_1, k_2) &= w_T^{(+)}(k^2, k_1^2, k_2^2) t_{\lambda\mu\nu}^{(+)}(k_1, k_2) + w_T^{(-)}(k^2, k_1^2, k_2^2) t_{\lambda\mu\nu}^{(-)}(k_1, k_2) \\ &+ \tilde{w}_T^{(-)}(k^2, k_1^2, k_2^2) \tilde{t}_{\lambda\mu\nu}^{(-)}(k_1, k_2), \end{aligned}$$

The anomaly is associated to a longitudinal component, which has a pole: the anomaly pole (1/s). The transverse sector does not contribute to the anomaly.

In the on-shell case (two photons on shell)

$$\Delta^{\lambda\mu\nu}(s, 0, 0) = W_{\mu\nu\lambda}(s, 0, 0) = -\frac{i}{2\pi^2} \frac{k^\lambda}{s} \varepsilon[k_1, k_2, \mu, \nu].$$

