# Deeply Virtual Neutrino Scattering at Leading Twist

(Electroweak Nonforward Parton Distributions)

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Work in collaboration with **Marco Guzzi** (Lecce)

Hadronic Interactions mediated by weak currents

ENERGY

INCLUSIVE PROCESSES

DGLAP, "structure functions"

Generalized Bjorken region

NONFORWARD PARTON DISTRIBUTIONS

(weak)

(nonforward RG Evolution)

EXCLUSIVE PROCESSES

Distribution Amplitudes
(hadronic Wave Function)

ERBL

The link is COMPTON SCATTERING (WEAK)

Leading twist amplitudes for exclusive neutrino interactions in the deeply virtual limit.

**Phys.Rev.D71:053002,2005**, with M. Guzzi

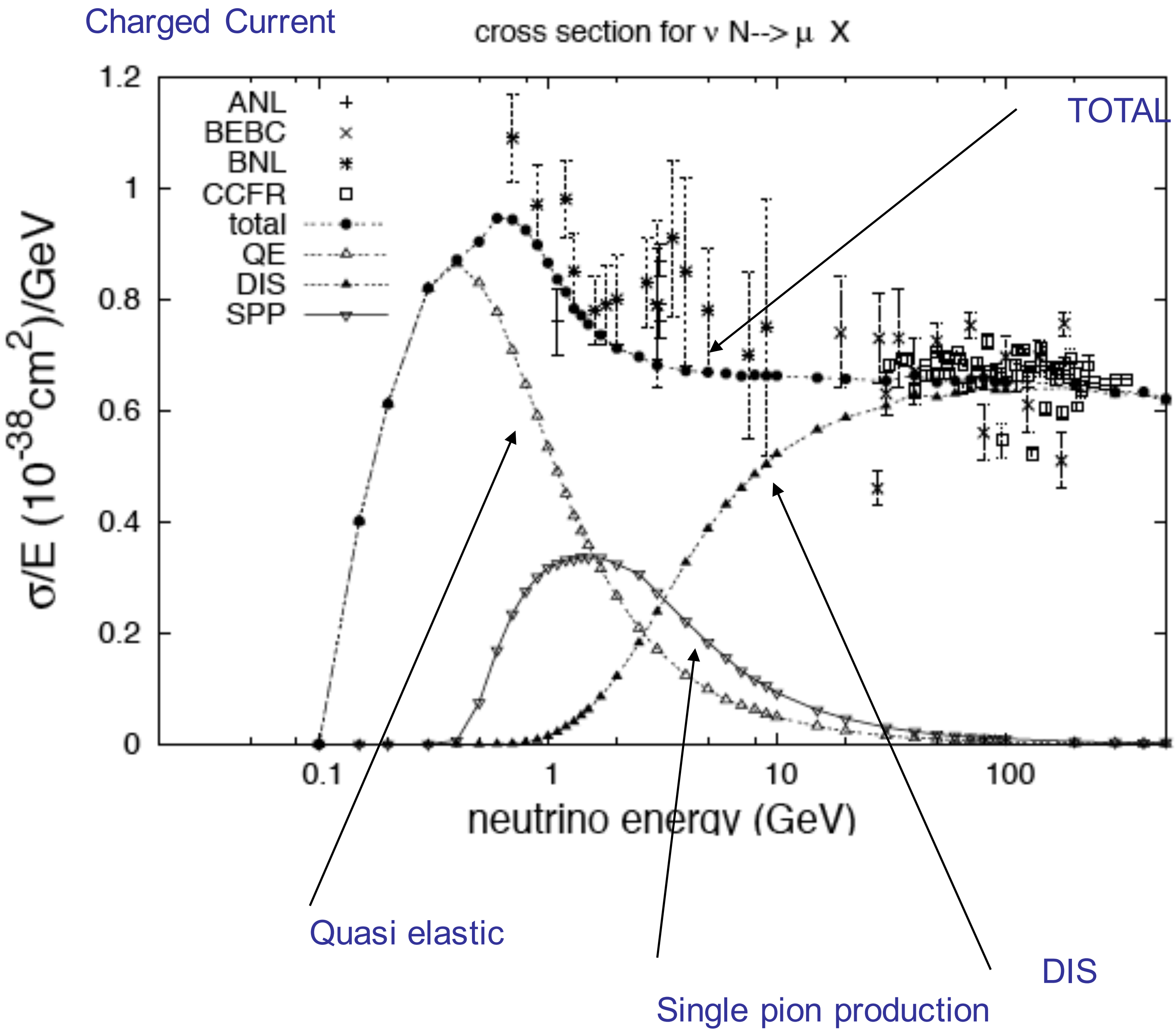
Deeply virtual neutrino scattering (DVNS), with M. Guzzi and P. Amore

**JHEP 0502:038,2005**

Generalized Bjorken region: more than 1 scaling variable:

1) Bjorken x

2) asymmetry parameter between the initial and final momenta of the nucleon



Charged Current
cross section for ν N--> μ X
TOTAL
ANL
BEBC
BNL
CCFR
total
QE
DIS
SPP
σ/E (10^-38 cm^2)/GeV
neutrino energy (GeV)
Quasi elastic
Single pion production
DIS

In neutrino factories the range of the interaction between the weak currents and the nucleons reaches the intermediate region of QCD “the Few GeV’s region”

This kinematical window, pretty large indeed (from 2-3 GEV^2 up to 20 GeV^2 or so) can be described by perturbative methods using FACTORIZATION THEOREMS.

Factorization means that

1) the theory is light-cone dominated
and in a given process
2) We can “separate” the non perturbative part of the interaction, due to confinement, from the “valence” part which is described by a standard perturbative expansion.

We can predict the intermediate energy behaviour of weak form factors and describe elastic processe with high accuracy

Factorization at intermediate energy is associated with a class of Renormalization Group Equations

EFREMOV-RADYUSHKIN-BRODSKY-LEPAGE (ERBL) RG Evolution of hadronic wave functions

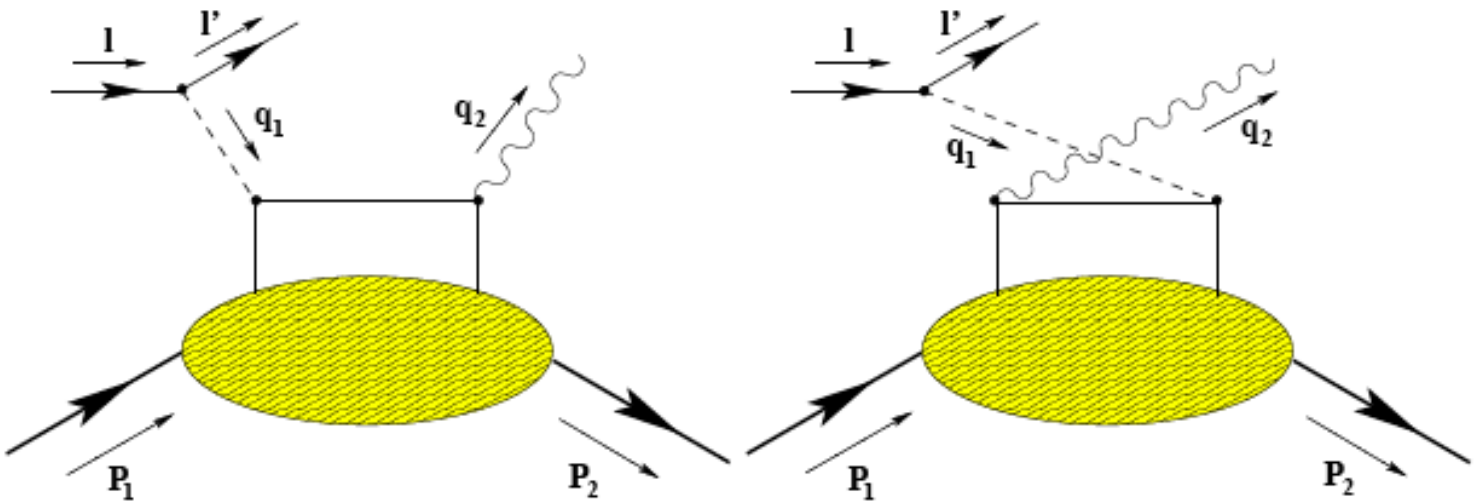
Complementary to the usual DGLAP Evolution in DIS

Both evolutions can be unified in a new class of evolution equations

Nonforward RG Evolution

The nonforward evolution summarizes both limits (DGLAP/ERBL)

# DVNS KINEMATICS

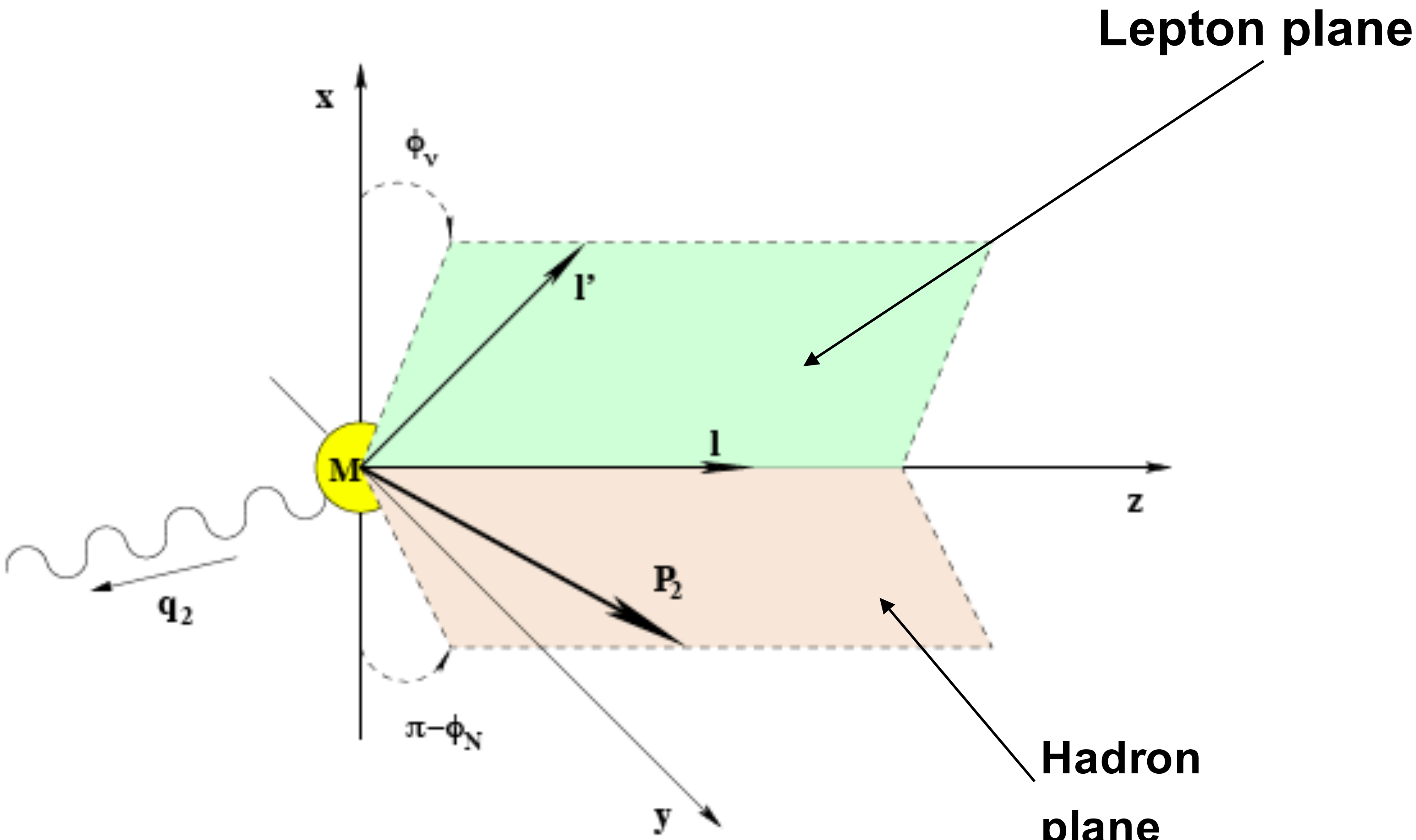



$$P_{1,2} = \bar{P} \pm \frac{\Delta}{2} \qquad q_{1,2} = \bar{q} \mp \frac{\Delta}{2}$$

with $-\Delta = P_2 - P_1$ being the momentum transfer. Clearly

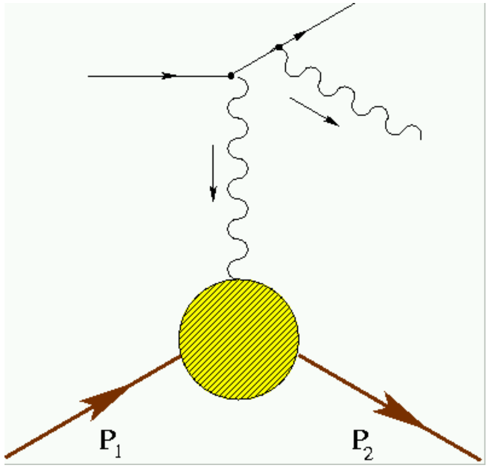
$$\bar{P} \cdot \Delta = 0, \quad t = \Delta^2 \quad \bar{P}^2 = M^2 - \frac{t}{4}$$

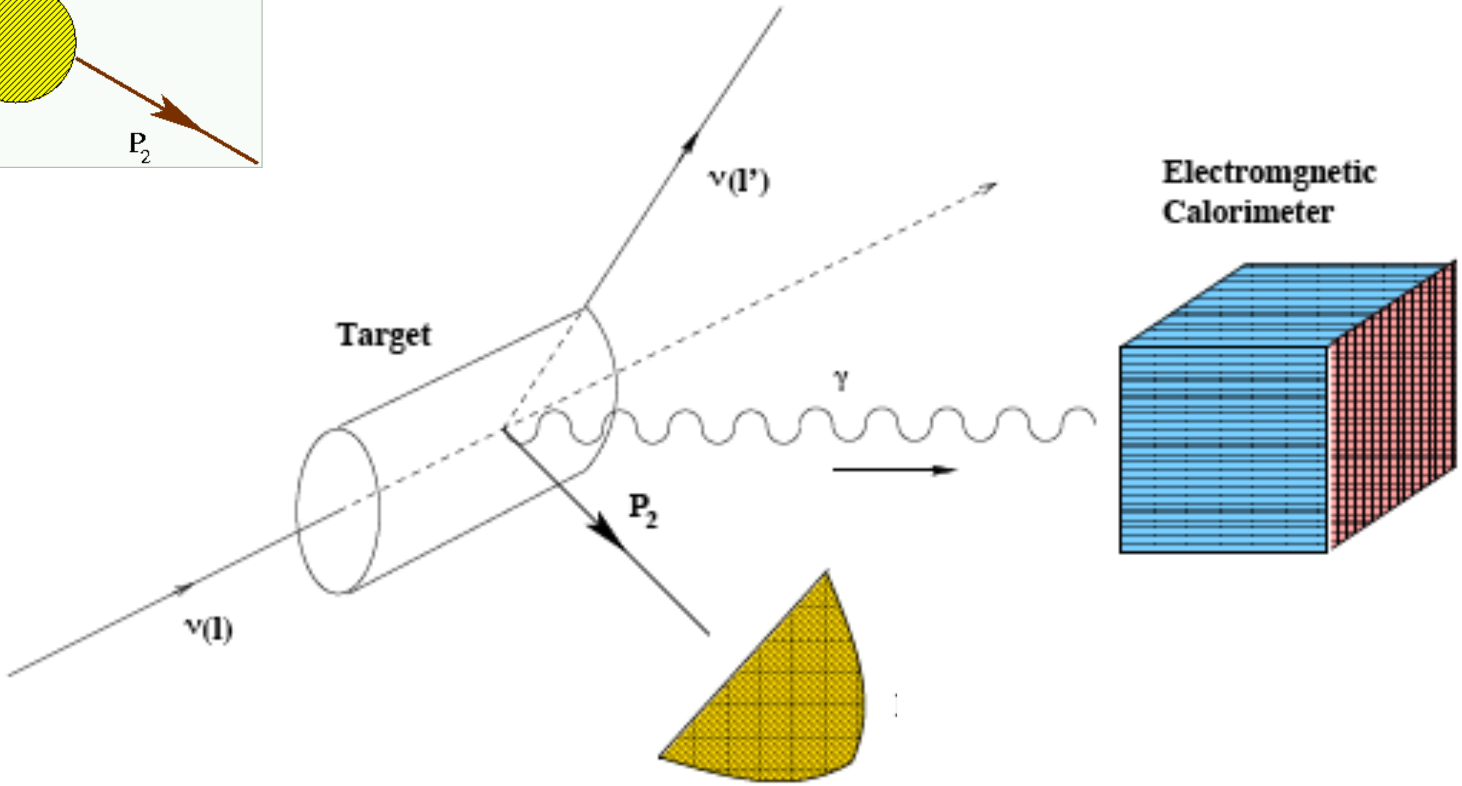
$$T_{\mu\nu}(q_1^2, \nu) = i \int d^4 z e^{iq\cdot z} \langle \bar{P} - \frac{\Delta}{2} | T(J_Z^{\mu}(-z/2) J_{\gamma}^{\nu}(z/2) | \bar{P} + \frac{\Delta}{2} \rangle.$$




Kinematics of the process $\nu(l)N(P_1) \to \nu(l')N(P_2)\gamma(q_2)$

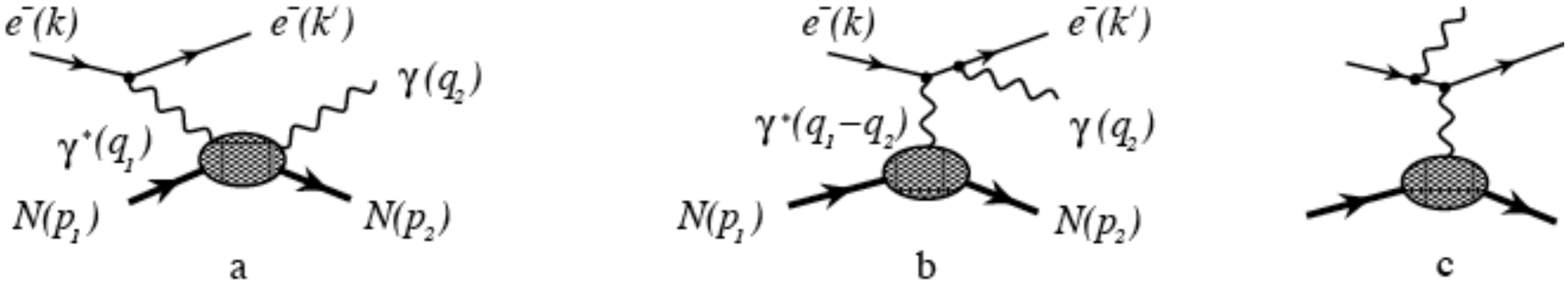
# No Bethe-Heitler (large) background for neutral currents




Recoiling nucleon

# Virtual Compton Amplitude




Bethe-Heitler

$$(pr) = 0, \qquad p^2 = m^2 - \frac{t}{4}, \qquad t \equiv r^2.$$

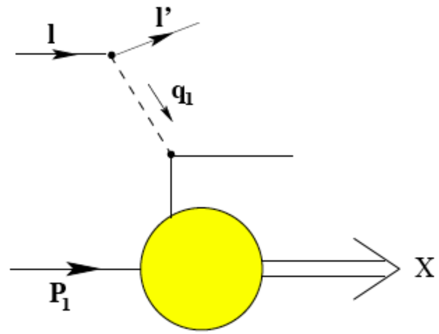
$$p_{1,2} = p \pm \frac{r}{2}, \qquad q_{1,2} = q \mp \frac{r}{2}.$$

$$T_{\mu\nu} = i \int d^4z \, e^{i(qz)} \langle p - r/2 | \mathrm{T}\, J_\mu(-z/2) J_\nu(z/2) | p + r/2 \rangle.$$

$$(q_1 = q_2 = q, p_1 = p_2 = p, r = 0)$$

unitarity

1) DIS Limit




$$q_1^2 \to \infty, (p_1 q_1) \to \infty, \text{ with } x_B \equiv -q_1^2/[2(p_1 q_1)]$$

2) DVCS/DVNS Limit

$q_2^2 = 0$, in the limit $q_1^2 \to \infty, (p_1 q_1) \to \infty$, again with $x_B \equiv -q_1^2/[2(p_1 q_1)]$ fixed.

$(p_1 - p_2 \equiv r \neq 0)$, t fixed.

$(rq_1) \to \infty$ proportional to $q_1^2$.

where $t \equiv r^2 < 0$ does not grow with $q_1^2$.

$$\xi \equiv \frac{-q^2}{2(pq)}, \qquad \eta \equiv \frac{(rq)}{2(pq)},$$

$$q_1^2 = \left(1 + \frac{\eta}{\xi}\right) q^2 + \frac{t}{4},$$
$$q_2^2 = \left(1 - \frac{\eta}{\xi}\right) q^2 + \frac{t}{4},$$

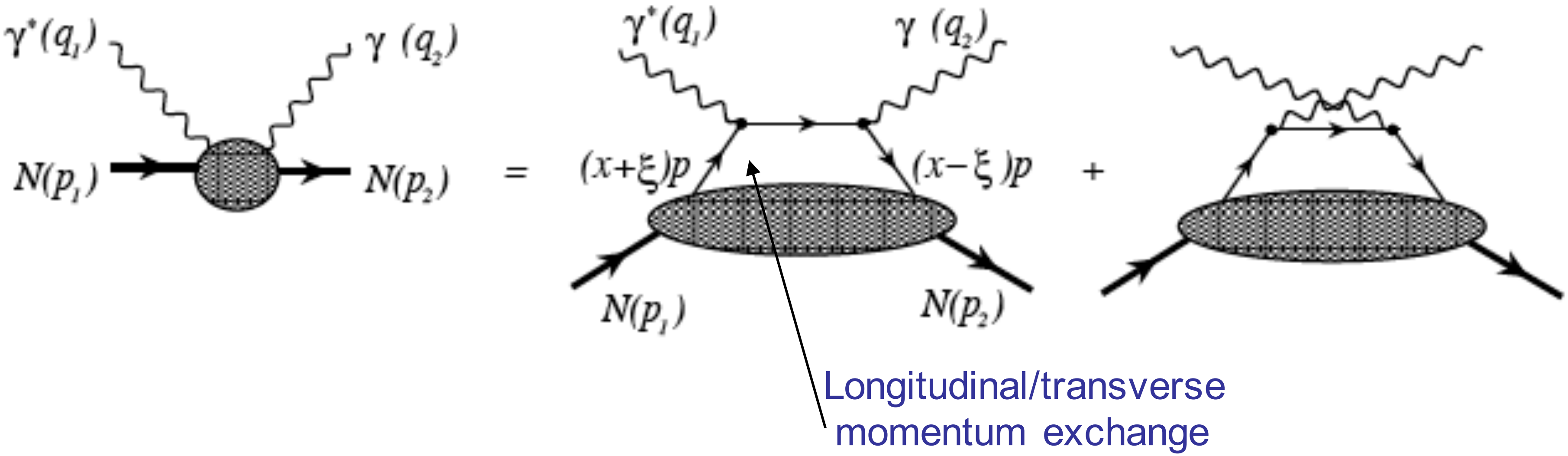
$$x_B = \frac{-q_1^2}{2(p_1 q_1)} \equiv \frac{\xi + \eta}{1 + \eta}.$$

$$\text{DIS:} \quad \eta = 0, \quad x_B = \xi,$$

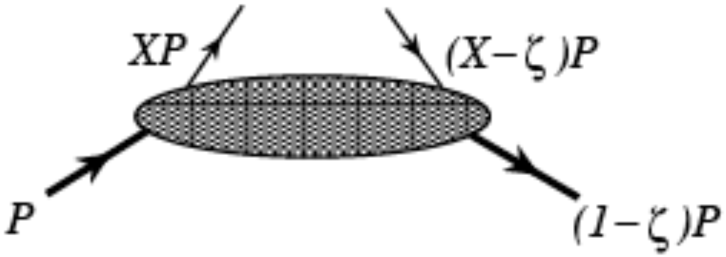
$$\text{DVCS:} \quad \eta = \xi, \quad x_B = \frac{2\xi}{1+\xi}.$$

**DVNS kinematics remains invariant w.r.t. DVCS**

## Nonforward (Radyushkin) vs Off-forward (Ji)




$$X = \frac{x+\xi}{1+\xi} \qquad \zeta = \frac{2\xi}{1+\xi}.$$





Nonforward pdf's: 0<X<1

Off forward -1< x <1

$$p = (p_1 + p_2)/2,$$

$$\int \frac{d\lambda}{(2\pi)} e^{i\lambda z} \langle P' | \overline{\psi}\left(-\frac{\lambda n}{2}\right) \gamma^\mu \psi\left(\frac{\lambda n}{2}\right) | P \rangle =$$

$$H(z,\xi,\Delta^2)\overline{U}(P')\gamma^\mu U(P) + E(z,\xi,\Delta^2)\overline{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P) + .....$$

$$\int \frac{d\lambda}{(2\pi)} e^{i\lambda z} \langle P' | \overline{\psi}\left(-\frac{\lambda n}{2}\right) \gamma^\mu \gamma^5 \psi\left(\frac{\lambda n}{2}\right) | P \rangle =$$

$$\tilde{H}(z,\xi,\Delta^2)\overline{U}(P')\gamma^\mu \gamma^5 U(P) + \tilde{E}(z,\xi,\Delta^2)\overline{U}(P')\frac{\gamma^5 \Delta^\mu}{2M}U(P) + .....$$

$$\begin{aligned}
H^i(z,\xi,\Delta^2,Q^2) &= F_1^i(\Delta^2) q^i(z,\xi,Q^2) \\
\tilde{H}^i(z,\xi,\Delta^2,Q^2) &= G_1^i(\Delta^2)\,\Delta q^i(z,\xi,Q^2) \\
E^i(z,\xi,\Delta^2,Q^2) &= F_2^i(\Delta^2)\, r^i(z,\xi,Q^2)
\end{aligned}$$

$$q(z,\xi,Q^2) = \int_{-1}^{1} dx' \int_{-1+|x'|}^{1-|x'|} dy' \delta(x' + \xi y' - z) f(y', x', Q^2)$$

**Double distributions**

$$f(y,x) = \pi(y,x) f(x),$$

$$\pi(x,y) = \frac{\Gamma(2b+2)}{2^{2b+1}\Gamma^2(b+1)} \frac{[(1-|x|)^2 - y^2]^b}{(1-|x|)^{2b+1}}$$

**Profile function (Radyushkin)**

$$\mathcal{M}_{fi} = J_Z^{\mu}(q_1) D(q_1) \epsilon^{\nu *}(q_1 - \Delta)$$

$$\times \Big\{ \frac{i}{2} \tilde{g} g_u U_v \int_{-1}^{1} dz \, (\tilde{n}^\mu n^\nu + \tilde{n}^\nu n^\mu - g^{\mu\nu})$$

$$\alpha(z) \left[ H^u(z,\xi,\Delta^2) \overline{U}(P_2) \not{n} \; U(P_1) + E^u(z,\xi,\Delta^2) \overline{U}(P_2) \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} U(P_1) \right] +$$

$$\beta(z) i\epsilon^{\mu\nu\alpha\beta} \tilde{n}_\alpha n_\beta \left[ \tilde{H}^u(z,\xi,\Delta^2) \overline{U}(P_2) \not{n} \; \gamma^5 U(P_1) + \tilde{E}^u(z,\xi,\Delta^2) \overline{U}(P_2) \gamma^5 \, (\Delta \cdot n) \, U(P_1) \right] +$$

$$\frac{i}{2} \tilde{g} g_d D_v \int_{-1}^{1} dz \, \{u \to d\} -$$

$$\frac{i}{2} \tilde{g} g_u \int_{-1}^{1} dz \, (-\tilde{n}^\mu n^\nu - \tilde{n}^\nu n^\mu + g^{\mu\nu})$$

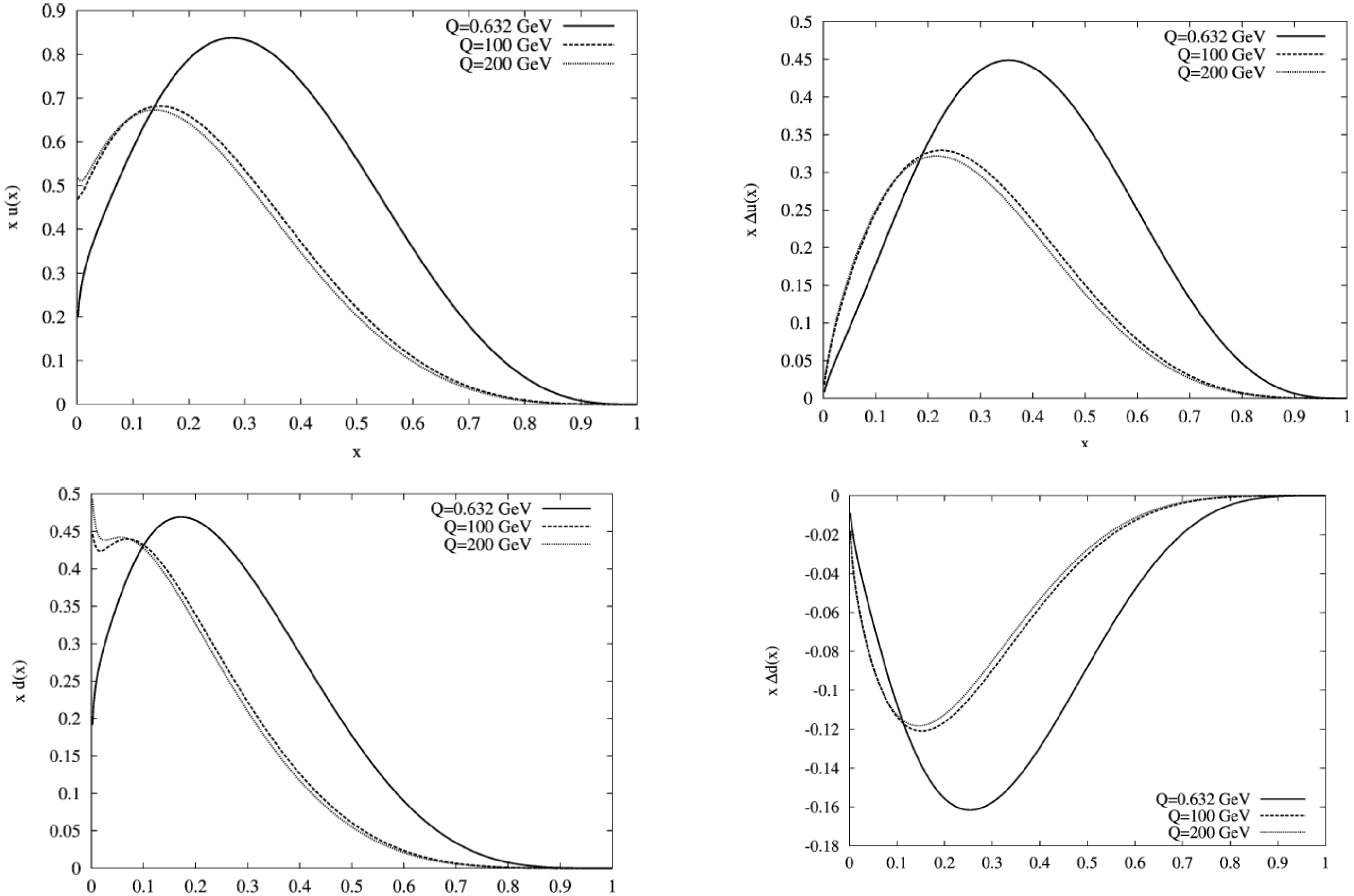
$$\alpha(z) \left[ \tilde{H}^u(z,\xi,\Delta^2) \overline{U}(P_2) \not{n} \; \gamma^5 U(P_1) + \tilde{E}^u(z,\xi,\Delta^2) \overline{U}(P_2) \frac{i\gamma^5 \Delta \cdot n}{2M} U(P_1) \right] +$$

$$\beta(z) i\epsilon^{\mu\nu\alpha\beta} \tilde{n}_\alpha n_\beta \left[ H^u(z,\xi,\Delta^2) \overline{U}(P_2) \not{n} \; U(P_1) + E^u(z,\xi,\Delta^2) \overline{U}(P_2) \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} (P_1) \right] -$$

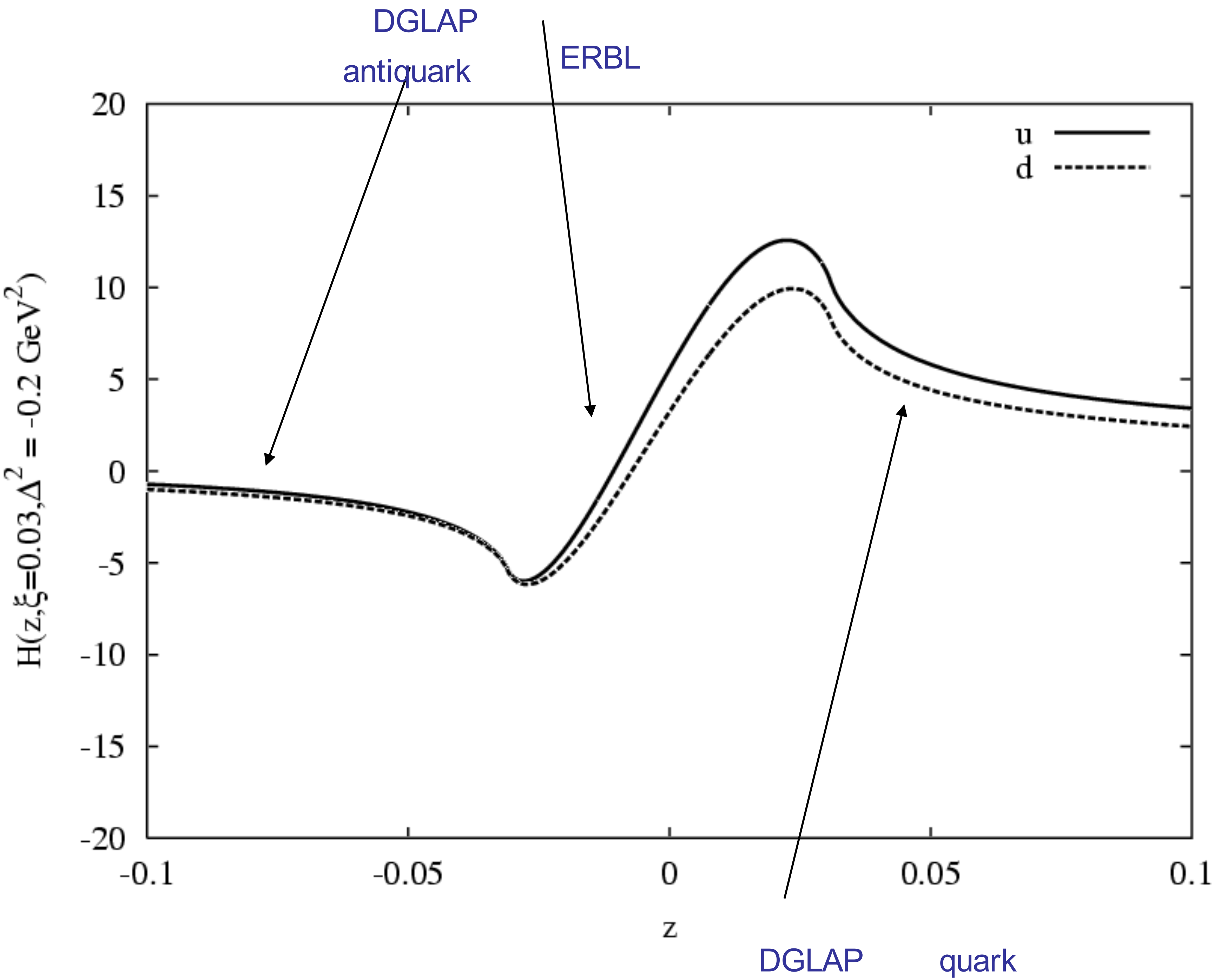
$$\frac{i}{2} \tilde{g} g_d \int_{-1}^{1} dz \, \{u \to d\} \Big\}.$$

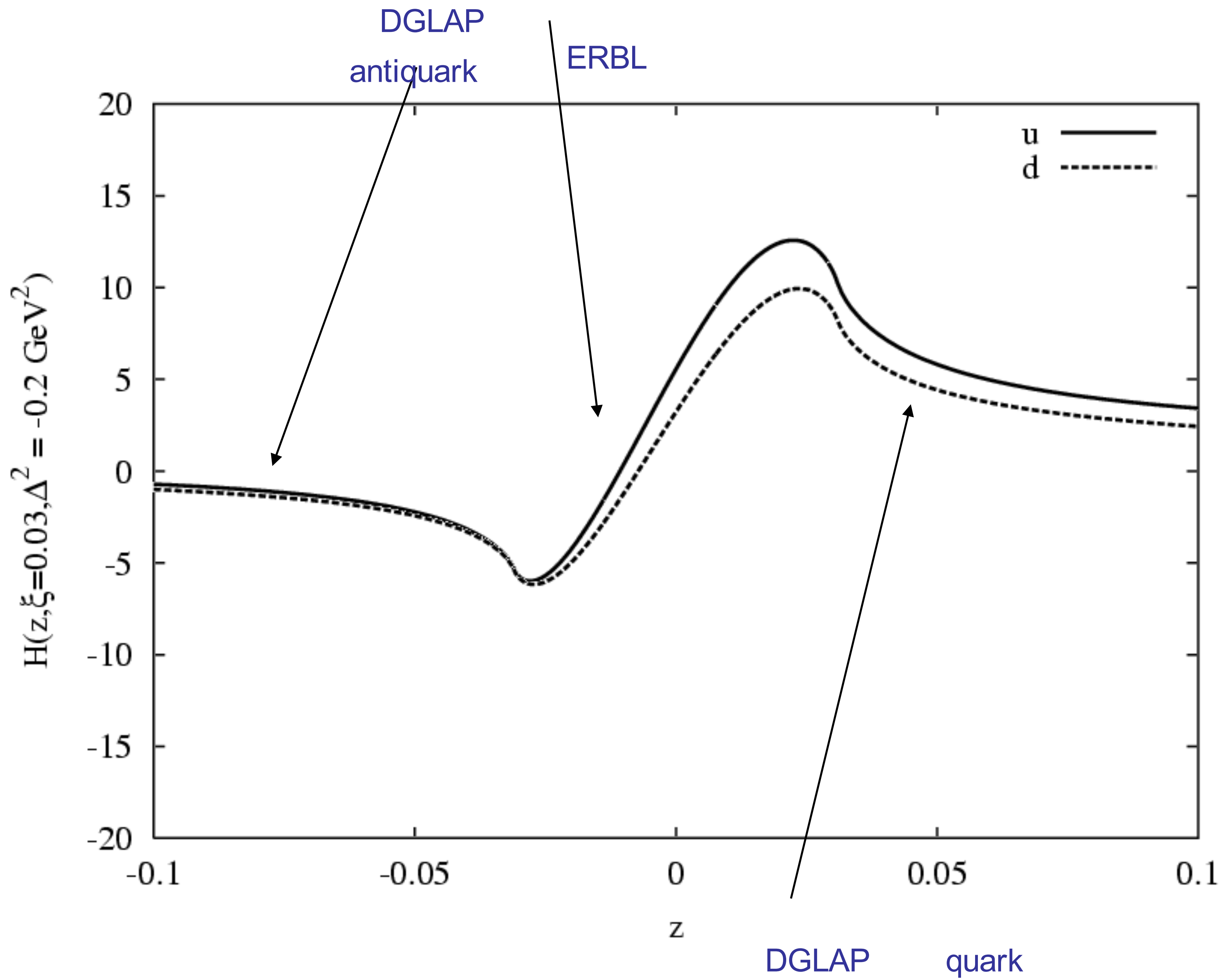
$$
\begin{aligned}
|\mathcal{M}|^2 &= P.V. \int_{-1}^{1} dz \int_{-1}^{1} dz' \left( K_1(z,z')\alpha(z)\alpha^*(z') + K_2(z,z')\beta(z)\beta^*(z') \right) \\
&+\pi^2 \left( K_1(\xi,\xi) - K_1(\xi,-\xi) - K_1((-\xi,\xi) + K_1(-\xi,-\xi) \right) \\
&+\pi^2 \left( K_2(\xi,\xi) + K_2(\xi,-\xi) + K_2((-\xi,\xi) + K_2(-\xi,-\xi) \right)
\end{aligned}
$$

$$
\begin{aligned}
A_1(z, z'x, t, Q^2) &= \tilde{g}^4 Q^2 \Big[ 4g_d^2 [\tilde{E}_d' (4\tilde{H}_d M^2 + \tilde{E}_d t) x^2 \\
&+ 4\tilde{H}_d' M^2 (4\tilde{H}_d (x-1) + \tilde{E}_d x^2)] \\
&+ 4g_d g_u [(4\tilde{E}_u' \tilde{H}_d M^2 + 4\tilde{E}_d' \tilde{H}_u M^2 + \tilde{E}_u' \tilde{E}_d t + \tilde{E}_d' \tilde{E}_u t) x^2 \\
&+ 4\tilde{H}_u' M^2 (4\tilde{H}_d (x-1) + \tilde{E}_d x^2) + 4\tilde{H}_d' M^2 (4\tilde{H}_u (x-1) + \tilde{E}_u x^2)] \\
&+ D_v U_v g_d g_u [4E_u' E_d t + 4E_d' E_u t - 4E_u' E_d t x - 4E_d' E_u t x + 4E_u' E_d M^2 x^2 \\
&+ 4E_d' E_u M^2 x^2 + 4E_u' H_d M^2 x^2 + 4E_d' H_u M^2 x^2 + E_u' E_d t x^2 + E_d' E_u t x^2 \\
&+ 4H_u' M^2 (4H_d (x-1) + E_d x^2) + 4H_d' M^2 (4H_u (x-1) + E_u x^2)]
\end{aligned}
$$



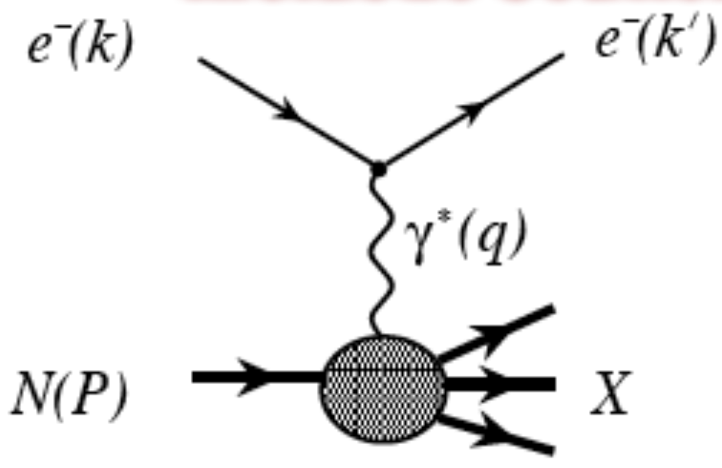

A. Cafarella, M. Guzzi, C.C.

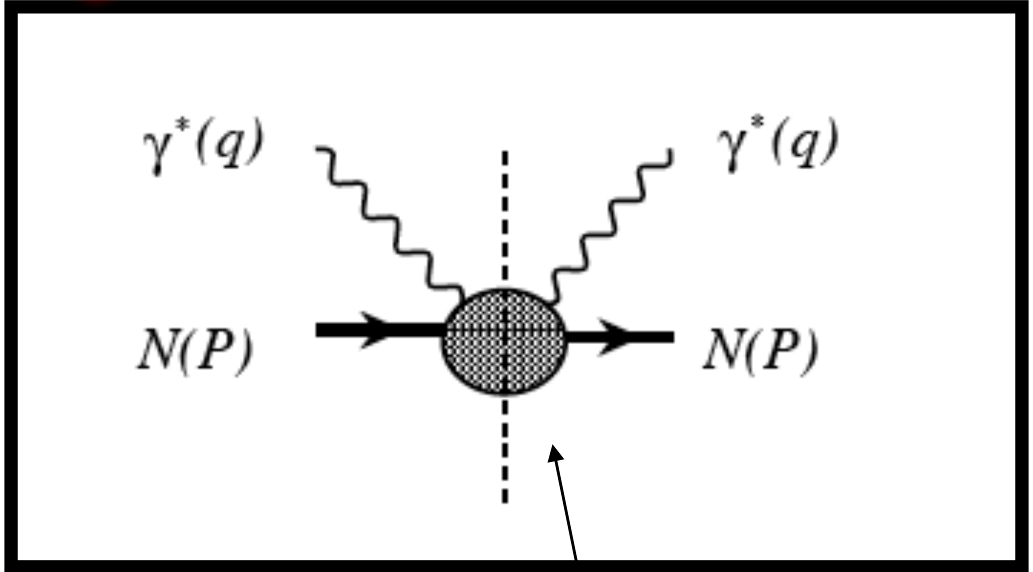

DGLAP
antiquark
ERBL
DGLAP
quark
u
d
$H(z,\xi=0.03,\Delta^2=-0.2\ GeV^2)$
z
20
15
10
5
0
-5
-10
-15
-20
-0.1
-0.05
0
0.05
0.1



DGLAP
antiquark
ERBL
u
d
20
15
10
5
0
-5
-10
-15
-20
H(z,ξ=0.03,Δ2 = -0.2 GeV2)
-0.1
-0.05
0
0.05
0.1
z
DGLAP
quark

# Inelastic Scattering




$$W^2 \equiv (P+q)^2 = M^2 + 2(P\cdot q) - Q^2.$$

unitarity

$$W^{\mu\nu} = \frac{1}{2\pi M}\Im T^{\mu\nu}$$

$$\frac{d^2\sigma_{(eN)}}{d\Omega d\omega'} = \frac{\alpha^2}{Q^4}\frac{\omega'}{\omega}L^{(e)}_{\mu\nu}W^{\mu\nu}$$

hadronic tensor

$$W^{\mu\nu} = \frac{1}{4\pi M}\int d^4z\, e^{i(q\cdot z)}\frac{1}{2}\sum_S \langle N(P,S)|\, J^{\mu}_{EM}(z)\, J^{\nu}_{EM}(0)\,|N(P,S)\rangle .$$

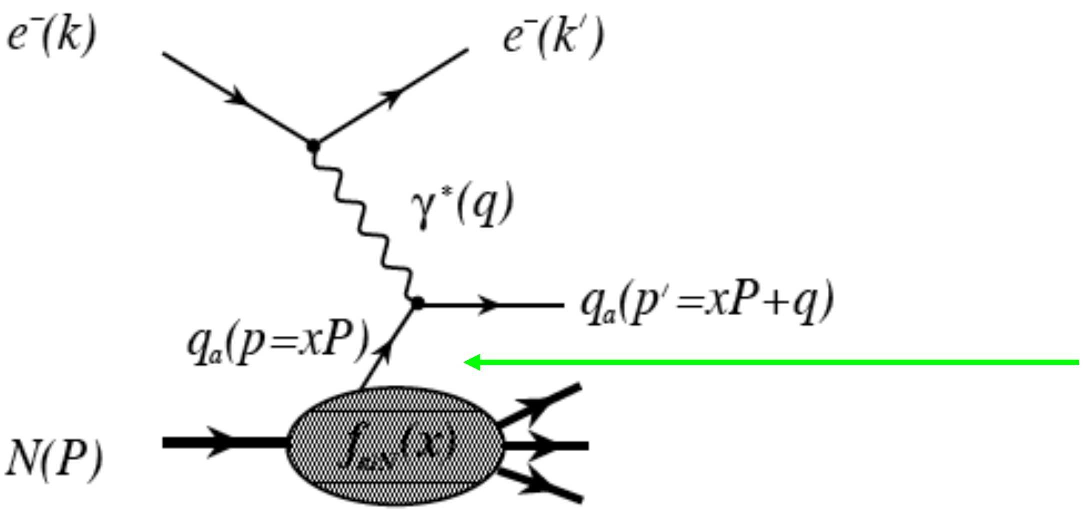
$$T^{\mu\nu} = i\int d^4z\, e^{i(q\cdot z)}\langle N(P,S)|\, T\left\{J^{\mu}_{EM}(z)\, J^{\nu}_{EM}(0)\right\}|N(P,S)\rangle$$

# Deep Inelastic limit

scale-less

$$\begin{cases} F_1\left(x_B, Q^2\right) & \equiv & MW_1\left(\nu, Q^2\right), \\ F_2\left(x_B, Q^2\right) & \equiv & \nu W_2\left(\nu, Q^2\right). \end{cases}$$

$$\begin{aligned} \frac{d^2\sigma_{(eN)}}{dx_B dy} &= \left(\frac{2\pi M\omega y}{\omega'}\right)\frac{d^2\sigma_{(eN)}}{d\Omega d\omega'} \\ &= \frac{4\pi\alpha^2}{Q^2 y}\left[y^2 F_1\left(x_B, Q^2\right) + \left(\frac{1-y}{x_B} - \frac{M^2 y}{s-M^2}\right)F_2\left(x_B, Q^2\right)\right] \end{aligned}$$




emission of a parton from a light-cone dominated process

## gauge invariance

$$q_\mu W^{\mu\nu} = 0 \quad \text{and} \quad W^{\mu\nu} q_\nu = 0,$$

$$\partial_\mu J^\mu_{EM} = 0,$$

## structure functions

$$W^{\mu\nu} = W_1\left(\nu, Q^2\right)\left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right] + \frac{W_2\left(\nu, Q^2\right)}{M^2}\left[P^\mu - q^\mu \frac{(P\cdot q)}{q^2}\right]\left[P^\nu - q^\nu \frac{(P\cdot q)}{q^2}\right]$$

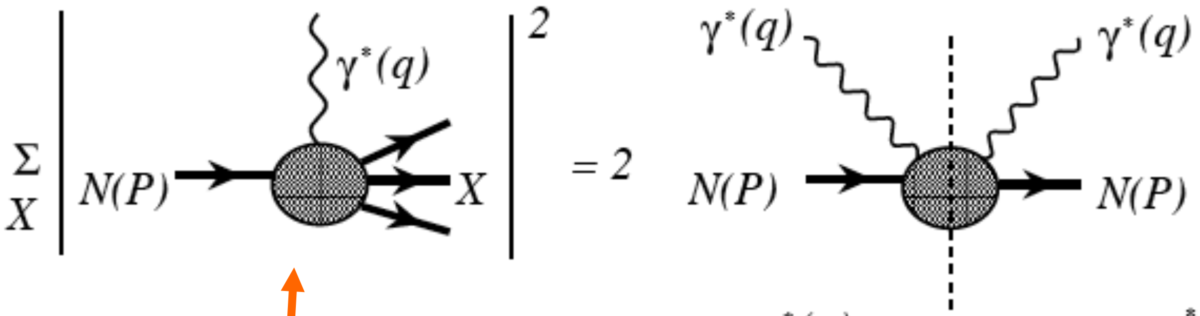
$$x_B = \frac{Q^2}{W^2 - M^2 + Q^2} = \frac{1}{1 + \left(W^2 - M^2\right)/Q^2}$$

$$s \equiv (k+P)^2$$

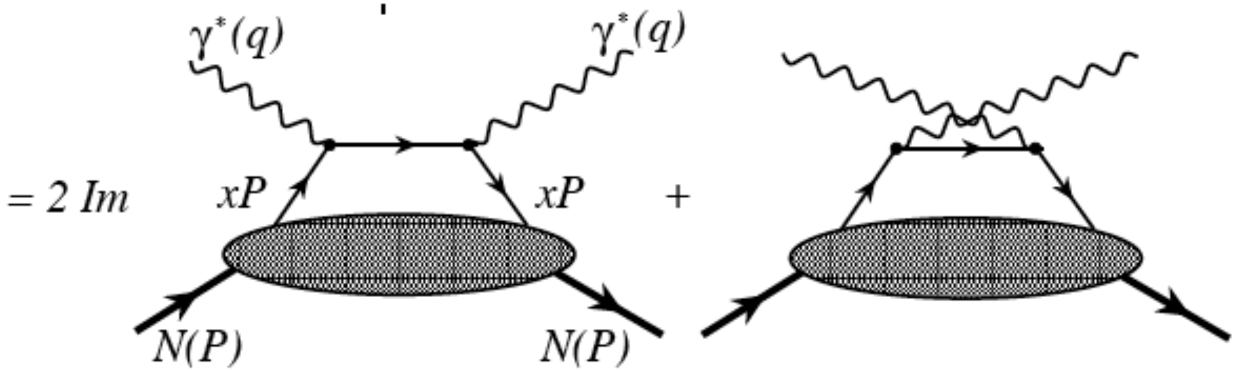
## inelasticity

$$y \equiv \frac{(P\cdot q)}{(P\cdot k)}.$$

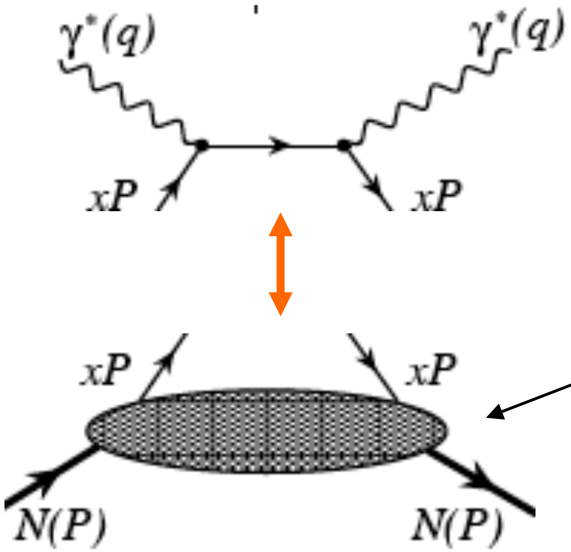
$$Q^2 = x_B y\left(s - M^2\right)$$




unitarity

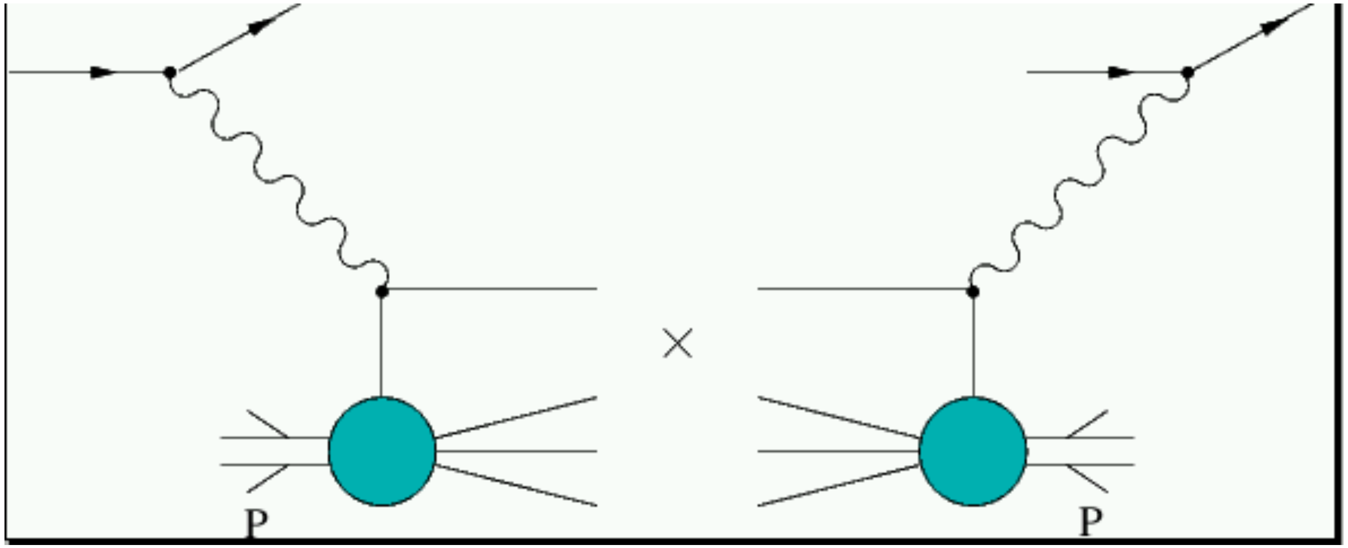



Total cross section

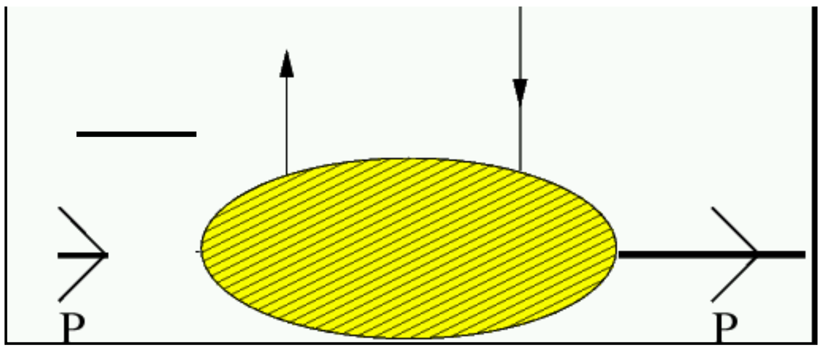



Forward parton distributions

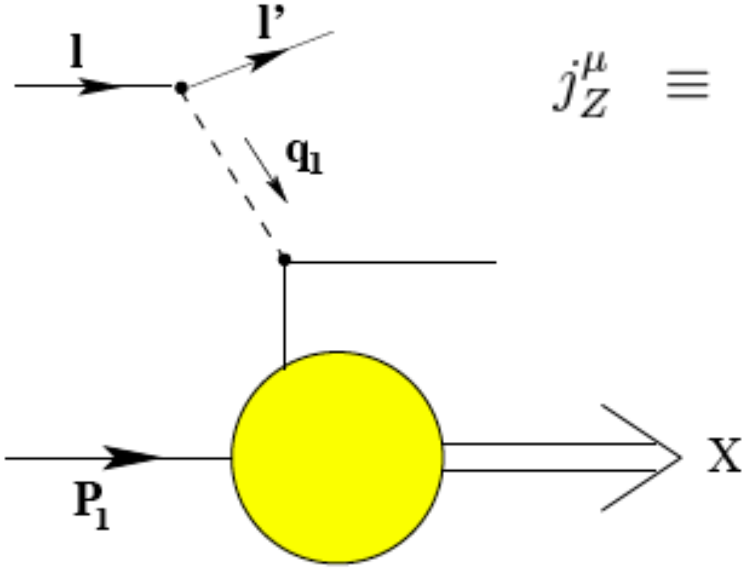
Partons are emitted and re-absorbed on the light-cone with momentum x P




$$
\begin{aligned}
f_1(x) &= \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P | \bar{\psi}(0) \not{n} \psi(\lambda n) | P \rangle \\
g_1(x) &= \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P S_{\|} | \bar{\psi}(0) \not{n} \gamma_5 \psi(\lambda n) | P S_{\|} \rangle \\
h_1(x) &= \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P S_{\perp} | \bar{\psi}(0) [\not{S}_{\perp}, \not{n}] \gamma_5 \psi(\lambda n) | P S_{\perp} \rangle
\end{aligned}
$$



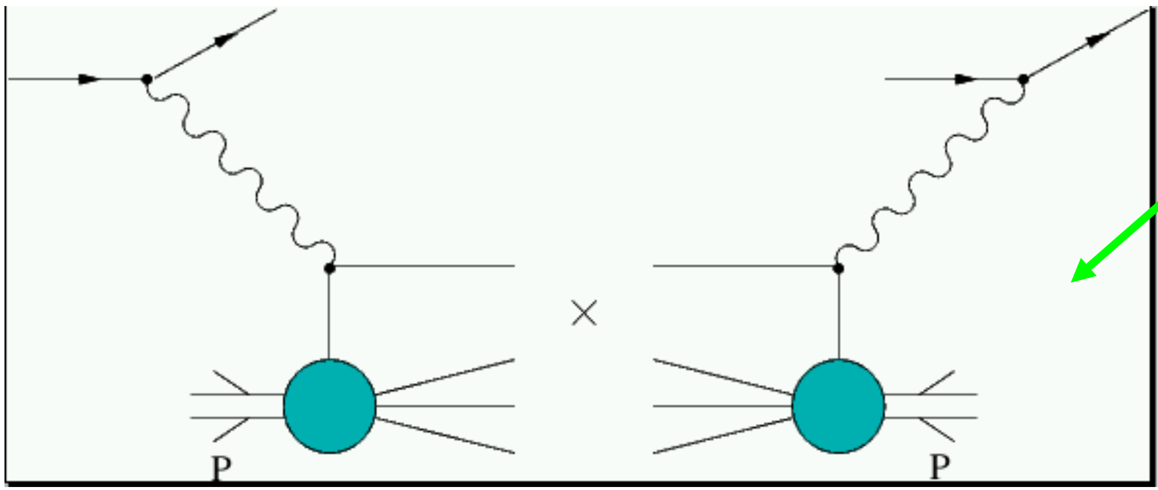
# Neutral current




$$j_Z^\mu \equiv \overline{u}(l')\gamma^\mu\left(-1+4\sin^2\theta_W+\gamma_5\right)u(l)$$

weak hadronic tensor
neutral current

$$T_{\mu\nu}(q_1^2,\nu) = i\int d^4z\, e^{iq\cdot z}\langle P_1|T(J_Z^\mu(\xi)J_Z^\nu(0)|P_1\rangle$$




unitarity

## leading twist

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_{1\mu}q_{1\nu}}{q_1^2}\right) W_1(\nu, Q^2) + \frac{\hat{P_1}^{\mu}\hat{P_1}^{\nu}}{P_1^2}\frac{W_2(\nu, Q^2)}{M^2} - i\epsilon_{\mu\nu\lambda\sigma}q_1^{\lambda}P_1^{\sigma}\frac{W_3(\nu, Q^2)}{2M^2}$$

$$\hat{P_1}^{\mu} = P_1^{\mu} - q_1^{\mu}P_1 \cdot q_1/q_1^2.$$

$$\begin{aligned} MW_1(Q^2,\nu) &= F_1(x,Q^2) \\ \nu W_2(Q^2,\nu) &= F_2(x,Q^2) \\ \nu W_3(Q^2,\nu) &= F_3(x,Q^2), \end{aligned}$$

## Higher twists

The analysis at higher twists is far more involved and one isolates 14 structures
If spin and mass effects are included

Use: Lorenz covariance

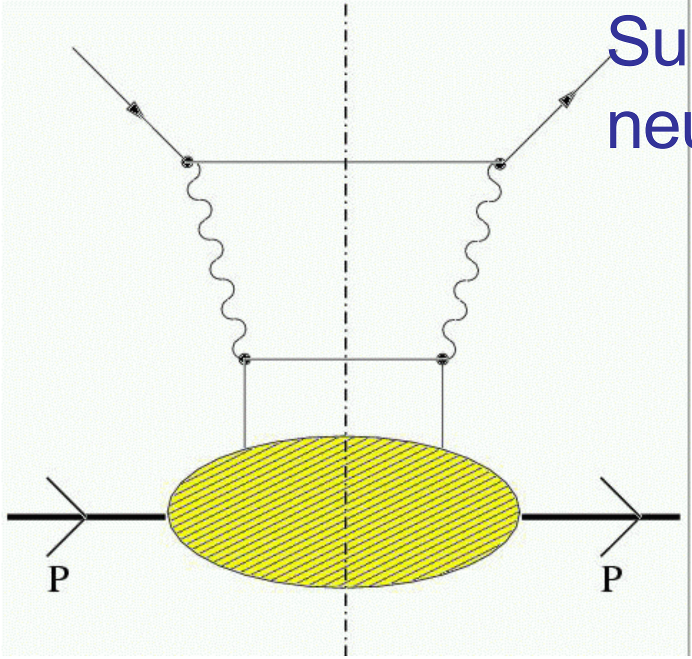
T- invariance

neglect CP violating efffects from CKM matrix

If we impose Ward Identities (current conservation)
we reduce them to 8.

Ward identities are broken in a spontaneously broken theory, so this is equivalent to set to zero the quark masses.

# Summary: Weak Unitarity for neutral currents




$$\hat{P}_1^{\mu} = P_1^{\mu} - q_1^{\mu} P_1 \cdot q_1 / q_1^2.$$

$$\eta^{|\gamma|^2}(Q^2) = 1,$$
$$\eta^{|\gamma Z|}(Q^2) = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \frac{Q^2}{Q^2 + M_Z^2},$$
$$\eta^{|Z|^2}(Q^2) = (\eta^{|\gamma Z|})^2 (Q^2).$$

EM

P-violation

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_{1\mu} q_{1\nu}}{q_1^2}\right) W_1(\nu, Q^2) + \frac{\hat{P}_1^{\mu} \hat{P}_1^{\nu}}{P_1^2} \frac{W_2(\nu, Q^2)}{M^2} - i\epsilon_{\mu\nu\lambda\sigma} q_1^{\lambda} P_1^{\sigma} \frac{W_3(\nu, Q^2)}{2M^2}$$

$$L^i_{\mu\nu} = \sum_{\lambda'} \left[\bar{u}(k', \lambda') \gamma_\mu (g_V^{i_1} + g_A^{i_1} \gamma_5) u(k, \lambda)\right]^* \bar{u}(k', \lambda') \gamma_\nu (g_V^{i_2} + g_A^{i_2} \gamma_5) u(k, \lambda).$$

Generic em/weak cross section $\longrightarrow$

$$\frac{d^3\sigma}{dx\,dy\,d\theta} = \frac{y\alpha^2}{Q^4} \sum_i \eta_i(Q^2) L_i^{\mu\nu} W_i^{\mu\nu},$$

$$\begin{aligned} M W_1(Q^2, \nu) &= F_1(x, Q^2) \\ \nu W_2(Q^2, \nu) &= F_2(x, Q^2) \\ \nu W_3(Q^2, \nu) &= F_3(x, Q^2), \end{aligned}$$

$$\begin{aligned} g_V^{\gamma} &= 1, & g_A^{\gamma} &= 0, \\ g_V^{Z} &= -\tfrac{1}{2} + 2\sin^2\theta_W, & g_A^{Z} &= \tfrac{1}{2}, \end{aligned}$$

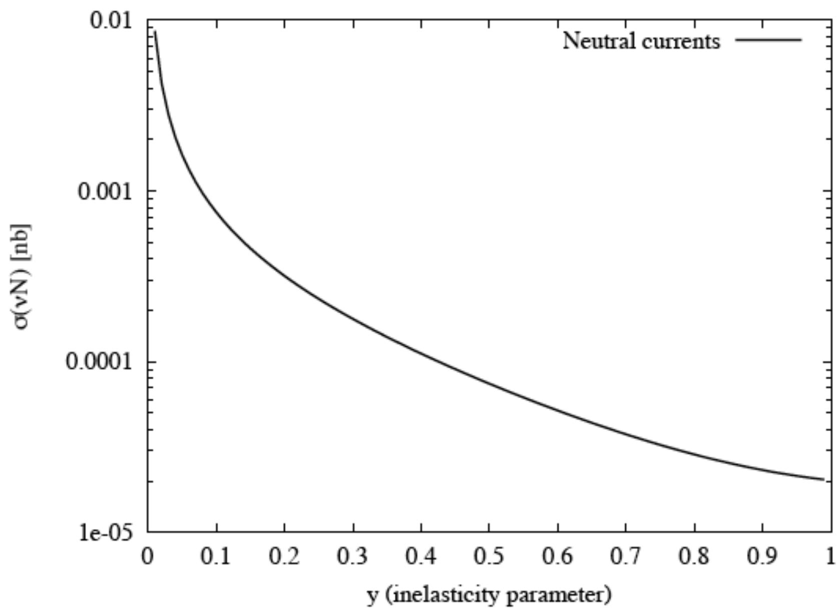
$$
\begin{aligned}
q^0(x,Q^2) &= \left[\frac{u_v(x,Q^2)+d_v(x,Q^2)}{2} + \frac{\bar{u}(x,Q^2)+\bar{d}(x,Q^2)}{2}\right]\left(L_u^2+L_d^2\right) \\
&+ \left[\frac{\bar{u}(x,Q^2)+\bar{d}(x,Q^2)}{2}\right]\left(R_u^2+R_d^2\right) \\
\bar{q}^0(x,Q^2) &= \left[\frac{u_v(x,Q^2)+d_v(x,Q^2)}{2} + \frac{\bar{u}(x,Q^2)+\bar{d}(x,Q^2)}{2}\right]\left(R_u^2+R_d^2\right) \\
&+ \left[\frac{\bar{u}(x,Q^2)+\bar{d}(x,Q^2)}{2}\right]\left(L_u^2+L_d^2\right)
\end{aligned}
$$

Parton distributions

$$L_u = 1 - \frac{4}{3}\sin^2\theta_W\,, \qquad L_d = -1 + \frac{2}{3}\sin^2\theta$$

$$R_u = -\frac{4}{3}\sin^2\theta_W\,, \qquad R_d = \frac{2}{3}\sin^2\theta_W$$

M. Guzzi, C.C.




$$x \approx 0.1$$

Quark-antiquark distributions using H(x)

$$q(x)-\bar{q}(x)=H_q(x)+H_q(-x)\equiv H_q^V(x)$$

$$\sum_q [q(x)+\bar{q}(x)]=\sum_q [H_q(x)-H_q(-x)]\equiv H^S(x)$$

In a similar way we may introduce $H_g(x)\equiv xg(x)$ where $g(x)$ is the familiar gluon distribution.

$$H_g(x)=\frac{1}{P^+}\int\frac{dy^-}{2\pi}e^{-ixP^+y^-}$$

$$\times\langle P|F^{+\nu}(0,y^-/2,\mathbf{0})F_\nu^{+}(0,-y^-/2,\mathbf{0})|P\rangle,$$

quark

where $F^{\mu\nu}$ is the gluon field strength tensor

$$H_q(x)=\frac{1}{2P^+}\int\frac{d^2k_T}{2x(2\pi)^3}\sum_\lambda [\langle P|b_\lambda^\dagger(xP^+,\boldsymbol{k}_T)b_\lambda(xP^+,\boldsymbol{k}_T)|P\rangle\theta(x)$$

$$-\langle P|d_\lambda^\dagger(-xP^+,\boldsymbol{k}_T)d_\lambda(-xP^+,\boldsymbol{k}_T)|P\rangle\theta(-x)]$$

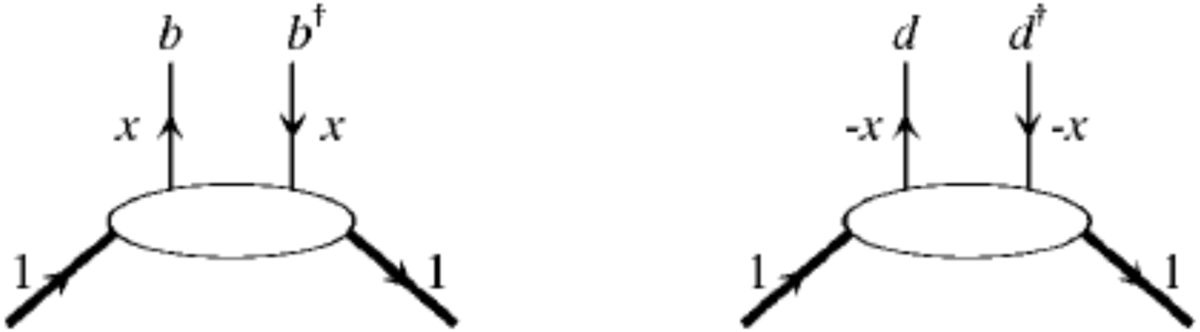
antiquark

In order to introduce off-diagonal distributions it is most convenient to first recall the definition of the conventional (diagonal) parton distributions in terms of light-cone coordinates $(x^{\pm}=(x^0 \pm x^3)/\sqrt{2}, x^1, x^2)$ and in the light-cone gauge $(A^+=0)$

$$H_q(x)=\frac{1}{2}\int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle P|\bar{\psi}_q(0,y^-/2,\mathbf{0})\,\gamma^+\psi_q(0,-y^-/2,\mathbf{0})|P\rangle.$$

Note that the matrix element is diagonal in the four momentum $P$ of the proton.

**(a)** $x>0$: $q(x) = H_q(x)$ **(b)** $x<0$: $\bar{q}(-x) = -H_q(x)$




$$H_q(x)=\begin{cases} q(x) & \text{for } x>0 \\ -\bar{q}(-x) & \text{for } x<0. \end{cases}$$

Hadronic Interactions mediated by weak currents

ENERGY

INCLUSIVE PROCESSES

DGLAP, "structure functions"

NONFORWARD PARTON DISTRIBUTIONS

(weak)

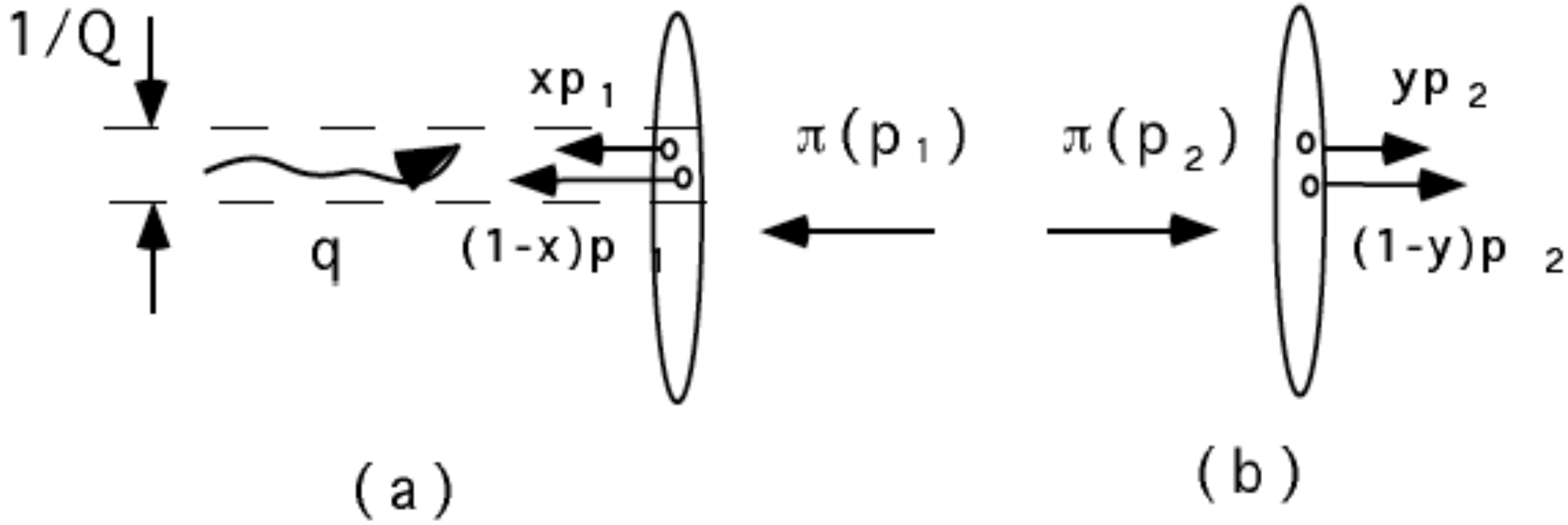
(nonforwar RG Evolution)

EXCLUSIVE PROCESSES

Distribution Amplitudes
(hadronic Wave Function)

ERBL

The link is COMPTON SCATTERING (WEAK)




$$(p_2 + p_1)_\mu \, F_\pi(Q^2) = \langle \pi(p_2) | J_\mu(0) | \pi(p_1) \rangle$$

$$p_1^+ = Q/\sqrt{2},\ p_1^- = 0, \qquad p_2^- = Q/\sqrt{2},\ p_2^+ = 0$$

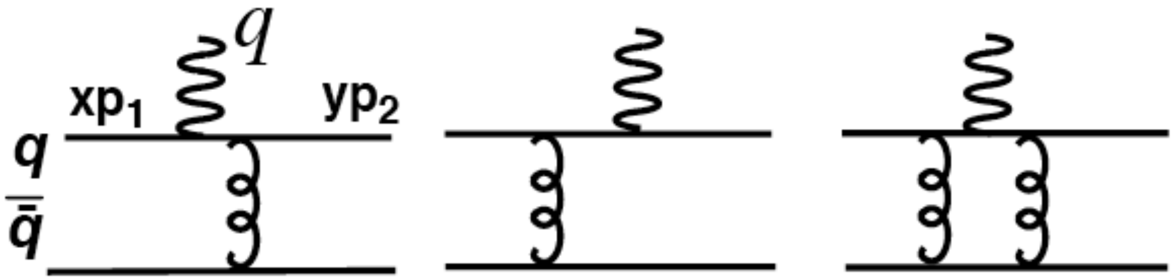
$$(p_2 - p_1)^2 = -2 p_1^+ p_2^- = -Q^2.$$

$$F_\pi(Q) \sim (1/Q^2)$$

$$F_\pi(Q^2) = \int_0^1 dx\, dy\, \phi_\pi(y, \mu^2)\, T(y, x, Q^2, \mu^2)\, \phi_\pi(x, \mu^2)$$

## Hard scattering Coefficient

$$T_H = 16\pi C_F \alpha_s(\mu^2)\left[\frac{2}{3}\frac{1}{xyQ^2} + \frac{1}{3}\frac{1}{(1-x)(1-y)Q^2}\right]$$



(a) (b)

## Inclusion of transverse momentum

$$F_\pi(Q^2) = \int_0^1 dx dy \int \frac{d^2b_1}{(2\pi)^2}\frac{d^2b_2}{(2\pi)^2}\,\mathcal{P}(y,b_2,p_2,\mu) \times T(y,x,p_i,b,\mu)\,\mathcal{P}(x,b_1,p_1,\mu)\,,$$

$$\mathcal{P}(x,b=1/\mu,p_i,\mu) \sim \phi(x,\mu^2)\,,$$

At small separation b, the hadronic Wave function reproduces the collinear one

The inclusion of transverse momentum allows to lower the validity of the Factorization picture.

## Evolution

$$\mu \frac{d}{d\mu} F_\pi(Q^2) = 0 .$$

$$0 = \int_0^1 dx\, dy \Big[ \frac{d\phi_\pi(y)}{d\mu} T \phi_\pi(x) + \phi_\pi(y) \frac{dT}{d\mu} \phi_\pi(x) + \phi_\pi(y) T \frac{d\phi_\pi(x)}{d\mu} \Big] .$$

$$\mu \frac{d\phi(y,\mu^2)}{d\mu} = \int_0^1 dz\, V(y, z, \alpha_s(\mu^2)) \phi_\pi(z, \mu^2)$$ ERBL

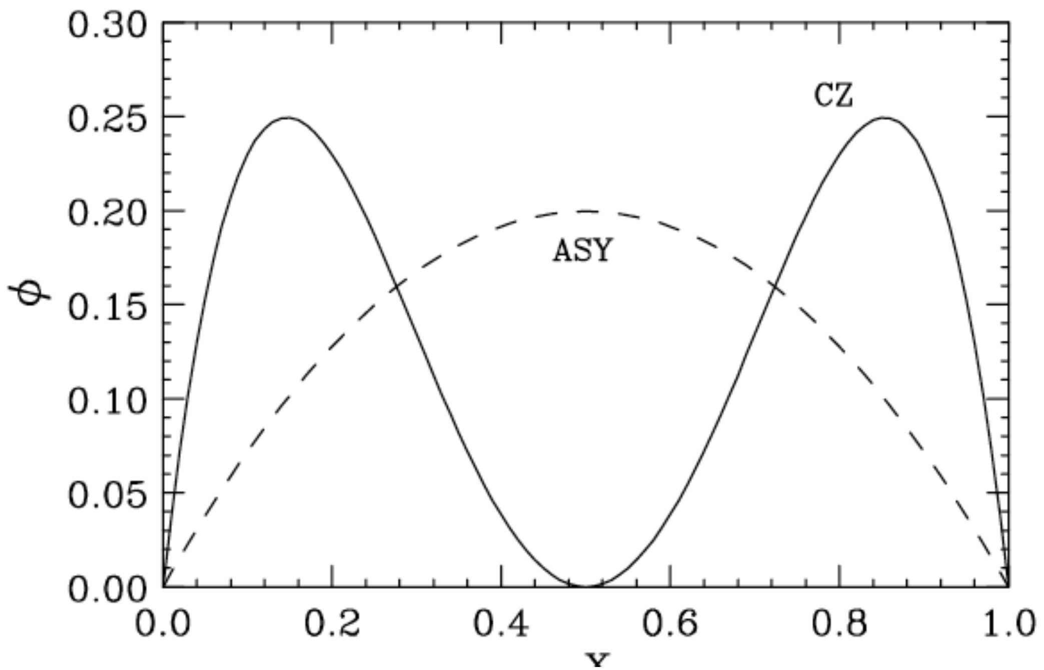
$$\phi_\pi(x,\mu^2) = x(1-x) \sum_{n\geq 0} a_n C_n^{3/2}(2x-1) \Big( \ln \frac{\mu^2}{\Lambda^2} \Big)^{-\gamma_n/2\beta_2}$$

# Inclusion of transverse momentum

$$Q\frac{\partial}{\partial Q}\mathcal{P}(x,b,p,\mu) = [K(b\mu) + G(x,Q/\mu)]\mathcal{P}(x,b,p,\mu)$$

$$\mathcal{P}(x,b;p,\mu) = \mathrm{e}^{-S(x,b,Q,\mu)}\left(\phi_\pi(x,1/b^2) + \mathcal{O}\left(\alpha_s^2(1/b)\right)\right)$$

Sudakov suppression




Asymptotic Solution

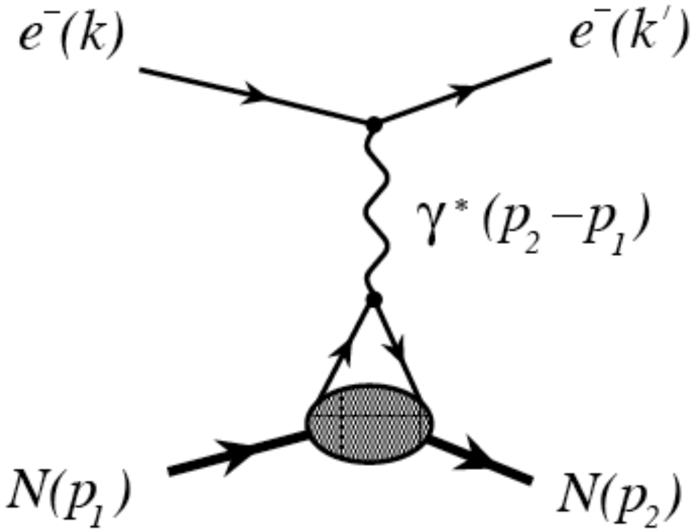
$$\phi_\pi(x,\mu^2) \rightarrow \sqrt{3}f_\pi x(1-x)$$

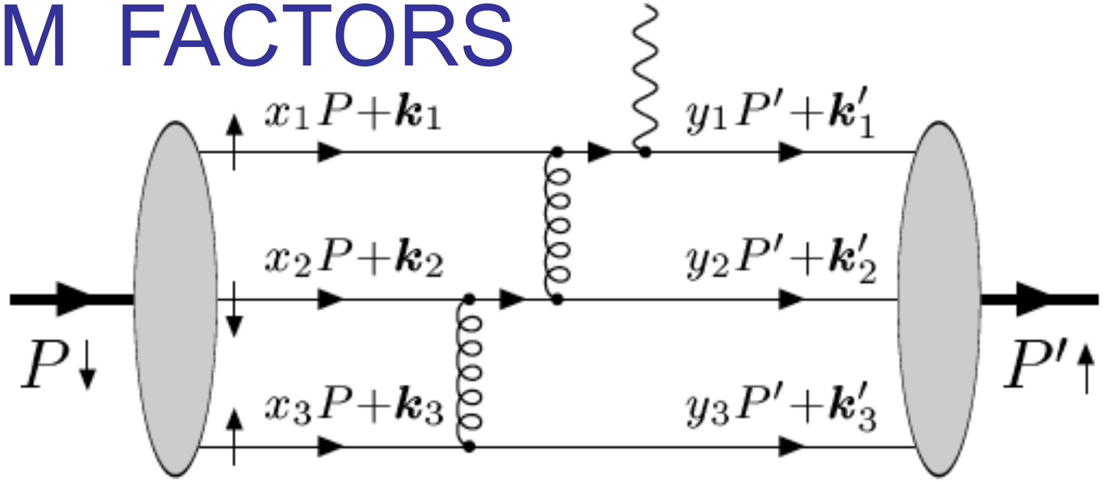
But….where is "asymptotia"?

$$\phi_{CZ}(x,\mu_0^2) = 5\sqrt{3}f_\pi\, x(1-x)(1-2x)^2$$

**Nothing prevents us from applying these consideration to weak processes**

# FORM FACTORS



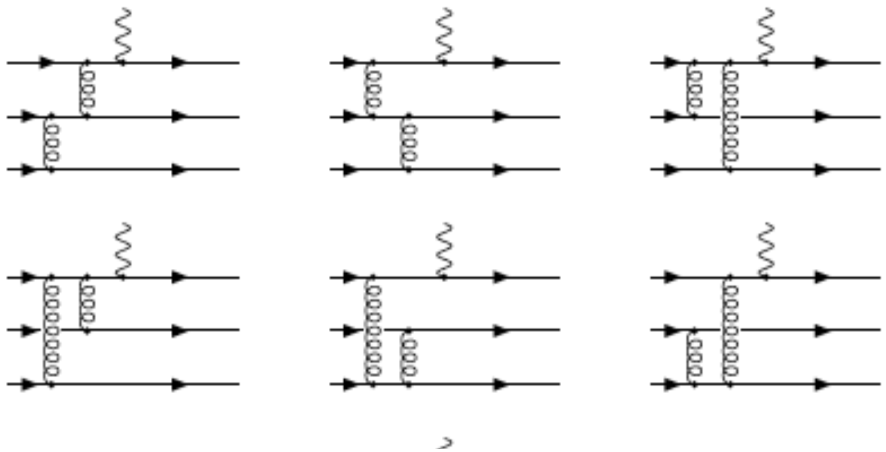




Transverse momentum dependence Sudakov suppression (Li-Sterman)

$$|P_{\uparrow}\rangle_{1/2} = \frac{1}{12}\int \frac{[dx][d^2\boldsymbol{k}]}{\sqrt{x_1x_2x_3}}\,\psi_1(\kappa_1,\kappa_2,\kappa_3)$$

$$\times\varepsilon^{abc}\,u^{\dagger}_{a\uparrow}(\kappa_1)\left\{u^{\dagger}_{b\downarrow}(\kappa_2)d^{\dagger}_{c\uparrow}(\kappa_3)-d^{\dagger}_{b\downarrow}(\kappa_2)u^{\dagger}_{c\uparrow}(\kappa_3)\right\}|0\rangle$$

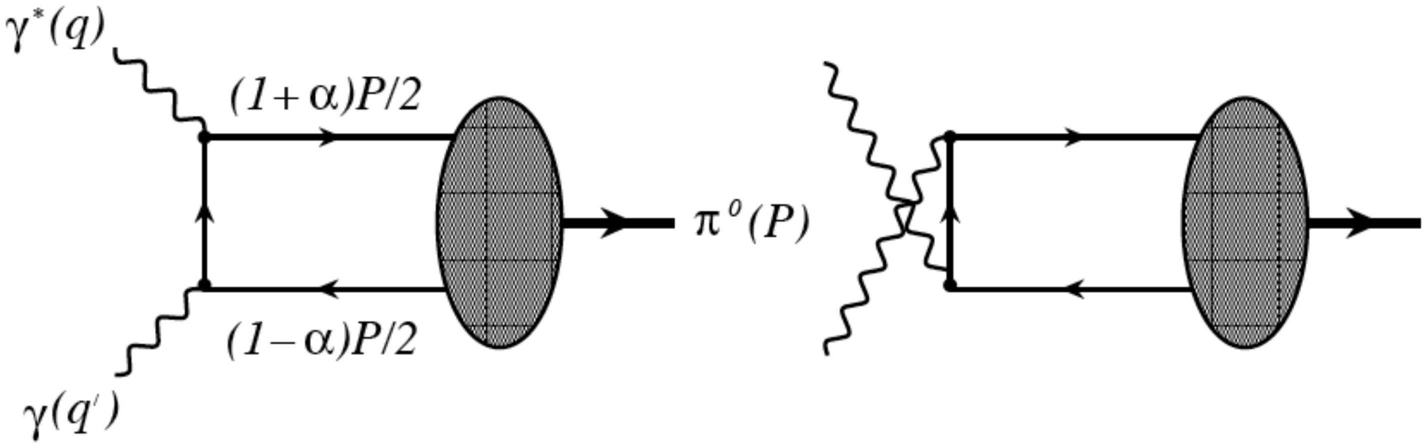
Fock vacuum



**hard scatterings**

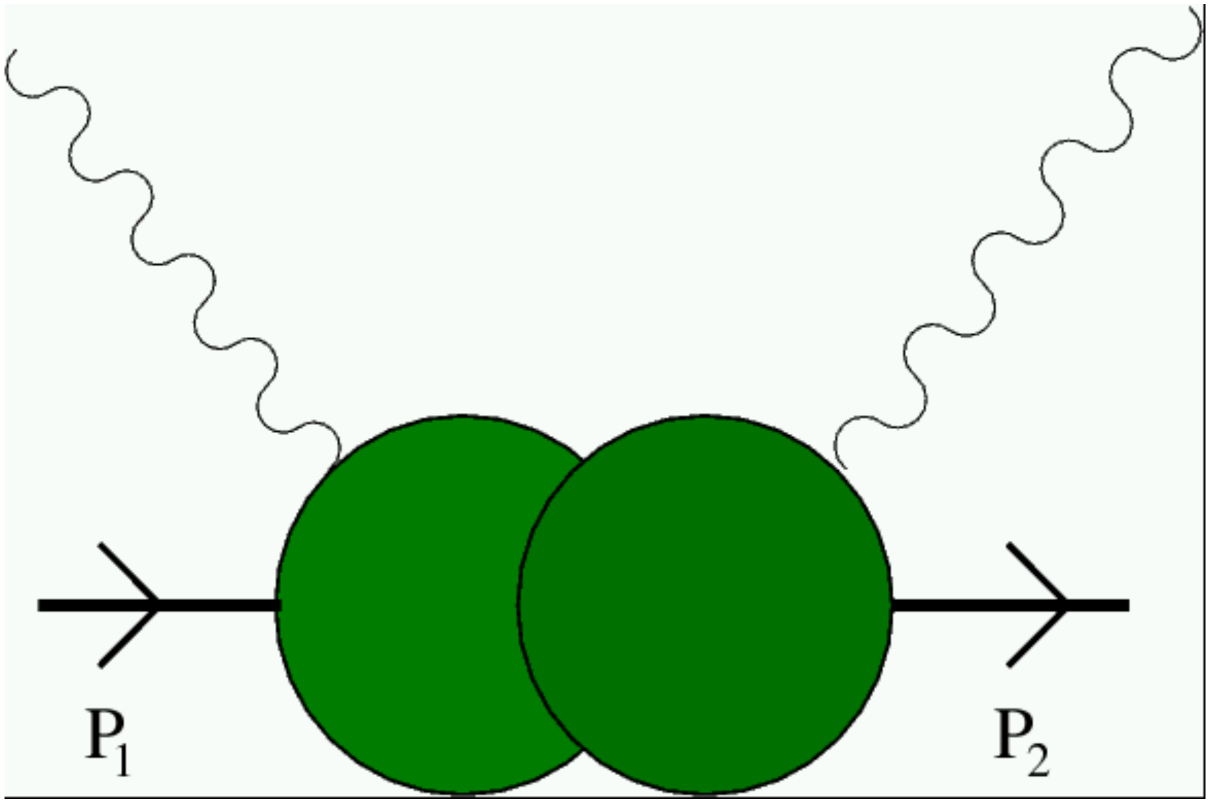
# DISTRIBUTION AMPLITUDES

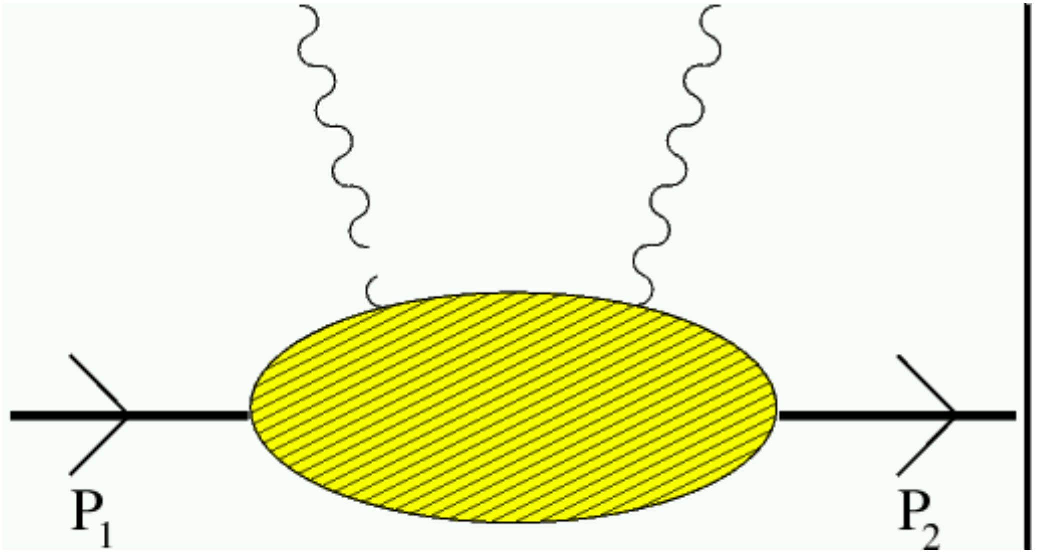
$$\langle 0|\,\bar{\psi}_d\,(-z/2)\,\gamma^\mu\gamma_5\psi_u\,(z/2)\left|\pi^+\,(P)\right\rangle_{z^2=0} \;=\; iP^\mu f_\pi \int_{-1}^{1} d\alpha\; e^{i\alpha(P\cdot z)/2}\varphi_{\pi^+}\,(\alpha)$$



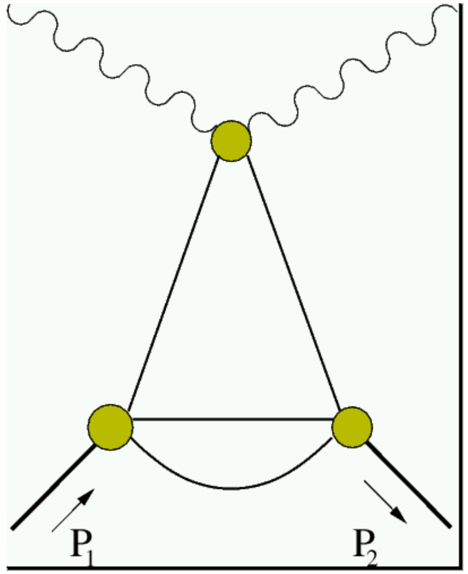

The fractions of the pion momentum carried by the quarks are $(1 \pm \alpha)\,/2$.

The Feynman mechanism
we may be unable to resolve the partonic structure of the nucleon,
Overlap of wave functions

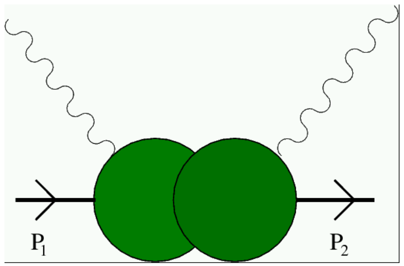





However: CS has a life of its own



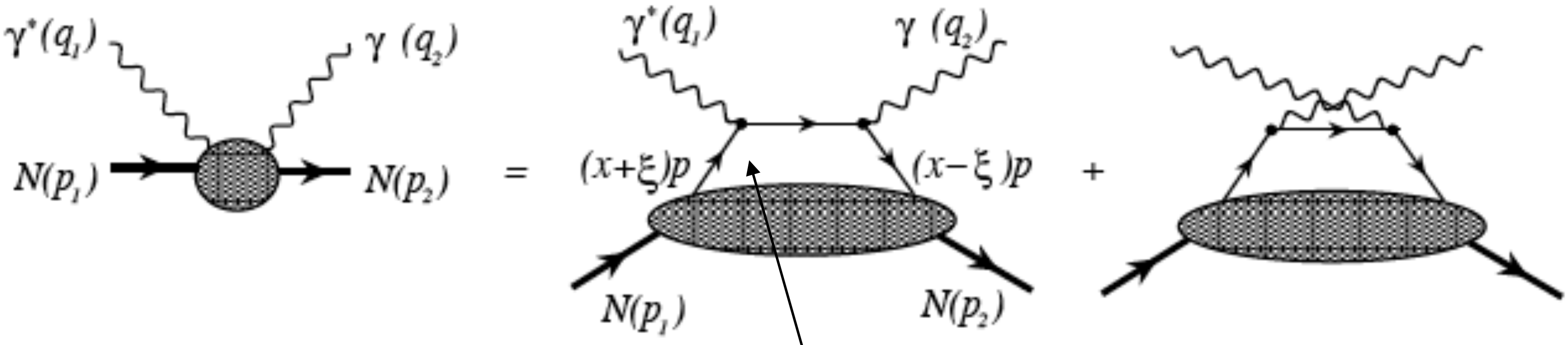

Soft




Feynman mechanism of overlapping wave functions

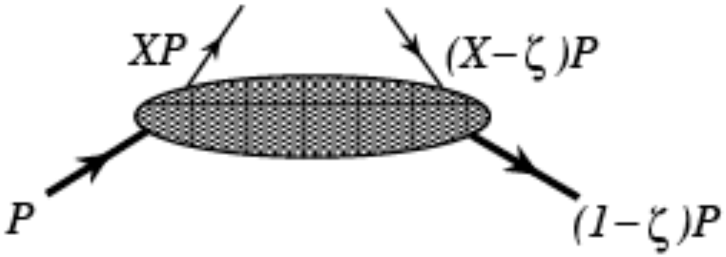
Intrinsically SOFT, not factorizable .
Use interpolating currents (Dispersive description)

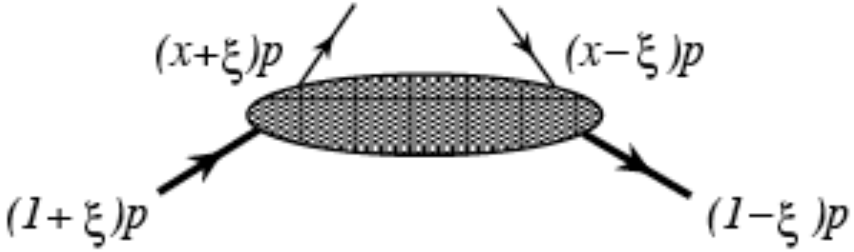
## Nonforward (Radyushkin) vs Off-forward (Ji)




Longitudinal/transverse
momentum exchange

$$X = \frac{x+\xi}{1+\xi} \qquad\qquad \zeta = \frac{2\xi}{1+\xi}.$$




Nonforward pdf's: 0<X<1

Off forward -1< x <1

$$p = (p_1 + p_2)/2,$$

$F_\zeta^f(X,t)$ is the probability amplitude that the initial fast-moving

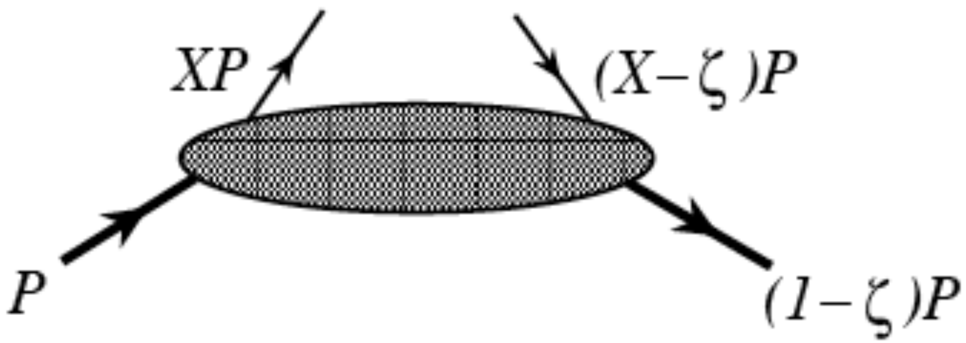
hadron, having longitudinal momentum $P^+$,

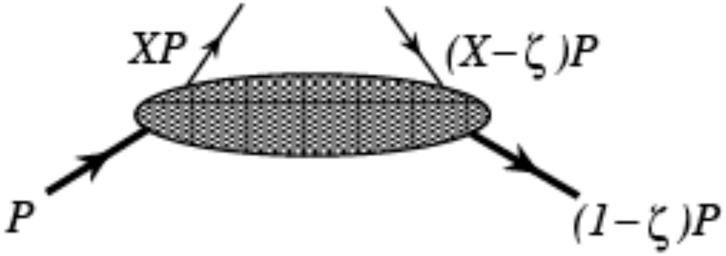
emits a parton of flavor $f$ carrying the

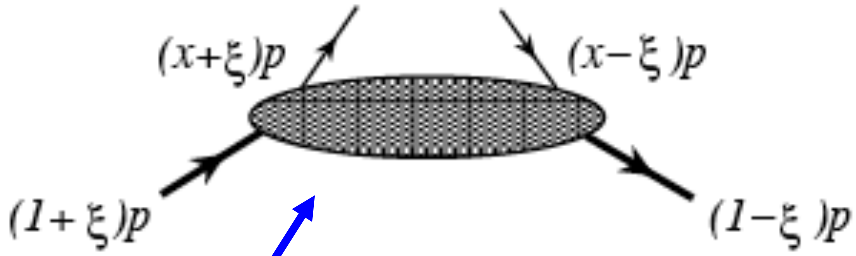
momentum $XP^+$ while the final hadron,

having longitudinal momentum $(1-\zeta)\,P^+$,

absorbs a parton of flavor $f$ carrying the momentum $(X-\zeta)\,P^+$.






$p = (p_1 + p_2)/2,$

$$H_q(x,\xi,t) = \frac{1}{2}\int \frac{dy^-}{2\pi} e^{-ix\bar{P}^+y^-}\langle P'|\bar{\psi}_q(0,y^-/2,\mathbf{0})$$
$$\times \gamma^+ \psi_q(0,-y^-/2,\mathbf{0})|P\rangle,$$

Averaged momentum

The variable $t$ is the usual $t$-channel invariant, $t=\Delta^2$,

$$\Delta \equiv (P-P') = \xi(P+P')$$

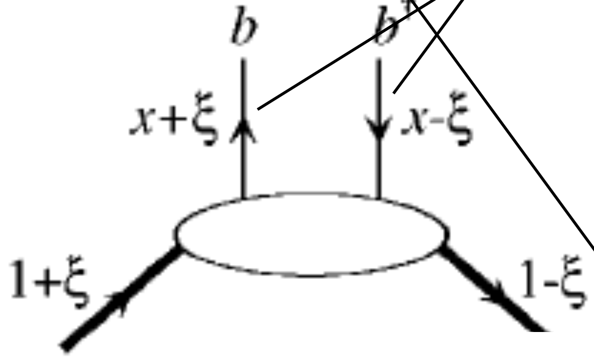
the distribution $H_q(x,\xi,t)$ now contains two extra scalar variables, in addition to the Bjorken $x$ variable.

$$H_q(x,\xi) = \frac{1}{2\bar{P}^+}\int \frac{d^2k_T}{2\sqrt{|x^2-\xi^2|}(2\pi)^3}\sum_{\lambda} [\langle P'|b^{\dagger}_{\lambda}((x-\xi)\bar{P}^+, \boldsymbol{k}_T-\boldsymbol{\Delta}_T) b_{\lambda}((x+\xi)\bar{P}^+,\boldsymbol{k}_T)|P\rangle \theta(x\geqslant\xi)$$

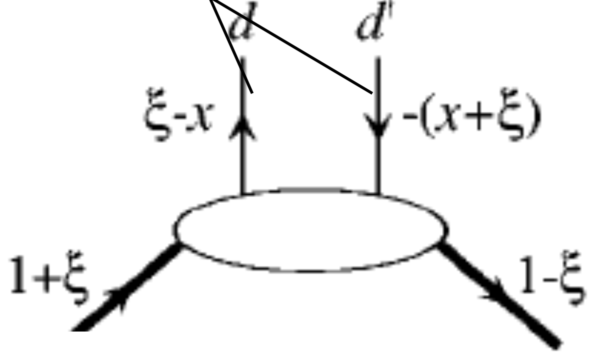
$$-\langle P'|d^{\dagger}_{\lambda}((-x-\xi)\bar{P}^+, \boldsymbol{k}_T-\boldsymbol{\Delta}_T)d_{\lambda}((-x+\xi)\bar{P}^+,\boldsymbol{k}_T)|P\rangle\theta(x\leqslant-\xi)].$$

$$+\langle P'|d_{\lambda}((-x+\xi)\bar{P}^+,-\boldsymbol{k}_T+\boldsymbol{\Delta}_T) b_{-\lambda}((x+\xi)\bar{P}^+,\boldsymbol{k}_T)|P\rangle\theta(-\xi<x<\xi)$$

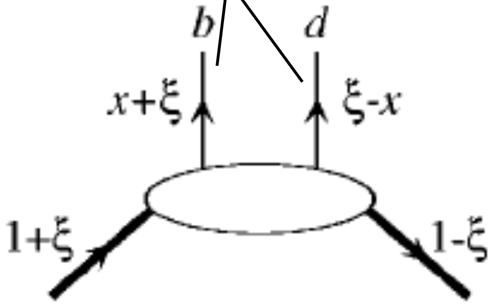
(a) $x>\xi$: DGLAP-type region for the quark distribution



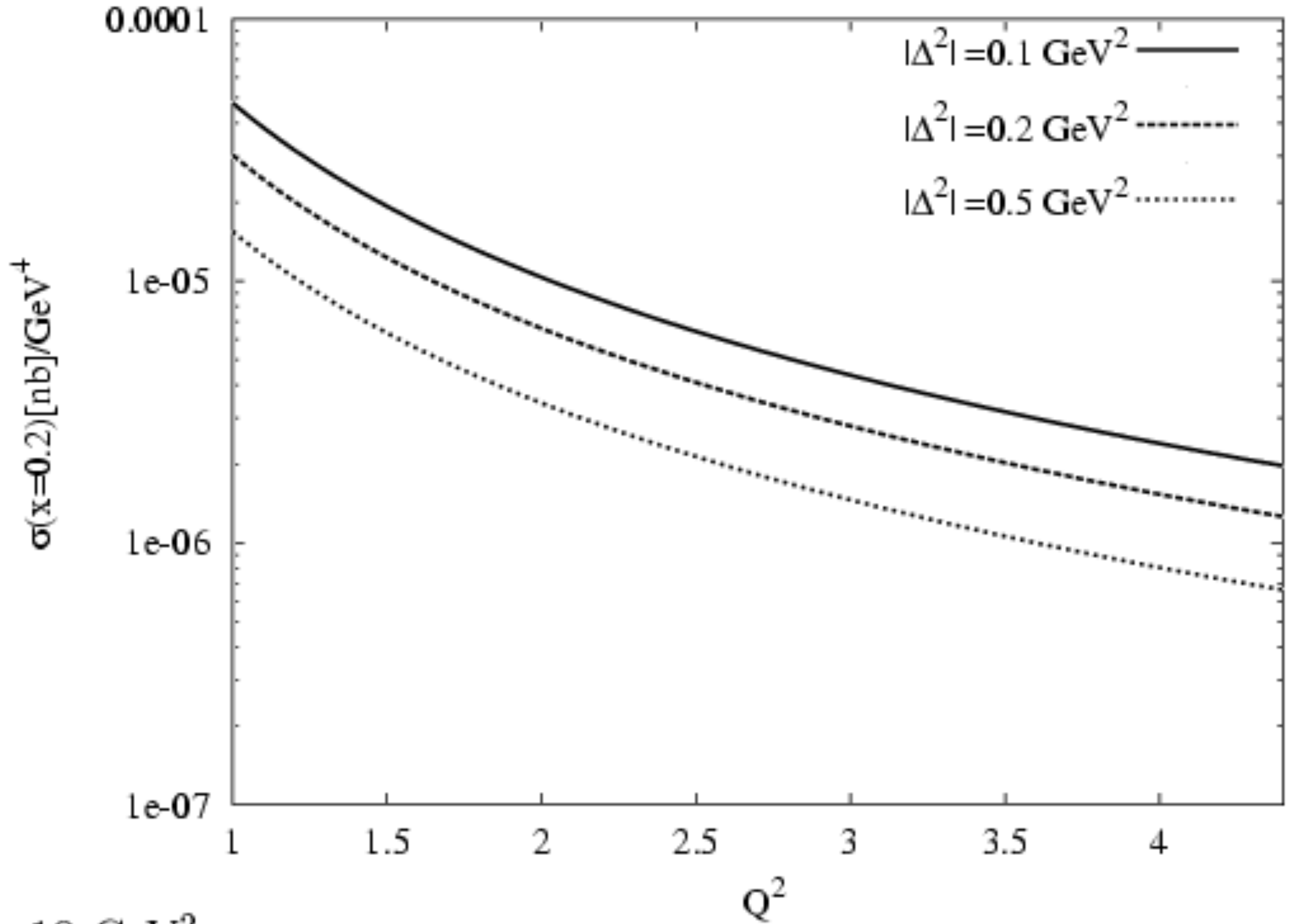

(c) $x<-\xi$: DGLAP-type region for the antiquark distribution




(b) $-\xi<x<\xi$: ERBL-type probability amplitude



$$\frac{d\sigma}{dxdQ^2d|\Delta^2|d\phi_r} = \frac{y}{Q^2}\frac{d\sigma}{dxdyd|\Delta^2|d\phi_r} = \frac{xy^2}{8\pi Q^4}\left(1+\frac{4M^2x^2}{Q^2}\right)^{-\frac{1}{2}}|\mathcal{M}_{fi}|^2.$$




$ME = 10\ \mathrm{GeV}^2$

## Charged Currents

$$
\begin{aligned}
T_{\mathrm{BH}}^{W^+} = & -|e| \frac{g}{2\sqrt{2}} \frac{g}{\sqrt{2}} \overline{u}(l') \left[ \gamma^\mu \frac{(\not{l} - \not{\Delta})}{(l - \Delta)^2 + i\epsilon} \gamma^\nu (1 - \gamma^5) \right] \\
& \times u(l) \frac{D^{\nu\delta}(q_1)}{\Delta^2 - M_W^2 + i\epsilon} \epsilon_\mu^*(q_2) \overline{U}(P_2) \\
& \times \left[ [F_1^u(\Delta^2) - F_1^d(\Delta^2)] \gamma^\delta + [F_2^u(\Delta^2) \right. \\
& \left. - F_2^d(\Delta^2)] i \frac{\sigma^{\delta\alpha} \Delta_\alpha}{2M} \right] U(P_1),
\end{aligned}
$$

$$
T_{\mu\nu} = i \int d^4 x e^{iqx} \langle P_2 | T[J_\nu^\gamma(x/2) J_\mu^{W^\pm, Z_0}(-x/2)] | P_1 \rangle,
$$

$$
\begin{aligned}
T_{\mu\nu}^{W^+} = i \int d^4 x & \frac{e^{iqx} x^\alpha U_{ud}}{2\pi^2 (x^2 - i\epsilon)^2} \langle P_2 | [i S_{\mu\alpha\nu\beta} (\tilde{O}_{ud}^\beta + O_{ud}^{5\beta}) \\
& + \epsilon_{\mu\alpha\nu\beta} (O_{ud}^\beta + \tilde{O}_{ud}^{5\beta})] | P_1 \rangle, \qquad (17)
\end{aligned}
$$

$$\tilde{O}_a^{\beta}(x/2,-x/2)=\overline{\psi}_a(x/2)\gamma^{\beta}\psi_a(-x/2)+\overline{\psi}_a(-x/2)\gamma^{\beta}\psi_a(x/2),$$
$$\tilde{O}_a^{5\beta}(x/2,-x/2)=\overline{\psi}_a(x/2)\gamma^5\gamma^{\beta}\psi_a(-x/2)-\overline{\psi}_a(-x/2)\gamma^5\gamma^{\beta}\psi_a(x/2),$$
$$O_a^{\beta}(x/2,-x/2)=\overline{\psi}_a(x/2)\gamma^{\beta}\psi_a(-x/2)-\overline{\psi}_a(-x/2)\gamma^{\beta}\psi_a(x/2),$$
$$O_a^{5\beta}(x/2,-x/2)=\overline{\psi}_a(x/2)\gamma^5\gamma^{\beta}\psi_a(-x/2)+\overline{\psi}_a(-x/2)\gamma^5\gamma^{\beta}\psi_a(x/2),$$

$$\tilde{O}_{ud}^{\beta}(x/2,-x/2)=g_u\overline{\psi}_u(x/2)\gamma^{\beta}\psi_d(-x/2)+g_d\overline{\psi}_u(-x/2)\gamma^{\beta}\psi_d(x/2),$$
$$\tilde{O}_{ud}^{5\beta}(x/2,-x/2)=g_u\overline{\psi}_u(x/2)\gamma^5\gamma^{\beta}\psi_d(-x/2)-g_d\overline{\psi}_u(-x/2)\gamma^5\gamma^{\beta}\psi_d(x/2),$$
$$O_{ud}^{\beta}(x/2,-x/2)=g_u\overline{\psi}_u(x/2)\gamma^{\beta}\psi_d(-x/2)-g_d\overline{\psi}_u(-x/2)\gamma^{\beta}\psi_d(x/2),$$
$$O_{ud}^{5\beta}(x/2,-x/2)=g_u\overline{\psi}_u(x/2)\gamma^5\gamma^{\beta}\psi_d(-x/2)+g_d\overline{\psi}_u(-x/2)\gamma^5\gamma^{\beta}\psi_d(x/2),$$

$$\begin{aligned}
&\langle P_2|\overline{\psi}_a(-kx)\gamma^5\gamma^{\mu}\psi_a(kx)|P_1\rangle^{\text{twist-2}}\\
&\quad=\int Dz e^{-ik(x\cdot P_z)}F^{5a(\nu)}(z_1,z_2,P_i\cdot P_jx^2,P_i\cdot P_j)\\
&\qquad\times\overline{U}(P_2)[\gamma^5\gamma^{\mu}-ikP_z^{\mu}\gamma^5\not{x}]U(P_1)\\
&\qquad+\int Dz e^{-ik(x\cdot P_z)}G^{5a(\nu)}(z_1,z_2,P_i\cdot P_jx^2,P_i\cdot P_j)\\
&\qquad\times\overline{U}(P_2)\gamma^5\left[\frac{(i\sigma^{\mu\alpha}\Delta_{\alpha})}{M}-ikP_z^{\mu}\frac{(i\sigma^{\alpha\beta}x_{\alpha}\Delta_{\beta})}{M}\right]U(P_1).
\end{aligned}$$

**Expressed in terms of nfpd's**

## CONCLUSIONS

Plenty of new applications of pQCD at intermediate energy

1) Perturbative analysis of weak form factors
2) Study of coherence effects
3) Will be able to explore hadronic/weak interactions
in a new territory