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Simonetta Abenda

Dipartimento di Matematica e CIRAM, Università degli Studi di Bologna
Bologna, Italy

Closed geodesics and billiards on quadrics related elliptic KdV solutions

We consider algebraic geometrical properties of the integrable billiard on a quadric Q with elastic impacts along another quadric confocal to Q. These properties are in sharp contrast with those of the ellipsoidal Birkhoff billiards in $\mathbb{R}^n$. Namely, in the case we consider, generic complex invariant manifolds are not Abelian varieties and the billiard map is no more algebraic. A Poncelet-like theorem for such system is known. We give explicit sufficient conditions both for closed geodesics and periodic billiard orbits on Q and discuss their relation with the elliptic KdV solutions and elliptic Calogero system. This research was done in collaboration with Yuri Fedorov (UPC, Barcelona and Moscow State University.

Mark J. Ablowitz

Department of Applied Mathematics, University of Colorado
UCB 526, Boulder 80309 CO

Solitary waves in nonlinear optics and fluid dynamics

Nonlinear optics involves quite naturally the study of nonlinear waves, and in particular their localized wave solutions, such as solitary waves, or solitons. Important examples are optical communications, where asymptotic analysis leads to the “classical” and “dispersion managed” nonlocal nonlinear Schrodinger (NLS) equations. These equations contain solitary waves, or solitons, as special solutions. Recent research has shown that similar dispersion managed equations occur in mode-locked lasers. A numerical method is introduced to find these and other solitary waves in nonlinear optics and fluid dynamics. In fluid dynamics examples include multi-fluid systems and water waves. The above numerical method is employed to find two dimensional water wave lumps to the fully nonlinear water wave equations with sufficient surface tension.

George Alekseev

Steklov Mathematical Institute, Russian Academy of Sciences
Gubkina St. 8, 119991, Moscow, Russia
e-mail: G.A.Alekseev@mi.ras.ru

Integrable reductions of Einstein-Maxwell equations: hierarchies of solutions with rational monodromy data and their applications to a black hole dynamics

Integrability of Einstein-Maxwell equations for stationary axisymmetric electrovacuum fields allows to construct in explicit form infinite hierarchies of solutions solving the system of linear singular integral equations which determine the solutions in terms of the monodromy data of the fundamental solution of associated linear system. These integral equations admit an infinite hierarchy of explicit solutions for arbitrary rational analytically matched monodromy data which extends considerably an hierarchy of Einstein - Maxwell solitons generated on the Minkowski background. We present a general construction of such solutions and describe some applications of these solutions to the analysis of nonlinear interaction of black holes with external electromagnetic and gravitational fields.

Victor Atanasov

INRNE
Sofia, Bulgaria

The emergence of the Toda chain for the N-kink (breather) train of the Sine-Gordon equation

A method for the description of the N–kink interaction, which generalizes in a natural way the Karpman–Solov'ev one for the nonlinear Schrödinger (NLS) equation is proposed. Using it we derive a nonlinear system of equations describing the dynamics of the parameters of N well separated kinks with nearly equal amplitudes and velocities. Under certain approximations we show, that the N–kink interaction for the unperturbed sine-Gordon (SG) equation is described by a special reduction of the complex Toda chain (CTC) with N–nodes, which is a completely integrable dynamical system. Results for the double sine-Gordon equation are also presented.

Ilkay Bakirtas

ITU Faculty of Science and Letters
Department of Mathematics
34469 Maslak Istanbul, Turkey

Wave collapse in nonlocal nonlinear Schrödinger equation

Wave collapse is investigated in nonlocal nonlinear Schrödinger systems, where a nonlocal potential is coupled to an underlying mean term. Such system are referred to NLSM systems or Davey Stewartson type and they arise in studies of shallow water waves and nonlinear optics. The role of the ground state in global existence theory is elucidated. The ground state is computed using a fixed point method. The critical

powers of collapse predicted by the Viral Theorem, Global Existence Theory and by direct simulations. The properties of the ground state profile is investigated for a wide range of parameters.

Gino Biondini

Department of Mathematics, State University of New York at Buffalo
Buffalo, NY - USA

Initial-boundary value problems for discrete linear and nonlinear Schrödinger equations

A methodology was recently developed to solve initial-boundary value problems for linear and integrable nonlinear partial differential equations. Here we show that similar ideas can also be used to solve initial-boundary value problems for linear and integrable nonlinear differential-difference equations. We do so by solving the initial-boundary value problem on the natural numbers for the discrete analogue of both the linear and the nonlinear Schrödinger equations.

Maciej Blaszak

Institute of Physics, A. Mickiewicz University
Poznam, Poland

Reciprocal transformations for classical Stackel systems and related dispersionless systems

Systematic construction of classical Stackel systems (these with all constants of motion quadratic in momenta) in coordinate free way is presented. With each class of such systems one can relates weakly nonlinear semi-hamiltonian dispersionless systems. It is shown that different classes of Stackel systems as well as connected dispersionless systems are related by an appropriate reciprocal transformations.

Leonid Bogdanov

Landau Institute
Moscow, Russia

On the heavenly equation hierarchy

The dbar-dressing method applicable to heavenly equation will be presented. Construction of solutions will be discussed. The heavenly equation hierarchy and its reductions will be considered.

Marco Boiti[a], **Flora Pempinelli**[a], **Andrei Pogrebkov**[b]
[a] *Dipartimento di Fisica - Università di Lecce*
Lecce, Italy
[b] *Steklov Math. Institute, Moscow, Russia*

Extended resolvent of the nonstationary Schrödinger operator for a Darboux transformed potential

The Nonstationary Schrödinger operator with a potential obtained by "superimposing" via a Darboux transformation N solitons to a generic smooth potential decaying at large space is considered. The corresponding extended resolvent is explicitly constructed and its properties are discussed.

S. Boscolo, S.K. Turitsyn
Photonics Research Group, School of Engineering and Applied Science,
Aston University, Birmingham B4 7ET, United Kingdom

On the theory of self-similar parabolic optical pulses

Self-similar optical pulses (or "similaritons") of parabolic intensity profile can be found as asymptotic solutions of the nonlinear Schrödinger equation in a gain medium such as a fiber amplifier or laser resonator. These solutions represent a wide-ranging significance example of dissipative nonlinear structures in optics.

Here, we address some issues related to the formation and evolution of parabolic pulses in a fiber gain medium by means of semi-analytic approaches. In particular, the effect of the third-order dispersion on the structure of the asymptotic solution is examined. Our analysis is based on the resolution of ordinary differential equations, which enable us to describe the main properties of the pulse propagation and structural characteristics observable through direct numerical simulations of the basic partial differential equation model with sufficient accuracy.

Francesco Calogero
Dipartimento di Fisica, Università di Roma "La Sapienza"
Istituto Nazionale di Fisica Nucleare, Sezione di Roma

Isochronous systems and their quantization

A (classical) dynamical system is called isochronous if it features an open (hence *fully dimensional*) region in its phase space in which *all* its solutions are *completely periodic* (i. e., periodic in *all* degrees of freedom) with the *same* fixed period (independent of the initial data, provided they are inside the *isochronicity* region). A trick is presented associating to an (autonomous) dynamical system an (also autonomous) "ω-modified" system depending on a parameter ω so that when $\omega = 0$ the original system is reproduced while when $\omega > 0$ the ω-modified system is isochronous (typically with period $T = 2\pi/\omega$). This technique is applicable to large classes of dynamical systems, justifying the statement that **isochronous systems are**

not rare. An analogous technique, even more widely applicable – for instance, to *any* translation-invariant (classical) many-body problem – transforms a Hamiltonian system to an isochronous Hamiltonian system (this finding is joint work with Francois Leyvraz). This opens the possibility to investigate the quantization of many isochronous systems, and to thereby find out whether, and to what extent, such systems exhibit equispaced spectra. A very recent result, applicable to an entire, if quite special, category of such systems states that any isochronous Hamiltonian which is linear in the momenta yields, after quantization, a spectrum at least part of which is equispaced: note that this result holds even if the system in question behaves isochronously only in an (open) subregion of its (natural) configuration space, but possibly in a quite complicated manner (even chaotically) elsewhere (also this finding is joint work with Francois Leyvraz).

Jean-Guy Caputo[1,2] **and L. Loukitch** [1]
[1]*Laboratoire de Mathématiques, INSA de Rouen*
B.P. 8, 76131 Mont-Saint-Aignan cedex, France
[2]*Laboratoire de Physique théorique et modelisation, Université de Cergy-Pontoise and C.N.R.S.*

Statics and dynamics of point nonlinear oscillators in a linear cavity

We introduce a continuous/discrete model to describe a linear system containing small nonlinear impurities. The initial motivation is to analyse the behavior of parallel arrays of small Josephson junctions between two superconducting films forming a linear cavity. These are used as magnetic field detectors (static regime), Terahertz oscillators and signal mixers.
Our model is a wave equation with delta distributed (sine) nonlinearities

$$\phi_{tt} - \phi_{xx} + \sum_{i=1}^{n} d_i \delta(x - a_i)(\sin(\phi) + \alpha\phi_t) = \nu \frac{\gamma}{l} , \tag{1}$$

with the boundary conditions $\phi_x(0;t) = H - (1-\nu)\gamma/2$, $\phi_x(l;t) = H + (1-\nu)\gamma/2$, where H, γ and $0 < \nu < 1$ are respectively a magnetic field, a current applied to the system and the type of current feed. Compared to the standard "lumped element" approach, our new long wave model allows to describe wave propagation between the elements and is completely consistent with a full numerical description using partial differential equations for the junctions and the cavity. The great benefit is the possibility of analysing the solutions and give simple expressions.

We consider the static problem of (1) and search for the maximal current $\gamma_{max}(H)$ which gives a solution for a given magnetic field (H). This is important for magnetic field detectors (SQUID). We analyse the periodicity of $\gamma_{max}(H)$, the influence of the type of current feed ν and of separating a junction from the others. For a few small junctions such that $|\sum_i d_i| << 1$ we show that $\gamma_{max}(H)$ curve tends to a simple function. This allows us to design a device in order to obtain a given $\gamma_{max}(H)$ curve.

The dynamical problem of (1), consists in computing the voltage (time average of ϕ_t) in the device for a given current (γ) to obtain the $I-V$ curve. Since (1) is non linear, all the cavity modes can be excited so that there are many solutions for the same γ. The dynamics of one junction in a 1D microstrip [2] showed two limiting behaviors, the ohmic mode where the junction acts as a wave stopper in the cavity and the junction mode where it drives the whole domain. These two analytical solutions bound $I - V$ curve. These results indicate how to select or kill a specific resonance of the $I - V$ curve, by choosing the position and area of the junctions.

These results show good agreement with the experiments done by the group of Morvan Salez [1] at the Observatoire de Paris. This long wave approach, valid when there are localized nonlinearities, small with respect to the wave-length, can be extended to other fields.

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Robert Carrol

University of Illinois
Urbana IL, USA

Metric fluctuations, entropy, and the WDW equation

A Schroedinger equation for the wave function in polar form with quantum potential Q can be shown to arise from a classical system via momentum fluctuations described via Fisher information. Under suitable conditions this can be extended to Wheeler-deWitt (WDW) equations and certain metric fluctuations. The quantum potential can be expressed via Fisher information and is related to a differential entropy.

Jen Hsu Chang

National Defense University
Tauyuan County, Taiwan

On the bi-Hamiltonian structure of waterbag model of dKP

One investigates the bi-Hamiltonian structure of waterbag model of dKP. In the two-component case, we establish the bi-Hamiltonian structure. The free energy is found. One also discusses the four-component case.

Roy Choudhury

Department of Mathematics University of Central Florida
Orlando, FL 32816-1364
choudhur@longwood.cs.ucf.edu

Solitary Wave Families of the Ostrovsky Equation:

An Approach via Reversible Systems Theory and Normal Forms

The Ostrovsky equation is an important canonical model for the unidirectional propagation of weakly nonlinear long surface and internal waves in a rotating, inviscid and incompressible fluid. Limited functional analytic results exist for the occurrence of one family of solitary wave solutions of this equation, as well as their approach to the well-known solitons of the famous Korteweg-de Vries equation in the limit as the rotation becomes vanishingly small. Since solitary wave solutions often play a central role in the long-time evolution of an initial disturbance, we consider such solutions here (via the normal form approach) within the framework of reversible systems theory. Besides confirming the existence of the known family of solitary waves and its reduction to the KdV limit, we find a second family of multihumped (or N-pulse) solutions, as well as a continuum of delocalized solitary waves (or homoclinics to small-amplitude periodic orbits). On isolated curves in the relevant parameter region, the delocalized waves reduce to genuine embedded solitons. The second and third families of solutions occur in regions of parameter space distinct from the known solitary wave solutions and are thus entirely new. Directions for future work are also mentioned.

Olga Choustova

International Center for Mathematical Modeling in Physics Engineering Economy and Cognitive Sciences
MSI, University of Växjö, S-35195, Sweden

Applications of Hamilton-Jakobi dynamics and
Bohmian mechanics to description of financial processes[1]

We apply methods of quantum mechanics for mathematical modeling of price dynamics at the financial market. The Hamiltonian formalism on the price/price-change phase space describes the classical-like evolution of prices. This classical dynamics of prices is determined by <<hard>> conditions (natural resources, industrial production, services and so on). These conditions are mathematically described by the classical financial potential V depending on the vector of prices of various shares. But the information exchange and market psychology play important (and sometimes determining) role in price dynamics. We propose to describe such behavioral financial factors by using the pilot wave (Bohmian) model of quantum mechanics. The theory of financial behavioral waves takes into account the market psychology. The real trajectories of prices are determined (through the financial analogue of the second Newton law) by two financial potentials: classical-like $V(q)$ (<<hard>> market conditions) and quantum-like $U(q)$ (behavioral market conditions).

Robert Conte

Service de Physique de l'etat condense CEA-Saclay
Gif-sur-Yvette, France

Reduction of the resonant three-wave interaction to the sixth Painlevé equation
(Joint work with A.M. Grundland and M. Musette)

[1]Poster

The system of three resonant waves ($k_1 + k_2 + k_3 = 0, \omega_1 + \omega_2 + \omega_3 = 0$) in one space dimension can be mathematically described (Zakharov and Manakov 1973) by six coupled partial differential equations in six complex amplitudes,

$$\left\{ \begin{array}{l} u_{j,t} + c_j u_{j,x} - i\bar{u}_k \bar{u}_l = 0, \\ \bar{u}_{j,t} + c_j \bar{u}_{j,x} + i u_k u_l = 0, \ i^2 = -1, \end{array} \right. \tag{1}$$

in which (j, k, l) denotes any permutation of $(1, 2, 3)$, c_j are the constant values of the group velocities, with $(c_2 - c_3)(c_3 - c_1)(c_1 - c_2) \neq 0$.

Among the reductions to six-dimensional differential systems, there is one for which several authors (Fokas, Leo, Martina and Soliani 1986, Kitaev 1990) noticed a link with particular sixth Painlevé equations P6. We derive this link explicitly and prove that, as expected from the completeness of the system (1), it involves the generic P6.

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R. Conte, A. M. Grundland, and M. Musette, J. Phys. A **39** to appear (2006). Special issue "One hundred years of Painlevé VI"
http://arXiv.org/abs/nlin.SI/0604011

S. De Lillo and A. Fokas

Dipartimento di Matematica e Informatica - Università di Perugia
Perugia, Italy

The Spectral Method for Inverse Moving Boundary Problems

An inverse moving boundary problem for the linear heat equation is discussed. Through the spectral method the problem is reduced to a linear Volterra integral equation of II type. Moreover, the solution of the inverse problem for the heat equation is shown to be a fundamental step toward the solution of similar problems for nonlinear evolution equations of diffusive type.

Valeri V. Dvoeglazov

Universidad de Zacatecas
Zacatecas, Mexico

Antisymmetric tensor fields and 4-vector fields

We generalize the Stueckelberg formalism in the (1/2,1/2) representation of the Lorentz group. Some relations to other modern-physics models are found.

Eugene Ferapontov

Department of Mathematical Sciences, Loughborough University,
Loughborough, LE11 3TU, UK

On integrability of hydrodynamic chains: the Haantjes tensor

The integrability of systems of hydrodynamic type by the generalized hodograph method requires the vanishing of the corresponding Haantjes tensor. We generalize this approach to hydrodynamic chains — infinite-component systems of hydrodynamic type for which the corresponding infinite matrix is 'sufficiently sparse'. For such systems the Haantjes tensor is well-defined, and the calculation of its components involves finite summations only. We illustrate our approach by classifying broad classes of conservative and Hamiltonian hydrodynamic chains with the zero Haantjes tensor. We prove that the vanishing of the Haantjes tensor is a necessary condition for a hydrodynamic chain to possess an infinity of semi-Hamiltonian hydrodynamic reductions, thus providing an easy-to-verify necessary condition for the integrability.

Thanasis Fokas

University of Cambridge
Cambridge, UK

Imaging of the brain, boundary value problems and Integrability in 4+2 and 3+1

The following three developments will be summarised:

1) There exists a method, based on the spectral analysis of a single eigenvalue equation, capable of inverting interesting integrals. Such integrals arise in imaging techniques, as well as in the characterisation of the Dirichlet to Neumann map.

2)There exists a method, based on the simultaneous analysis of two eigenvalues equations and on the exact inversion of the so-called global relation, which can be used for the effective analysis of boundary value problems.

3)There exists integrable nonlinear PDES in the 4+2 and 3+1 dimensions, as well as a method for solving their initial-value problem.

Lazar Friedland

The Hebrew University of Jerusalem
Jerusalem, Israel

Emergence and control of multi-phase nonlinear waves by resonant perturbations

Many extended physical systems are modelled by classical soliton equations (KdV, SG, NLS etc.), possessing, far from equilibrium, a variety of nontrivial solutions. The question of reaching and controlling a particular stable solution (excitation) in this solutions set by starting from simple initial/boundary conditions is fundamental to many applications. I will describe a recent approach to formation of multiplicity of nonlinear excitations based on capturing the system into resonance with slow external perturbations followed by a continuing self-synchronization (autoresonance) in space and/or time. The synchronization means excursion in system's solutions space with possible emergence of a desired nontrivial state. Applications of this recent paradigm exist in vorticity dominated flows, plasmas, as well as in planetary dynamics, atomic and molecular physics. I will present main ideas of autoresonance in extended systems and new developments on formation and control of multi-phase nonlinear states. The relation between the multi-phase autoresonance and the Inverse Scattering Transform will be outlined.

Maria Luz Gandarias

Departamento de Matemáticas, Universidad de Cádiz,
PO.BOX 40,11510 Puerto Real, Cádiz, Spain
E.mail: marialuz.gandarias@uca.es

New solutions through weak symmetries of the Schwarzian-Korteweg-de Vries

In this paper we consider the (2+1)-dimensional integrable Schwarzian-Korteweg-de Vries equation. By using weak symmetries we obtain a system of partial differential equations in (1+1) dimensions. By further reductions we obtain second order ordinary differential equations which provide new solutions expressible in terms of known functions. These solutions depend up to three arbitrary functions. Hence, some of the solutions of the Schwarzian-Korteweg-de Vries equation we have found exhibit a wide variety of qualitative behavior.

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P. G. Estévez[a], **J. Prada** [b] **and J. Villarroel** [c]

[a]*Area de Fisica Teórica - Universidad de Salamanca*
[b] *Departamento de Matemáticas - Universidad de Salamanca*
[c] *Departamento de Estadistica - Universidad de Salamanca*
Salamanca 37008, Spain

Algorithmic construction of Lumps

The Singular Manifold Method is used to generate lump solutions of a Schrödinger equation in $2 + 1$ dimensions. Three different types of lumps are presented as well as the two-lump solution. Connection between this method and the Ablowitz-Villarroel scheme is also analyzed.

Vladimir Gerdjikov

Institute for Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences,
72 Tsarigradsko chaussee, 1784 Sofia, Bulgaria

Multicomponent solitons: properties, stability, interactions.

An attempt is made towards classification of different types of soliton solutions for the N-wave and for the multicomponent nonlinear Schrödinger equations. The relevant Lax operator is related to the simple Lie algebra $\mathfrak{g}$ takes the form:

$$L\psi(x,\lambda) \equiv i\frac{d\psi}{dx} + (q(x,t) - \lambda J)\psi(x,t,\lambda) = 0, \tag{1}$$

Here J is a constant element of the Cartan subalgebra $\mathfrak{h}$ of $\mathfrak{g}$, the potential $q(x,t)$ takes values in $\mathfrak{g}\backslash\mathfrak{h}$. The dressing Zakharov-Shabat method allows us to construct reflectionless potentials related to each semi-simple subalgebra $\mathfrak{g}_0 \in \mathfrak{g}$.

We also briefly discuss the problems of the interactions and linear stability of the solitons.

Subir Ghosh

Indian Statistical Institute
Kolkata, W. Bengal, India

Current Algebra and Non-Linear sigma-Model

The current algebra of a particular form of non-linear sigma-model will be reported. The algebra has a non-abelian form with the structure functions being field dependent. The connection of the model with noncommutative spacetime will be commented upon.

Rossen Dandoloff[a] **and** **Georgi G. Grahovski**[b]

[a] *Laboratoire de Physique Théorique et Modélisation, Université de Cergy-Pontoise*
2 avenue A. Chauvin, F-95302 Cergy-Pontoise Cedex, France
[b]*Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences,*
72 Tsarigradsko chaussee, 1784 Sofia, Bulgaria

An Exactly Solvable Case for a Thin Elastic Rod

It is shown that for a thin elastic rod (with an asymmetric cross-section) the twist angle is not constant along the rod and satisfies the static sine-Gordon equation in the case of constant curvature and torsion. Furthermore it is proved that the periodic (soliton-lattice) solution for the static sine-Gordon model is compatible with the statics of such rods (called also “Kirchhoff rods”).

A mapping of a XY Heisenberg spin chain model with two perpendicular spins par site onto a Kirchhoff thin elastic rod is constructed. It is shown that the single asymmetric elastic Kirchhoff rod model can be mapped onto a 2-spin XY Heisenberg chain and the spin vectors must have a different lengths.

The soliton-like solutions and the nonlinear ecxitations are briefly analyzed.

P.G. Grinevich and I.A. Taimanov

L.D. Landau Institute
Moscow, Russia

Infinitesimal Darboux transformations of the spectral curves of tori in the four-space

We resume the study of relations between the geometric properties of surfaces in $\mathbb{R}^3$ and $\mathbb{R}^4$ and the spectral properties of the corresponding Dirac operators. In the paper we study the behavior of the spectral curve of a torus in $\mathbb{R}^4$ under conformal transformations of $\bar{\mathbb{R}}^4$ and, in particular, prove that the conformal transformations of $\bar{\mathbb{R}}^4$ which map a torus $T \subset \mathbb{R}^4$ into a compact torus preserve all Floquet multipliers of the corresponding Dirac operator.

Alfred Michel Grundland

Universite du Quebec
Montreal, Canada

Rank-k solutions described by quasilinear systems of PDEs

In this talk, I will present a comprehensive analysis of rank-k solutions in terms of Riemann invariants obtained from interrelations between two concepts of the symmetry reduction method and the generalized method of characteristics for first order systems of PDEs in many dimensions. A variant of the conditional symmetry method for obtaining this type of solutions will be presented. A Lie module of vector fields which are symmetry of an overdetermined system defined by the initial system of equations and certain first order differential constraints is constructed. It is shown that this overdetermined system admits rank-k solutions expressible in terms of Riemann invariants. Finally, examples of applications of the proposed approach to fluid dynamics equations in $(n + 1)$-dimensions are discussed in detail.

I.TY. Habibullin

Institute of mathematics of Russian Academy of Science
Ufa, Russia

Characteristic algebras and the problem of classification of discrete chains

We consider discrete equations of the form $t(n+1, m+1) = f(t(n, m), t(n+1, m), t(n, m+1))$ and of the form $t_x(n + 1) = f(t(n), t(n + 1), t_x(n))$, where $t_x(n)$ means the derivative of $t(n)$ with respect to x. We introduce the notion of characteristic Lie algebra for these type equations (see also I.T.Habibullin, Characteristic Algebras of Fully Discrete Hyperbolic Type Equations, arXiv:nlin.SI/0506027). Some properties of this algebra are studied. The problem of using characteristic algebras for the classification purposes will be discussed. Some classification results are represented.

John Harnad

Universite de Montreal
Quebec, Canada

Correlators of characteristic polynomials in two-matrix models

We give two new methods for the evaluation of a class of integrals of rational symmetric functions in N pairs of variables $\{(x_a, y_a)\}_{a=1,...N}$ representing averaged ratios of products of characteristic polynomals in 2-matrix models, valid for a broad class of two-variable measures. The result is expressed as the determinant of a matrix whose entries consist of the associated biorthogonal polynomials, their Hilbert transforms, evaluated at the zeros and poles of the integrand, and bilinear expressions in these. The first method is elementary and direct, using only standard determinantal identities, partial fraction expansions and the property of biorthogonality. The second is based on interpreting the result as the vacuum state expectation value of a product of fermionic operators in the sense of Jimbo-Miwa tau functions, and using dressing transformations to convert monomials into biorthogonal polynomials.*

(*This talk is based on joint work with Alexander Orlov)

Boaz Ilan

University of Merced,
California, USA

Two-dimensional solitons in irregular lattices

Solitons (localized modes) are obtained for the two-dimensional nonlinear Schrodinger equation with external potentials that posses large variations from periodicity, i.e., vacancy defects, edge dislocations, and quasicrystal structure. The solitons are obtained by employing a spectral fixed-point computational scheme. Their stability is studied and confirmed numerically.

L.A. Kalyakin

Institute of Mathematics RAS,
Ufa, Russia

Autoresonance phenomenon in magnetic reversal of the weak ferromagnetic

We give an asymptotic analysis of the Sin-Gordon equation under a special perturbation. The perturbed equation is interpreted as a mathematical model of a weak ferromagnetic at presence of both a weak dissipation and an external magnetic field. The differential equation is completed by an initial data in the form of Sine-Gordon breather with a small amplitude. Such type initial function represents a seed domain of magnetic reversal. The leading order term of an asymptotic solution is constructed in the form of the breather with slow varying parameters. The principal features of the considering problem is an initial resonance and a slow variation of the pumping frequency. Due to these properties the perturbed breather sometimes rises so that the amplitude increases up to order of unity at large times. But it is occurred just only under some restrictions on input data. Such type of solutions have a long time resonance between breather and pumping frequencies. This automatic phase locking is referred to as autoresonance. We give a construction of such autoresonance asymptotic solutions and find conditions under which they exist. These results are crucial for the theory of magnetic reversal.

David J. Kaup and Thomas K. Vogel

Department of Mathematics, University of Central Florida
Orlando, FL

The Quantitative Estimation of Variational Approximations

The Variational Principle of mechanics, and the resulting Euler-Lagrange equations, are powerful means for studying and understanding mathematical and physical evolution systems. This principle has long been used, albeit irregularly, as a means for obtaining approximate solutions in such areas as classical mechanics, quantum mechanics, applied mathematics and for our interests, nonlinear waves. In the latter case, it has frequently been used for obtaining approximate solutions of solitary waves, particularly in those systems where one cannot find exact closed-form solutions. However the variational approximation has its Achilles' heel, for which it has often been criticized. The Achilles' heel is simply that one cannot readily judge how valid the approximation is. And this occurs in spite of the fact that a common observation is when the variational approximation is used, more often than not, its predictions are more valid than one would normally estimate. In this presentation, we shall provide a simple means for obtaining a quantitatively estimation of the validity of a variational approximation. The techniques involved are rather simple and only involve linear methods.

Ramaz Khomeriki

Tbilisi State University
Tbilisi, Georgia

Coexistence of Josephson oscillations and novel self-trapping regime in optical waveguide arrays

Considering the coherent nonlinear dynamics between two weakly linked optical waveguide arrays, we find the first example of coexistence of Josephson oscillations with a novel self-trapping regime. This macroscopic bistability is explained by proving analytically the simultaneous existence of symmetric, antisymmetric and asymmetric stationary solutions of the associated Gross-Pitaevskii equation. The effect is,

moreover, illustrated and confirmed by numerical simulations. This property allows to conceive an optical switch based on the variation of the refractive index of the linking central waveguide.

Andrei Khrennikov

International Center for Mathematical Modeling in Physics, Engineering, Economy and Cognitive Sciences
MSI, University of Växjö, S-35195, Sweden
Email:Andrei.Khrennikov@msi.vxu.se

Infinite dimensional Hamilton dynamics and quantum mechanics

We show that QM can be represented as a natural projection of a classical statistical model on the phase space $\Omega = H \times H$, where H is the real Hilbert space. Statistical states are given by Gaussian measures on Ω having zero mean value and dispersion of very small magnitude alpha (which is considered as a small parameter of the model). These Gaussian random variables with values in Ω can be interpreted as fluctuations of the vacuum field. Physical variables (e.g., energy) are given by maps $f : \Omega \to R$ (functions of classical fields). The crucial point is that statistical states and variables are symplectically invariant. The conventional quantum representation of our prequantum classical statistical model is constructed on the basis of the Taylor expansion (up to the terms of the second order at the vacuum field point of variables f with respect to the small parameter. A Gaussian symplectically invariant measure (statistical state) is represented by its covariation operator (von Neumann statistical operator). A symplectically invariant smooth function (variable) is represented by its second derivative at the vacuum field point. Equations of Schrödinger, Heisenberg and von Neumann are images of dynamics on Ω with a symplectically invariant Hamilton function. In particular, the nonlinear Schrödinger equations arise as a special class of infinite dimensional Hamilton equations.

Karima Khusnutdinova

Department of Mathematical Sciences, Loughborough University,
Loughborough, LE11 3TU, UK

On two classes of integrable second order quasilinear equations in 3D

We characterize non-degenerate Lagrangians of the form $\int f(u_x, u_y, u_t)\, dx\, dy\, dt$ such that the corresponding Euler-Lagrange equations $(f_{u_x})_x + (f_{u_y})_y + (f_{u_t})_t = 0$ are integrable by the method of hydrodynamic reductions (see [1]). The integrability conditions constitute an over-determined system of fourth order PDEs for the Lagrangian density f, which is in involution. Familiar examples include the dispersionless Kadomtsev-Petviashvili (dKP) and the Boyer-Finley Lagrangians, $f = u_x^3/3 + u_y^2 - u_x u_t$ and $f = u_x^2 + u_y^2 - 2e^{u_t}$, respectively. A complete description of integrable cubic and quartic Lagrangians is obtained.

We prove that the Euler-Lagrange equations are integrable by the method of hydrodynamic reductions if and only if they possess a scalar pseudopotential playing the role of a dispersionless 'Lax pair'.

Integrability conditions (by the method of hydrodynamic reductions) are also obtained for a class of three-dimensional dispersionless Hirota-type equations $u_{tt} = f(u_{xx}, u_{yy}, u_{tx}, u_{ty}, u_{xy})$. Earlier, we considered the integrable equations of the form $u_{tt} = f(u_{xx}, u_{xt}, u_{xy})$ (see [2]). In both cases the integrability conditions constitute an over-determined system of third order PDEs for the function f, which is in involution.

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On a hierarchy of weakly nonlinear models for multi-phase wavetrains and energy exchange in a two-component system[2]

We consider the dynamics of weakly nonlinear multi-phase wavetrains within the framework of two pairs of counter-propagating waves in a system of two coupled Sine-Gordon equations. The emphasis is on the generic case when the system is not integrable, and the group velocities of each pair of waves are arbitrary and usually different from each other. Using an asymptotic multiple-scales expansion we obtain a hierarchy of *asymptotically exact* coupled evolution equations describing the amplitudes of the waves. Although each set of amplitude equations can be used to describe a range of dynamical phenomena, we focus here on stability of plane-wave solutions, and show that they may be modulationally unstable.

We study these instabilities in the context of solutions exhibiting an energy exchange between the two physical components of the system. We also perform numerical simulations of the original unapproximated coupled Sine-Gordon equations to assess the accuracy of the instability predictions derived from the various asymptotic models, and consider the range of validity of each asymptotic model. We show that the instabilities can lead to the formation of localized structures, and to a modification of the linear energy exchange, which then continues for some time into the nonlinear regime as an energy exchange between these localized structures. When the system is close to being integrable, the time-evolution is distinguished by a remarkable almost periodic sequence of energy exchange scenarios, with spatial patterns alternating between approximately uniform wavetrains and localized structures, which invites further theoretical study of this special case.

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[2]Poster

Arthemy V. Kiselev

Department of Higher Mathematics, Ivanovo State Power University
Rabfakovskaya str. 34, Ivanovo 153003, Russia
kiselev@math.ispu.ru

Algebraic aspects of Gardner's deformations for integrable systems

We outline the algebraic properties of the Gardner deformations for Hamiltonian PDE that preserve the complete integrability of the systems. A description of the Gardner deformations in terms of the Frölicher–Nijenhuis bracket is obtained using the homological formalism by Krasil'shchik (2003). We show that the Gardner deformations, being dual to the Bäcklund transformations, are inhomogeneous generalizations of the infinitesimal symmetries.

A distinction between the Gardner deformations and extensions of the Magri schemes is underlined; the Lax approach by Kupershmidt and Wilson (1983) is augmented. Both Gardner's deformations and extensions of bi-Hamiltonian systems result in new (adjoint) completely integrable scaling (non-)invariant equations.

Two deformations of the Boussinesq equation and an extension of the Kaup–Boussinesq equation are constructed, and integrability of the adjoint systems is established.

Partially supported by TUBITAK 2005/6 postdoctoral fellowship (NSF, Turkey)

Oleg Kiselev

Institute of Mathematisc of RAS
Ufa, Russia

Slow passage through resonance for a weakly nonlinear dispersive wave

Solution of the nonlinear Klein-Gordon equation perturbed by small external force is investigated. The frequency of perturbation varies slowly and passes through a resonance. The resonance generates a solitary packets of waves. Full asymptotic description of this process is described.

Boris G. Konopelchenko (joint work with F. Magri)

Dipartimento di Fisica and Sezione INFN Università di Lecce
Via per Arnesano, Lecce (Italy)

Associative algebras, coisotropic manifolds and dispersionless hierarchies

It is shown that the universal Whitham hierarchy of zero genus represents itself the coisotropic deformations of certain associative algebras. In such an approach the dispersionless Hirota equations are just the associativity conditions for corresponding structure constants.

Josiph Krasil'shchik
Independent University of Moscow
Moscow, Russia

Nonlocal aspects of integrability

We present a geometrical framework to construct recursion operators, Hamiltonian and symplectic structures, both local and nonlocal, for nonlinear PDEs of general nature. The scheme is based on the concept of natural coverings over infinitely prolonged PDE. The results are illustrated by by several particular equations (two-dimensional associativity, short capillary-gravity waves, dispersionless Boussinesq).

Martin Kruskal
Dept. Mathematics - Rutgers University
Rutgers (USA)

Surreal Numbers

There is a revolutionary (relatively new) number system discovered by John Conway, which encompasses the usual real numbers as well as vastly more, such as a plethora of infinitely large and infinitely small numbers, in a strikingly simple and natural way. It may be expected to play an increasing role in physics, and to be of comparable interest to the attendees of this workshop.

Anjan Kundu
Saha Institute of Nuclear Physics, Theory Group,
1/AF Bidhan Nagar, Calcutta 700 064, India
Email: anjan@theory.saha.ernet.in

Yang-Baxter algebra and generation of integrable models

Particular Yang-Baxter (YB) algebra underlies each integrable systems. Quantum algebra which introduced a new concept of q-parametric deformation (e.g $sl_q(2)$) was supposed to be the most general YB algebra within certain class. We find however that YB equation allows actually more general YB algebra, we call ancestor algebra, which is giving yet another completely new concept of 'operator-deformed' algebra (together with q-deformation).

After exploiting the nontrivial Hopf algebra structure of this algebra we show three different types of applications of this ancestor algebra and the related ancestor model for generating known models as well as new classes of integrable models.

Though the setup is aimed for quantum integrable systems, at the classical limit we get as well new results concerning classical integrable models.

Integrable nonlinearity generated only from Linear dispersion[3]

Can we generate nonlinear part of an Integrable equation uniquely knowing only its linear dispersive part.

The problem seems to be an impossible one. However motivated by the fact that soliton stability in integrable systems is due to balance between linear disperssion (LD) and Integrable nonlinearity (IN), we are able to derive from the LD part of the equation only, both the Lax operators $U(q, r, \Delta)$ and $V(q, r, \Delta)$, exploiting their scaling properties (Δ is a parameter with dimension of length) and consequently using the flatness condition uniquely generate the IN part.

Moreover we find that the difference in the scaling dimension of the fields q, r is responsible for yielding two different types of IN from the same LD part yielding NLS and DNLS type integrable hierarchies.

A curious conjecture relating the trace of the V-matrix with conserved quantities is put forward.

Evgenii Kuznetsov
L.D. Landau Institute for Theoretical Physics
Moscow, Russia

Vortex line representation

In this paper we give a brief review of the recent results obtained by the author and his co-authors for description of three-dimensional vortical incompressible flows in the hydrodynamic type systems. For such flows we introduce a new mixed Lagrangian-Eulerian description - the so called vortex line representation (VLR), which corresponds to transfer to the curvilinear system of coordinates moving together with vortex lines. Introducing the VLR allows to establish the role of the Cauchy invariants from the point of view of the Hamiltonian description. In particular, these (Lagrangian) invariants, characterizing the property of frozenness of the generalized vorticity into fluids, are shown to represent the infinite (continuous) number of Casimirs for the so-called non-canonical Poisson brackets. The VLR allows to integrate partially the equations of motion, to exclude the infinite degeneracy due to frozenness of the primitive Poisson brackets and to establish in new variables the variational principle. It is shown that the original Euler equations for vortical flows coincides with the equations of motion of a charged *compressible* fluid moving due to a self-consistent electromagnetic field. Transition to the Lagrangian description in a new hydrodynamics is equivalent to the VLR. The VLR, as a mapping, turns out to be compressible that gives a new opportunity for collapse in fluid systems - breaking of vortex lines, resulting in infinite vorticity. It is shown that such process is possible for three-dimensional integrable hydrodynamics with the Hamiltonian $\mathcal{H} = \int |\mathbf{\Omega}| d\mathbf{r}$ where $\mathbf{\Omega}$ is the vorticity. We also discuss some arguments in the favor of existence of such type of collapses for the Euler hydrodynamics, based on the results of some numerics.

Ye. Larionova, O. Egorov and C.O. Weiss
Physikalisch-Technische Bundesanstalt

[3]Poster

Braunschweig, Germany

Dark solitons in passive and active semiconductor microresonators

The properties of dark spatial solitons are investigated in a passive to active quantum-well semiconductor resonator. We find experimentally and confirm theoretically that the solitons show local light amplification. The amount of amplification as well as its parameter dependence changes with pump and with the character of the nonlinearity (absorptive, dispersive, mixed). By pumping, a reduction of soliton sustaining power (up to a factor of 300) is observed, and the influence of pumping on dynamics of solitons and the background on which they exist are investigated. When the pump is around laser threshold, we find the behaviour of solitons in line with the qualitative picture of "laser injection locking", as theoretically treated in [G. Tissoni et al., Proc. SPIE 5452, 335-343 (2004).]

Sergei Leble
Gdansk University of Technology
Gdansk, Poland

Interacting matter-radiation systems: dressing and quantization in generalized Jaynes-Cummings models

Some generalization of matrix dressing and auto-dressing fomalism are presented. It links sets of parameters of linear matrix-differential equations of similar form. The form of the link joins classic Darboux Transformations and a specified form of the gauge one. The results applies to a class of Hamiltonians that contains Jaynes-Cummings and Dicke model as well as its generalizations that include non-rotating wave version. Quantization via zeta-function formalizm is based on the dressing scheme. One-loop corrections to action and mass are evaluated.

Jyh-Hao Lee and O.K. Pashaev
Institute of Mathematics, Academica Sinica
Taipei 11529, Taiwan

Solitons of the Resonant Nonlinear Schrödinger Equation with Nontrivial Boundary Condition by the Hirota Method

We will make a brief report on the resonant nonlinear Schrödinger equations and reaction-diffusion systems and the application[1], [2]. A novel integrable version of the NLS equation namely [1],

$$i\frac{\partial\psi}{\partial t}+\frac{\partial^2\psi}{\partial x^2}+\frac{\Lambda}{4}|\psi|^2\psi = s\frac{1}{|\psi|}\frac{\partial^2|\psi|}{\partial x^2}\psi. \tag{1}$$

has been termed the *resonant nonlinear Schrödinger equation* (RNLS). It can be regarded as a third version of the NLS, intermediate between the defocusing and focusing cases. Even though the RNLS is integrable for arbitrary values of the coefficient s, the critical value $s = 1$ separates two distinct regions of behaviour. Thus, for $s < 1$ the model is reducible to the conventional NLS, (focusing for $\Lambda > 0$ and defocusing for $\Lambda < 0$). However, for $s > 1$ it is not reducible to the usual NLS, but rather to a reaction-diffusion system. In this case, the model exhibits novel solitonic phenomena [1].

The RNLS can be interpreted as an NLS-type equation with an additional 'quantum potential' $U_Q = |\psi|_{xx}/|\psi|$. Very recently it was shown that RNLS naturally appears in the plasma physics, where it describes the propagation of one-dimensional long magnetoacoustic waves in a cold collisionless plasma subject to a transverse magnetic field [3]. A Hirota bilinear representation of the Reaction-Diffusion system with non-zero boundary condition is given.Here one dissipaton and two-dissipaton exact solutions are obtained by Hirota bilinear method and their mutual interactions are studied. Some plots will be also presented here.

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Gaia Lupo

Università di Perugia
Perugia, Italy

A Chauchy problem in nonlinear heat conduction

A Cauchy problem on the semiline for a nonlinear diffusion equation is considered, with a boundary condition corresponding to a prescribed thermal conductivity at the origin. The problem is mapped into a moving boundary problem for the linear heat equation with a Robin-type boundary condition. Such problem is then reduced to a linear integral Volterra equation of II type which admits a unique solution.

Franco Magri

Dipartimento di Matematica ed Applicazioni - Università di Milano Bicocca
Milano, Italy

On the art of choosing the right coordinates in a problem of Mechanics

In the talk I present a new reading of the Kowalevski's paper of 1889, where she linearizes the equations of motion of the Kowalevski's top on the Jacobian of a hyperelliptic curve, and I derive from it a rule that may be of help in the difficult art of choosing the right coordinates in a mechanical problem.

Tina Maiolo[a], **Fabio Della Sala**[b], **Luigi Martina**[a] **and Giulio Soliani**[a]

[a]*Dipartimento di Fisica and Sezione INFN Università di Lecce*
Via per Arnesano, Lecce (Italy)
[b]*NNL and CNR–INFM of Lecce, 73100 Lecce, Italy*

Entanglement of electrons in interacting molecules[4]

Quantum entanglement is a concept commonly used with reference to the existence of certain correlations in quantum systems that have no classical interpretation. It is a useful resource to enhance the mutual information of memory channels or to accelerate some quantum processes as, for example, the factorization in Shor's Algorithm. Moreover, entanglement is a physical observable directly measured by the von Neumann entropy of the system. We have used this concept in order to give a physical meaning to the electron correlation energy, which is not directly observable, since it is defined as the difference between the Hartree–Fock energy and the exact ground state energy of the Schrödinger equation. Thus, for the H_2 molecule, firstly we have calculated the correlation energy and compared with the entanglement, as functions of the nucleus–nucleus separation. The Configuration Interaction method has been used for the wave function calculations. Then, in the same spirit, we have analyzed a dymer of ethylene, changing the relative orientation and distance of the molecules, in order to obtain the configuration corresponding to maximum entanglement.

Sergey Manakov

Landau Institute for Theoretical Physics
Chernogolovka, Russia

Vector fields and integrable nonlinear PDEs in multidimensions. Part II.

We solve the inverse scattering problem for vector fields in arbitrary dimensions, and we use this result to construct the formal solution of the Cauchy problems for the second heavenly equation of Plebanski and for the dispersionless Kadomtsev-Petviashvili equations. Part II.

Stefan C. Mancas and S. Roy Choudhury

University of Central Florida
Orlando, FL

Spatiotemporal structure of dissipative solitons

[4]Poster

in the cubic-quintic Ginzburg-Landau equation

Comprehensive numerical simulations of pulse solutions of the Cubic-Quintic Ginzburg-Landau Equation (CGLE) reveal various intriguing and entirely novel classes of solutions. In particular, there are five new classes of pulse or solitary waves solutions, viz. pulsating, creeping, snake, erupting, and chaotic solitons that are not stationary in time. Rather, they are spatially confined pulse-type structures whose envelopes exhibit complicated temporal dynamics. The numerical simulations also reveal very interesting bifurcations sequences of these pulses as the parameters of the CGLE are varied. In this talk, we focus on the conditions for the occurrence of the five categories of dissipative solitons, as well the dependence of both their shape and their stability on the various parameters of the CGLE, viz. the system parameters. We develop and discuss a variational formalism within which to explore the various classes of dissipative solitons. Given the complex dynamics this formulation is significantly generalized over all earlier approaches. Also, the Euler-Lagrange equations are treated in a completely novel way. Rather than consider the stable fixed points which correspond to the well-known stationary solitons or plain pulses, we use dynamical systems theory to focus on more complex attractors viz. periodic, quasiperiodic, and chaotic ones. This approach, which has been partially explored, fails only to address the fifth class of dissipative solitons, viz. the exploding or erupting solitons. Results will be presented for the pulsating and snake soliton cases.

Margarita A. Man'ko

P. N. Lebedev Physical Institute, Moscow
Email: mmanko@sci.lebedev.ru

Uncertainty relations for tomographic entropy of solitons

The notion of tomographic entropy for functions depending on continuous variables both for one and many degrees of freedom is discussed. The function called tomographic-entropy uncertainty is introduced as a characteristic of solitons. The known in quantum mechanics and signal analysis entropic uncertainty relations are generalized and formulated as positivity property of the entropic uncertainty function. The generic construction is illustrated by an example of the soliton solution of Gross-Pitaevskii equation for Bose-Einstein condensate.

Giuseppe Marmo

Dipartimento di Scienza Fisiche - Università di Napoli Federico II
Napoli, Italy

Geometry of Quantum Systems. Density States and Entanglement.

There are many reasons to achieve a geometrization of Quantum Mechanics. We will address the problem of the geometrization of the space of density states. In particular we shall consider canonical Poisson and Riemann-Jordan tensor,along with the corresponding foliation into Kaehler submanifolds.We will show that the space of density states is naturally a stratified manifols with the stratification induced by the rank of the state.We shall comment on the behaviour of these geometrical structures for composite systems and comment on their use for the study of separability and entanglement.

Luigi Martina

Dipartimento di Fisica - Università di Lecce
via per Arnesano, I73100 Lecce, Italy

Semiclassical dynamics of wave - packets in generalized mechanics

Semiclassical wave packets for electrons in crystals, subject to external electromagnetic field, satisfy Hamiltonian equations. In (2+1)-dimensions and in the limit of uniform fields, the symmetry group resulting is a two-folded Galilei algebra, incorporating an "exotic" central charge. It has the physical meaning of the Berry-phase curvature. In the Hamiltonian scheme, we discuss possible deformations of that algebra and its physical meaning. Generalization to (3+2) dimensions are discussed.

Eugene M. Maslov

Pushkov Inst. of Terrestrial Magnetism
Troitsk, Russia

Breather resonant phaselocking by external perturbation

We consider the sine-Gordon equation perturbed by a small oscillating term of a general form with slowly varying frequency. Using the perturbation theory based on the inverse scattering transform [V.I. Karpman, E.M. Maslov, V.V. Solov'ev, Sov. Phys.-JETP 57 (1983) 167] we investigate the effect of this perturbation on the isolated breather. We obtain the system of equations describing evolution of the breather's parameters and analyze it by the generalized Krylov-Bogoliubov asymptotic expansion method. As a result, we have found that, under certain conditions, the amplitude of the breather begins to grow, while its frequency follows the varying frequency of the external perturbation. This phenomenon (named phaselocking) is resonant by its nature and provides the efficient transfer of the energy from the perturbation to the breather. It can occur only for certain relations between parameters of the perturbation and initial data. The critical value of the ratio of the perturbation frequency variation rate to the perturbation amplitude is calculated, below of which the domains of phaselocking appear in an initial data space. The results are in agreement with numerical simulation.

Vladimir Matveev

Universite de Bourgogne
Dijon, France

Cylindrical KP equation revisited

We present a review of some results concerning the integration of axially symetric KP equation also known as Johnson equation including the description of the solutions depending on any number of functional parameters, rational solutions and the finite gap solutions.

Elena B. Medina
Universidad de Cadiz
Cadiz, Spain

Classification of Integrable Deformations of Algebraic Curves

We study some problems arising in the analysis of integral deformations of hydrodynamic type of algebraic curves based on Lenard relations. The general solution of the consistency condition for the degrees of the potentials is obtained, and the expressions for the solutions of the corresponding Lenard relations are provided.

Oleg Mokhov
Centre for Nonlinear Studies, L.D.Landau Institute for Theoretical Physics, Russian Academy of Sciences
Kosygina 2, Moscow, 117940, Russia
Dept of Geometry and Topology, Fac of Mechanics and Mathematics, M.V.Lomonosov Moscow State University
Moscow, 119992, Russia

Theory of Submanifolds, Associativity Equations in 2D Topological Quantum Field Theories, and Frobenius Manifolds

Theorem 1. *Each N-dimensional Frobenius manifold can locally be represented as a flat N-dimensional submanifold with flat normal bundle in a $2N$-dimensional pseudo-Euclidean space. This submanifold is uniquely determined up to motions.*

Consider an arbitrary N-dimensional flat submanifold with flat normal bundle in an $(N+L)$-dimensional pseudo-Euclidean space. Let $g_{ij}(u)du^i du^j$ be the first fundamental form of this flat submanifold, where $g_{ij}(u)$ is a flat metric. Then the following corollary of the theory of submanifolds and the Bonnet theorem is true. There locally exist functions $\Phi_n(u)$, $1 \leq n \leq L$, such that the second fundamental forms of the submanifold in question are given by $\Omega_n(u) = \nabla_i \nabla_j \Phi_n du^i du^j$, where ∇_i is the covariant differentiation operator defined by the Levi-Civita connection of the metric $g_{ij}(u)$. The functions $\Phi_n(u)$, $1 \leq n \leq L$, satisfy the Ricci equations and the Gauss equations for submanifolds. (The Peterson–Codazzi–Mainardi equations are satisfied automatically in this case). Each solution of the system of the Ricci and Gauss equations defines a unique (up to motions) N-dimensional submanifold in an $(N+L)$-dimensional pseudo-Euclidean space with flat normal bundle, first fundamental form $g_{ij}(u)du^i du^j$, and second fundamental forms $\Omega_n(u)$.

Theorem 2. *If $L = N$ and there exists a function* (*a potential*) $\Phi(u)$ *such that* $\Phi_n(u) = \partial\Phi/\partial u^n$ *for all n, then the corresponding Ricci and Gauss equations coincide in flat coordinates of the metric* $g_{ij}(u)$ (*for a suitable $2N$-dimensional pseudo-Euclidean space*). *Moreover, in this case both the Ricci and Gauss* *equations coincide with the associativity equations in two-dimensional topological quantum field theories for the potential* $\Phi(u)$ (*the Witten–Dijkgraaf–Verlinde–Verlinde and Dubrovin equations*).

Each solution $\Phi(u)$ of the associativity equations (with the corresponding constant metric η_{ij}) defines a unique (up to motions) N-dimensional submanifold of a $2N$-dimensional pseudo-Euclidean space with flat normal bundle, first fundamental form $\eta_{ij} du^i du^j$, and second fundamental forms

$$\omega_n(u) = (\partial^3 \Phi / \partial u^n \partial u^i \partial u^j) du^i du^j \ .$$

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Antonio Moro

Department of Mathematical Sciences, Loughborough University,
Loughborough, LE11 3TU, UK

High frequency integrable regimes in nonlocal nonlinear optics

Recently, it has been shown that nonocality plays a crucial role in the propagation of stable light beams in 3D. We evaluate the high frequency limit of nonlocal nonlinear Schroedinger equation for a light beam propagating in a Cole-Cole medium. In this regime nonlocal perturbations can be seen as a class of phase deformation along one direction. The equations of the phase for different degrees of nonlocality are described by the dispersionless Veselov-Novikov (dVN) hierarchy. The dVN hierarchy is integrable by the quasiclassical Dbar-dressing method and by the hydrodynamic reductions method. Finally, we demonstrate that for a general form of nonlinearity there exist selfguided light beams and discuss singular wavefronts and nonlocal perturbations.

Folkert Müller-Hoissen

Max-Planck-Inst. for Dynamics & S. O.
Goettingen, Germany

From Gelfand-Dickey-type hierarchies to functional representations

The Zakharov-Shabat (zero curvature) conditions of a linear system of the form $\partial_{t_n} W = L_n W$, $n = 1, 2, \ldots$, can be expressed as a *discrete* zero curvature equation:

$$E(\lambda_2)_{-[\lambda_1]} \, E(\lambda_1) = E(\lambda_1)_{-[\lambda_2]} \, E(\lambda_2)$$

with a formal power series $E(\lambda)$ in an indeterminate λ, and $E_{-[\lambda]}(t_1, t_2, \ldots) := E(t_1 - \lambda, t_2 - \lambda^2/2, \ldots)$ (which is a 'Miwa shift').

If $L_n = (L^n)_+$ with a projection $(\,)_+$ to a subalgebra of the algebra in which the 'Lax operator' L lives, we talk about a 'Gelfand-Dickey-type hierarchy', for which there is a simple recursion formula which determines the coefficients of $E(\lambda)$ in terms of L.

Generalizing examples like AKNS and (discrete) KP, we consider in particular the case where L is a formal series in some (formally) invertible operator D with certain properties. Using the aforementioned result, there is a systematic way to obtain functional representations of GD-type hierarchies, i.e., expressions of the hierarchy equations as functional equations formulated directly in terms of the relevant dependent variables. The hierarchies reached by this method also include, via associative deformations of the usual matrix product, derivative NLS hierarchies.

The formalism still works in the case where the dependent variables of the hierarchy equations are generalized to matrices of functions, or even matrices with entries in a (typically noncommutative) associative algebra. This covers cases of systems of coupled equations (which, in turn, are sources of integrable equations via reductions) and is, in particular, a necessary step in the 'operator approach' to solutions of soliton equations (cf. V. Marchenko, Nonlinear Equations and Operator Algebras (1988), and, for example, B. Carl and C. Schiebold, Nonlinearity **12** (1999) 333).

We also demonstrate that Bäcklund transformations are easily obtained in this framework. All this is jointly with A. Dimakis (nlin.SI/0603018, nlin.SI/0603048, and work in progress).

Victor Novokshenov

Institute of Mathematics Ufa Scientific Center Russian Academy of Sciences
Ufa, Russia

Autoresonant excitation and control of nonlinear waves in the integrable systems

Autoresonance is a powerful technique for controlling the amplitude of nonlinear modes. It is a robust method because, over a broad range of parameters, it does not depend on the details of the system, nor on the amplitude or exact range of of the sweeping drive.

Autoresonance is usually associated with single frequency mode excitations due to the synchronization and phase lock of various nonlinear modes with the driving force. Despite this we propose a model of multifrequency autoresonance which occur in completely integrable systems. This phenomenon is due to a number stable invariant tori governed by integrals of motion of the integrable system. The basic autoresonant effect of phase locking appears here as Whitham deformations of the invariant tori. This provides also a possibility to transfer a certain initial -periodic motion to a given -periodic motion as a final state.

Maria Clara Nucci

Dipartimento di Matematica e Informatica
Università di Perugia, Italy

Jacobi last multiplier, Lie symmetries, and hidden linearity: what's up

A novel application of Jacobi last multiplier in order to find Lie symmetries of ordinary differential equations algorithmically is presented. For example, it is demonstrated how to find Lie symmetries of equation $y'' = \frac{y'^2}{y} + f'(x)y^{p+1} + pf(x)y'y^p$ for any $f = f(x)$. Also the hidden linearity of certain nonlinear equations of relevance in Physics may be unveiled thanks to Lie symmetries. Examples will be provided.

Alexander Orlov

Oceanology Institute RAS
Moscow, Russia

Fermionic tau function and random processes

We shall show that the fermionic construction of tau function suggested by Kyoto school provides a natural link between integrable systems and random processes. In particular well-known processes like ASEP, polynuclear growth, vicious walkers may be described in this way. We shall show that the Wick's theorem yields determinant formulae known in descriptions of random processes as Gessel-Vientot formulae.

Carmine Ortix and Matteo Beccaria

Dipartimento di Fisica dell'Università and Sezione INFN
73100 Lecce, Italy

Anomalous dimensions of $N = 4$ super Yang-Mills from the Hubbard model[5]

The dilatation operator in the $N = 4$ super Yang-Mills (SYM) theory measures the anomalous dimensions of single trace operators. At one-loop, the dilatation operator can be identified with the Hamiltonian of the integrable Heisenberg XXX spin $1/2$ lattice model [J. A. Minahan and K. Zarembo, *JHEP* **0303**, 013 (2003)]. As the loop order increases, the dilatation operator can be identified with the Hamiltonian of a spin $1/2$ chain with an increasing range of the spin interaction [N. Beisert, V. Dippel and M. Staudacher, *JHEP* **0407**,075 (2004)]. Since the spin model is nothing but the strong coupling expansion of an itinerant fermion model, it has been proposed [A. Rej, D. Serban and M. Staudacher, *JHEP* **0603**, 018 (2006)] the one-dimensional Hubbard model hamiltonian as the correct all loops dilatation operator of the $N = 4$ SYM. The analysis of the Hubbard states can be performed via the Lieb-Wu (LW) equations [E. H. Lieb and F. Y. Wu, *Phys. Rev. Lett.* **20**, 1445 (1968)] which encode integrability. In the thermodynamic limit, the Bethe roots of LW equations accumulate on a discrete set of non trivial curves in the complex plane. The integral equations for the root density are quite difficult due to the unknown shape (and numbers) of the contours. For finite spin chains, specific states of the Hubbard model are analyzed with the numerical exploration of the complete LW equations. This allows to investigate the weak-strong coupling behavior of anomalous dimensions.

G. L. Alfimov[1], V. V. Konotop[2], P. Pacciani[2]

[5]Poster

[1]*Moscow Institute of Electronic Engineering, Zelenograd, Moscow, 124498, Russia*
[2]*Centro de Física Teórica e Computacional and Departamento de Física, Faculdade de Ciências Universidade de Lisboa, Portugal*

Localized solutions of the quintic nonlinear Schrödinger equation with a periodic potential

We consider one-dimensional quintic nonlinear Schrödinger equation with a periodic potential. In the case of attractive nonlinearity we deduce sufficient condition for collapse. We show that there exist spatially localized solutions with arbitrarily large number of particles. We study such solutions in the semi-infinite gap (attractive case) and in the first gap (attractive and repulsive cases), and show that a nonzero minimum value of the number of particles is necessary for a mode to be created. In the limit of large negative frequencies (attractive case) we observe "quantization" of the number of particles of the stationary modes. Such solutions can be interpreted as coupled "Townes" solitons and appear to be stable. The modes in the first gap have numbers of particles infinitely growing with frequencies approaching one of the gap edges, what is explained by the power decay of the modes. Gap solitons are found to be unstable.

Marcella Palese
Department of Mathematics - University of Torino
Via C. Alberto 10 I-10123 TORINO - ITALY

Towers with infinite-dimensional skeletons and prolongation structures[6]

We propose an interpretation of prolongation structures as infinitesimal skeletons of truncated towers associated with admissible Bäcklund-Cartan connections.

Rajesh Parwani
National University of Singapore
Singapore

Solutions of an information-theoretic motivated nonlinear Schrodinger equation

A nonlinear generalisation of Schrodinger's equation had previously been obtained using information-theoretic arguments. The equation is equivalent to one with higher derivative nonlinearities at all orders. Here I summarise some perturbative and nonperturbative properties of the equation in 1+1 dimensions. An intriguing feature of some nonperturbative solutions is their induced periodicity and infinite degeneracy: for a given energy, there exists a very large arbitrariness in the normalisable wavefunctions.

Oktay K. Pashaev and Zeynep Nilhan Gurkan
Department of Mathematics, Izmir Institute of Technology,

[6]Poster

Urla-Izmir, 35430, Turkey
oktaypashaev@iyte.edu.tr, nilhangurkan@iyte.edu.tr

Galilean Group, Vortex Generation Techniques and Complex Burgers Hierarchy

The Abelian Chern-Simons Gauge Field Theory in 2+1 dimensions and its relation with Complex Burgers Hierarchy is considered. It is shown that the relation between complex potential and the complex gauge (velocity) field has meaning of the analytic Cole-Hopf transformation, linearizing the Burgers Hierarchy in terms of the analytic Schrodinger Hierarchy. Then the motion of planar vortices in Chern-Simons theory, appearing as pole singularities of the gauge field, corresponds to motion of zeroes of the analytic Schrodinger Hierarchy. Using boost transformations of the complex Galilean group of the hierarchy, the vortex generation techniques is proposed. It allows us to construct a rich set of exact solutions, describing integrable dynamics of planar vortices and vortex lattices in terms of the generalized Hermite polynomials. The results are applied to the holomorphic reduction of the Ishimori model and the corresponding hierarchy, describing dynamics of magnetic vortices and corresponding lattices in terms of complexified Calogero-Moser models.

Maxim Pavlov
FIAN RAN
Moscow, Russia

New Hamiltonian formalism for hydrodynamic type systems

At the beginning of 80-ties the local Hamiltonian formalism (associated with a flat Riemann metric) for hydrodynamic type systems was constructed by B.A. Dubrovin and S.P. Novikov. At the end of 80-ties this Hamiltonian formalism was extended to nonlocal case (associated with a flat normal bundle). We consider new local and nonlocal Hamiltonian formalism (also associated with the Riemann geometry). Plenty well known hydrodynamic type systems are presented. Their multi-Hamiltonian structures are described.

Marco Boiti[a], Flora Pempinelli[a], Andrei Pogrebkov[b]
[a]*Dipartimento di Fisica - Università di Lecce*
Lecce, Italy
[b] *Steklov Mathematical Institute, Moscow, Russia*

Scattering transform for the nonstationary Schrödinger operator with a bidimensionally perturbed N-soliton potential

In the framework of the extended resolvent approach the direct and inverse scattering problems for the nonstationary Schrödinger equation with a potential being a perturbation of the N-soliton potential by means of a generic bidimensional smooth function decaying at large spaces are introduced and investigated. The initial value problem of the Kadomtsev-Petviashvili I equation for a solution describing N wave solitons on a generic smooth decaying background is then linearized, giving the time evolution of the spectral data.

Barbara Prinari, Mark J. Ablowitz, Gino Biondini
Dipartimento di Fisica and Sezione INFN - Università di Lecce
Lecce, Italy

Inverse Scattering Transform for vector defocusing Nonlinear Schrödinger equation with nonvanishing boundary conditions

We construct the inverse scattering transform for the defocusing vector nonlinear Schrodinger equation with non-vanishing boundary conditions at space infinity. We discuss the solitonic sector, which includes the vector generalizations of the usual dark and gray solitons of the scalar defocusing nonlinear Schrodinger, as well as dark-bright soliton pairs.

A.P. Protogenov[1] and V. A. Verbus[2]
[1]*Institute of Applied Physics of the RAS,*
[2]*Institute for Physics of Microstructures of the RAS,*
Nizhny Novgorod, Russia

Decomposition of variables and duality in non-Abelian models

We will discuss the problems of the inhomogeneous current states (PV) in the systems, which are characterized by spatial separation of the spin and charge order. In particular, we will consider the consequences of Iwasawa factorization, $f = g \cdot h$, of spin $(g \in G)$ and charge $(h \in H)$ degrees of freedom of the order parameter $f \in F = G/H$ in non-Abelian models. Our attention will be focused on the comparison of dual pictures for the spinor Ginzburg-Landau model and Yang-Mills theory in the case of the cosets $F_1 = SU(2)/U(1) = CP^1 \cong S^2$ (see (FN), (BF)), $F = SU(3)/U(2) = SU(3)/(SU(2) \times U(1)) = CP^2$, as well as in the case of the flag space $F_2 = SU(3)/(U(1) \times U(1))$.
This work was partially supported by the RFBR (Grant No. 06-02-16561) and by the programme RNP 2.1.1. (Grant No. 2369).

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Colin Rogers
School of Mathematics - University of NSW
Sidney, Australia

On a novel class of nonlinear elastic materials. Soliton connections

A class of infinitesimal Bäcklund transformations is used to construct model constitutive laws for which the nonlinear system describing uniaxial wave propagation may be reduced to a canonical form associated with Hooke's law in an asymptotic limit. The stress-strain laws are given parametrically in terms of elliptic functions. Importantly, these models can allow a change in concavity in the stress-strain law. Such behaviour can occur, for instance, in the unloading regimes of a superelastic nickel-titanium material (nitinol) with widespread application in medical devices such as stents.

Vladimir Rosenhaus

California State University
Chico, CA 95929, USA

On infinite conservation laws for systems with infinite symmetries

We consider partial differential equations of a variational problem admitting infinite-dimensional Lie symmetry algebras parametrized by arbitrary functions of dependent variables and their derivatives.

Infinite symmetry algebras with arbitrary functions of independent variables were shown to lead to a finite number of local conservation laws, where each conservation law is determined by a specific form of boundary conditions (V.Rosenhaus "Infinite Symmetries and Conservation Laws", J. Math. Phys. 43, 6129 (2002)).

The present work is an extension of this approach for Lagrangian differential equations whose symmetry algebras contain arbitrary functions of dependent variables and their derivatives. We will show that unlike differential systems with arbitrary functions of independent variables, these equations possess infinite sets of local conservation laws. We will discuss conditions necessary for the existence of infinite number of conservation laws and give examples of such systems.

Giovanna Ruggeri

Dipartimento di Fisica and Sezione INFN Università di Lecce
Via per Arnesano, Lecce (Italy)

Quantum Gaussian Memory Channels[7]

We are interested in the understanding of the classical capacities of noisy quantum channels and, especially, in the exploitation of quantum effects, as entanglement, for information processing and communication. In fact, in information theory, a key open question concerns what happens when entangled states are used as signals in a noisy environment.

Many interesting results have been obtained for the discrete case. It has been proved that, for memoryless channel, in which noise acts independently on each symbol, encoding of messages by means of entangled states instead of product states is not advantageous, since it does not improve the capacity of such a channel.

The scenario changes when the channel is memory, i.e. the noise acting on consecutive uses is partially correlated. For several kinds of memory channels, it has been demonstrated that entanglement can enhance the amount of classical information which can be reliably transmitted through these channels. This is the fundamental reason of the importance of quantum memory channels in information theory.

[7]Poster

However, we deal with continuous variables. The main motivation of this choice originates in a more practical observation: efficient preparation, manipulation and measurement of quantum states are achievable in quantum optics utilizing continuous quadrature amplitudes of the quantized electromagnetic field. Therefore, continuous-variable entanglement can be efficiently produced using squeezed light.

Our main result, derived by the analysis of an additive Gaussian channel and a lossy bosonic memory channel, is that entanglement due to squeezed light is a useful resource to improve the quantity of classical information reliably transferred in a environment with correlated noise.

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Paolo Santini

Dipartimento di Fisica - Università "La Sapienza"
Roma, Italy

Vector fields and integrable nonlinear PDEs in multidimensions. Part I.

We solve the inverse scattering problem for vector fields in arbitrary dimensions, and we use this result to construct the formal solution of the Cauchy problems for the second heavenly equation of Plebanski and for the dispersionless Kadomtsev-Petviashvili equations. Part I.

Giuseppe Scolarici

Dipartimento di Fisica - Università di Lecce
via per Arnesano, I73100 Lecce, Italy

Completely positive quaternionic maps

We show that the complex projection of completely positive quaternionic maps of quaternionic density matrices is a complex positive positive map. In order to illustrate this result we discuss the complex projection of a one-parameter quaternionic unitary dynamics of a spin-$\frac{1}{2}$ quantum system.

M. Segev, B. Freedman, T. Schwartz, G. Bartal, S. Fishman [a]
R. Lifhshitz[b], J. Fleischer[c], D. Christodoulides[d]
[a] *Physics Department, Technion - Israel Institute of Technology, Haifa 32000, Israel*
[b] *Tel Aviv University, Tel Aviv, Isreael*
[c] *Princeton University, Princeton USA*
[d] *CREOL, University of Central Florida, Orlando, Florida 32816, USA*

Nonlinear waves and solitons in photonic quasi-crystals, Anderson localization, etc.

We will describe the recent theoretical and experimental progress on nonlinear waves in periodic photonic media, including nonlinear quasi-crystals, Anderson Localization of light, etc

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Matteo Sommacal
Laboratoire J.-L. Lions, Universit P.-M. Curie,
Paris VI 175 rue du Chevaleret 75013, Paris (France)

The Transition from Regular to Irregular Motions, Explained as Travel on Riemann Surfaces[8]

We introduce and discuss a simple Hamiltonian dynamical system, interpretable as a three-body problem in the (complex) plane and providing the prototype of a mechanism explaining the transition from regular to irregular motions as travel on Riemann surfaces. The interest of this phenomenology - illustrating the onset in a deterministic context of irregular motions - is underlined by its generality, suggesting its eventual relevance to understand natural phenomena and experimental investigations. The results reported herein have been obtained in collaboration with F. Calogero (Università di Roma "La Sapienz"), P. Santini (Università di Roma "La Sapienza") e D. Gomez-Ullate (Universitat Politecnica de Catalunya).

Sandro Sozzo
Dipartimento di Fisica and Sezione INFN Università di Lecce
Via per Arnesano, Lecce (Italy)

On the physical interpretation of partial traces: two alternative approaches[9]

Mixed states, or *mixtures*, are usually introduced in physics as expressing our ignorance about the actual state of a physical system (or *entity*) and are represented in the standard formulation of quantum mechanics (QM) by density operators. Such operators also appear whenever one considers a (pure) entangled state of a compound entity S made up by some subentities $S_1, S_2, \ldots$ and performs partial traces on the density operator (actually, a projection operator) representing it. Yet, the density operators obtained in this way cannot be interpreted as representing true mixed states of the subentities, since the probabilities appearing in the convex sum expressing them are nonepistemic (hence some authors write that they represent *improper mixtures*

[8] Poster
[9] Poster

[See, *e.g.*, [1]]). The problem of attributing states to the subentities of a compound physical entity in QM is known as the *subentity problem* and is controversial in standard QM. In particular, the *Brussels approach* and the *Lecce approach*, though sharing a number of common features, propound different solutions of this problem, as follows.

(i) Improper mixtures are considered pure states in the Brussels approach [see *e.g.*, [2]] though they are represented by density operators. Hence, the set of pure states must be enlarged, which implies some kind of *nonlinearity of QM* (more precisely, the validity of the superposition principle is limited in QM).

(ii) Nonepistemic probabilities disappear in the Lecce approach that provides an *objective* interpretation of QM according to which all properties of a physical entity can be considered as possessed or not possessed by the entity (*Semantic Realism* [see *e.g.* [3]].) Improper mixtures become true mixed states, and QM is considered as an incomplete theory, which needs not be modified but can be embedded within a broader (possibly nonlinear) theory.

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Martin Speight

University of Leeds
Leeds, UK

Magnetic bubble refraction in inhomogeneous antiferromagnets

We will show that the spin dynamics of an inhomogeneously doped 2 dimensional isotropic antiferromagnet is described by the O(3) sigma model on a curved space. This model supports topological solitons, interpreted as magnetic bubbles, whose trajectories are remarkably similar to light rays in geometric optics. In particular, we will argue that they scatter off doping domain walls in accordance with Snell's law of refraction, allowing the possibility to construct, by controlled doping, bubble analogues of lenses and fibre optic cables.

Elaine Spiller

Department of Mathematics, State University of New York at Buffalo

Buffalo, NY - USA

Rare events in dispersion-managed nonlinear lightwave systems

The nonlinear Schrödinger equation (NLS) with a periodic, varying dispersion coefficient models the dynamics of light in dispersion-managed communication systems and mode-locked lasers. The dispersion-managed nonlinear Schrödinger equation (DMNLS) is an averaged version of NLS which restores some symmetries that are lost in NLS when the dispersion coefficient is not constant. I will discuss these symmetries, the corresponding conservation laws, and modes of the linearized DMNLS. I will also discuss how these linearized modes can be utilized to guide importance-sampled Monte-Carlo simulations of rare events in dispersion-managed lightwave systems subject to noise. This study is pertinent because the performance of lightwave systems is limited by the occurrence of rare events, i.e., noise-induced errors.

Piergiulio Tempesta
Scuola Internazionale Superiore di Studi Avanzati (SISSA)
via Beirut 4, Trieste, Italy

Frobenius Manifolds and integrable hierarchies of Toda type
joint work with B. Dubrovin

We study hierarchies of differential-difference equations of Toda type in the geometric framework of Frobenius manifolds. We prove that these hierarchies possess a Lie algebra of linear operators isomorphic to half of the Virasoro algebra and acting as Lie symmetries on the tau function of the hierarchies.

In analogy with the classical work of Kontsevich-Witten on the KdV hierarchy and the topology of moduli spaces of stable algebraic curves, we propose a new connection between Toda hierarchies and the theory of Gromow-Witten invariants for the weighted projective space $CP1(a, b)$.

David Trubatch
Dept. of Math. Sciences - United States Military Academy, West Point,
NY, USA

Recurrence in the KdV equation?

The repeated near-recurrence of the initial state observed in numerical simulations of the periodic Korteweg-de Vries (KdV) equation has, historically, been explained as a consequence of the "elastic" soliton interaction of KdV. However, simulations with higher-order methods and/or smaller grid sizes do not manifest the well-known recurrence, despite the fact that these schemes do capture the individual solitons. These results suggest that the recurrence phenomenon observed in some simulations is not completely explained by the whole-line (soliton) dynamics of KdV. Additionally, some discretizations of KdV suffer from finite-time blow up if the l^2 norm of the solution is not preserved by the scheme. The blow up is a result of a nonlinear aliasing instability not present in schemes that preserve the l^2 norm.

Caroline Verhoeven
Vrije Universiteit Brussel
Brussel, Belgium

Spiderweb solutions for the CKP equation

In [1], the authors describe a method to analyse the behaviour of multisoliton solutions on a finite domain for $2+1$-dimensional equations and apply it on the so-called cKP equation. They show how the soliton equations of this equation can give rise to rich patterns, namely spider-web solutions. This type of interactions seems not only to exist for the cKP equation but also for the KP equation [2]. We here give an argument that those interactions of solitons is common between (2+1)-dimensional equations, as we extend the method of [1] to the CKP equation, which possesses soliton solutions of a different type as those of KP, and show that we also find spider-web solutions.

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Gaetano Vilasi
Dipartimento di Fisica Università di Salerno
Salerno, Italy

Spin-1 gravitational waves

Solutions of Einstein field equations, for metrics admitting a non Abelian 2-dimensional Lie algebra of Killing fields, are explicitly described. When the distribution orthogonal to the orbits is integrable and the metric is not degenerate along the orbits, these solutions are parameterized either by solutions of a transcendental equation (the tortoise equation), or by solutions of a linear second order partial differential equation in two independent variables. Metrics, corresponding to solutions of the tortoise equation, are characterized as those that admit a 3-dimensional Lie algebra of Killing fields with 2-dimensional leaves. It is shown that the remaining metrics, corresponding to the case in which the commutator of the two Killing fields is isotropic, represent nonlinear gravitational impulsive waves obeying to a nonlinear superposition law. The energy and the polarization of this family of waves are explicitly evaluated; it is shown that they have spin-1 and their sources are also described.

Alexander Vinogradov

Salerno University and INFN
Salerno, Italy

Solution singularities and the reconstruction problem

There will be explained what geometric singularities of solutions of (nonlinear) PDEs are, their simplest classification and the associated equations that describe their behaviour. The reconstruction problem is to reconstruct the original equation by observing the behaviour of singularities of its solution. This problem will be discussed and its solution will be given for some classes of equations.

G. Manno, F. Oliveri, Raffaele Vitolo
Dipartimento di Matematica Università di Lecce
Lecce, Italy

On differential equations characterized by their Lie point symmetries[10]

We consider the problem of finding under which conditions a given PDE is uniquely determined by the Lie algebra of its infinitesimal point symmetries. More precisely, if r is the order and l is the dimension of the given PDE (as a submanifold of the r-th order jet space) we find conditions under which the equation is the only one of order r and dimension l admitting at least the same point symmetries. We show that in some cases this property is related to the existence of many invariant solutions. We provide many examples of such equations: Monge-Ampere equations of various types, minimal submanifolds equations, geodesic equations.

[10]Poster

Carl Otto Weiss

Physikalisch-Technische Bundesanstalt
Braunschweig, Germany

Spatial solitons in nonlinear optical resonators

The existence and properties of spatial solitons in laser-type and wavemixing-type resonators were studied experimentally.

We observe writeable/erasable, moveable, moving, and "self-replicating" solitons. Solitons can interact in velocity space and have quantized velocities. Large number of solitons can coexist as required for optical information processing.

Appropriate for applications are spatial solitons in semiconductor microcavities. We show the existence of bright and dark solitons, and their (coherent and incoherent) switching. The sustaining powers for such solitons are reduced to the microwatt range, favourable for technical systems, using population inversion (created by optical pumping.)

Hexagonal space-filling patterns disintegrate, with increasing nonlinearity, into arrangements of loosely bound solitons.

This formation of "building blocks" out of rigid patterns results in a high degree of spatial multistability. It illustrates how nonlinearity makes a system "autonomous", i.e. allows it to, independently from boundaries or spatial parameter distributions, choose its spatial structures. Brain-like processing schemes like pattern recognition suggest themselves.

Pavel Winternitz

Centre de Recherches mathematiques - Universite de Montreal
Montreal, Canada

Symmetry preserving discretization of differential equations with applications to computing

The talk will consist of two parts. In the "group theory" part, we will show how to discretize an ordinary differential equation and obtain difference schemes invariant under the same Lie point symmetry group. In the "numerical" part, we will consider several second and third order nonlinear differential equations and show that the invariant schemes provide a much higher precision than standard schemes, without significantly increasing the the complexity of the computations. Moreover, we will show on an example how invariant schemes can provide solutions in the neighbourhood of singularities and beyond singularities, where standard numerical methods fail.

Ekkehart Winterroth

Department of Mathematics - University of Torino
Via C. Alberto 10 I-10123 TORINO - ITALY

On the relation between the Hessian and the Jacobi morphisms in gauge-natural field theories[11]

We generalize a classical result due to Goldschmidt and Sternberg relating the Hessian with the Jacobi morphism for first order field theories to higher order gauge-natural field theories. In particular we define a generalized gauge-natural Jacobi morphism where the variation vector fields are Lie derivatives of sections of the gauge-natural bundle with respect to gauge-natural lifts of infinitesimal principal automorphisms. As a consequence of the Second Noether Theorem, there exists a covariant conserved current associated with the Lagrangian obtained by contracting the Euler–Lagrange morphism with a suitable defined gauge-natural Jacobi vector field. We show that Hamilton equations for the corresponding Hamiltonian connection - equivalent to the kernel of the Euler lagrange morphism of the above mentioned Lagrangian - are identically satisfied and can be suitably interpreted as generalized Bergmann-Bianchi identities.

Vladimir Zakharov
LD Landau Theoret Phys Inst, Moscow 119334, Russia
Univ Arizona, Tucson, AZ 85721, USA

Spaces of Diagonal Curvature, Spaces of Flat Connection and Problem of N-Orthogonal Coordinate Systems

Several classical and new problems of Differential Geometry can be solved by the Dressing Method.

Yair Zarmi
Ben Gurion University of the Negev
Midreshet Ben-Gurion, 84990, Israel

Perturbed Evolution Equation and Asymptotic Integrability

The equations governing complex dynamical systems are often approximated by integrable evolution equations. When the small terms that have been omitted in the derivation of the latter are reinstated, the perturbed equations lose integrability. The perturbation contains or generates terms that are obstacles to asymptotic integrability. (Obstacles do not emerge only for a single- soliton or single-front zero-order solution.) Two ways to account for obstacles have been presented in the literature:

(i) Modify the zero-order approximation, u_0, by the effect of the obstacles. The gain is that the higher-order terms in the expansion of the solution are bounded automatically. The loss is that the Normal Form (the evolution equation for u_0) is not asymptotically integrable, and u_0 does not have the closed-form expression and the simple multiple-wave nature of the solution of the unperturbed equation.

(ii) Allow for non-polynomial corrections in higher-orders in the expansion to account for the obstacles. The gain is that the Normal Form is asymptotically integrable, and u_0 has the closed-form expression and the simple multiple-wave nature of the solution of the unperturbed equation. The loss is that the higher-order corrections in the expansion of the solution contain terms, for which there are no closed-form expressions.

[11] Poster

Moreover, to guarantee boundedness of these terms, the freedom in the expansion must be exploited in a specific manner.

A new approach, which avoids the emergence of obstacles to integrability, will be presented. Exploiting the freedom that is inherent in the original derivation of the evolution equation, the perturbed equation can be made free of obstacles to integrability order-by-order. The example of the first-order obstacle encountered in the perturbed Burgers equation will be presented.

Index

Ablowitz MJ, 1
Alekseev G, 2
Atanasov V, 2

Bakirtas I, 2
Biondini G, 3
Blaszak M, 3
Bogdanov L, 3
Boiti M, 4, 31
Boscolo S, 4

Calogero F, 4
Caputo J-G, 5
Carroll R, 6
Chang JH, 6
Choudhury R, 6
Choustova O, 7
Conte R, 7

De Lillo S, 8
Dvoeglazov VV, 8

Ferapontov E, 9
Fokas A, 9
Friedland L, 9

Gandarias ML, 10
Garcia Estévez P, 11
Gerdjikov V, 12
Ghosh S, 12
Grahovski GG, 12
Grinevich P, 13
Grundland AG, 13

Habibullin I TY, 13
Harnad J, 14

Ilan B, 14

Kalyakin LA, 14
Kaup DJ, 15
Khomeriki R, 15
Khrennikov A, 16
Khusnutdinova K, 16
Kiselev AV, 18
Kiselev O, 18
Konopelchenko BG, 18
Krasil'shchik J, 19
Kruskal M, 19
Kundu A, 19
Kuznetsov E, 20

Larionova E, 20
Leble S, 21
Lee J-H, 21
Lupo G, 22

Müller-Hoissen F, 27
Magri F, 22
Maiolo T, 23
Man'ko MA, 24
Manakov S, 23
Mancas SC, 23
Marmo G, 24
Maslov EM, 25
Matveev V, 25
Medina EB, 26
Mokhov O, 26
Moro A, 27

Novokshenov V, 28
Nucci MC, 28

Orlov A, 29
Ortix C, 29

Pacciani P, 29
Palese M, 30
Parwani R, 30
Pashaev OK, 30
Pavlov M, 31
Prinari B, 32
Protogenov AP, 32

Rogers C, 32
Rosenhaus V, 33
Ruggeri G, 33

Santini P, 34
Scolarici G, 34

Sommacal M, 35
Sozzo S, 35
Speight JM, 36
Spiller E, 36

Tempesta P, 37
Trubatch AD, 37

Verhoeven C, 38
Vilasi G, 38
Vinogradov A, 38
Vitolo R, 39

Weiss O, 40
Winternitz P, 40
Winterroth E, 40

Zakharov V, 41
Zarmi Y, 41