# Abstract booklet for
# Nonlinear Physics: Theory and Experiment. V

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**Algebraically closed real geodesics on n-dimensional ellipsoids are dense in the parameter space and related to hyperelliptic tangential coverings**

The closedness condition for real geodesics on $n$–dimensional ellipsoids is in general transcendental in the parameters (semiaxes of the ellipsoid and constants of motion). We show that it is algebraic in the parameters if and only if both the real and the imaginary geodesics are closed and we characterize such double–periodicity condition via real hyperelliptic tangential coverings. We prove the density of algebraically closed geodesics on $n$–dimensional ellipsoids with respect to the natural topology in the $2n$–dimensional real parameter space. In particular, the approximating sequence of algebraic closed geodesics on the approximated ellipsoids may be chosen so to share the same values of the length and of the real period vector as the limiting closed geodesic on the limiting ellipsoid.

Finally, for real doubly–periodic geodesics on triaxial ellipsoids, we show how to evaluate algebraically the period mapping and we present some explicit examples of families of algebraically closed geodesics.

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**Mark J. Ablowitz**

*Department of Applied Mathematics, University of Colorado*
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**Reformulation and asymptotic reductions of interfacial waves**

Nonlocal spectral reformulations of classic water and multi-fluid interfacial waves are developed in terms of data only on the free surfaces. From the nonlocal spectral equation Dirichlet-Neumann series, asymptotic reductions and integral relations are found. In certain limits lump type solutions are obtained numerically.

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**Complete integrability and monodromy preserving structure of the symmetry reduced heterotic string effective equations in any dimensions**

Completely integrable and monodromy preserving structure of the equations, which govern the space-time symmetry reduced (two-dimensional) low-energy dynamics of gravity coupled bosonic sector of the heterotic string effective action with arbitrary number of Abelian gauge fields in the space-time of arbitrary dimensions, is revealed. It is assumed that all field components and potentials depend only on time and one spatial coordinate (e.g., plane or cylindrical waves, cosmological solutions) or on two spatial coordinates (e.g., stationary axisymmetric fields). We show that this dynamics preserves the monodromy of the normalized fundamental solution of associated spectral problem, and this allows to reduce the problem of constructing of solutions of these equations to solution of some system of linear singular integral equations. We describe also a construction of an infinite hierarchy of explicitly calculating solutions which correspond to arbitrarily chosen analytically matched, rational monodromy data. This hierarchy includes, in particular, the solutions for black holes with various charges which generalize different already known solutions of this kind.

This work was supported in parts by the Russian Foundation for Basic Research (grants 08-01-00501, 08-01-00618, 06-01-92057-CE) and the programs “Mathematical Methods of Nonlinear Dynamics” of the Russian Academy of Sciences and “Leading Scientific Schools” of Russian Federation (grant NSh-1959.2008.1).

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**N.V. Alexeeva**(*) **and E.V. Zemlyanaya**(†)

(*) *Department of Mathematics, University of Cape Town, South Africa*
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**Spatially Localised, Temporally Periodic Solutions of the Parametrically Driven Damped Nonlinear Schrödinger Equation**

We consider the parametrically driven damped nonlinear Schrödinger equation,

$$i\psi_t + \psi_{xx} + 2|\psi|^2\psi - \psi = h\psi^* - i\gamma\psi. \tag{1}$$

The stationary soliton solution to equation (1) is known to lose its stability to a localised time-periodic solution as the driver's strength $h$ is increased past some critical value. Numerical simulations demonstrated that on further increase of $h$, the soliton undergoes a period-doubling transition to temporal chaos or yields to a spatio-temporal chaotic state — depending on the value of $\gamma$. The purpose of this work is to follow the transformations of temporally periodic solitons of equation (1), study their stability and bifurcation in order to explain the attractor chart for this equation which was compiled using direct numerical simulations. We also add missing details to this chart such as alternative attractors in the case of bistability.

Instead of direct numerical simulations, we study the time-periodic solitons by means of the numerical continuation of solutions to the boundary-value problem on a two-dimensional cylindric surface: $\psi(x,t) \to 0$ as $x \to \pm\infty$; $\psi(x,t+T) = \psi(x,t)$. The advantage of this approach is that it furnishes both stable and unstable solutions.

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**Hynek Baran**

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**To be announced**

(Abstract not provided)

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**Igor Barashenkov and Oliver Oxtoby**

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**Damped driven wobbling kinks in the $\phi^4$ theory**

An asymptotic method is used to construct the wobbling kink of the $\phi^4$ equation. The amplitude of the free wobbling is found to decay as $t^{-1/2}$ due to the emission of the second-harmonic radiation. We study the compensation of these radiation losses (as well as dissipative losses in case those are present in the system) by the resonant driving of the kink. Both the direct

$$\frac{1}{2}\phi_{tt} - \frac{1}{2}\phi_{xx} + \gamma\phi_t - \phi + \phi^3 = h\cos(\Omega t)$$

and parametric

$$\frac{1}{2}\phi_{tt} - \frac{1}{2}\phi_{xx} + \gamma\phi_t - [1 + h\cos(\Omega t)]\,\phi + \phi^3 = 0$$

driving are considered, at a range of resonance driving frequencies. In each case, we derive the amplitude equations which describe the time evolution of the kink's wobbling amplitude and velocity. In the case of the parametric driving, the strongest resonance is found to occur when the driving frequency equals the natural wobbling frequency itself (and not the double frequency), and provide a qualitative explanation for this atypical behaviour. For the direct driving, we confirm the existence of the superharmonic resonance at half the natural frequency, and show that a weaker resonance at the natural frequency itself does exist, contrary to the recent claims. This resonance is found to be accompanied by translational motion of the kink. We find no evidence for the chaotic dynamics reported in recent literature.

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**Matteo Beccaria**

*Dipartimento di Fisica - Università del Salento*
*Lecce, Italy*

**Integrability in $N = 4$ SYM: A tool for QCD?**

We review recent applications of integrability techniques to $N = 4$ super Yang-Mills as a theoretical tool for perturbative QCD.

**Julia Bernatska and Holod Petro**
*National University of Kiev-Mohyla Academy*
*Department for Physical and Mathematical Sciences*
*Kiev, Ukraine*

**Nonlinear excitations in 2D spin systems with high spin $s \geq 1$**

We investigate ordered states of 2D spin systems with high spins ($s \geq 1$). The systems are described by quantum Heisenberg models with high powers of biquadratic exchange. We associate each quantum model with a classical model (as a continuous classical analogue). The classical models are generalizations of the 1D integrable hierarchy of Landau-Lifshits equation. One can naturally find effective Hamiltonians for the classical models. In the case of spin $s$, the classical vector fields obviously live on coadjoint orbits of the unitary group $\mathrm{SU}(2s+1)$.

In order to describe large-scale excitations in the considered systems we use variational methods, namely we minimize the corresponding effective Hamiltonian. An investigation shows that topological excitations appear at a low temperature and cause a destruction of the long range ordering in 2D systems. We consider possible ways to stabilize ordered states by taking into account the finite thickness of real 2D magnetics.

**Gino Biondini**
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**The dispersion-managed Ginzburg-Landau equation and its applications to femtosecond lasers**

The complex Ginzburg-Landau equation is a model that has been used extensively to study various non-equilibrium phenomena. In the context of lasers, it describes the evolution of a pulse by averaging over the changes that take place in a cavity roundtrip. Pulses produced by Ti:sapphire femtosecond lasers, however, undergo large dynamical changes in various parts inside the cavity during each roundtrip, and are therefore not adequately described by an average model that does not take these changes into account.

In this talk I will present the derivation the dispersion-managed Ginzburg-Landau equation (DMGLE) in a general context as an average model that describes the long-term dynamics of systems characterized by rapid variations of dispersion, nonlinearity and gain loss dynamics. After discussing various properties of the equation, I will then show in particular how the DMGLE arises in the description of Ti:sapphire femtosecond lasers, and I will characterize its solutions.

**M. Blaszak and A Sergeyeyev**
*Institute of Physics, A. Mickiewicz University*
*Poznam, Poland*

**Reciprocal transformations for a Class of Weakly Nonlinear Hydrodynamic-Type Systems**

We present a coordinate-free construction for a large class of weakly nonlinear semi-Hamiltonian hydrodynamic-type systems – the dispersionless Killing systems – along with general solutions thereof. These systems are constructed from the seed systems using suitably chosen reciprocal transformations and are intimately related to the class of Stäckel separable metrics. The seed systems are constructed from the so called L-tensors and are related to the Stäckel separable metrics of Benenti type.

**Leonid Bogdanov**

*Landau Institute*
*Moscow, Russia*

**On a class of multidimensional integrable hierarchies and their reductions**

A class of multidimensional integrable hierarchies connected with commutation of vector fields is considered. They are represented in the form of generating equation, as well as in the Lax-Sato form. A dressing scheme based on nonlinear vector Riemann problem is developed for this class. Some reductions, including waterbag-type reductions, are studied. Dunajski equation, which is an integrable generalization of Plebansky second heavenly equation, is considered as a characteristic example.

The research was partially supported by RFBR grants 07-01-00446 and 06-01-92053 (RFBR - Consortium E.I.N.S.T.E.IN, Italy)

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**Marco Boiti, F. Pempinelli, A.K. Pogrebkov, B. Prinari**

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*Lecce, Italy*

**Extended resolvent of the heat operator for a Darboux transformed potential**

In the framework of the resolvent approach so called twisting operators for the heat operator are introduced. They are used, at the same time, to superimpose à la Darboux an arbitrary number of solitons to a generic smooth decaying potential and to generate the extended resolvent and Green's function corresponding to such potential. Jost solutions are introduced and an explicit bilinear representation of the related extended resolvent in their terms is derived.

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**Sonia Boscolo and Sergei K. Turitsyn**

*Photonics Research Group, School of Engineering and Applied Science*
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**Nonlinear optical pulse transformations in fibre-based systems**

Optical fibre systems are not only of great practical importance, but can also provide useful experimental testbeds for the study of fundamental physical phenomena of widespread interest. A typical example is the formation of autosolitons in fibre lines with periodically inserted nonlinear optical devices. In this context, the term "autosolitons" or "dissipative solitons" describes robust localized pulses with parameters prescribed by properties of the system, which can propagate in the system thanks to an equilibrium condition between nonlinearity, dispersion, amplification and nonlinear dissipation. Considering the transformation of a pulse after propagation in a segment of the transmission line as the mapping of an input pulse into an output one, the problem of finding such steady-state solutions is equivalent to the analysis of the fixed points of the corresponding mapping problem. Another important example of a fundamental process that has attracted much recent interest is the occurrence of self-similar dynamical effects in the nonlinear pulse propagation in optical fibre amplifiers. Recent results have demonstrated a fundamentally new operating regime where nonlinear propagation is exploited to generate a particular class of parabolic pulse that evolves self-similarly. The self-similar parabolic pulse is an asymptotic, approximate solution of the nonlinear Schrödinger equation with gain in the semiclassical (large-amplitude/small-dispersion) limit, representing a type of nonlinear "attractor" towards which an arbitrarily shaped input pulse of given, sufficiently high energy converges with sufficient distance. Techniques for the

control and manipulation of the shape of optical pulses, which can be regarded as one-path transformations of an input pulse into a target output pulse, have become increasingly important in a range of photonic applications.

In this talk, we overview our recent advances in the development of models and techniques for the nonlinear transformations of optical pulses in optical fibre systems. A theory of dissipative solitons in high-speed fibre communication lines is reviewed. Theoretical and experimental aspects of the generation and characterization of parabolic pulses in active and passive fibres are considered. A method for passive nonlinear pulse shaping based on pulse pre-chirping and propagation in the normal dispersion regime is presented. The approach provides a simple way of generating various temporal waveforms of fundamental and practical interest, ranging from a parabolic profile to a flat-top profile and a triangular profile. Promising applications of the parabolic (flat-top) pulses in nonlinear all-optical signal pre-processing at the receiver and regeneration in optical communications are also covered. Novel applications of the triangular pulses in signal copying and time-to-wavelength mapping of multiplexed signals are presented as well.

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**Francesco Calogero**

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**Isochronous systems are not rare**

A (classical) dynamical system is called isochronous if it features an open (hence fully dimensional) region in its phase space in which all its solutions are completely periodic (i. e., periodic in all degrees of freedom) with the same fixed period (independent of the initial data, provided they are inside the isochrony region). When the isochrony region coincides with the entire phase-space one talks of entirely isochronous systems. A trick is presented associating to a dynamical system a modified system depending on a parameter so that when this parameter vanishes the original system is reproduced while when this parameter is positive the modified system is isochronous. This technique is applicable to large classes of dynamical systems, justifying the title of this talk. An analogous technique, developed with François Leyvraz and even more widely applicable - for instance, to any translation-invariant (classical) many-body problem - transforms a real autonomous Hamiltonian system into an entirely isochronous real autonomous Hamiltonian system. The modified system is of course no more translation-invariant, but in its centre-of-mass frame it generally behaves quite similarly to the original system over times much shorter than the isochrony period T (which may be chosen at will). Hence, when this technique is applied to a "realistic" many-body Hamiltonian yielding, in its centre of mass frame, chaotic motions with a natural time-scale much smaller than (the chosen) T, the corresponding modified Hamiltonian shall yield a chaotic behavior (implying statistical mechanics, thermodynamics with its second principle, etc.) for quite some time before the entirely isochronous character of the motion takes over hence the system returns to its initial state, to repeat the cycle over and over again. We moreover show that the quantized versions of these modified Hamiltonians feature infinitely degenerate equispaced spectra. Analogous techniques are applicable to nonlinear evolution PDEs, but in this talk there will be no time to cover this aspect. The material presented is a synthesis of work done over the last 10 years with several collaborators, as reviewed in the 264-page monograph entitled Isochronous systems, just published by Oxford University Press.

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**Roberto Camassa**

*Department of Mathematics - University of North Carolina*
*Chapel Hill, NC (USA)*

**Spinning rods, microfluidics, and propulsion by cilia in biological systems**

An important component in understanding and modeling how human lungs function lies in the hydrodynamics of the mucus fluid layers that coat lung airways. In healthy subjects, the beating of cilia is thought to be the primary mechanism for moving mucus. With the aim of establishing a quantitative benchmark of how cilia motion propels the surrounding fluid, we study the idealized situation of a single rod spinning in a fluid obeying the Stokes approximation, the appropriate limit for a Newtonian fluid with typical dimensions and time scales of cilia dynamics. New approximate – for cylindrical rods pinned to a flat plane boundary, and exact – for ellipsoidal rods freely spinning around their center – solutions for the fluid motion will be presented and compared with the experimental data collected with spinning magnetic nano-rods in water. In order to assess the influence of Brownian perturbations in this micro-scale experiment, data from an experimental set-up scaled by dynamical similarity to macroscopic (table-top) dimensions will also be presented and compared to the theoretical predictions.

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**Jen-Hsu Chang and Yu-Tung Chen**

*Department of Computer Science, National Defense University*
*Taiwan*

**Solutions for Real Dispersionless Veselov-Novikov Hierarchy**

We investigate the dispersionless Veselov-Novikov (dVN) equation based on the framework of dispersionless two-component BKP hierarchy. Symmetry constraints for real dVN system are considered. It is shown that under the symmetry constraints, the conserved densities are therefore related to the associated Faber polynomials and can be solved recursively. Moreover, the method of hodograph transformations as well as the expressions of Faber polynomials are used to find exact real solutions of the dVN hierarchy.

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*Perugia, Italy*

**An analytical model for synthetic- and bio- polymers based on continuous elastic rods: general framework**

The study of elastic deformations in thin rods has recently seen renewed interest due to the close connection between these systems and coarse-grained models of widespread application in life- and material- sciences. Until now, the analysis has been restricted to the solution of equilibrium equations for continuous models characterized by constant bending and twisting elastic moduli and/or by isotropic rod section. However, more realistic models often require more general conditions: indeed this is the case whenever microscopic information issuing from atomistic simulations is to be transferred to analytic or semi-analytic coarse-grained or macroscopic models. In this paper we will show that integrable, indeed solvable, equations are obtained under quite general conditions and that regular (e.g. circular helical) solutions merge from reasonable choices of elastic stiffnesses.

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**Andrew Docherty**
*School of Mathematics and Statistics, University of Sydney, NSW*
*Sidney, Australia*

**Techniques for solving for leaky modes in optical fiber structures**

There is much current research in novel fiber designs which combine high bandwidths with good light acceptance properties and easy coupling. These optical fibers support mode fields that are not tightly restricted to the core and propagate outwards into the cladding causing a loss. To find these so-called leaky modes it is vital to consider the outward going fields must be solved on the infinite domain.

There are numerous techniques for the reduction of wave problems on an infinite domain onto finite domains so that they can be solved using numerical methods. A common method is to impose an transparent boundary on the restricted domain. A certain class of these boundary conditions is investigated which results in a nonlinear eigenvalue problem. The problems of this technique using numerical techniques to solve the nonlinear eigenvalue problem is investigated. The resultant problem is be solved with a range of methods for the nonlinear eigenproblem, and future directions for efficient algorithms for finding many modes are discussed.

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**V.G. Dubrovsky** and A. V. Gramolin
*Novosibirsk State Technical University*
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**Gauge-invariant description of some $(2+1)$-dimensional integrable nonlinear evolution equations**

Ideas of gauge invariance in the theory of integrable nonlinear equations are discussed. New manifestly gauge-invariant forms of two-dimensional generalized dispersive long wave and Nizhnik-Veselov-Novikov systems of integrable nonlinear equations are presented. It is shown how in different gauges from such forms famous two-dimensional generalization of dispersive long wave system of equations, Nizhnik-Veselov-Novikov and modified Nizhnik-Veselov-Novikov equations and other known and new integrable nonlinear equations arise. Miura-type transformations between nonlinear equations in different gauges are considered.

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**Pilar G. Estévez and J. Prada**

*Area de Fisica Teórica Universidad de Salamanca*
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**Reciprocal transformations and reductions for a 2+1 integrable equation**

Different reciprocal transformations for a spectral problem in $2+1$ dimensions are investigated. Reductions of the transformed spectral problems are presented. The Vakhnenko and Degasperis-Procesi equation appear as two of the different possible reductions.

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**Marianna Euler**

*Department of Mathematics*
*Lulea University of Technology - SE-97187 Lulea, Sweden*

**Higher Order Adjoint Symmetries of Evolution Equations and Conservation Laws**

We discuss the use of higher order adjoint symmetries for the construction of conservation laws and reciprocal transformations. Some third order evolution equations will be described.

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**Norbert Euler**

*Department of Mathematics*
*Lulea University of Technology - SE-97187 Lulea, Sweden*

**On Proper Differential Sequences of ODEs**

We define and describe Proper Differential Sequences of Ordinary Differential Equations and methods of integrability. Several examples will be discussed.

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**Margarida Facao**[1] **and M.I. Carvalho**[2]

[1]*Departamento de Física, Universidade de Aveiro*
[2]*Departamento de Engenharia Electrotécnica e de Computadores*
*Faculdade de Engenharia, Universidade do Porto*

**Unstable modes of dark photorefractive solitons**

Biased photorefractive media are known to admit bright and dark solitons. Their bright solitons are always stable, nevertheless the stability criterion for dark solitons shows that the black and almost black photorefractive solitons are unstable. Here, we determine and characterize the unstable region using the actual physical parameters and determine the unstable modes using the Evans function model.

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**E.V. Ferapontov**

*Loughborough University*
*Leicestershire, UK*

**Moduli spaces of dispersionless integrable PDEs in three dimensions**

It will be demonstrated that the moduli space of integrable equations of the dispersionless Hirota type and the moduli space of first order integrable lagrangians are finite dimensional (of dimensions 21 and 15, respectively). The action of the corresponding equivalence groups (Sp(6, R) and Sl(4, R), respectively) on these moduli spaces are shown to possess an open orbit, manifesting the existence of a universal master-equation generating the whole moduli space via appropriate (singular) limits.

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**E.V. Ferapontov, A. Moro, V.V. Sokolov**

*School of Mathematics - Loughborough University*
*Leicestershire, UK*

**Hamiltonian systems of hydrodynamic type in 2+1 dimensions**

We investigate multi-dimensional Hamiltonian systems associated with constant Poisson brackets of hydrodynamic type. A complete list of two- and three-component integrable Hamiltonians is obtained. All our examples possess dispersionless Lax pairs and an infinity of hydrodynamic reductions.

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**E. Fersino, G. Mussardo, A. Trombettoni**

*SISSA*
*Trieste, Italy*

**One-dimensional Bose gases with N-body attractive interactions**

We study the ground state properties of a one-dimensional Bose gas with $N$-body attractive contact interactions. By using the explicit form of the bright soliton solution of a generalized nonlinear Schrödinger equation, we compute the chemical potential and the ground state energy. For $N = 3$, a localized soliton wave-function exists only for a critical value of the interaction strength: in this case the ground state has an infinite degeneracy that can be parameterized by the chemical potential. The stabilization of the bright soliton solution by an external harmonic trap is also discussed, and a comparison with the effect of $N$-body attractive contact interactions in higher dimensions is presented.

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### Conditional symmetries for ordinary differential equations

In this work we derive conditional symmetries for ordinary differential equations. By using these conditional symmetries we find that the order of the ODE can be reduced even if this equation does not admit point symmetries. Moreover, in the case for which the ODE admits a group of point symmetries, we find that the conditional symmetries allow us to perform further reductions than its point symmetries.

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**Vladimir Gerdjikov, N.A. Kostov**

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### Reductions of multicomponent NLS and mKdV Equations with Nonvanishing Boundary Conditions

Mikhailov's methods [1] for imposing reductions on the multicomponent NLS and mKdV equations related to simple Lie algebras $\mathfrak{g}$ and symmetric spaces [2] will be given. We start by formulating the spectral properties of the corresponding Lax operators $L$

$$L\psi \equiv \left( i\frac{d}{dx} + q(x,t) - \lambda J \right) \psi(x,t,\lambda) = 0, \qquad (2)$$

on a generic symmetric space and generic boundary conditions. This means that $J \in \mathfrak{h}$ takes values in the Cartan subalgebra of the simple Lie algebra $\mathfrak{g}$ and is such that the set of positive roots $\Delta^+$ of $\mathfrak{g}$ is split into $\Delta^+ = \Delta_0^+ \cup \Delta_1^+$ where $\alpha(J) = 0$ for all $\alpha \in \Delta_0^+$ and $\beta(J) = \pm 1$ for all $\beta \in \Delta_1^+$ [3]. Then the potential $q(x,t)$ is parametrized by

$$q(x,t) = \sum_{\alpha \in \Delta_1^+} (q_\alpha(x,t) E_\alpha + q_{-\alpha}(x,t) E_{-\alpha}), \qquad \lim_{x \to \pm\infty} q(x,t) = q_\pm$$

where the boundary values $q_\pm$ of $q(x,t)$ may have generic set of complex-valued eigenvalues $\rho_j$ restricted only by the condition $q_+^2 = q_-^2$. The corresponding inverse scattering problem for $L$ is reduced to a Riemann-Hilbert problem on a Riemannian surface which is determined by $\rho_j$.

Next we analyze the effects of algebraic reductions on the Riemannian surface and on the discrete spectrum of $L$. Special attention is paid to the reductions, obtained by elements of the Weyl group of $\mathfrak{g}$, see [4]. We also outline their consequences for the soliton solutions and the hierarchies of Hamiltonian structures. The results are illustrated by explicit examples for several particular choices of low-rank symmetric spaces. A number of peculiarities such as singularities at the end-points of the spectrum, regularization of the integrals of motion, classical $R$-matrices etc will be discussed thus extending the results in [5,6].

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*Glasnevin, Dublin 9, Ireland*

**On Nonlinear Schrödinger Equations on Symmetric Spaces and their Gauge Equivalent**

The fundamental properties of the multi-component nonlinear Schrödinger (MNLS) equations on symmetric spaces and their gauge equivalent Heisenberg ferromagnetic type equations are reviewed.

This includes: the properties of the fundamental analytical solutions for these models, the corresponding minimal set of scattering data, the description of the class of nonlinear evolutionary equations, solvable by the inverse scattering method and the recursion operator, related to such systems, the hierarchies of Hamiltonian structures.

The results are illustrated on the example of MNLS model related to $so(5,\mathbb{C})$ algebra, describing $\mathcal{F} = 1$ spinor Bose-Einstein condensate, and its gauge equivalent multi-component Heisenberg ferromagnetic type model.

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**Piotr Grinevich, Paolo M. Santini**

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*Moscow, Russia*

**Integration of 2-d discrete analogs of the Novikov-Veselov hierarchy with double-periodic boundary conditions**

Recently A. Doliwa, M. Nieszporski and the authors suggested an integrable at one energy elliptic discretization of the 2-D Schrodinger operator. The associated hierarchy can be treated as an integrable discretization of the Novikov-Veselov hierarchy with discrete spatial variables and continuous times.

We study this hierarchy on the space of the double-periodic in spatial variables potentials. We show, that in contrast with the continuous case, the discretized hierarchy is well-defined only at a distinguished submanifold in the phase space. This manifolds has a simple natural spectral characterization. Therefore only operators with special potentials possess the full algebra of symmetries.

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**Itay Hen**

*Tel Aviv University*
*Raymond and Beverly Sackler School of Physics and Astronomy*
*Tel Aviv, Israel*

**Hexagonal structure of baby Skyrmion lattices**

The Skyrme model is a non-linear theory for pions in (3+1) dimensions, serving as a low-energy effective theory for QCD in the large number of colors limit. Within this model, topological solitons, known as Skyrmions, are identified with nucleons and the topological charge is identified with the baryon number. Hence, the properties of its solutions are of considerable physical interest since they correspond almost directly to the low-energy excitations of nucleons.

Under proper settings, the Skyrme model can also be used to study the behavior of nuclear matter at finite densities, so the lattice structure of Skyrmions at low or zero temperatures has attracted a lot of attention.

In this talk I will present new results on the zero-temperature crystalline structure of "baby Skyrmions", which are the two-dimensional solitonic versions of Skyrmions. As it turns out, their minimal-energy crystalline structure is a hexagonal lattice, in which the baby Skyrmions split into quarter-Skyrmions.

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**Robert Conte, Benoit Huard and A.M. Grundland**

*Universite de Montreal*
*Montreal, Quebec, Canada*

**Elliptic solutions of isentropic ideal compressible fluid flow in (3+1) dimensions**

A modified version of the conditional symmetry method, together with the classical method, is used to obtain new classes of elliptic solutions of the isentropic ideal compressible fluid flow in (3+1) dimensions. We focus on those types of solutions which are expressed in terms of the Weierstrass $\wp$-functions of Riemann invariants. These solutions are of special interest since they remain bounded even when the invariants admit the gradient catastrophe. We describe in detail a procedure for constructing such classes of solutions. Finally, we present several examples of an application of our approach which includes bumps, anti-bumps and multi-wave solutions.

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**Boaz Ilan**

*School of Science - UC Merced*
*Merced, CA USA*

**Stability and instability dynamics of lattice solitons**

Stability of solitons in the multi-dimensional nonlinear Schrodinger equations with lattice-type potentials is investigated. Using a combination of rigorous, asymptotic and numerical methods, it is shown that the solitons are stable precisely whenever they satisfy the power-slope (Vakhitov-Kolokolov) condition and the lesser-known spectral condition. Violation of the power-slope condition induces an amplitude instability, whereas violation of the spectral condition induces a transverse drift instability. This unified approach can be used to predict the strength of the (in)stability as well. The results are elucidated by examples with periodic, defect, and quasi-crystal lattice structures.

Joint works with Mark Ablowitz, Gadi Fibich, Yonatan Sivan, and Michael Weinstein.

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**Theodora Ioannidou**
*School of Mathematics, Physics and Computational Sciences*
*Aristotele University of Thessaloniki*
*Thessaloniki, Greece*

**Discrete Skyrmions in 2+1 and 3+1 Dimensions**

A lattice version of the Skyrme model in $2+1$ and $3+1$ dimensions. In particular, the existence and stability of the skyrmion solutions existing on the lattice is investigated. One consequence of the proposed models is that the corresponding discrete skyrmions have a high degree of stability, similar to their continuum counterparts.

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**Rossen Ivanov**
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**Two component integrable systems modelling shallow water waves**

The motion of inviscid fluid is described by Euler's equations. In the case of shallow water, one can consider a perturbative asymptotic expansion of Euler's equations to a certain order of smallness of the scale parameters. The so obtained asymptotic equations can be matched to certain integrable equations. The best known example in this regard is the KdV equation.

The main aim of the talk is to presentsome recent results, concerning the use of integrable equations, like a two-component generalization of the Camassa-Holm equation, in modeling the motion of shallow water waves.

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**Leonid A. Kalyakin**
*Institute of Mathematics RAS*
*Ufa, Russia*

**Asymptotic analysis of the cyclotron autoresonance phenomenon**

The theory of cyclotron accelerator is based on the equations [1]:

$$\frac{dP_z}{dt} = \frac{d\gamma}{dt} = \frac{P_\perp}{2\gamma} a\cos\varphi,$$

$$\frac{dP_\perp}{dt} = \frac{1}{2}\Big(1 - \frac{P_z}{\gamma}\Big) a\cos\varphi,$$

$$\frac{d\theta_c}{dt} = -\frac{\Omega}{\gamma} + \frac{1}{2P_\perp}\Big(1 - \frac{P_z}{\gamma}\Big) a\sin\varphi,$$

$$\frac{d\theta}{dt} = -1 + \frac{P_z}{\gamma},\ \varphi = \theta - \theta_c, \quad (\Omega, a = \mathrm{const}).$$

The cyclotron autoresonance phenomenon corresponds to the solutions with increasing component $P_z(t) \to \infty$, as $t \to \infty$. Such type of solutions are obtained by analysis of the first integrals. The known class of the autoresonance solutions is very poor. We find an additional first integral and analyze the general solution.

The research was supported by RFBR grants nos. 06-01-00124, 06-01-92052.

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**David Kaup**

*Dept. of Mathematics and Institute for Simulation and Training*
*University of Central Florida*
*Orlando, FL*

**Perturbation Expansions for Integrable PDE's and the "Squared Eigenfunctions"**

We will review perturbation expansions for integrable systems from a general point of view and also address the structure of the "squared eigenfunctions." A general approach to solving the question of closure for these eigenfunctions will be addressed as well.

Research supported in part by the USA National Science Foundation and the USA Air Force Office of Scientific Research.

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**Arthemy V. Kiselev and Johan W. van de Leur**

*Mathematical Institute, University of Utrecht*
*Budapestlaan 6, 3584CD Utrecht, The Netherlands*

**Operator-valued involutive distributions of evolutionary vector fields**

Involutive distributions of evolutionary vector fields that belong to images of matrix operators in total derivatives are considered and some classifications of the operators are obtained. In particular, we describe endomorphisms of evolutionary derivations whose images are closed w.r.t. the commutation and which induce hierarchies of Lie algebra structures on the infinite jet spaces.

Analysis of the deformation problem for the operators suggests to distinguish their homotopy compatibility as the analogue of the Poisson pencils for Hamiltonian structures, and the strong compatibility as the commutation closure of sums of images for $N$-tuples of the operators. This allows to endow the spaces of totally compatible operators, which may not be the endomorphism solutions of the classical Yang–Baxter equations, with a Lie algebra structure specified by bi-differential structural constants. We show that the operators with closed images determine flat non-Cartan connections on the modules of evolutionary vector fields such that the structural constants of the Lie algebras are bi-differential Christoffel's symbols, and we conclude that completely integrable hierarchies are the geodesics. Also, differential complexes are constructed using the operators, which leads to a realization of Lie algebroids over jet spaces in terms of homological vector fields.

A class of matrix operators, whose images are closed w.r.t. the commutation, and the Koszul brackets induced in their pre-images are assigned to integrable KdV-type hierarchies of symmetry flows on the hyperbolic Euler–Lagrange Liouville-type systems (e.g., on the open 2D Toda lattices associated with semi-simple Lie algebras).

Reference: Kiselev A. V., van de Leur J. W. (2008) *arXiv:math-ph/0703082v3*.

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**Oleg M. Kiselev**
*Institute of Mathematics of RAS*
*UFA, Russia*

### Slow passage through parametric resonance for a weakly nonlinear dispersive wave

A solution of the nonlinear parametric driven Klein-Gordon equation is investigated. The frequency of the parametric driver varies slowly and passes through a resonant value. The asymptotic description of the passage through the resonance is presented. The formula for connection of the solution before and after the parametric rasonance is obatined for main term of asymptotic expansion.

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**Boris Konopelchenko**
*Dipartimento di Fisica - Università del Salento*
*Lecce, Italy*

### Deformations of algebras and integrable systems

Coisotropic, quantum and discrete deformations of associative algebras are discussed. It is shown that such deformations are governed by integrable systems like oriented associativity equation, WDVV equation, KP equation and KP hierarchy and so on and their discrete versions.

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**Lubomir Kovachev**
*Institute of Electronics, Bulgarian Academy of Sciences*
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### Nonparaxiality and femtosecond optics

We use the method of amplitude envelopes to obtain class of nonlinear nonparaxial amplitude equations, governing the propagation of short laser pulses in media with dispersion, dispersionless media and vacuum. After writing the equations in dimensionless form three dimensionless parameters, governing different regimes of propagation, are obtained. The normalized amplitude equations for media with dispersion and the amplitude equations in dispersionless media and vacuum are equal with precise a constant, ratio between diffraction and dispersion lengths. In linear regime, we solving the equations using the method of Fourier transform. One unexpected new result is the relative stability of light pulses with spherical and spheroid spatial form, when we compare their enlargement with the light filaments and dynamics of pulses in paraxial approximation. It is important to emphasize here the case of light disks, i.e. pulses whose longitudinal size is small with respect to the transverse one, which turns out to be practically diffractionless over distances of more than hindered kilometers. New formula for diffraction length of optical pulses is found. The nonlinear regime of femtosecond pulses with initial power, slightly above critical for self-focusing, is investigated in the framework of same non-paraxial amplitude equation. In case of long pulse, the dynamics is closer to nonlinear paraxial dynamics of a laser beam, and the difference consists in large spectral and longitudinal spatial modulation of the long pulse. The non-paraxial terms play an important role on the evolution of light bullets and light disks. In regime of light bullets arrest of the collapse with weak self-focusing without large base is obtained. Non-collapsed regime of light disks is also observed. Our results are in good agreement with the recent experiments on nonlinear propagation of femtosecond pulses. We demonstrated that such non-paraxial model could explain effects as spectral broadening, collapse arrest and nonlinear wave-guide behavior.

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**Ajit Kumar**

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**Sub-cycle pulse propagation in a Kerr optical fiber including delayed nonlinear response**

Due to some extra-ordinary developments in experimental techniques for generating and stabilizing ultra-short pulses [1-3], presently there is a great deal of interest in the study of a few-cycle and sub-cycle pulses. Evidently, theoretical analysis of propagation characteristics of such pulses, in linear as well as nonlinear media, is very important due to the enormous potential of such pulses for practical applications. Such ultra-short pulses have found applications in diverse areas of physics and technology including nonlinear optical devices, all-optical communication, medical diagnostics and imaging, controlled manipulation of chemical reactions and bond formation, coherent quantum control of microscopic dynamics etc [4-10]. Theoretical modeling of dynamics of such pulses has shown that the usual nonlinear Schrödinger equation is inadequate for this purpose because of the breakdown of envelope approximation for pulses that contain a single or a few optical cycles. This has motivated researchers to look for novel model equations that can describe the pulse dynamics of ultra-short pulses in linear and nonlinear media more accurately [11-15]. For this purpose, researchers have used the concept of a pulse envelope [12,13] for a few cycle pulses, or approximated the form of the susceptibility function $\chi(\omega)$ by assuming the frequency range of the pulse to be not only far from the material resonance frequency but also much larger than it [14] etc. In the given talk I present a nonlinear evolution equation governing a few- and sub-cycle pulse dynamics in a cubic nonlinear medium which, following the work of Tsurumi [15], is derived from the nonlinear wave equation without using the concept of pulse envelope. The delayed nonlinear response of the medium is also taken into account. The results of numerical calculations, for the propagation of a half-cycle pulse at $\lambda = 1.55\mu m$ in a Kerr fiber, are presented and discussed.

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*St. Petersburg, 194021 Russia*

**Transmission and reflection of a strain solitary wave in a delaminated bar**

We study the dynamics of a longitudinal bulk solitary wave in a delaminated, symmetric layered elastic bar. First, we consider a two-layered bar and assume that there is a perfect interface when $x < 0$ and complete debonding (splitting) when $x > 0$, where the axis $Ox$ is directed along the bar. We derive the so-called Doubly Dispersive Equation (DDE) for a long nonlinear longitudinal bulk wave propagating in an elastic bar of rectangular cross section. We formulate the problem for a delaminated two-layered bar in terms of the DDE with piecewise constant coefficients, subject to continuity of longitudinal displacement and normal stress across the “jump” at $x = 0$. We find the weakly nonlinear solution to the problem and consider the case of an incident solitary wave. The solution describes both the reflected and transmitted waves in the far field, as well as the diffraction in the near field (in the vicinity of the jump). We generalize the solution to the case of a symmetric $n$-layered bar. We establish that delamination can lead to the fission of an incident solitary wave, and obtain explicit formulae for the secondary solitary waves propagating in each layer of the split waveguide. We show that generally there is a higher-order reflected wave even when the leading order reflected wave is absent. We also discuss first experimental results on soliton fissioning in solid waveguides.

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**Giulio Landolfi**

*Dipartimento di Fisica - Università del Salento*
*Lecce, Italy*

**An outlook on two different approaches to the quantum phase problem**

The problem of constructing likely quantum phase operators has been tackled in the years by resorting to very different physical and mathematical perspectives. We shall mention two among the ways to proceed that we are currently investigating. The first one is basically concerned with the concept of feasible phase. We shall provide examples of experimental schemes we have devised for the simultaneous measurement of quadratures of linear nonnormal multimode currents, in fact. In the second part of the talk, we shall focus instead on the possibility to formally construct a quantum angle operator associated with general time-dependent quadratic hamiltonians through a Weyl-ordered expansion in terms position and inverse momentum operators.

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**Sergey Leble**

*Gdansk University of Technology*
*Gdansk, Poland*

**Quantum corrections to static solutions of Nahm equation and Sin-Gordon models via generalized zeta-function**

One-dimensional Yang-Mills Equations are considered from a point of view of a class of nonlinear Klein-Gordon-Fock models. The case of self-dual Nahm equations and non-self-dual models are discussed. A quasiclassical quantization of the models is performed by means of generalized zeta-function and its representation in terms of a Green function diagonal for a heat equation with the correspondent potential. It is used to evaluate the functional integral and

quantum corrections to mass in the quasiclassical approximation. Quantum corrections to a few periodic (and kink) solutions of the Nahm as a particular case of the Ginzburg-Landau (phi-in-quadro) and and Sin-Gordon models are evaluated in arbitrary dimensions. The Green function diagonal for heat equation with a finite-gap potential is constructed by universal description via solutions of Hermit equation. An alternative approach based on Baker-Akhiezer functions for KP equation is proposed . The generalized zeta-function and its derivative at zero point as the quantum corrections to mass is expressed in terms of elliptic integrals.

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**Margarita A. Man'ko**

*P.N. Lebedev Physical Institute, Moscow, Russia*
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### New inequalities for Renyi entropy associated to tomographic probabilities of linear and nonlinear radiation beams in waveguides

New entropic inequalities for Renyi entropy are discussed. The inequalities are presented for functions of one continuous variable and functions of two continuous variables which are mapped onto tomographic-probability distributions using fractional Fourier transforms in one and two dimensions, respectively. The Shannon entropy and the entropic inequalities for the tomograms associated to solutions of dynamic equations are studied. The minimum value of the Shannon entropy associated to saturation of entropic inequality is discussed. The discrete Fourier transform is used to obtain some inequalities for entropies associated to unitary matrices. The radiation beam modes in optical waveguides are described by tomographic-probability distributions containing complete information on the modes. The partial case of Gaussian-Hermite modes is studied in detail. The entropic inequalities are also considered for beam profiles corresponding to solitons.

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**Sergei Manakov**

*Landau Inst. Theoretical Physics*
*Moscow, Russia*

### Inverse Spectral Transform for the Vector Fields and the 2-dimensional dispersionless Toda Equation

Using the recently developed IST methods for the vector fields we solve the initial-value problem for the above mentioned equation, formulate the dressing procedure and present exact implicit solutions. We also study the long-time behavior of the solutions and describe the arising singularities. (Together with P.M.Santini)

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**Luigi Martina**

*Dipartimento di Fisica - Università del Salento*
*Lecce, Italy*

### Knotted field distributions of order parameters in pseudogap phase states

We considered quasi-1D configurations of the gauged two-component 3D Ginzburg-Landau model. They are characterized by the Hopf invariant and by the mutual linking number of the order parameter field distributions. A possible inhomogeneous order in such system is described by a net of linked knots. An analysis of the possible configurations and conditions for their existence are described.

**L. Martina, A.P. Protogenov[1], V.A. Verbus**

[1] *Institute of Applied Physics of the RAS, 603950 Nizhny Novgorod, Russia*
*Leibnitz Institute for Solid State and Materials Research*
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### Bloch Hamiltonians and topologically ordered states

The degree of the ground state degeneracy jumps abruptly at quantum phase transitions between different topological orders [1] caused by continuous changes of parameters. One of challenging topological quantum order nowadays is condensate in the form of the string-net [2]. The attractive topic of research in this field is involved also non-Abelian statistics of the Majorana fermion zero modes inside vortex-like defects of topological insulators. We will discuss this type of universal behaviour of the strongly correlated two-dimensional electron systems in the case of the Bloch matrix of effective Hamiltonians $H = \mathbf{h}\,(\mathbf{p}, g_i) \cdot \boldsymbol{\sigma}$. The field $\mathbf{h}$ here depends in general on the components of the momentum $\mathbf{p}$ and on a set of the coupling constants $g_i$; $\boldsymbol{\sigma}$ is the vector of Pauli matrices. As an example, we will consider the Hamiltonians which are relevant for chiral Dirac fermions in graphene [3,4], for the phase states with a $p_x + ip_y$-wave pairing as well as for exact solvable spin models possessing anyon excitations.

This work was supported in part by the E.I.N.S.T.EIN grant (L.M.), RFBR grants (A.P., V.V.) Nos. 06-02-16561, 06-02-92052 and the program (A.P.) "Problems of nonlinear dynamics" of Presidium of the Russian Academy of Sciences.

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**Kenichi Maruno(1), Yasuhiro Ohta, Baofeng Feng**

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### An integrable difference scheme for the Camassa-Holm equation and numerical computation

The Camassa-Holm (CH) equation has attracted considerable interest since it has been derived as a model equation for shallow-water waves. Originally, the CH equation has been found in a mathematical search of recursion operators connected with the integrable partial differential equations by Fuchssteiner and Fokas. The CH equation is shown to be completely integrable, admitting peakon, cuspon and soliton solutions.

It is extremely difficult to perform the numerical computations of the CH equation due to the singularities of the cuspon and peakon solutions. So far, none of the numerical methods for the CH equations gives a satisfactory result.

In this talk, we present an integrable semi-discretization of the CH equation. Determinant formulas for N-soliton solutions of the continuous and semi-discrete CH equations are presented. Based on determinant formula, we can generate multi-soliton and cusped-soliton solutions for both continuous and semi-discrete CH equations. The numerical computations using the integrable semi-discrete CH equation are performed. We show that the integrable scheme gives very accurate numerical results even for the soliton-solition and cuspon-cuspon interactions. This is a joint work with Yasuhiro Ohta (Kobe Univ.) and Baofeng Feng (UT-Pan American).

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**Vladimir Novikov**

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*Loughborough, Leicestershire LE11 3TU, UK*

**On classification of Camassa-Holm type equations**

We present the classification of generalised Camassa-Holm type equations which possess infinite hierarchies of higher symmetries. We also present the corresponding Lax representations or linearisation transformations for these equations.

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**Victor Novokshenov**

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**Padé approximations of the Painlevé Transcendents**

All the solutions to the Painlevé equations are meromorphic functions. However, no reliable algorithm is known to approximate Painlevé transcendents in the complex plane. It is natural to choose the algorithm as the (main diagonal) Padé approximation, since the lattice of poles relates to the lattice of zeros by a Möbius transformation.

A version of the Fair-Luke algorithm has been applied to find the Padé approximate solutions to the Painlevé I and II equations. The distributions of poles for the famous Hastings-McLeod and Tracy-Widom solutions to PII equation was found. The Boutroux tritronquée solution to PI equation was shown to have poles only in the critical sector of the complex plane. The algorithm allows to check further conjections on other analytic properties of the Painlevé transcendents in the complex plane.

This work has been partially supported by the RFBR grants 07-01-00081 and 06-01-92052.

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**Maria Clara Nucci**

*Dipartimento di Matematica e Informatica - Università di Perugia*
*Perugia, Italy*

**Searching (and finding) Lagrangians**

It is well-known that a Lagrangian always exists for any second-order ordinary differential equation, the key being the Jacobi last multiplier. Is it possible to find Lagrangians for ordinary differential equations of higher order? We show that the Jacobi last multiplier plays again the rôle of “deus ex machina”. Naturally, the connection between Jacobi last multiplier and Lie symmetries is fully exploited, and even in the apparently useless happenstance, when they cannot be related, a very useful result is indeed obtained. Finally, we explain what to do with a nil Jacobi last multiplier.

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**Aleksander Yu Orlov**

*Nonlinear Studies Laboratory, Oceanology Institute*
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**Phase transitions in random turn walk of hard core particles**

We consider a lattice gas model describing random hops of particles, which was suggested by M. E. Fisher. We find connection of this model to classical integrable systems. We have established that the tau function - the central object in integrability - on the one hand generates transition probabilities between configurations of the hard core particles, and, on the other hand, generates “partition functions” for this random model. After finding long time asymptotics, we identify a phase transition w.r.t. a hopping rate of the particles. (A joint work with J. Harnad and J. van de Leur.)

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**Oktay K. Pashaev**

*Department of Mathematics, Izmir Institute of Technology,*
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**q-Calculus in Action. From Vortex Images to Relativistic Integrable NLS**

In the talk two applications of the q-calculus would be considered. In the first one the problem of vortex images in the annular domain between two coaxial cylinders is solved by the q-elementary functions. We show that all images are determined completely as poles of the q-logarithmic function, and are located at sites of the q-lattice, where a dimensionless parameter q is given by the square ratio of the cylinder radii. Resulting solution for the complex potential is represented in terms of the Jackson q-exponential function. The Schottky-Klein prime function for annular domain, the Hamiltonian, the Kirchhoff-Routh and the Green functions are constructed. The uniformly rotating exact N-vortex polygon solutions with the rotation frequency expressed in terms of q-logarithms at N-th roots of unity are found. Motion of a single vortex with constant angular velocity, given as the q-harmonic series and vortex images in two particular geometries with only one cylinder as the $q \to \infty$ limit are studied in detail.

The second application is related with NLS hierarchy and information measures. Motivated by information theory we propose higher order information measures which include the Fisher measure as their first member. Then properly combining NLS hierarchy we introduce semi-relativistic Nonlinear Schrodinger equation

$$i\sigma_3 \left( \begin{array}{c} \psi \\ \bar{\psi} \end{array} \right)_t = mc^2 \sqrt{1 + \frac{1}{mc^2}\mathcal{R}^2} \left( \begin{array}{c} \psi \\ \bar{\psi} \end{array} \right)$$

where $\mathcal{R}$ is recursion operator. Non-relativistic expansion of this equation provides NLS and relativistic corrections to it, integrable at any order of $1/c^2$. For the relativistic quantum mechanics written in the rapidity variables we find another integrable model and its relation with the Wootters type measure. The AKNS linear problems for these equations are compactly written in terms of the operator q-number $[N]_q$ where $q = \mathcal{R}/p$.

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**Maxim Pavlov**

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### Universality of the Gibbons–Tsarev system

Three distinct integrable hydrodynamic chains are considered. They are not related to each other. Nevertheless, their hydrodynamic reductions are described by the unique Gibbons–Tsarev system. It means, that any solution of one such a hydrodynamic chain simultaneously can be recalculated into solutions of two other hydrodynamic chains.

Corresponding local Hamiltonian structures of these hydrodynamic chains are found.

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**Beatrice Pelloni**

*Department of Mathematics, University of Reading*
*Reading, UK*

### The elliptic sine-Gordon equation in the quarter plane

I will present preliminary results on the analysis of the elliptic sine-Gordon equation, $q_xx + q_yy = \sin q$. This equation in integrable, in the sense of admitting a Lax pair. However a solution requires also the characterisation of the Dirichlet to Neumann map. I follow the ideas of Fokas for the analysis of boundary value problems for the usual evolution integrable PDEs in one dimension, and adapt them to this case. I will review the solution of the modified Helmholtz equation, which is the linear limit of elliptic sine-Gordon, in the quarter plane, and then discuss the nonlinear problem.

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**Andrei K. Pogrebkov**

*Steklov Mathematical Institute*
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### Algebraic origins of integrability of nonlinear evolution equations

Commutator identities on associative algebras generate solutions of linearized versions of integrable equations. We realize elements of associative algebra as operators in the sense of the resolvent approach to IST. In this class of integral operators we impose some generic conditions on the representation and develop a dressing procedure that enables to derive both: nonlinear integrable equation itself and associated Lax pair if the commutator identity is given. Thus problem of construction of new integrable nonlinear evolution equations is reduced to the problem of construction of commutator identities on associative algebras. Different examples of this approach that involves (1+1) and (2+1) nonlinear differential equations, as well as difference-differential equations, are considered.

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**Barbara Prinari**

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**Inverse scattering transform for the vector NLS equation with non-vanishing boundary conditions**

The inverse scattering transform for the vector defocusing vector nonlinear Schrödinger NLS equation with non-vanishing boundary values at infinity is constructed. The direct scattering problem is formulated on a two-sheeted covering of the complex plane. In the two component case, two out of the six scattering eigenfunctions do not admit an analytic extension on either sheet of the surface. Two additional analytic solutions are constructed by considering *adjoint* eigenfunctions. The discrete spectrum, bound states and symmetries of the direct problem are discussed. In general a discrete eigenvalue corresponds to a quartet of zeros (poles) of certain scattering data. The inverse scattering problem is formulated in terms of a Riemann-Hilbert (RH) problem in the upper/lower half planes of a suitable uniformization variable. Special soliton solutions, which have dark solitonic behavior in both components and ones which have one dark and one bright component are constructed from the poles in the RH problem. The generalization to vector NLS with an arbitrary number of components will also be discussed.

Joint work with: M.J. Ablowitz, G. Biondini and A.D. Trubatch.

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**Zhijun (George) Qiao**

*University of Texas Pan American*
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**Integrable peakon equations**

In my talk, I will present some integrable equations that have peaked solitons, cusp solitons, and M-shape/W-shape peak solitons. These nonlinear equations are shown integrable through Lax representations.

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**Andrea Raimondo**

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**Hamiltonian Structure of Reductions of the Benney System**

We show how to construct the Hamiltonian structures of any reduction of the Benney chain (dispersionless Kadomtsev-Petviashvili) starting from the family of conformal maps associated to it. This is a joint work with John Gibbons and Paolo Lorenzoni.

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**Colin Rogers**

*School of Mathematics - University of NSW*
*Sidney, Australia*

### On the kinematics of the 2+1-dimensional motion of a fibre-reinforced fluid. An integrable connection

A class of 2+1-dimensional motions under a condition of constant divergence is linked to the scattering problem for the integrable modified Korteweg -de Vries hierarchy.This result is applied to a set of kinematic relations which arise in the theory of ideal fibre-reinforced fluids.In particular,it is established that the fibres constitute generalised tractrices.

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### On infinite conservation laws for systems of differential equations

We consider Lagrangian systems of partial differential equations admitting infinite-dimensional Lie symmetry algebras parametrized by arbitrary functions of dependent variables and their derivatives.

Infinite symmetry algebras with arbitrary functions of independent variables were demonstrated in a number of examples to lead to a finite number of local conservation laws, where each conservation law is determined by a specific form of boundary conditions.

The present work is an extension of the approach for scalar differential equations with infinite symmetry algebras that were shown to lead to infinite conservation laws (V.Rosenhaus ”On infinite set of conservation laws for infinite symmetries”, Theor. Math. Phys. 151, 869-878 (2007)).

We will show that unlike differential systems with arbitrary functions of independent variables, our systems possess infinite sets of local conservation laws. We will discuss conditions for the existence of infinite conservation laws as well as classes and examples of such systems.

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**Paolo Maria Santini**

*Dipartimento di Fisica - Unibersità “La Sapienza”*
*Roma, Italy*

### Commuting vector fields, integrable multidimensional PDEs and the analytic description of the gradient catastrophe of 2D water waves near the shore

We make use of the recently developed Spectral Transform for one-parameter families of commuting vector fields to study the dynamics of localized waves evolving according to the heavenly equation of Plebansky, describing self-dual Einstein fields, and to the dispersionless Kadomtsev-Petviashvili (dKP) equation, describing the evolution of two-dimensional shallow water waves near the shore. In particular, in the dKP case, we obtain the exact analytic description of the gradient catastrophe of 2D water waves near the shore.

Joint work with S. V. Manakov.

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**Giuseppe Scolarici**

*Dipartimento di Fisica - Università del Salento*
*Lecce, Italy*

**Bi-hamiltonian descriptions for composite quantum systems**

We discuss bi-hamiltonian quantum descriptions when composite systems and interaction among them are considered. Some examples are also exhibited.

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**Artur Sergyeyev**

*Mathematical Institute, Silesian University in Opava*
*Opava, Czech Republic*

**Integrable systems in (1+1) dimensions: nonlocal recursion operators vs. local symmetries**

We present simple and universal sufficient conditions for a weakly nonlocal recursion operator in (1+1) dimensions to generate a hierarchy of local symmetries. These conditions are satisfied by the majority of known today recursion operators and are less restrictive than those found in the earlier work on the subject.

Using this result we prove, under some natural assumptions, the MaltsevNovikov conjecture stating that higher Hamiltonian, symplectic and recursion operators of integrable systems in (1+1) dimensions are weakly nonlocal, i.e., the coefficients of these operators are local and these operators contain at most one integration operator in each term.

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**Silvana De Lillo and Matteo Sommacal**

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*Perugia, Italy*

**An analytical model for synthetic- and bio- polymers based on continuous elastic rods: circular helix solutions**

The study of elastic deformations in thin rods has recently seen renewed interest due to the close connection between these systems and coarse-grained models of widespread application in life- and material- sciences. Until now, the analysis has been restricted to the solution of equilibrium equations for continuous models characterized by constant bending and twisting elastic moduli and/or by isotropic rod section. However, more realistic models often require more general conditions: indeed this is the case whenever microscopic information issuing from atomistic simulations is to be transferred to analytic or semi-analytic coarse-grained or macroscopic models. In this paper we will show that integrable, indeed solvable, equations are obtained under quite general conditions and that regular (e.g. circular helical) solutions merge from reasonable choices of elastic stiffnesses.

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**Craig M. Sooman**

*University of Glasgow, Department of Mathematics*
*Glasgow, UK*

**Quasideterminant solutions of a noncommutative mKP equation**

In this talk, I will present two families of solutions of a noncommutative mKP (ncmKP) equation which can be expressed as quasideterminants. I will show that these solutions originate from Darboux and binary Darboux

transformations. As an example, we will look at a matrix version of the ncmKP equation. This is based on joint work with Claire Gilson and Jon Nimmo.

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**Sandro Sozzo**

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*Lecce, Italy*

**Generalized Observables, Heisenberg's Inequalities and the Measurement Process in the ESR Model**

The *extended semantic realism* (*ESR*) model recently worked out in Lecce [1-4] embodies the mathematical formalism of standard quantum mechanics (SQM) into a general framework in which *objectivity of physical properties* holds, which avoids the quantum measurement problem and some quantum paradoxes, and reinterprets quantum probabilities as *conditional* instead of *absolute*. As a consequence, the ESR model provides some predictions that differ from those of SQM and are, at least in principle, falsifiable [3,5]. The ESR model thus constitutes a new theoretical scheme that opens the way towards the construction of a new theory, possibly nonlinear, going beyond SQM and saying more than it. We show here that the expectation values and the root mean squares of the observables of the model (*generalized observables*) depend on *detection probabilities* that are not predicted by SQM, which implies, in particular, that some relations on joint measurements of compatible observables that hold in SQM are not necessarily satisfied in the model. Moreover, *Heisenberg's uncertainty relations* must be replaced by modified relations in the ESR model, which sheds new light on the possibility of performing joint measurements of noncompatible observables, already suggested by some scholars in the framework of unsharp QM [6-8]. Finally, we propose a mathematical representation of generalized observables in the ESR model that uses the Hilbert space formalism but is different from the standard representation and from its unsharp generalizations [9], and provide a formal treatment of the measuring process within the new perspective.

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**Piergiulio Tempesta**

*Universidad Complutense*
*Madrid, Spain*

**Formal Groups, Integrable Systems and L-series**

In the last years, new connections emerged between mathematical physics and number theory. In this talk, I will discuss how relevant number theoretical structures, i.e. L-series and Bernoulli-type polynomials, are related to symmetry preserving discretizations of integrable systems as well as to Toda-type systems. A major role in the construction I propose is played by the so called delta operators, introduced by G. C. Rota, which in turn are connected with the formal groups of algebraic topology.

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**Takayuki Tsuchida**

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**Integrability of vector derivative NLS-type systems**

A complete list of vector derivative nonlinear Schrödinger-type systems possessing a higher symmetry of third order was presented by V. V. Sokolov and T. Wolf [J. Phys. A: Math. Gen. **34** (2001) 11139–48]. The list consists of the following six systems ($\alpha$ and $\beta$ are constants):

$$\begin{cases} U_t = U_{xx} + 2\alpha\langle U,V\rangle U_x + 2\alpha\langle U,V_x\rangle U - \alpha\beta\langle U,V\rangle^2 U, \\ V_t = -V_{xx} + 2\beta\langle V,U\rangle V_x + 2\beta\langle V,U_x\rangle V + \alpha\beta\langle V,U\rangle^2 V, \end{cases} \tag{1}$$

$$\begin{cases} U_t = U_{xx} + 2\alpha\langle U,V\rangle U_x + 2\beta\langle U,V_x\rangle U + \beta(\alpha-2\beta)\langle U,V\rangle^2 U, \\ V_t = -V_{xx} + 2\alpha\langle V,U\rangle V_x + 2\beta\langle V,U_x\rangle V - \beta(\alpha-2\beta)\langle V,U\rangle^2 V, \end{cases} \tag{2}$$

$$\begin{cases} U_t = U_{xx} + 2\alpha\langle U,V\rangle U_x + 2\beta\langle U,V_x\rangle U + 2(\beta-\alpha)\langle U_x,V\rangle U \\ \qquad - \alpha\beta\langle U,V\rangle^2 U, \\ V_t = -V_{xx} + 2\alpha\langle V,U\rangle V_x + 2\alpha\langle V,U_x\rangle V + \alpha\beta\langle V,U\rangle^2 V, \end{cases} \tag{3}$$

$$\begin{cases} U_t = U_{xx} + 2\alpha\langle U,V\rangle U_x + 2\alpha\langle U,V_x\rangle U + 2\beta\langle U_x,V\rangle U \\ \qquad - \alpha(\alpha-\beta)\langle U,V\rangle^2 U, \\ V_t = -V_{xx} + 2\alpha\langle V,U\rangle V_x + 2\alpha\langle V,U_x\rangle V + 2\beta\langle V_x,U\rangle V \\ \qquad + \alpha(\alpha-\beta)\langle V,U\rangle^2 V, \end{cases} \tag{4}$$

$$\begin{cases} U_t = U_{xx} + 4\alpha\langle U,V\rangle U_x + 2(\alpha-\beta)\langle U,U\rangle V_x + 4\beta\langle U,V_x\rangle U \\ \qquad + 4\beta(\alpha-2\beta)\langle U,V\rangle^2 U - 2\beta(\alpha-\beta)\langle U,U\rangle\langle V,V\rangle U \\ \qquad - 4\beta(\alpha-\beta)\langle U,U\rangle\langle U,V\rangle V, \\ V_t = -V_{xx} + 4\alpha\langle V,U\rangle V_x + 2(\alpha-\beta)\langle V,V\rangle U_x + 4\beta\langle V,U_x\rangle V \\ \qquad - 4\beta(\alpha-2\beta)\langle V,U\rangle^2 V + 2\beta(\alpha-\beta)\langle V,V\rangle\langle U,U\rangle V \\ \qquad + 4\beta(\alpha-\beta)\langle V,V\rangle\langle V,U\rangle U, \end{cases} \tag{5}$$

$$\begin{cases} U_t = U_{xx} + 4\alpha\langle U,V\rangle U_x - 2\beta\langle U,U\rangle V_x + 4\alpha\langle U,V_x\rangle U \\ \qquad + 4\beta\langle U_x,V\rangle U - 4\beta\langle U,U_x\rangle V - 4\alpha(\alpha-\beta)\langle U,V\rangle^2 U \\ \qquad + 6\beta(\alpha-\beta)\langle U,U\rangle\langle V,V\rangle U - 4\beta(\alpha-\beta)\langle U,U\rangle\langle U,V\rangle V, \\ V_t = -V_{xx} + 4\alpha\langle V,U\rangle V_x - 2\beta\langle V,V\rangle U_x + 4\alpha\langle V,U_x\rangle V \\ \qquad + 4\beta\langle V_x,U\rangle V - 4\beta\langle V,V_x\rangle U + 4\alpha(\alpha-\beta)\langle V,U\rangle^2 V \\ \qquad - 6\beta(\alpha-\beta)\langle U,U\rangle\langle V,V\rangle V + 4\beta(\alpha-\beta)\langle V,V\rangle\langle V,U\rangle U. \end{cases} \tag{6}$$

We demonstrate that the two systems (1) and (3) are linearizable by a change of variables, while the other four systems (2), (4), (5) and (6) admit a Lax representation and can be solved by the inverse scattering method.

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**Vasyl Tyberkevych**

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**Stochastic theory of nonlinear auto-oscillator: Spin-torque nano-oscillator**

The universal "nonlinear auto-oscillator model" is used to study the stochastic dynamics of a generic auto-oscillatory system under the action of thermal fluctuations. The Fokker-Planck equation, that physically correctly describes stochastic dynamics of an arbitrary auto-oscillator, is derived. The stationary and non-stationary solutions of the derived Fokker-Planck equation are analyzed, yielding expressions for the averaged generated power, level of power and phase fluctuations, and generation linewidth of the auto-oscillator. The obtained general results are applied to the case of spin-torque oscillators - a novel class of microwave nano-oscillators, - and are compared to experimental data.

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**Vadim L. Vereschagin**

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**Generalized discrete Toda lattice**

We examine system of fully discrete "hyperbolic" equations

$$u_{i+1,j+1} = f(u_{i+1,j}, u_{i,j+1}, u_{i,j}, u_{i,j-1}),$$

where $i, j$ are integer indices, $u_{ij}$ is a vector of $n$ unknown sequences, $f$ is a vector of $n$ known functions. The problem is to find conditions for so-called C-integrability, or Darboux integrability which means existence of sufficient number of i- and j-integrals for initial system. The integrals are $I(u)$ such that $(T_i - 1)I(u) = 0$, ($T_i$ is shift in i) and the same for J: $(T_j - 1)J(u) = 0$. Conditions of C-integrability for linear systems were written out explicitly in the form of proven statements and were illustrated by appropriate examples. For the systems equality of generalized Laplace invariant to zero is the condition of integrability. System is called "C-integrable" if its lattice of generalized Laplace invariants breaks at both directions. For nonlinear systems the concept of C-integrability should be understood as C-integrability for appropriate "linearized" equation. There is a procedure for constructing complete set of integrals for those systems. Examples of C-integrable nonlinear models (discrete analog of the Liouville equation included) are shown.

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**Alexander Vinogradov**

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**What are really tensors?**

Exterior (de Rham) differential connects differential forms, i.e., skew-symmetric covariant tensors, while such a natural operator does not exist for symmetric covariant tensors. Why? The Levi-Civita connection accompanies a metric, i.e., a nondegenerate symmetric covariant 2-tensor. But this is no longer so for nondegenerate skew-symmetric covariant 2-tensors, i.e., 2-forms, etc. The answer on these and many similar questions can not be done on the basis of the standard approach, which is descriptive by its nature. A conceptual definition of tensors can be found in differential calculus over commutative algebras. Some consequences of this fact in general relativity and mechanics will be shown.

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### Madelung Fluid Description of gNLS Equation Special Solutions and their Stability

A Madelung's fluid approach is used to discuss the solitary wave solutions of an generalized nonlinear Schrödinger (gNLS) equation

$$i\frac{\partial \Psi}{\partial t} + \frac{1}{2}\frac{\partial^2 \Psi}{\partial x^2} + 2\Psi^2\frac{\partial \Psi^*}{\partial x} + \alpha|\Psi|^{4p}\Psi = 0$$

For $p = 2$ it is a completely integrable system. Remarks on the stability of the bright solitons are given.

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### The Cauchy problem of the Ward equation

The Ward equation (or the modified $2 + 1$ chiral model)

$$\partial_t \left(J^{-1}\partial_t J\right) - \partial_x \left(J^{-1}\partial_x J\right) - \partial_y \left(J^{-1}\partial_y J\right) - \left[J^{-1}\partial_t J, J^{-1}\partial_y J\right] = 0,$$

for $J : \mathbb{R}^{2,1} \to SU(n)$, $\partial_w = \partial/\partial w$, is obtained from a dimension reduction and a gauge fixing of the self-dual Yang-Mills equation on $\mathbb{R}^{2,2}$. It is an integrable system which possesses the Lax pair

$$\left[\lambda\partial_x - \partial_\xi - J^{-1}\partial_\xi J, \lambda\partial_\eta - \partial_x - J^{-1}\partial_x J\right] = 0$$

with $\xi = \frac{t+y}{2}$, $\eta = \frac{t-y}{2}$. Using the inverse scattering method and the theory of Backlund transformation, we solve the Cauchy problem of the Ward equation with both continuous

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