Nuclear Physics School “Raimondo Anni”, 5th course
Otranto, May 30 – June 4, 2011

The Physics of Neutron Star Interiors

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Plan of the lectures

1. The Pulsars
2. Neutron Stars' Structure
3. Quark Matter in neutron stars: astrophysical implications and possible signatures

Bibliography

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- S.L. Shapiro and S.A. Teukolsky, Black Holes, White Dwarfs, and Neutron Stars, Wiley & Sons 1983
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The Physics of Neutron Star Interiors

1st Lecture

Pulsars

1st Lecture: Pulsars (PSRs)

- **The basic observational properties of PSRs**

- **Pulsars as magnetized rotating Neutron Stars**
  The magnetic dipole model for PSRs

Pulsars (PSRs) **are astrophysical sources which emit** **periodic pulses of electromagnetic radiation.**

Number of known pulsars:

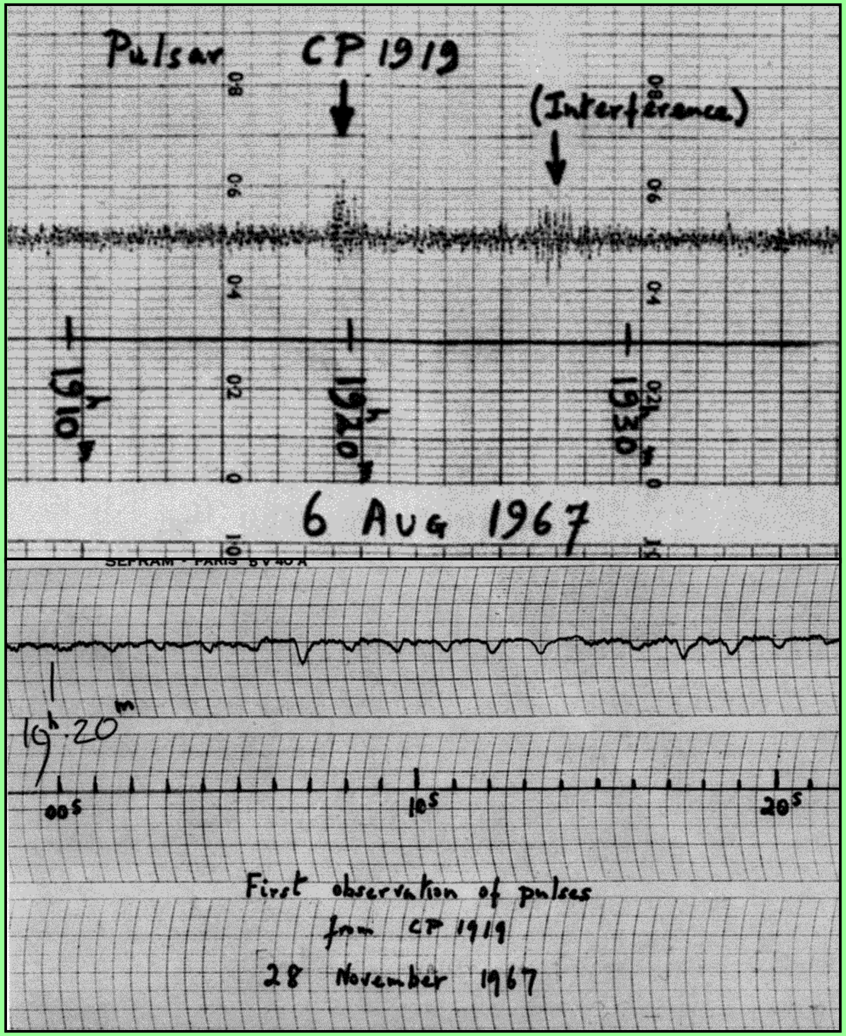
~ 1900	**Radio PSRs**	
~ 40	**X-ray PSRs**	(radio-quiet)
~ 60	**γ-ray PSR**	(most recent. discov. by **LAT/Fermi)**

1st discovered pulsar: PSR B1919 +21

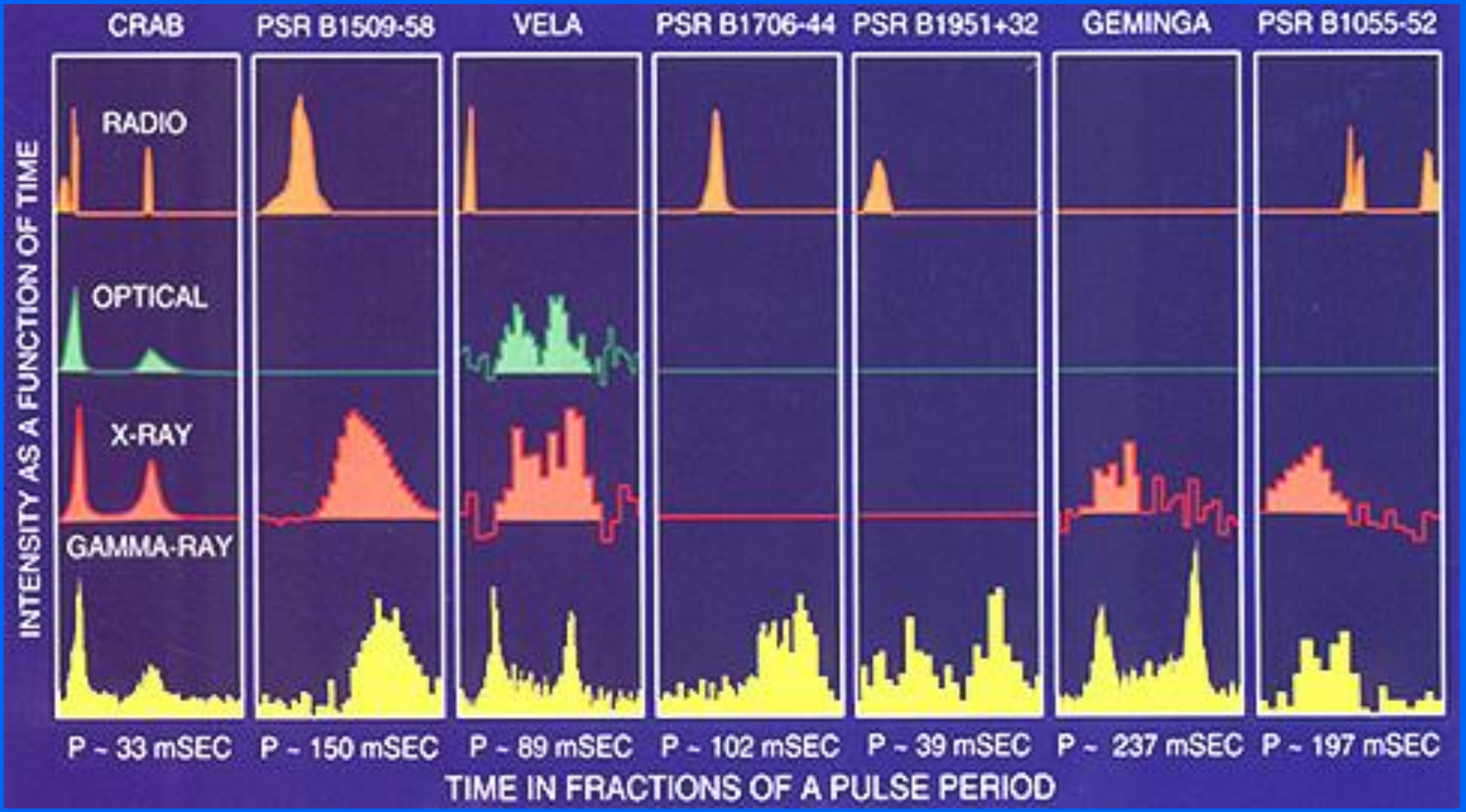
radio pulsar at **81.5 MHz** Pulse period **P = 1.337 s**

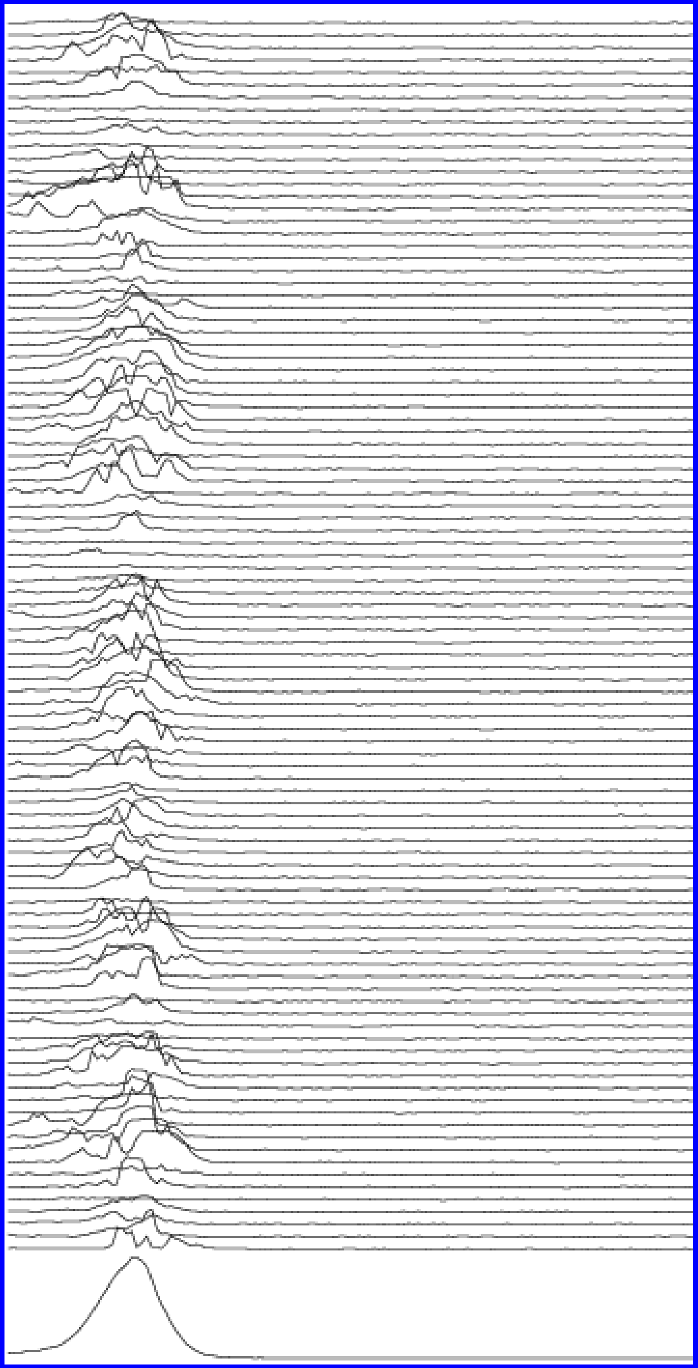
Hewish et al., 1968, Nature 217

Tony Hewish and Jocelyn Bell
(Bonn, August 1980)



Pulse shape at different wavelength





Top: 100 **single pulses** from the pulsar B0950+08 (P = 0.253 s), demonstrating the pulse-to-pulse variability in shape and intensity.

Bottom: **Cumulative profile** for this pulsar over 5 minutes (about 1200 pulses).
This averaged **“standard profile”** is reproducible for a given pulsar at a given frequency.

The large noise which masks the “true” pulse shape is due to the interaction of the pulsar elettro-magnetic radiation with the ionized **interstellar medium (ISM)**

Observations taken with the Green Bank Telescope (Stairs et al. 2003)

The Arecibo Radio Telescope

d = 304.8 m

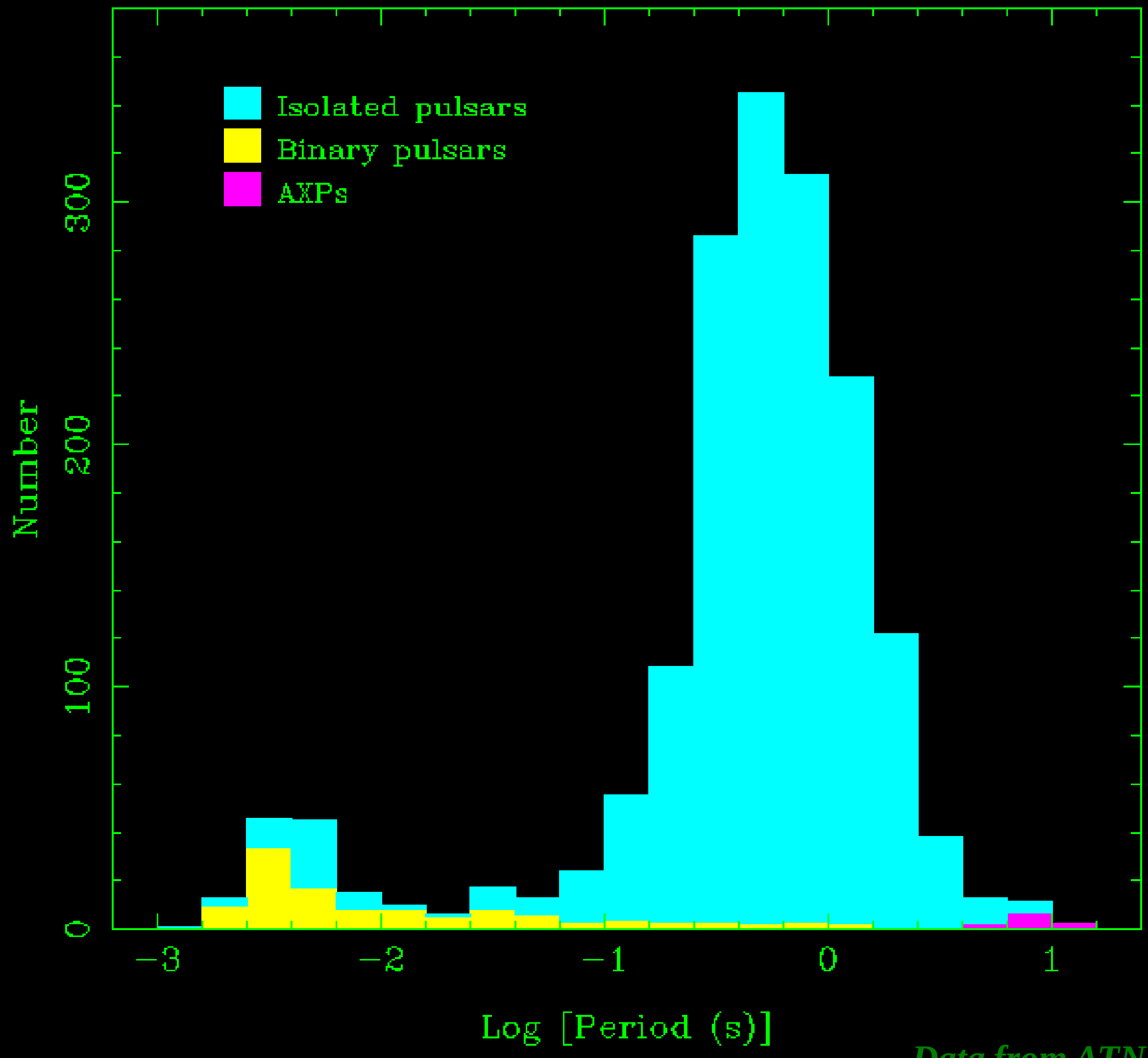
The Parkes Radio Telescope

d = 64 m

The Green Bank Radio Telescope

Pulsar Period Distribution

~ 10^{-3} seconds < P < *a few* seconds



Data from ATNF Pulsar Catalogue, V1.25

The “fastest” Pulsar”

PSR J1748 –2446ad (in the globular cluster Terzan 5)

P = 1.39595482(6) ms *i.e.* **ν = 716.3 Hz** **Fa# (F#)**

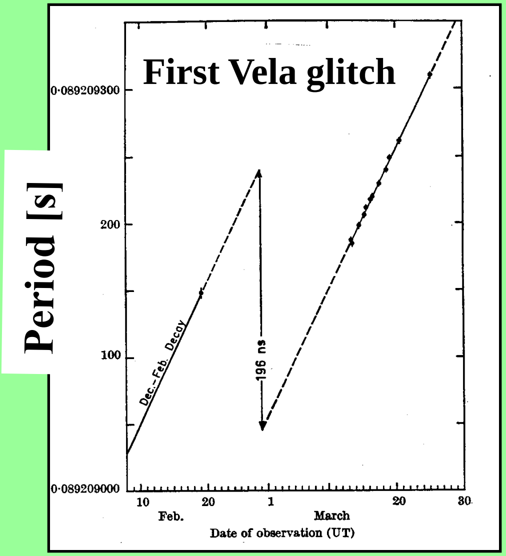
J.W.T. Hessel *et al*., march 2006, Science 311, 1901

PSR mame	**frequncy (Hz)**	**Period (ms)**
J1748 –2464ad	716.358	1.3959
B1937 +21	641.931	1.5578
B1957 +20	622.123	1.6074
J1748 –24460	596.435	1.6766

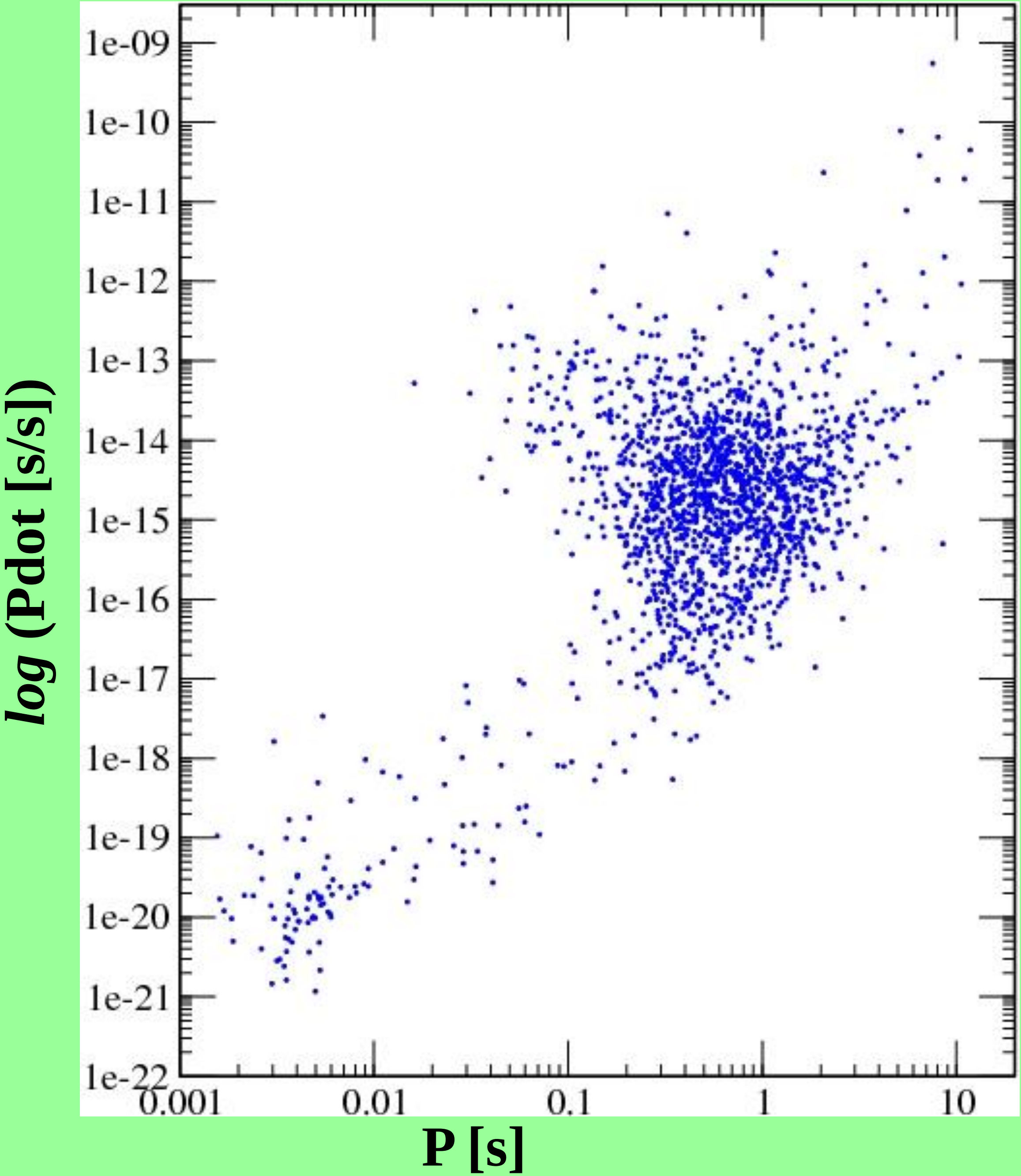
- **PSRs are remarkable astronomical clocks**
  extraordinary stability of the pulse period:
  P(sec.) can be measured up to 18 significant digits!
  e.g. on Jan 16, 1999, **PSR J0437-4715** had a period of:
  5.757451831072007 ± 0.000000000000008 ms

- **Pulsar periods always (*) increase very slowly**

$$\dot{P} \equiv dP/dt = 10^{-21} — 10^{-10} \text{ s/s} = 10^{-14} — 10^{-3} \text{ s/yr}$$



(*) except in the case of PSR "**glitches**", or **spin-up** due to **mass accretion**



Pulsars distribution in the P- Pdot plane

Data from: **ATNF Pulsar Catalogue** 1704 PSRs (Apr. 2011)

What is the nature of pulsars?

Due to the extraordinary stability of the pulse period the different parts of the source must be connected by **causality condition**

$$R_{source} \leq c\ P \sim 9900 \text{ km} \qquad (P_{crab} = 0.033 \text{ s})$$

Pulsars are compact stars

White Dwarfs ?

or

Neutron Stars ?

A famous whithe dwarf, **Sirius B**: $R = 0.0074\ R_{\odot} = 5150$ km

What is the nature of pulsars?

Due to the extraordinary stability of the pulse period the different parts of the source must be connected by **causality condition**

$$R_{source} \leq c\ P \sim 9900\ km \qquad (P_{crab} = 0.033\ s)$$

$$R_{source} \leq 450\ km \qquad (P \sim 1.5\ ms)$$

PSR B1937+21 (P ~ 1.5 ms) discovered in 1982

Pulsars are compact stars

White Dwarfs ?

or

Neutron Stars ?

A famous whithe dwarf, **Sirius B**: $R = 0.0074\ R_{\odot} = 5150\ km$

Pulsars as rotating white dwarfs

Mass-shed limit.

For a particle at the equator of homogeneus uniformely rotating sphere

$$G\frac{M}{R^2} = \Omega_{\lim}^2 R$$

$$\Omega \le \Omega_{\lim} = \sqrt{G\frac{M}{R^3}} = \sqrt{\frac{4\pi}{3} G\rho_{av}}$$

$P \geq P_{lim} = 2\pi / \Omega_{lim}$ ~ 6 *s* (ρ_{av} ~ 3.4×10^6 g/cm^3, Sirius B)

Pulsars can not be rotating white dwarfs

Earth: $P_{\lim}$ = 84 min.
Neutron Star (M = 1.4 $M_\odot$, R = 10 km): $P_{\lim}$ ~ 0.5 ms

Pulsars as vibrating white dwarfs

WD models $\Rightarrow$ $P \geq P_{lim} \sim 2$ s

In the case of **damped oscillations**:

- Decreasing oscillation amplitude
- Constant period ($dP/dt = 0$)

For **PSRs** $dP/dt > 0$

Pulsars can not be vibrating white dwarfs

Pulsars as rotating Neutron Stars

The Neutron Star idea: (Baade and Zwicky, 1934)

"With all reserve we advance the view that supernovae represent the transition from ordinary stars into neutron stars, which in their final stages consist of extremely closely packed neutrons."

1st calculation of Neutron Star properties:
(Oppenheimer and Volkov, 1939)

Discovery of Pulsars (Hewish et al. 1967)

Interpretation of PSRs as rotating Neutron Strar:
(Pacini, 1967, Nature 216), (Gold, 1968, Nature 218)

The “fastest” Pulsar

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Terrestial fast spinning bodies

Centrifuge of a modern washing machine.

$\Omega \cong$ 1,800 round/min = 30 round/s

P = 0.0333 s

Engine Ferrari F2004 (F1 world champion 2004)

$\Omega \cong$ 19,000 round/min = 316.67 round/s

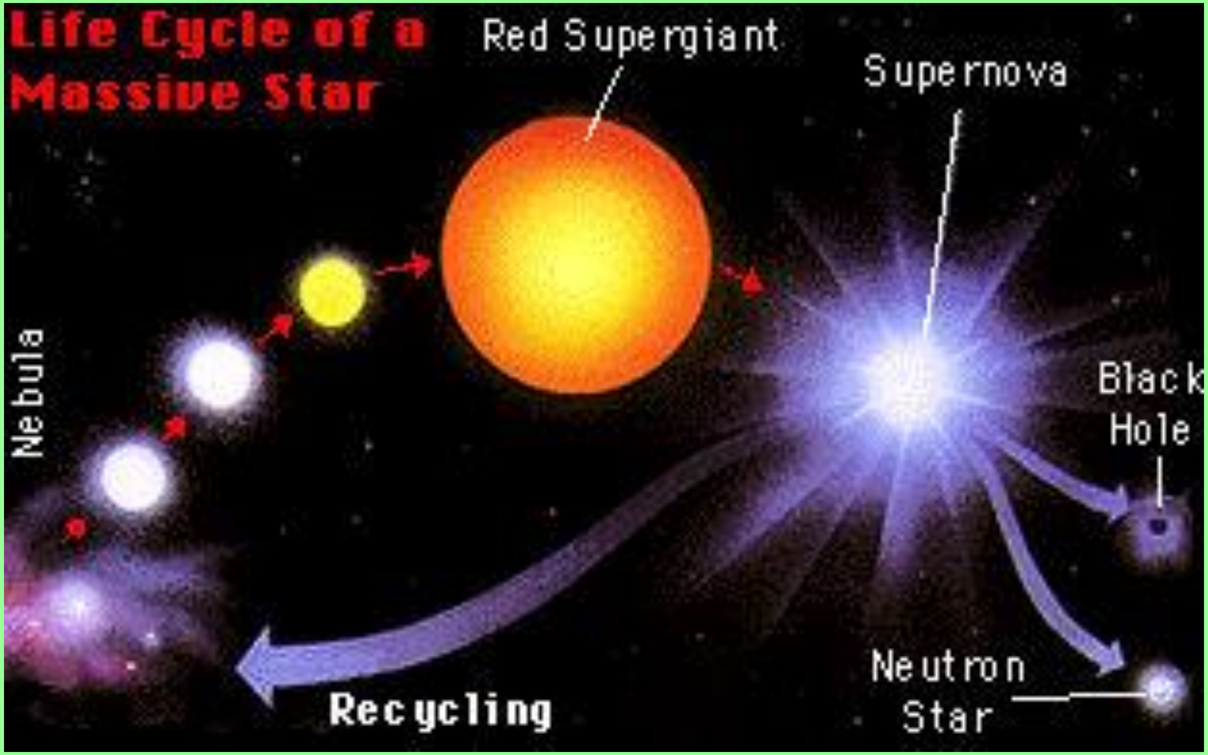
P = 3.158 ms

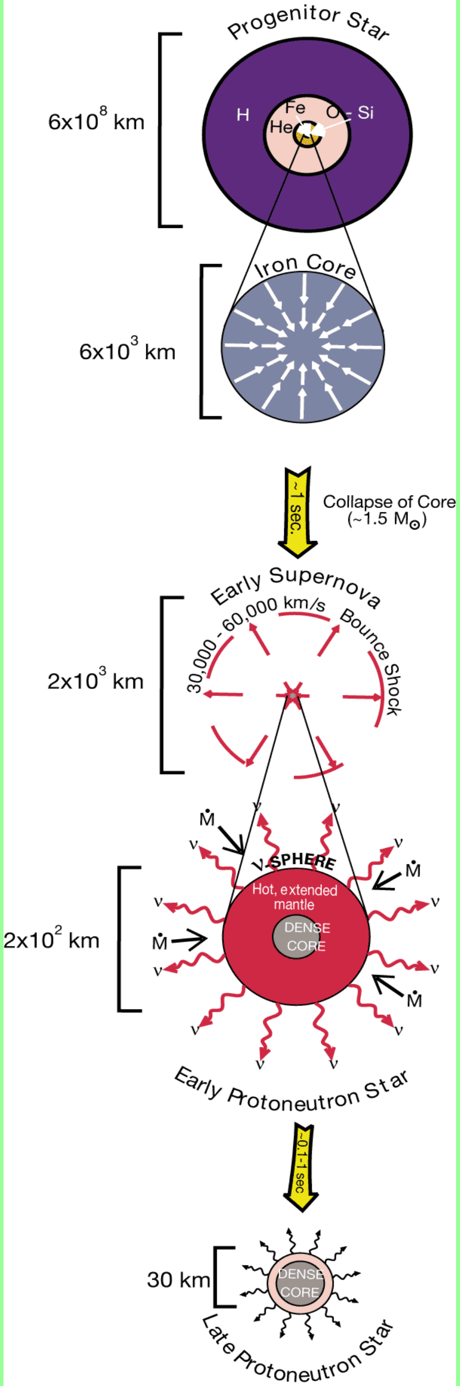
Ultracentrifuge (Optima L-100 XP, Beckman-Coulter)

$\Omega \cong$ 100,000 round/min = 1666.67 round/s

P = 0.6 ms

The birth of a Neutron Star





Neutron stars are the compact remnants of type II Supernova explosions, which occur at the end of the evolution of massive stars ($8 < M/M_\odot < 25$).

Supernova Classification

Type I

No H lines in the spectra

- Ia (strong Si)
- Ib (strong He)
- Ic (weak He)

Type II

H lines in the spectra

- II (normal: P- Cygni)
- II (peculiar:emission)

Thermonuclear explosions of white dwarfs (Ia)

Core collapse of massive stars (Ib, Ic, II)

“Historical” Supernovae

Table 1 Supernovae that have exploded in our Galaxy and the Large Magellanic Cloud within the last millennium

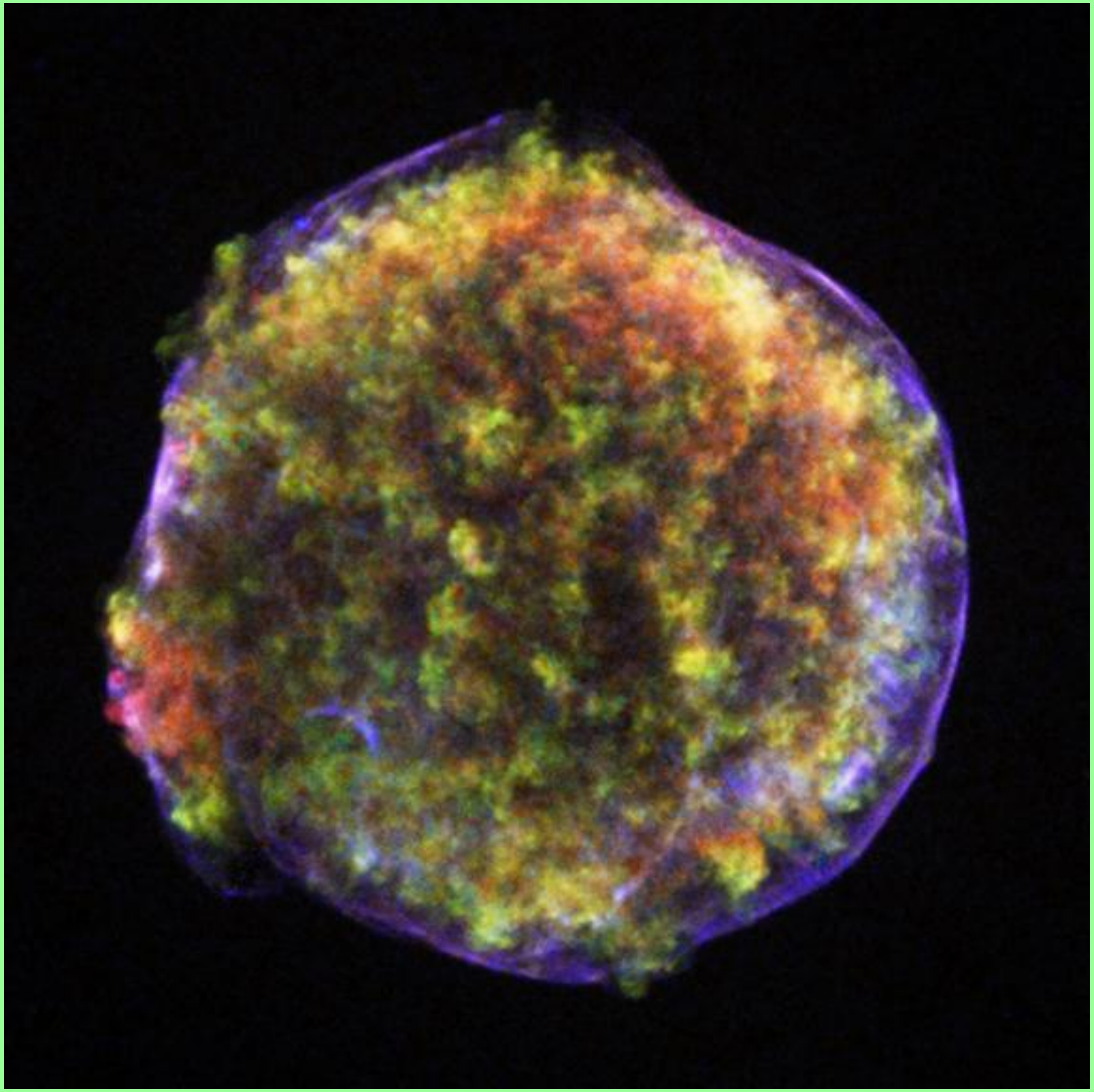
Supernova	Year (AD)	Distance (kpc)	Peak visual magnitude
SN1006	1006	2.0	−9.0
Crab	1054	2.2	−4.0
SN1181	1181	8.0	?
RX J0852-4642	~1300	~0.2	?
Tycho	1572	7.0	−4.0
Kepler	1604	10.0	−3.0
Cas A	~1680	3.4	~6.0?
SN1987A	1987	50 ± 5	3.0

New stars **(*guest stars*)** in the sky were considered by acient people as a possible signal for inauspicious events.

Aristotele – Ptolomy vision of the World
Supra-Lunar world: **perfect, incorruptible, immutable.**
new stars interpreted as **Sub-Lunar world** events

Tyco Brahe observed a ***new stars*** in the **Cassiopea constellation** in **1572** and using his **observational data** demonstrated that **the star was much farther that the Moon** **(T. Brahe, *De nova et nullius aevi memoria prius visa stella*, 1573)**

Tycho's Supernova Remnant

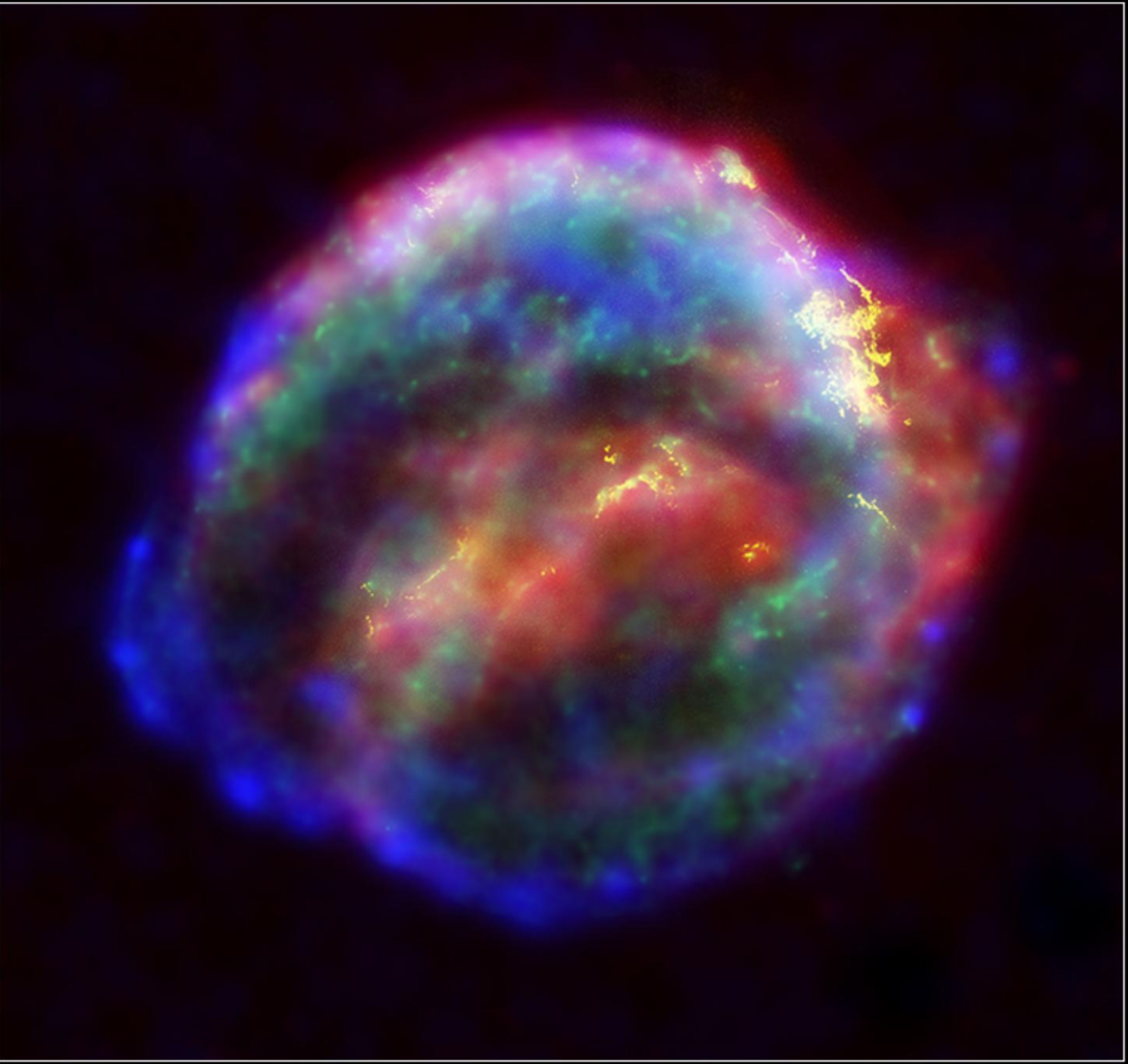


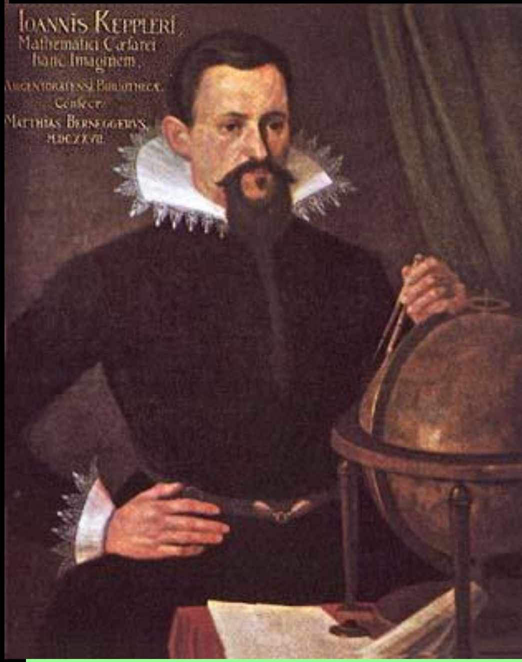
X-ray image (Chandra satellite, sept. 2005)

Supernova observed by Tycho Brahe in 1572

No central point source has been so far detected.**:**
Type Ia supernova

Kepler's supernova Remnant, SN1604

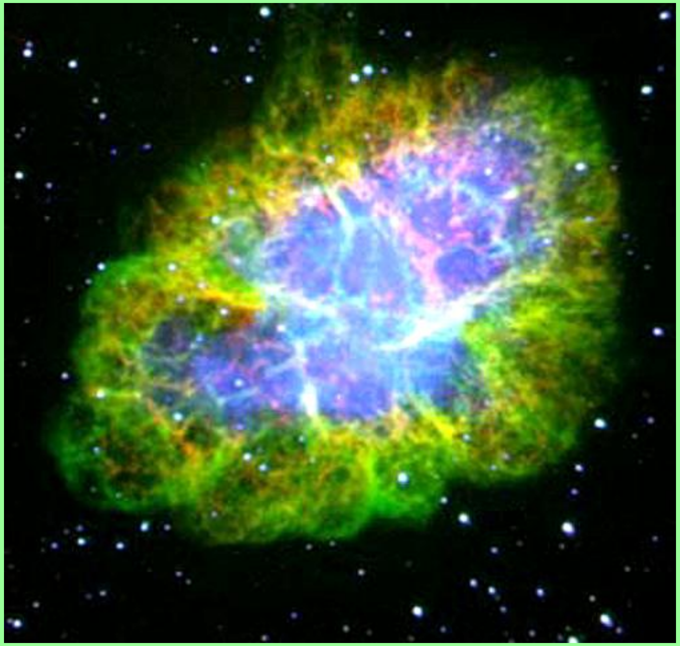


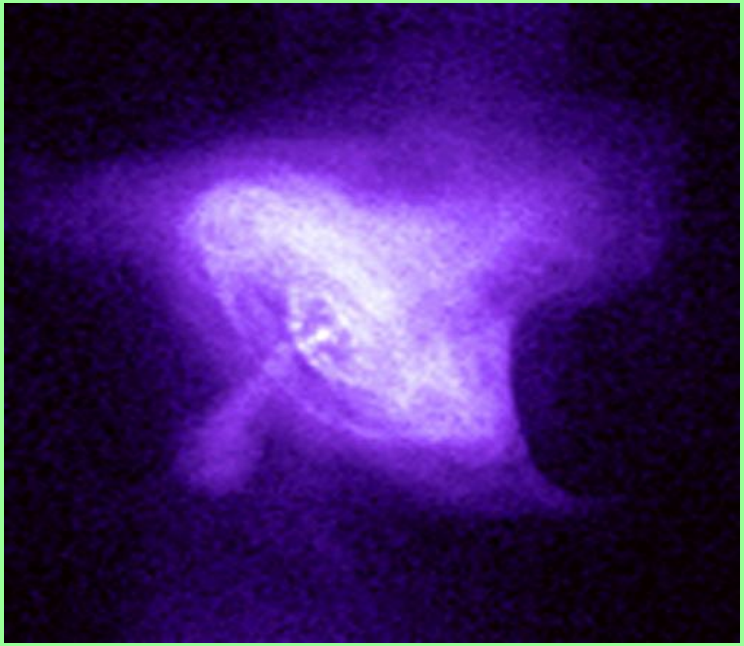


Supernova observed by Johannes Kepler in october 1604

Supernova type: unclear

The Crab Nebula





Optical (left) and X-ray (right) image of the Crab Nebula.

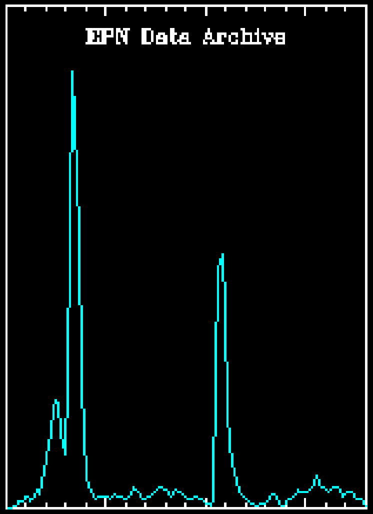
The Crab Nebula is the remnant of a supernova explosion that was seen on Earth in 1054 AD. Its distance to the Earth is 6000 lyr. At the center of the nebula is a pulsar which emits pulses of radiation with a period $P = 0.033$ seconds.

Multi wave lenght image of the Crab:

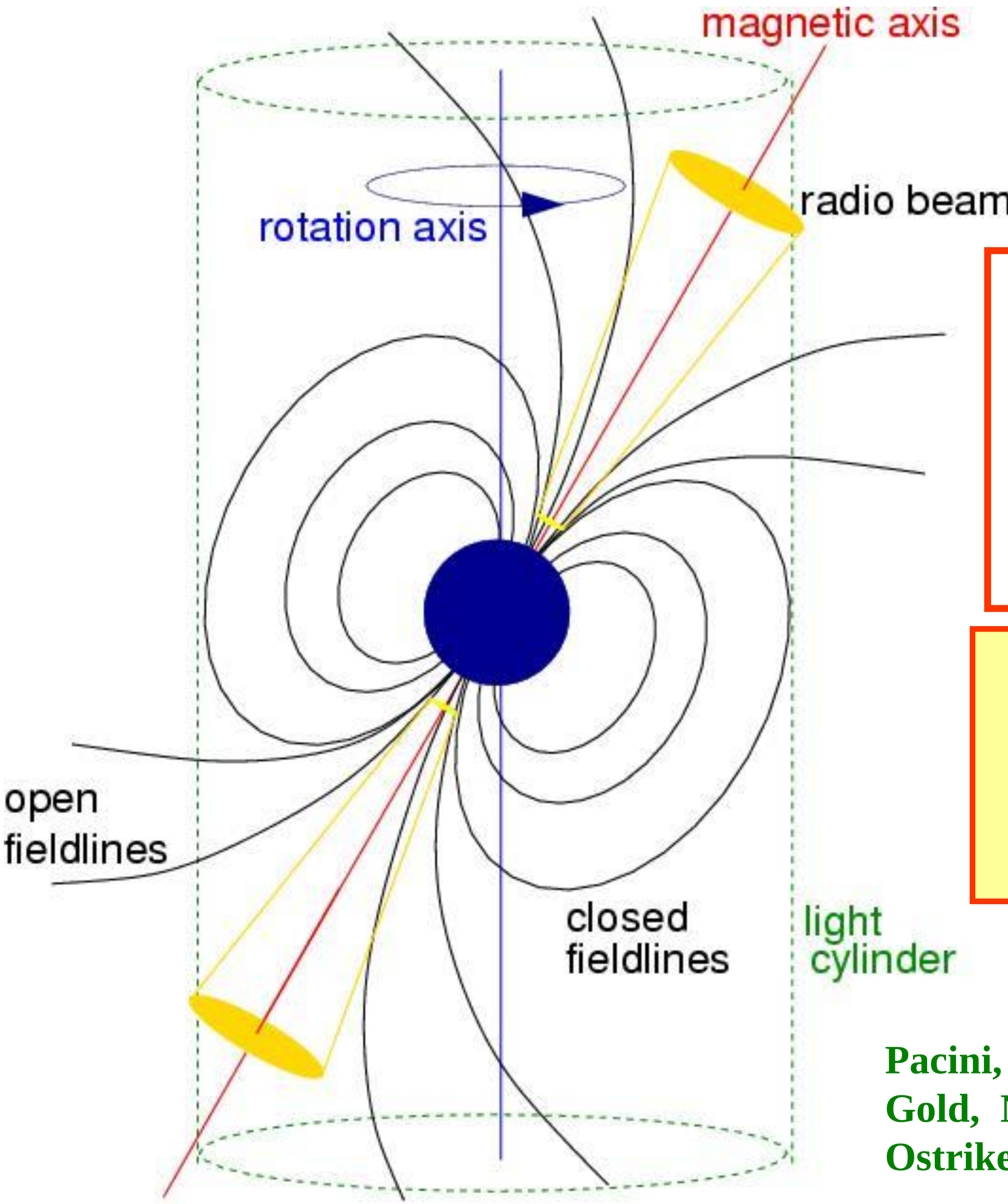
Blue: X-ray

Red: optical

Green: radio



The magnetic dipole model for pulsars



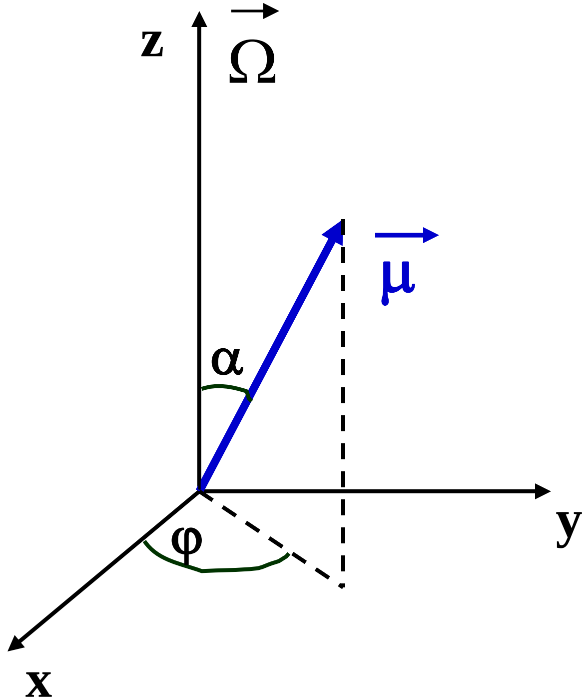
The lighthouse model

Pulsars are believed to be highly magnetized rotating Neutron Stars radiating at the expenses of their rotational energy

$$\dot{E}_{mag} = -\frac{2}{3c^3}\left|\ddot{\vec{\mu}}\right|^2$$

$\vec{\mu} \equiv$ **magnetic dipole moment**

Pacini, Nature 216 (1967), Nature 219 (1968)
Gold, Nature 218 (1968), Nature 221 (1969)
Ostriker and Gunn, ApJ 157 (1969)

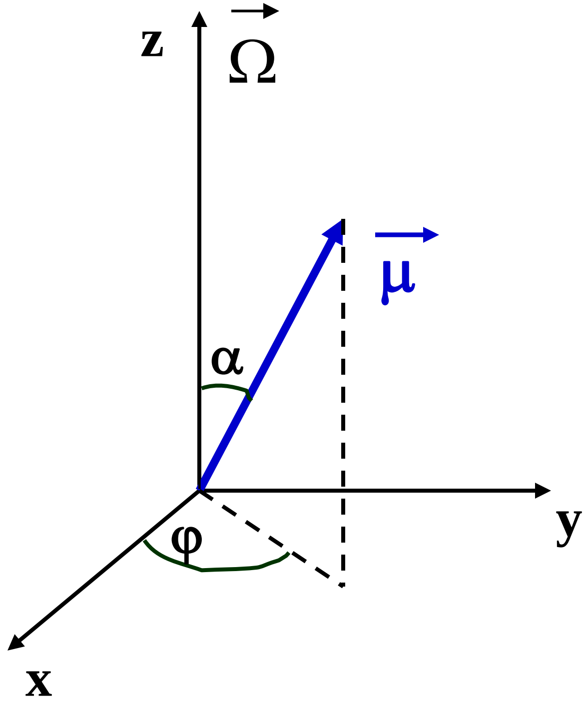


Suppose: α = const, $\mu \equiv |\vec{\mu}|$ = const

$$\Omega = \frac{d\varphi}{dt} \equiv \dot{\varphi}$$

$$\vec{\mu} = \mu\ \sin\alpha\cos\varphi\ \vec{e}_x + \mu\ \sin\alpha\sin\varphi\ \vec{e}_y + \mu\ \cos\alpha\ \vec{e}_z$$

Next one calculates $\dot{\vec{\mu}} = \frac{d}{dt}\vec{\mu}$ and $\ddot{\vec{\mu}}$



Suppose: $\alpha = \text{const}$, $\mu \equiv |\vec{\mu}| = \text{const}$

$$\left|\ddot{\vec{\mu}}\right|^2 = \mu^2 \sin^2\alpha \left(\Omega^4 + \dot{\Omega}^2\right)$$

$$\dot{\Omega}^2 \ll \Omega^4$$

$$\left|\ddot{\vec{\mu}}\right|^2 \approx \mu^2 \sin^2\alpha \cdot \Omega^4$$

$$\dot{E}_{mag} = -\frac{2}{3c^3}\mu^2 (\sin\alpha)^2 \Omega^4$$

For a **sphere** with a **pure magnetic dipole field:**

$$\mu = (1/2)\, B_p\, R^3$$

B_p = magnetic fiels at the poles,
R = radius of the sphere

The magnetic dipole model for pulsars

$$\dot{E}_{mag} = -\frac{1}{6c^3} R^6 B_p^2 \sin^2\alpha \, \Omega^4$$

Rotational kinetic energy

$$E_{rot} = \frac{1}{2} I \Omega^2 \quad \xrightarrow{\dot{I}=0} \quad \dot{E}_{rot} = I\Omega\dot{\Omega}$$

Energy rate balance:

$$\dot{E}_{rot} = \dot{E}_{mag}$$

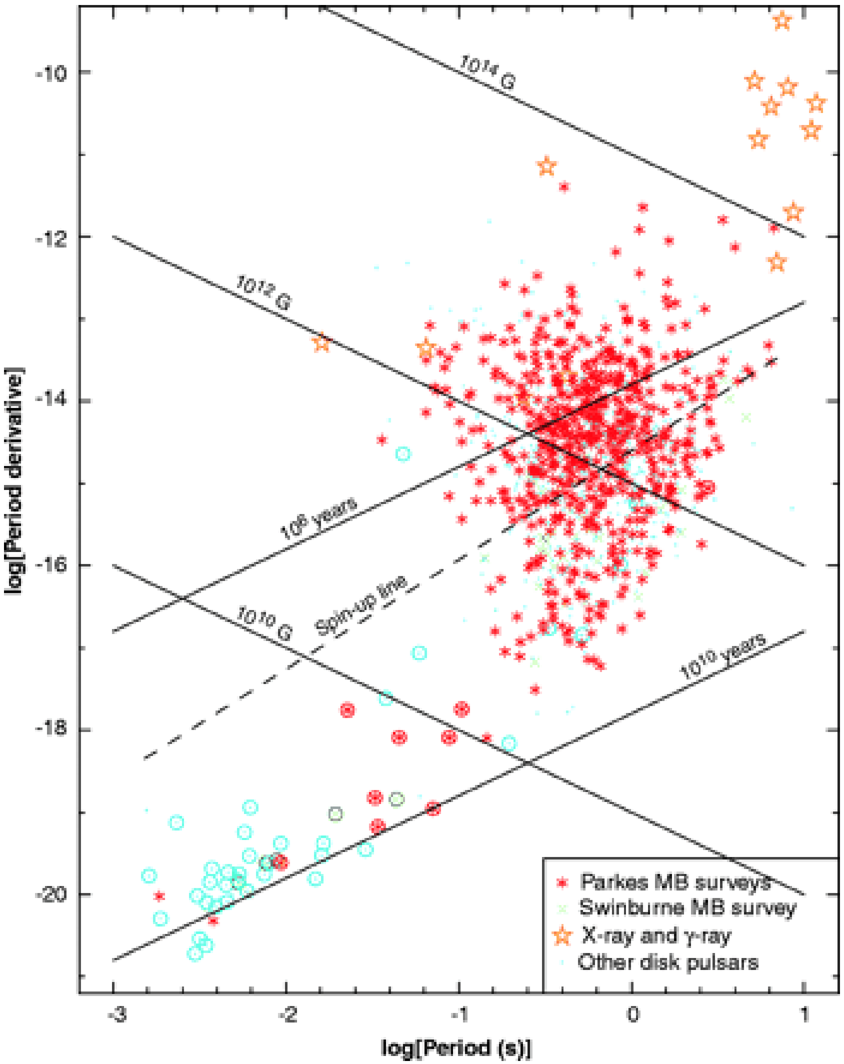
$$\dot{\Omega} = -K\Omega^3$$

$$P\dot{P} = (2\pi)^2 K$$

$$K \equiv \frac{1}{6c^3} \frac{R^6}{I} (B_p \sin\alpha)^2$$

Distribution of PSRs on the $P - \dot{P}$ plane

log(dP/dt)



log(P[sec.])

$$B_{\perp} = \frac{\sqrt{6c^3}}{2\pi}\frac{I^{1/2}}{R^3}\left(P\dot{P}\right)^{1/2} =$$

$$= 3.2\times10^{19}\left(P\dot{P}\right)^{1/2} Gauss$$

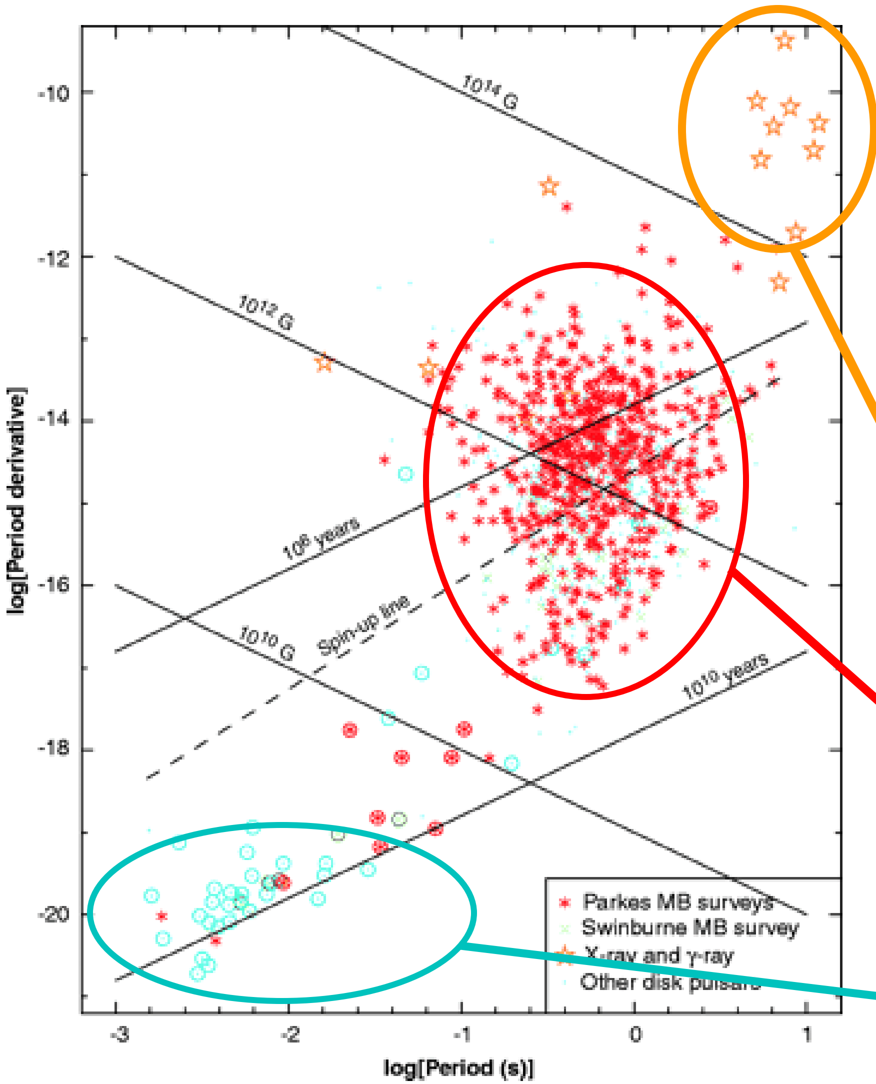
$B_{\perp} = B_p \sin\alpha$

$R = 10\ km$

$I = 10^{45}\ g\ cm^2$

Distribution of PSRs on the $P - \dot{P}$ plane

log(dP/dt)



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$$B_{\perp} = \frac{\sqrt{6c^3}}{2\pi} \frac{I^{1/2}}{R^3} \left(P \dot{P} \right)^{1/2} =$$
$$= 3.2 \times 10^{19} \left(P \dot{P} \right)^{1/2} Gauss$$

B ~ 10^{14}–10^{15} G "Magnetars"

B ~ 10^{12} G "normal" PSR

B ~ 10^{8}–10^{9} G millisecond PSR

The PSR evolution differential equation can be rewritten as:

$$\dot{\Omega} = -K\Omega^n$$

$$P^{n-2}\dot{P} = (2\pi)^{n-1}K$$

Differentiating this equation, with **K = *const***, one obtains:

braking index

$$n \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2 = 2 - P\ddot{P}/\dot{P}^2$$

***n* = 3** within the **magnetic dipole model**

The three quantities $\mathbf{P}$, $\dot{\mathbf{P}}$ and $\ddot{\mathbf{P}}$ have been measured for a few PSRs.

Measured value of the braking index n

PSR name	n	P (s)	P_{dot} $(10^{-15}$ s/s)	Dipole age (yr)
PSR B0531+21 (Crab)	**2.515 ± 0.005**	**0.03308**	**422.765**	**1238**
PSR B0833-45 (Vela)	**1.4 ± 0.2**	**0.08933**	**125.008**	**11000**
PRS B1509-58	**2.839 ± 0.005**	**0.1506**	**1536.5**	**1554**
PSR B0540-69	**2.01 ± 0.02**	**0.0505**	**478.924**	**1672**
PSR J1119-6127	**2.91 ± 0.05**	**0.40077**	**4021.782**	**1580**

The **deviation of the breaking index from 3** could probably be due
(*i*) to **torque** on the pulsar **from outflow of particles**;
(*ii*), **Change with time of the "constant"** ***K*****,** *i.e.* ***I(t),*** or/and **B(t)** or/and **α(t)**

Solutions of the PSR time evolution differential equation

$$\Omega(t) = \Omega_0 \left[(n-1)K\Omega_0^{n-1}\, t + 1\right]^{-1/(n-1)}$$

$$P(t) = P_0 \left[(n-1)K\Omega_0^{n-1}\, t + 1\right]^{1/(n-1)}$$

$n = 3$

$$\Omega(t) = \Omega_0 \left[2K\Omega_0^2\, t + 1\right]^{-1/2}$$

$$P(t) = P_0 \left[2K\Omega_0^2\, t + 1\right]^{1/2}$$

$t_0 = 0$ (NS birth), $P_0 = P(t_0)$, $\Omega_0 = \Omega(t_0)$; $K = const$

The Pulsar age

The solution of the PSR differential equation can be rewritten as:

$$t = -\frac{1}{n-1}\frac{\Omega(t)}{\dot{\Omega}(t)}\left[1-\left(\frac{\Omega(t)}{\Omega_0}\right)^{n-1}\right] \quad (*)$$

or,

$$t = \tau - \{(n-1)\,K\,\Omega_0^{\,n-1}\}^{-1}$$

"true" pulsar age

$$\tau \equiv -\frac{1}{n-1}\frac{\Omega}{\dot{\Omega}} = \frac{1}{n-1}\frac{P}{\dot{P}}$$

$n = 3$ →

dipole age

$$\tau = P/(2\dot{P}) = -\,\Omega/(2\dot{\Omega})$$

if $\Omega(t) << \Omega_0$

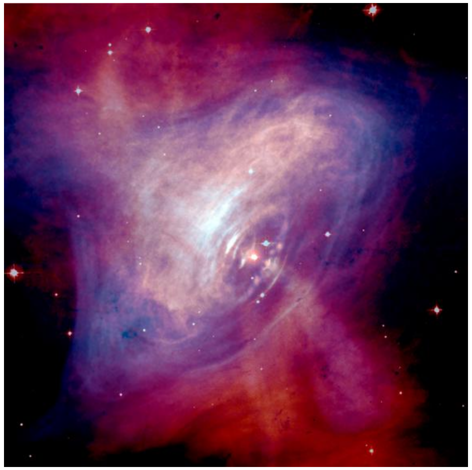
(t $\equiv$ *present time*)

$$t \approx \tau$$

The measure of P and $\dot{P}$ gives the pulsar dipole age

This determination of the PRS age is valid under the assumpion K = const.

Example: **the age of the Crab Pulsar**



SN explosion: 1054 AD

$P = 0.0330847$ s, $\dot{P} = 4.22765 \times 10^{-13}$ s/s

braking index: $n = 2.515 \pm 0.005$

t_{crab} = (2011 – 1054) yr = **957 yr ,** τ **= 1238 yr** (dipole age)

Assuming the validity of the PSR dipole model, using the previous equation **(*)** for the pulsar true age, we can infer the initial spin period of the Crab

$$P_0 = P\,(1 - t_{crab}/\tau)^{1/2} \cong 0.016 \text{ s}$$

But $n_{crab} \neq 3$

Pulsar evolutionary path on the P–$\dot{P}$ plane

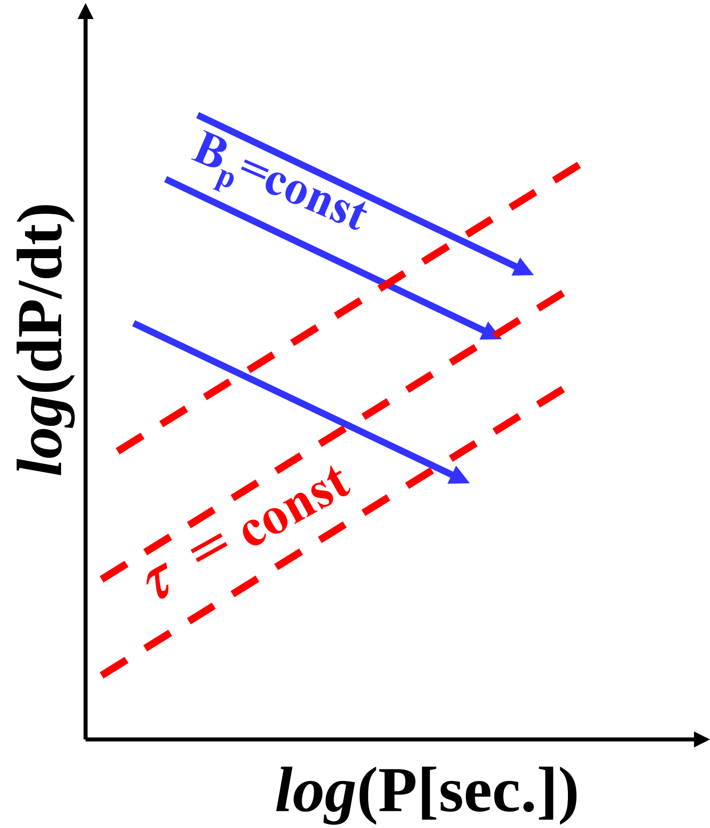
$$P\dot{P} = (2\pi)^2 K \qquad K \equiv \frac{1}{6c^3}\frac{R^6}{I}(B_p \sin\alpha)^2$$

Taking the logarithm of this equation we get:

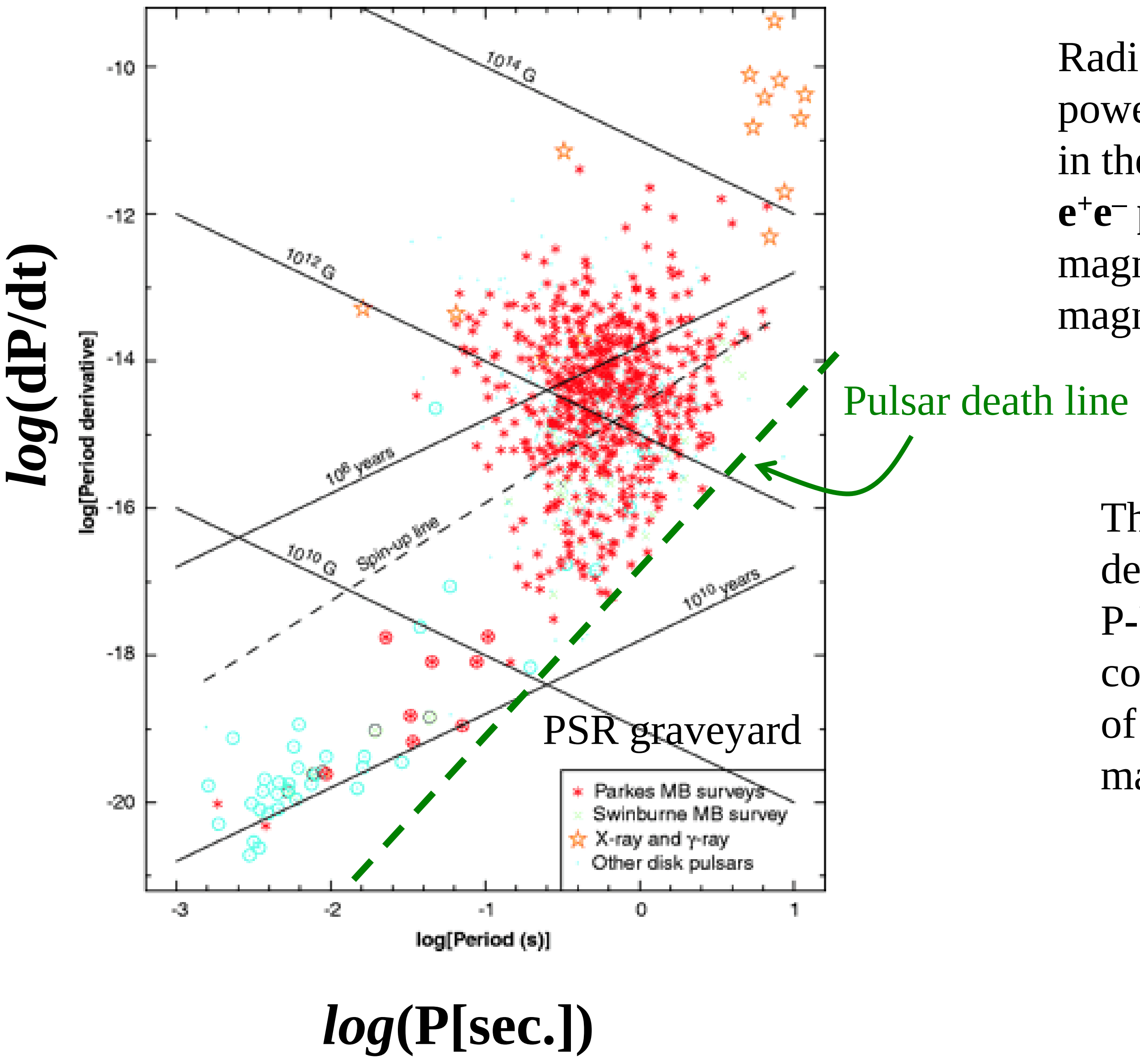
$$\log \dot{P} = \log\left[\frac{(2\pi)^2 R^6}{6c^3 I} B_p^2 \sin^2\alpha\right] - \log P$$

$$\tau = P/(2\dot{P})$$

$$\log P = \log P - \log(2\tau)$$



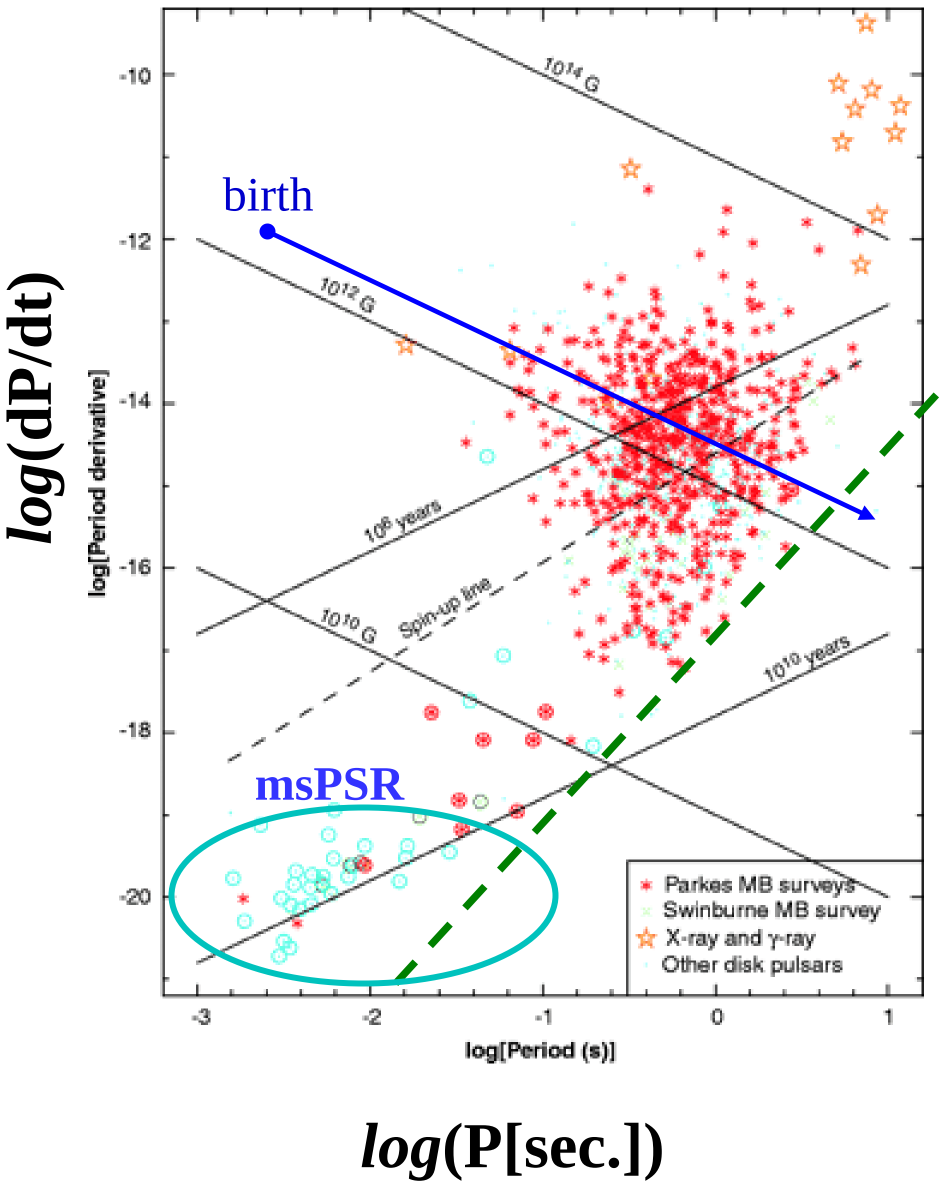
Pulsar evolutionary path on the P–$\dot{P}$ plane



Radio emission from rotating powered pulsars has its origin in the relativistic outflow of **e^+e^- pairs** along the polar magnetic fiel lines of the NS magnetic field.

The pulsar **"death line"** is defined as the line in the P-Pdot plane which correspond to the cessation of pair creation over the magnetic poles of the NS.

Pulsar evolutionary path on the P–$\dot{P}$ plane



millisecons PSRs have dipole ages in the range **10^8 — 10^{10} yr** thus they are **very old** pulsars.

What is the **origin** of millisecond pulsars?

Millisecond pulsar are believed to result from the **spin-up** of a “slow” rotating neutron star through **mass accretion** (and **angular momentum transfer**) from a companion star in a binary stellar system

The PSR/NS magnetic field

Based on the magnetic dipole model for PSRs: **$B \sim 10^{14}$–10^{15} G "Magnetars"**

$B \sim 10^{12}$ G "normal" PSR, **$B \sim 10^{8}$–10^{9} G millisecond PSR**

Key questions

1. **Where does the PSR/NS magnetic field come from?**
2. **Is the magnetic field constant in time? Or, does it decay?**

If **B decays in time** what are the **implications** for the determination of the **pulsar age** and **braking index** ?

Where does the NS magnetic field come from?

There is as yet no satisfacory theory for the generation of the magnetic field in a Neutron Star.

Traditional ***answer***: **"It is as it is, because it was as it was"**

- **Fossil remnant magnetic field from the progenitor star:**

Assuming **magnetic flux conservation** during the birth of the neutron star

$$\Phi(\mathbf{B}) \sim B\,R^2 = \text{const.}$$

Progenitor star: $R_* \sim 10^6$ km, $B_* \sim 10^2$ G

$$\mathbf{B_{NS} \sim (R_*/R_{NS})^2\, B_* \sim 10^{12}\ G}$$

Earth (at the magnetic poles): B = 0.6 G, Refrigerator magnet: B ~ 100 G

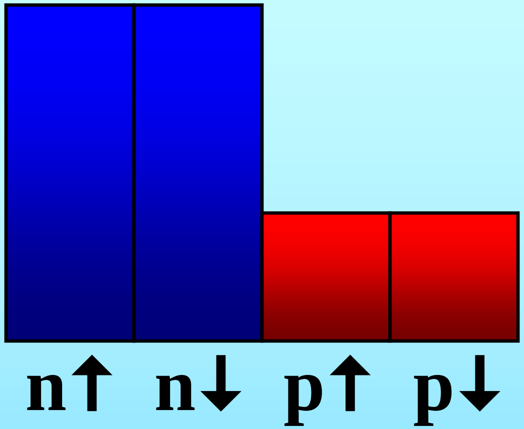
Where does the NS magnetic field come from?

- The field could be generated after the formation of the NS by some long living electric currents flowing in the highly conductive neutron star material.

- Spontaneus "ferromagnetic" transition in the neutron star core

Does the nuclear interaction leads to a spontaneus ferromagnetic transition in nuclear matter at some density and some isospin asymmetry?

Spin-unpolarized isospin-asymmetric MN



$S_n = 0$

$S_p = 0$

Baryon numb. densities

$$\rho_n = \rho_{n\uparrow} + \rho_{n\downarrow}$$
$$\rho_p = \rho_{p\uparrow} + \rho_{p\downarrow}$$
$$\rho = \rho_{n\uparrow} + \rho_{n\downarrow}$$

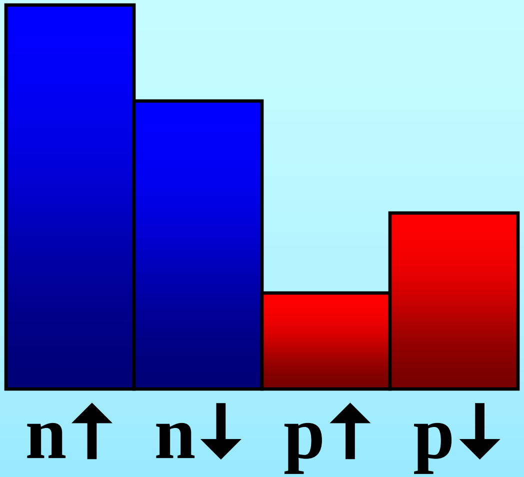
Spin polarization $S_n = (\rho_{n\uparrow} - \rho_{n\downarrow})/\rho_n$, $S_p = (\rho_{p\uparrow} - \rho_{p\downarrow})/\rho_p$

Isospin asymmetry $\beta = (\rho_n - \rho_p)/\rho$

$$\rho_{n\uparrow} = \frac{1+S_n}{2}\frac{1+\beta}{2}\rho \qquad \rho_{p\uparrow} = \frac{1+S_p}{2}\frac{1-\beta}{2}\rho$$

$$\rho_{n\downarrow} = \frac{1-S_n}{2}\frac{1+\beta}{2}\rho \qquad \rho_{p\downarrow} = \frac{1-S_p}{2}\frac{1-\beta}{2}\rho$$

Spin-polarized isospin-asymmetric MN



$S_n \neq 0$

$S_p \neq 0$

Baryon numb. densities

$$\rho_n = \rho_{n\uparrow} + \rho_{n\downarrow}$$
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$$\rho_{n\downarrow} = \frac{1-S_n}{2}\frac{1+\beta}{2}\rho \qquad \rho_{p\downarrow} = \frac{1-S_p}{2}\frac{1-\beta}{2}\rho$$

Brueckner–Bethe–Goldstone Theory

Bethe - Goldstone equation

$$\langle a;b|G(\omega)|c;d\rangle = \langle a;b|\mathrm{v}|c;d\rangle +$$

$$\sum_{i,j}\langle a;b|\mathrm{v}|i;j\rangle \frac{Q_{\tau_i\sigma_i\tau_j\sigma_j}}{\omega - e_{\tau_i\sigma_i} - e_{\tau_j\sigma_j}} \langle i;j|G(\omega)|c;d\rangle$$

$|a;b\rangle = |a\rangle \otimes |b\rangle$

$|a\rangle = |\vec{k}_a, \tau_a, \sigma_a\rangle$

$\tau_a =$ n, p — 3rd isospin component

$\sigma_a = \uparrow, \downarrow$ — 3rd spin component

$Q_{\tau_i\sigma_i\tau_j\sigma_j}$ Pauli operator

Single particle energy: BHF approximation

$$e_{\tau\sigma}(k) = \frac{\hbar^2 k^2}{2m_\tau} + U_{\tau\sigma}(k)$$

$$U_{\tau\sigma}(k) = \sum_{\tau'}\sum_{\sigma'} U_{\tau\sigma\tau'\sigma'}(k) =$$

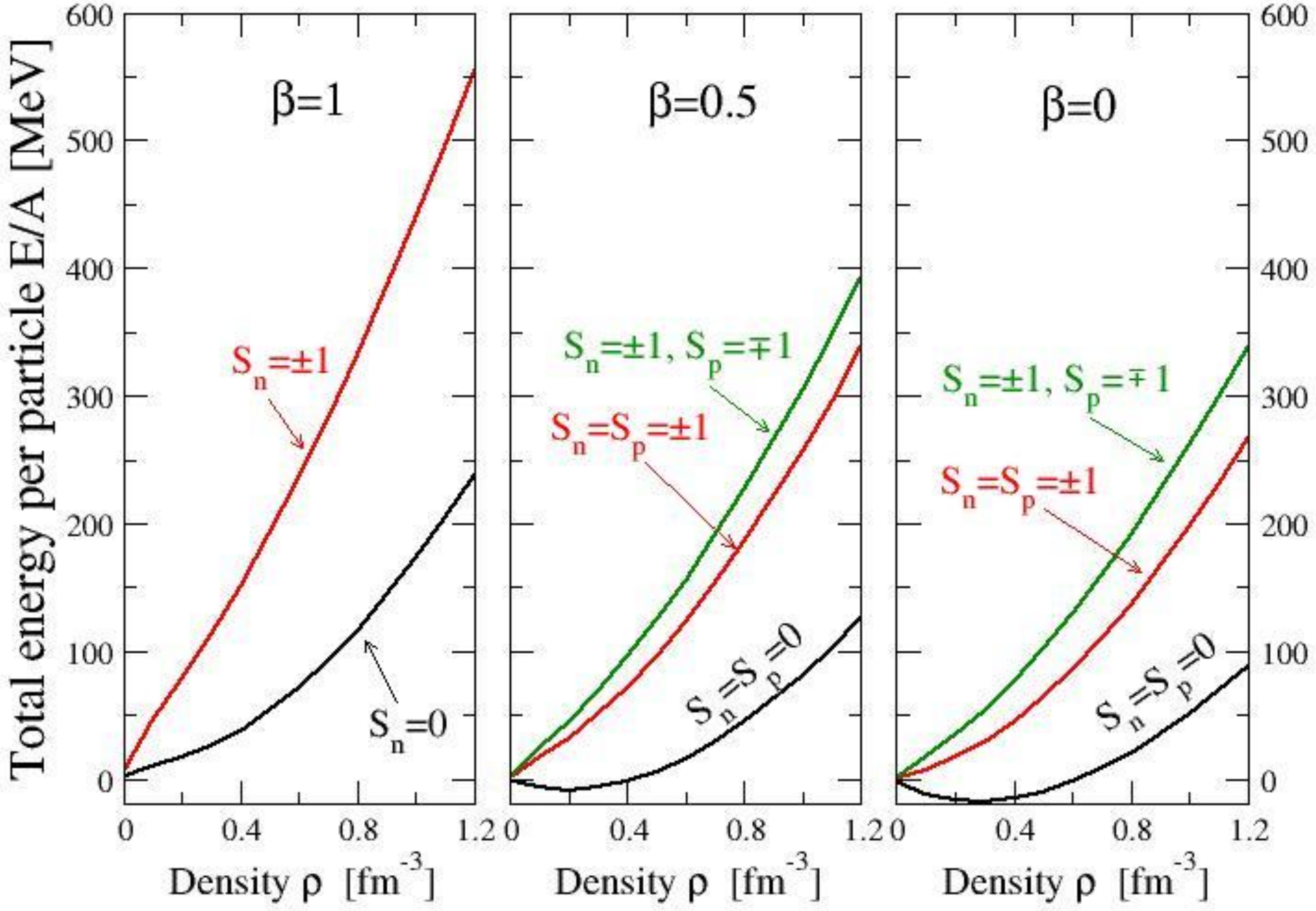
$$= \sum_{\tau'}\sum_{\sigma'}\sum_{k' \le k_F^{\tau'\sigma'}} \langle \vec{k}\tau\sigma; \vec{k}'\tau'\sigma' | G(e_{\tau\sigma} + e_{\tau'\sigma'}) | \vec{k}\tau\sigma; \vec{k}'\tau'\sigma' \rangle_{\mathcal{A}}$$

Total energy per particle energy: BHF approximation

$$\frac{E}{A} = \frac{1}{A}\sum_{\tau}\sum_{\sigma}\sum_{k \le k_F^{\tau\sigma}} \frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2A}\sum_{\tau}\sum_{\sigma}\sum_{k \le k_F^{\tau\sigma}} U_{\tau\sigma}(k)$$

Total energy per particle

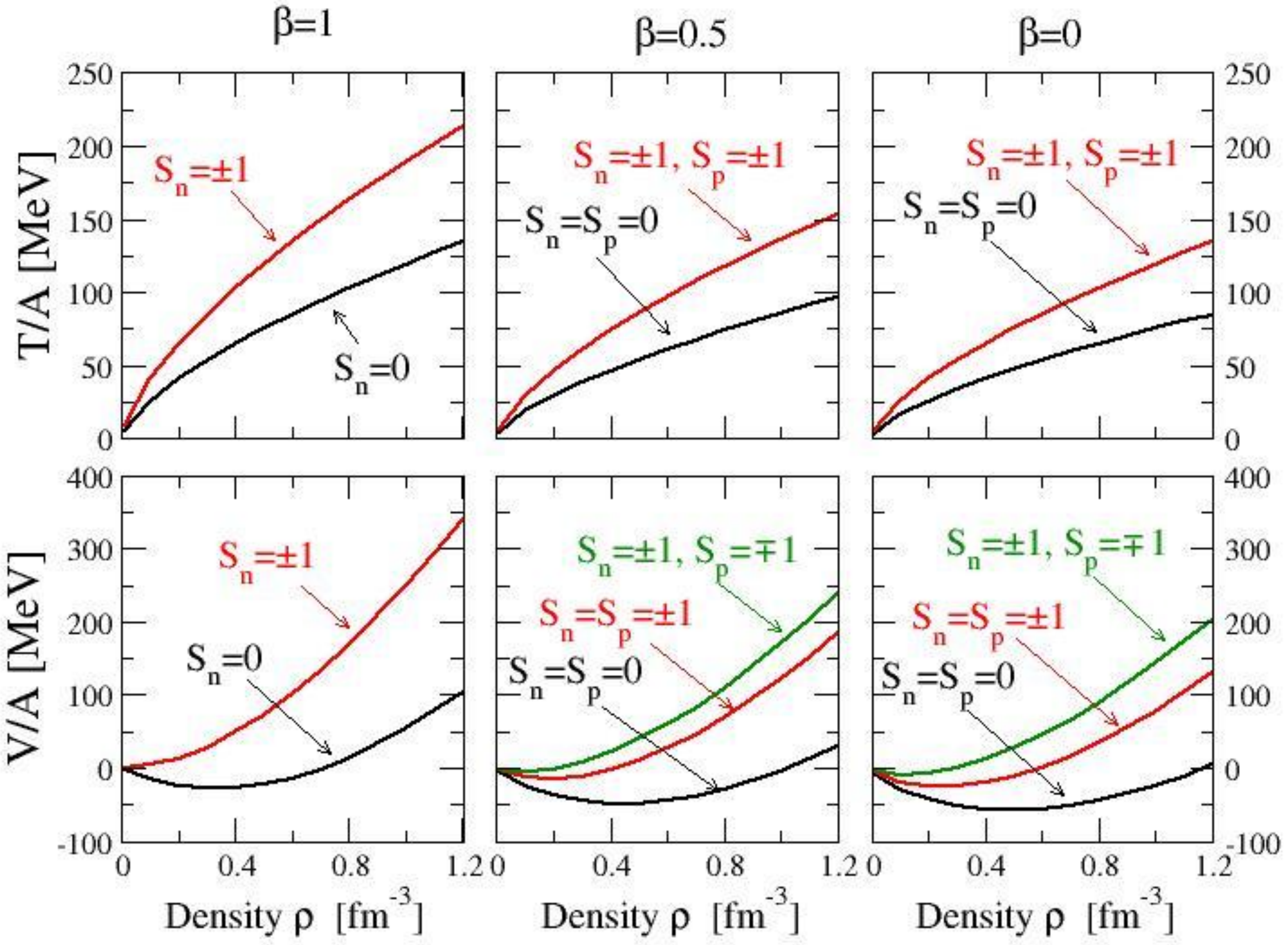
Nijmegen NSC97e interaction



I. Bombaci, I. Vidaña, Phys. Rev. C66 (2002) 045801

Kinetic and potential energy contributions to E/A

Nijmegen NSC97e interaction



I. Bombaci, I. Vidaña, Phys. Rev. C66 (2002) 045801

Magnetic susceptibility: pure Neutron Matter

The **magnetic susceptibility** **of a system characterize the response of the system to an external magnetic field $\mathcal{H}$**

$$\chi = \left(\frac{\partial \mathcal{M}}{\partial \mathcal{H}} \right)_{\mathcal{H}=0}$$

$\mathcal{M}$ is the **magnetization per uinit volume** of the system (i.e. the magnetic moment per unit volume of the material)

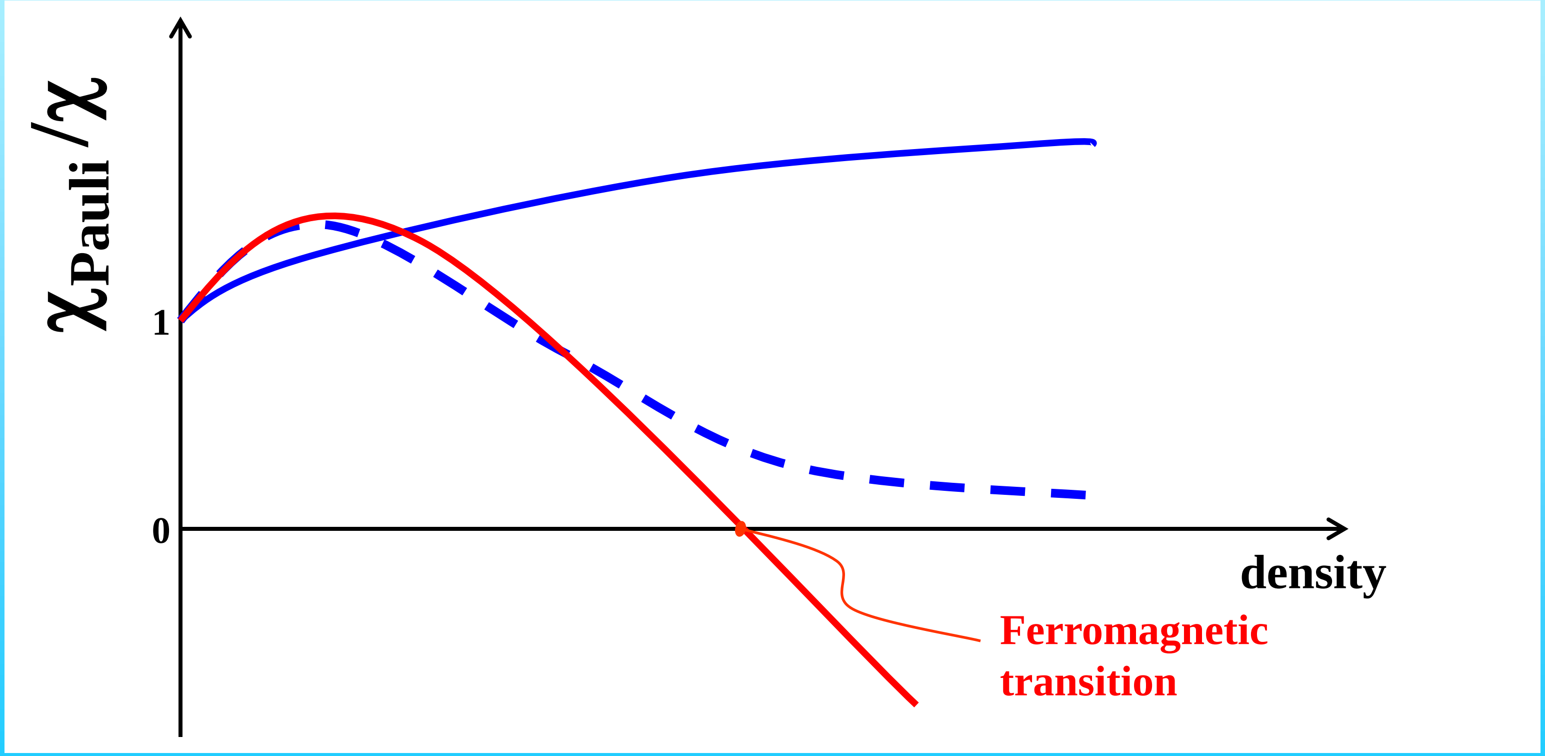
$$\mathcal{M} = \mu_n (\rho_{n\uparrow} - \rho_{n\downarrow}) = \mu_n \ \rho \ S_n$$

$$\chi = \frac{\mu_n^2 \rho}{\left(\frac{\partial^2 (E/N)}{\partial S_n^2} \right)_{S_n=0}}$$

μ_n = -1.913 μ_N = neutron magnetic dipole moment

Pauli magnetic susceptibility: free Fermi gas

$$\chi_{Pauli} = \frac{m\mu_n^2}{\hbar^2\pi^2} k_F$$



Magnetic susceptibility: asymmetric Nucl. Matter

$$\frac{1}{\chi} = \begin{pmatrix} 1/\chi_{nn} & 1/\chi_{np} \\ 1/\chi_{pn} & 1/\chi_{pp} \end{pmatrix}$$

$$\frac{1}{\chi_{ij}} = \frac{\partial \mathcal{H}_i}{\partial \mathcal{M}_j}$$

i, j* = *n, p

$\mathcal{M}_j$ is the **magnetization per uinit volume** of the component j (*i.e.* neutrons or protons)

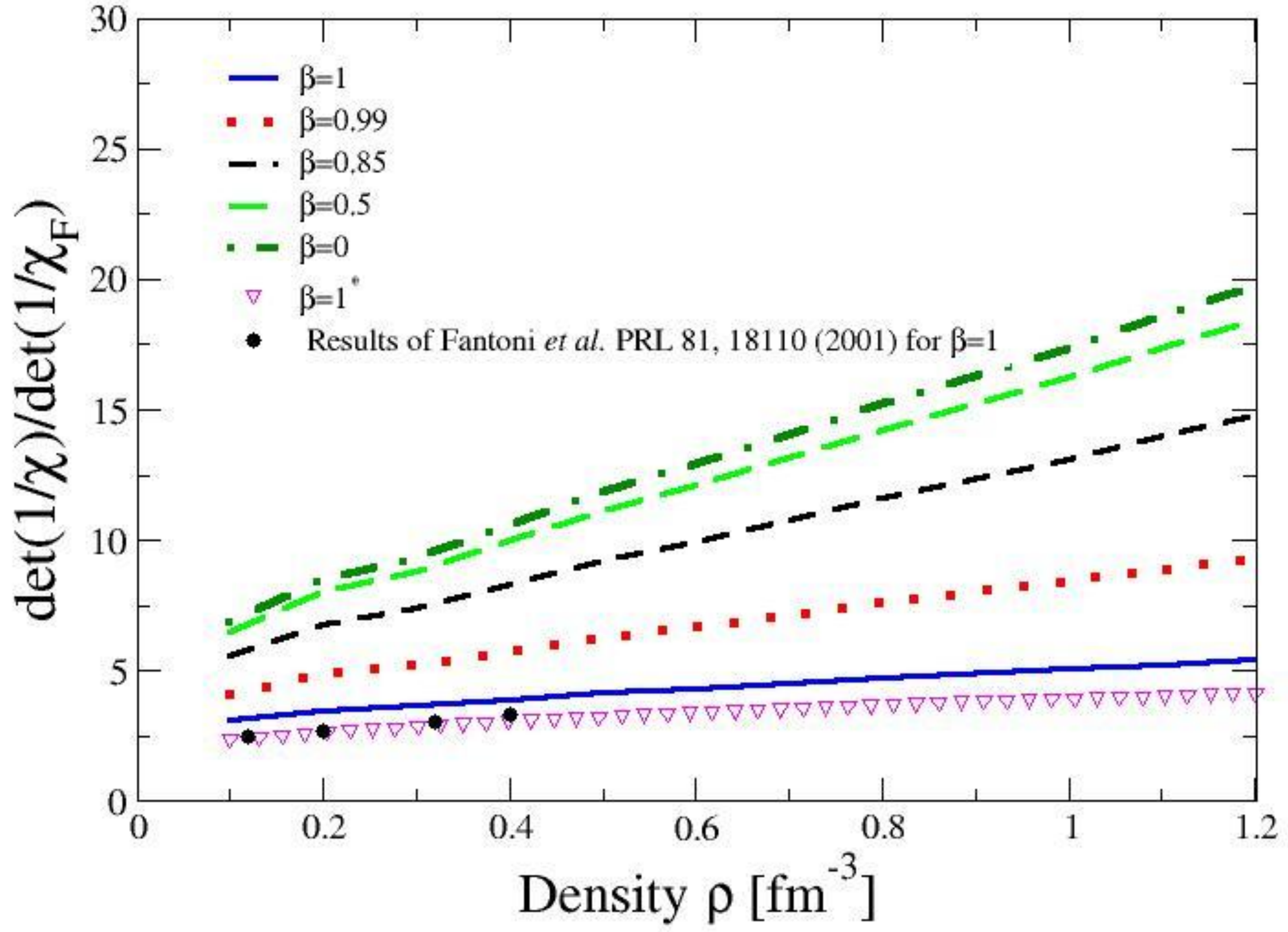
$$\mathcal{M}_j = \mu_j(\rho_{j\uparrow} - \rho_{j\downarrow})$$
$$= \mu_j \ \rho \ S_j$$

μ_j magnetic dipole moment: $\mu_n = -1.9130\ \mu_N$, $\mu_p = 2.7928\ \mu_N$

$$\frac{1}{\chi_{ij}} = \frac{\rho}{\mu_i \rho_i \mu_j \rho_j} \frac{\partial^2 (E/A)}{\partial S_i \partial_j S}$$

Magnetic susceptibility: asymmetric NM

Nijmegen NSC97e interaction



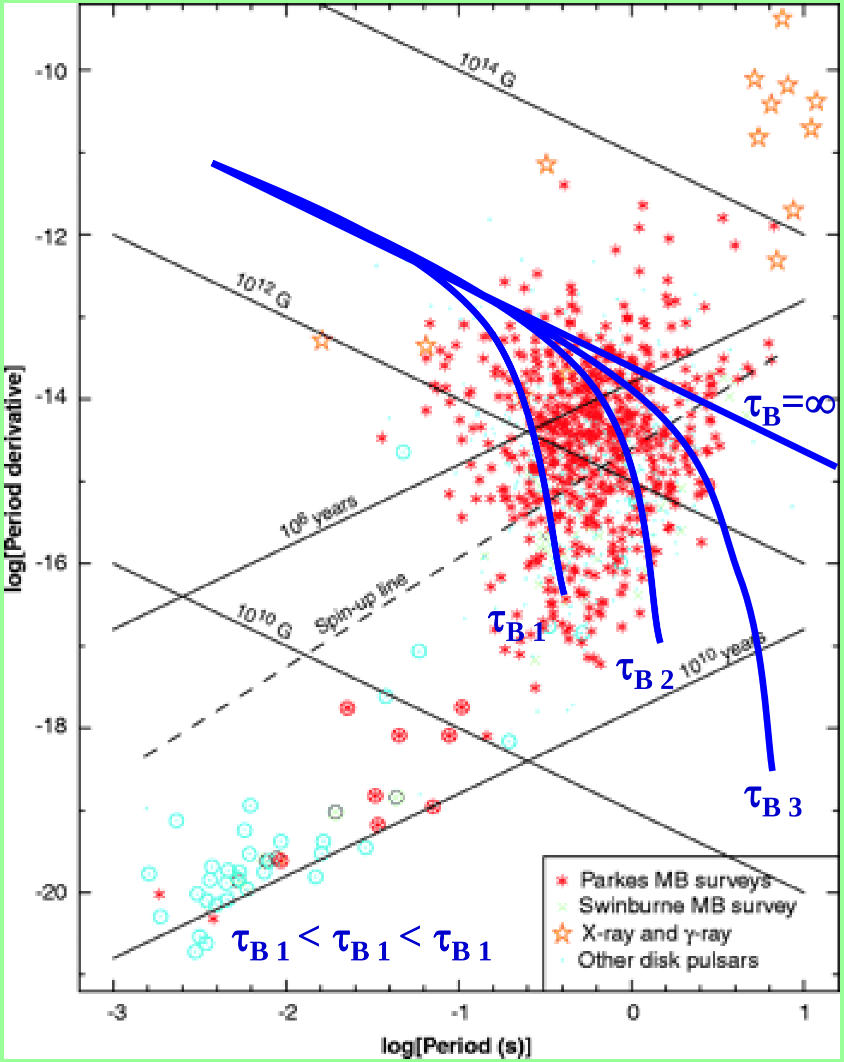
I. Bombaci, I. Vidaña, Phys. Rev. C66 (2002) 045801

Magnetic susceptibility: asymmetric NM

Miscroscopic calculations show
no indication of
a ferromagnetic transition
at any density and for any
isospin asymmetry
in nuclear matter

Magnetic field decay in Neutron Stars

There are strong theoretical and observational arguments which indicate a decay of the neutron star magnetic field. (Ostriker and Gunn, 1969)



$$B(t) = B_\infty + [B_0 - B_\infty]\, exp(-t/\tau_B)$$

B_∞ = residual magn. field

$\tau_B \sim 1$ — 10 Myr

B-field decay

Decrease with time of the magnetic braking

$$P\dot{P} = (2\pi)^2 K(t)$$

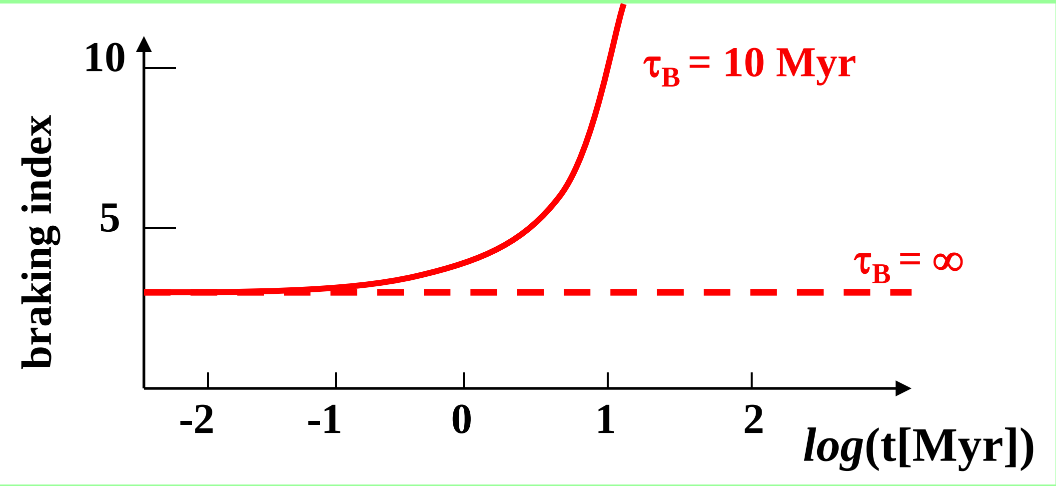
$$K(t) \equiv \frac{1}{6c^3}\frac{R^6}{I}\left(B_p(t)\sin\alpha\right)^2$$

$B_\infty = 0$

$$P(t) = P_0 \{\tau_B K_0 \Omega_0^2 [1 - exp(-2t/\tau_B)] + 1\}^{1/2}$$

braking index

$$n(t) \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2 = 3 - \frac{3c^3 I \dot{B}}{R^6 B^3 \sin^2\alpha\ \Omega^2}$$

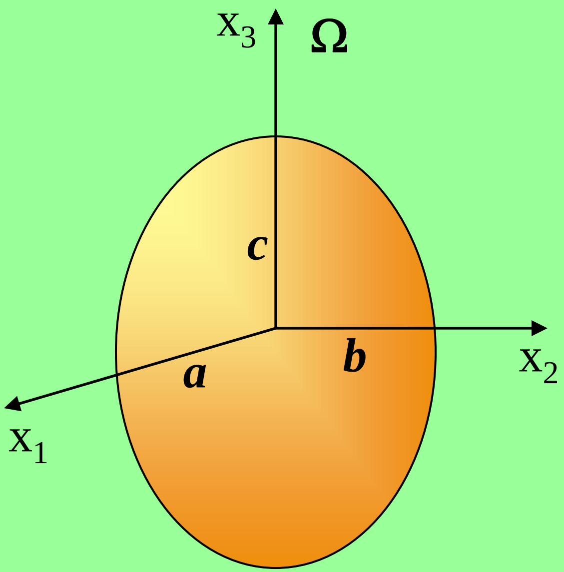


Tauris and Konar,
Astron. and Astrophys. 376 (2001)

Gravitational radiation from a Neutron Star

The **lowest-order gravitational radiation is quadrupole.** Thus in order to radiate gravitational energy a **NS** must have a **time-varying quadrupole moment**

Gravitational radiation from a spinning triaxial ellipsoid



$$a \neq b \neq c$$

$$I_1 \neq I_2 \neq I_3$$

ellipticity: $\varepsilon = \dfrac{a-b}{(a+b)/2}$

If: $\varepsilon \ll 1$

$$\dot{E}_{grav} = -\frac{32}{5}\frac{G}{c^5} I_3^2 \varepsilon^2 \, \Omega^6$$

$$\dot{E}_{rot} = I_3 \Omega \dot{\Omega}$$

$$\dot{\Omega} = -K_g\,\Omega^5$$

braking index for gravitational quadrupole radiation

$$n \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2 = 5$$

pulsar age

$$\tau_{n-1} \equiv -\frac{1}{n-1}\frac{\Omega}{\dot{\Omega}} = \frac{1}{n-1}\frac{P}{\dot{P}}$$

$$\tau_4 = P/(4\dot{P}) = -\Omega/(4\dot{\Omega})$$

An application to the case of the Crab pulsar

Suppose that the Crab Nebula is powered by the emission of gravitational radiation of a spinning Neutron Star (triaxial ellipsoid).

We want to calculate the deformation (ellipticity ε) of the Neutron Star.

$\mathbf{L_{crab} = 5 \times 10^{38}}$ **erg/s** $\mathbf{P = 0.033}$ **s** $\mathbf{\dot{P} = 4.227 \times 10^{-13}}$ **s/s**

$$L_{crab} = \left|\dot{E}_{grav}\right| = \frac{32}{5}(2\pi)^6 \frac{G}{c^5}\frac{I_3^2}{P^6}\varepsilon^2 \equiv A\ \varepsilon^2$$

assuming:

$\mathbf{I_3 = 10^{45}}$ $\mathbf{g\ cm^2}$

↓

$\mathbf{A = 8.38 \times 10^{44}}$ **erg/s**

⇒ $\mathbf{\varepsilon \sim 7.7 \times 10^{-4}}$ —— **R = 10 km** ⇒ $a - b \cong \varepsilon R \cong 7.7$ **m**

A rotating neutron star with a **8 meter high mountain at the equator** could power the **Crab nebula** via **gravitational wave emission**

Is it possible to have a 8 meter high mountain on the surface of a Neutron Star?

Is there a limit to the maximum possible height of a mountain on a planet?

On the Earth: **Mons Everest**: **h ~ 9 km** (~4 km high from the Tibet plateau)
Mauna Kea (Hawaii): **h ~ 10 km** (from the ocean botton to the peak)
$\mathbf{R_{\oplus} = 6380}$ **km** (equatorial terrestial radius)

$\mathbf{h_{max}}$ will depend on: *(i)* **inter-atomic forces (rock stress, melting point),**

(ii) the **planetary gravity acceleration g**

Pressure at the base of the mountain: $\mathbf{P \sim \rho\, g\, h < P_{max}}$ (ρ**=const, g = const**)

$\mathbf{g = G\, M/R^2}$, (R=planet's radius)

For a constant density planet ($M \propto R^3$), one has:

$$h_{max} = \frac{P_{max}}{\rho} \frac{1}{g} \propto \frac{1}{g} \propto \frac{R^2}{GM} \propto \frac{R^2}{R^3} = \frac{1}{R}$$

Assuming for the **Earth**: $\mathbf{h_{max\,\oplus} = 10\ km}$**,** using the previous eq. we can calculate the maximum height of a mountain in a terrestial-like planet (rocky planet):

$$\mathbf{h_{max} = (R_{\oplus}/R)\ h_{max\,\oplus}} \qquad (R_{\oplus} = 6380\ \text{km})$$

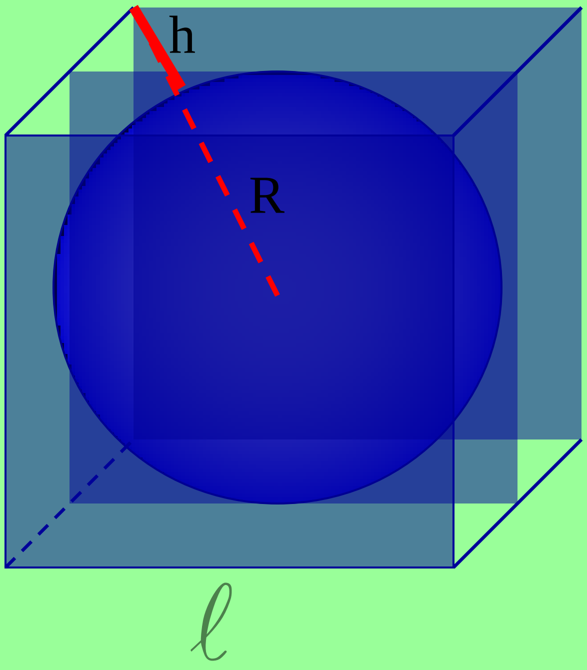
The planet **Mars:**

R = 3400 km = 0.53 R $\Rightarrow$ $\mathbf{h_{max} = 19\ km}$

mons Olympus h = 25 km

Exercise: using this simple argument, estimate the maximum size of a **cubic Earth-like planet**

$$\ell = 2R = 590 \text{ km}$$



Pressure at the base of the mountain: $\mathbf{P \sim \rho\, g\, h < P_{max}}$ $(\rho = \text{const}, \quad g = \text{const})$

$\mathbf{g = G\, M/R^2}$, (R=planet's radius)

For a constant density planet ($M \propto R^3$), one has:

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$$\mathbf{h_{max} = (R_{\oplus}/R)\ h_{max\,\oplus}} \qquad (R_{\oplus} = 6380\ \text{km})$$

For a **Neutron Star** this simple formula can **not** be used.

More reliable calculations give: $\mathbf{h_{max,NS} \sim 1\ cm}$

Crab pulsar: $\boldsymbol{n = 2.515 \pm 0.005}$

$\mathbf{t_{crab} = 957\ yr}$, $\boldsymbol{\tau_4} \mathbf{= 619\ yr}$ (quadrupole age)

Time dependent moment of Inertia

Up to now we supposed that the NS moment of inertia does nor depend on frequency and on time (Ω changes with time as the NS spins down).

Suppose now: $\boldsymbol{I = I(t) = I(\Omega(t))}$

Rotational kinetic energy

$$\dot{E}_{rot} = \frac{d}{dt}\left(\frac{1}{2} I \Omega^2\right) = I\Omega\dot{\Omega} + \frac{1}{2}\frac{dI}{d\Omega}\dot{\Omega}\Omega^2$$

We can write the energy rate radiated by the star due to some **general braking mechanism** as

$$\dot{E}_{brak} = -C\Omega^{n+1}$$

n braking index

Energy balance: $\dot{E}_{brak} = \dot{E}_{rot}$ ⇒

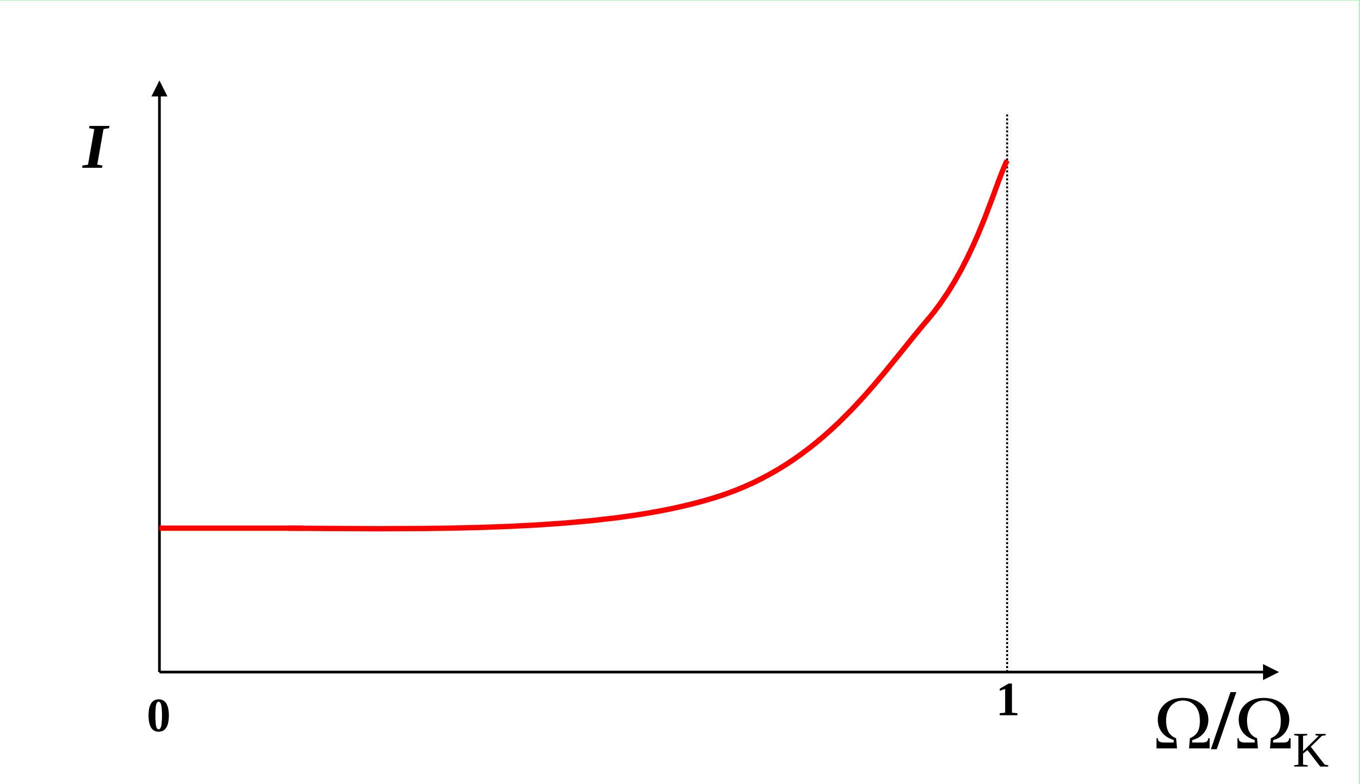
$$\dot{\Omega} = -K(t)\left(1 + \frac{I'\Omega}{2I}\right)^{-1}\Omega^n$$

$K(t) \equiv C\,/\,\boldsymbol{I}(t)$ $\qquad$ $\boldsymbol{I'}(t) \equiv dI\,/\,d\,\Omega$

In the case of a pure magnetic dipole braking mechanism (***n* = 3**), this eq. generalizes to the case of time-dependet moment of inertia, the “standard” magnetic dipol model differential eq.:

$$\dot{\Omega} = -K\Omega^3$$

$$K \equiv \frac{1}{6c^3}\frac{R^6}{I}(B_p \sin\alpha)^2$$



$I' \equiv dI/d\Omega > 0$

B-field determination form P and $\dot{P}$ in the case dI/dΩ≠0

The value of the magnetic field deduced from the **measured values of P and dP/dt**, when the proper frequency dependence of the moment of inertia is considerd, is given by

$$\tilde{B}_p = \left(1 + \frac{I'\Omega}{2I}\right)^{1/2} B_p$$

B_p being the value obatined for constant moment of inertia ***I***.

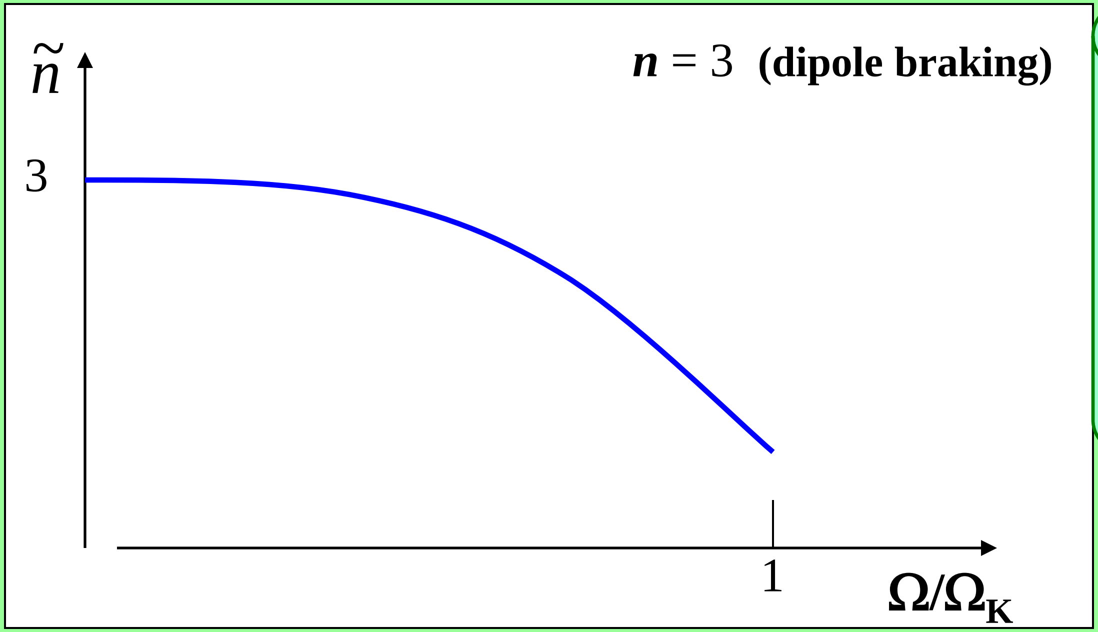
$$B_p \sin\alpha = \frac{\sqrt{6c^3}}{2\pi} \frac{I^{1/2}}{R^3} \left(P \dot{P}\right)^{1/2}$$

I ′ ≡ dI/dΩ > 0 , thus the **“true” value $\mathbf{B_p}$** of the magnetic field is **larger** than the value B_p deduced assuming I **′=** 0.

apparent braking index

$$\tilde{n}(\Omega) \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2 = n - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}$$

$\tilde{n}(\Omega) < n$ because I’ > 0 and I” > 0 (the moment of inertia increases with Ω and the centrifugal force grows with the equatorial radius).



Dramatic consequences on the apparent braking index when the stellar core undergoes a phase transition