Nuclear Physics School “Raimondo Anni”, 5th course
Otranto, May 30 – June 4, 2011

The Physics of Neutron Star Interiors

2nd Lecture

Neutron Stars' Structure

	Sun	White dwarf	Neutron Star	Black Hole
mass	$M_\odot$	1–1.4 $M_\odot$	1–2 $M_\odot$	arbitrary
radius	$R_\odot$	$\sim 10^{-2} R_\odot$	~ 10 km	$2GM/c^2$
R/R_g	2.4×10^5	$\sim 2 \times 10^3$	~ 2 – 4	1
av. dens	~ 1 g/cm^3	$\sim 10^{7\text{-}8}$ g/cm^3	$2\text{–}9 \times 10^{14}$ g/cm^3	=

$R_g \equiv 2GM/c^2$ (Schwarzschild radius)

$x \equiv R/R_g$ (compactness parameter)

$M_\odot = 1.989 \times 10^{33}$ g $R_\odot = 6.96 \times 10^5$ km $R_{g\odot} = 2.95$ km

$\rho_0 = 2.8 \times 10^{14}$ g/cm^3 (nuclear saturation density)

When x is "small" gravity must be described by the Einstein theory of **General Relativity**

Relativistic equations for stellar structure

Consider a **self-gravitating mass distribution** under the following assumpions:

- **Spherical symmetry**
- **Static (no time dependence: e.g. non-rotating configurations)**
- **No magnetic field ("weak" magnetic field)**

Coordinates: $x^0 = ct,\quad x^1 = r,\quad x^2 = \theta,\quad x^3 = \varphi$

Line element

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = e^{2\Phi(r)}c^2dt^2 - e^{2\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$\Phi = \Phi(r)$, $\lambda = \lambda(r)$ metric functions

$g_{\mu\nu}$ metric tensor

$$g_{\mu\nu} = \begin{pmatrix} e^{2\Phi} & 0 & 0 & 0 \\ 0 & -e^{2\lambda} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}$$

$$g^{\mu\alpha}g_{\alpha\nu} \equiv g^\mu_\nu = \delta^\mu_\nu$$

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$\Phi = \Phi(r)$, $\quad \lambda = \lambda(r)$ metric functions

One can introduce a new metric function $m(r)$ related to $\lambda(r)$ by:

$$e^{\lambda(r)} \equiv \frac{1}{\sqrt{1 - \frac{2G\, m(r)}{c^2 r^2}}}$$

$m(r)$ = gravitational mass contained inside a sphere of radial coordinate r

Proper radial lenght (fix t, θ, φ)

$$d\ell = e^{\lambda(r)}dr = \left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1/2} dr$$

$$\ell(r) = \int_0^r e^{\lambda(r')}dr'$$

Proper volume of a spherical shell with radial coorinate r ÷ r +dr

$$dV = 4\pi\, e^{\lambda(r)} r^2 dr = 4\pi \left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1/2} r^2 dr$$

Proper time

$$d\tau = e^{\Phi(r)}dt$$

Energy–momentum tensor of stellar matter

Perfect fluid (no shear stresses and heath transport)

$$T^{\mu\nu} = (P+\varepsilon)\, u^{\mu}u^{\nu} - P g^{\mu\nu}$$

P = pressure, ε = energy density, $u^{\mu} = \frac{dx^{\mu}}{ds}$ 4-velocity of the fluid element

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \rightarrow 1 = g_{\mu\nu} u^{\mu} u^{\nu}$$

For a **static star** : (fluid rest-frame) $u^{\mu} = \left(1/\sqrt{g_{00}},\ \vec{0} \right) = \left(e^{-\Phi},\ \vec{0} \right)$

$$T_{\mu}^{\ \nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

$$T_{\mu}^{\ \nu} = g_{\mu\alpha} T^{\alpha\nu}$$

One needs **P** and **ε**
i.e. the **stellar matter EOS**

Einstein equations

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \overline{R} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

$R^{\mu\nu}$ = Ricci tensor, $\overline{R} = g_{\mu\nu} R^{\mu\nu}$ = scalar curvature

for the present **static, spherical symmetric** case the Einstein's field equations take the form called the **Tolman – Oppenheimer – Volkov equations (TOV)**

$$\frac{dP}{dr} = -G \frac{m(r)\rho(r)}{r^2} \left(1 + \frac{P(r)}{c^2 \rho(r)}\right) \left(1 + 4\pi \frac{r^3 P(r)}{m(r)\; c^2}\right) \left[1 - \frac{2Gm(r)}{c^2\; r}\right]^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{d\Phi}{dr} = -\frac{1}{\rho(r)c^2} \frac{dP}{dr} \left(1 + \frac{P(r)}{\rho(r)c^2}\right)^{-1}$$

In the limit: $P << \rho c^2, \quad P r^3 << mc^2, \quad \frac{2Gm}{c^2} << r$

Newtonian case

$$\frac{dP}{dr} = -G \frac{m(r)\rho(r)}{r^2}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

$$c^2 \frac{d\Phi}{dr} = \frac{Gm}{r^2}$$

$$U(r) = c^2 \Phi(r) = -\frac{Gm}{r}$$

Boundary conditions:

$m(r=0) = 0$ (Density is finite at the star center)

$P(r=R) = P_{surf}$ define the stellar surface (surface area $4\pi R^2$)

R = stellar radius

The solutions of the TOV eq.s depend parametrically on the central density

$\rho_c = \rho(r=0)$

$$P = P(r, \rho_c)$$
$$m = m(r, \rho_c)$$

Role of the Equation of State (EOS)

The key input to solve the TOV equations EOS of dense matter.

In the following we assume matter in the Neutron Star to be a **perfect fluid** (this assumption has been already done to derive the TOV eq.) in a **cold (T = 0)** and **catalyzed** state (state of minimum energy per baryon)

$$\rho = \rho(n) = \rho_0 + \frac{\varepsilon'}{c^2} = \frac{\varepsilon}{c^2}$$

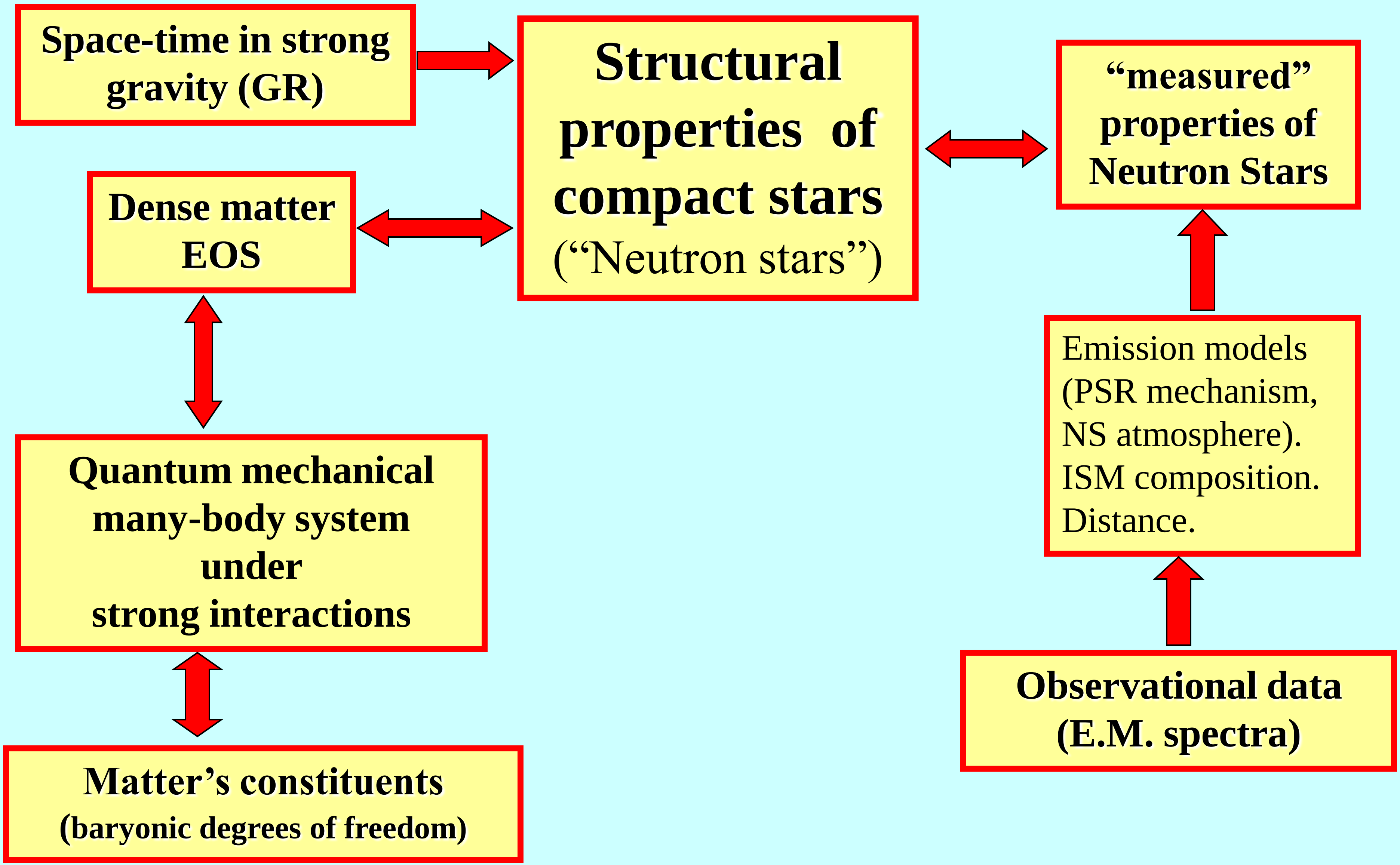
$$P = P(n) = n\left(\frac{\partial \varepsilon}{\partial n}\right) - \varepsilon$$

ρ = **total mass density**

ρ_0 = **rest mass density**

ε = **total energy density**

ε' = **internal energy density (**includes the kinetic plus the potential energy density due to interactions (not gravity)


Space-time in strong gravity (GR)
Structural properties of compact stars ("Neutron stars")
"measured" properties of Neutron Stars
Dense matter EOS
Emission models (PSR mechanism, NS atmosphere). ISM composition. Distance.
Quantum mechanical many-body system under strong interactions
Observational data (E.M. spectra)
Matter's constituents (baryonic degrees of freedom)

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

Gravitational mass

$$M_G \equiv m(R) = \int_0^R 4\pi\ r^2 \rho(r) dr$$

M_G is the mass measured by a distant keplerian observer

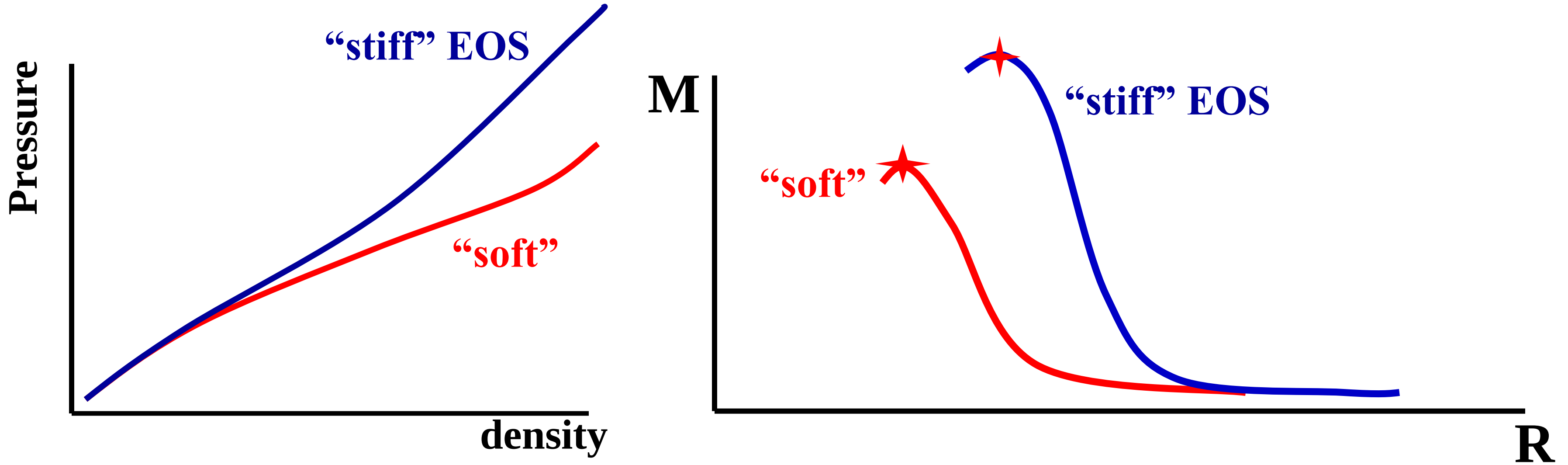
$M_G c^2$ = total energy in the star (rest mass + internal energy + gravitational energy)

The Oppenheimer-Volkoff maximum mass

There is a maximum value for the gravitational mass of a Neutron Star that a given EOS can support. This mass is called the Oppenheimer-Volkoff mass

$$M_{max} = (1.4 – 2.5)\ M_{\odot}$$

EOS dependent



The OV mass represent the key physical quantity to separate (and distinguish) Neutrons Stars from Black Holes.

Baryonic mass

is the **rest mass of the N_B baryons** (dispersed at infinity) which form the star

$$M_B = m_u \int n(r) dV$$

$$= m_u \int_0^R 4\pi \, r^2 n(r) \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-1/2} dr = m_u N_B$$

$N_B \sim 10^{57}$

m_u = baryon mass unit (average nucleon mass)
$n(r)$ = baryon number density

Proper mass

$$M_P = \int \rho(r) dV$$

$$= \int_0^R 4\pi \, r^2 \rho(r) \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-1/2} dr$$

is equal to the **sum of the mass elements** on the whole volume of the star, it includes the contributions of the **rest mass and internal energy** of the constituents of the star

- **Gravitational energy:** $E_G = (M_G - M_P)\, c^2 \le 0$

  Gravitational binding energy: $B_G = - E_G$

B_G **is the gravitational energy released moving the infinitesimal mass elements** ρdV **from infinity to form the star.**

In the Newtonian limit

$$E_G = -G\int_0^R 4\pi\ r^2 \frac{m(r)\rho(r)}{r} dr \equiv E_G^{Newt}$$

- **Internal energy:** $E_I = (M_P - M_B)\, c^2 = \int_0^R \varepsilon'(r) dV$

  Internal binding energy: $B_I = - E_I$

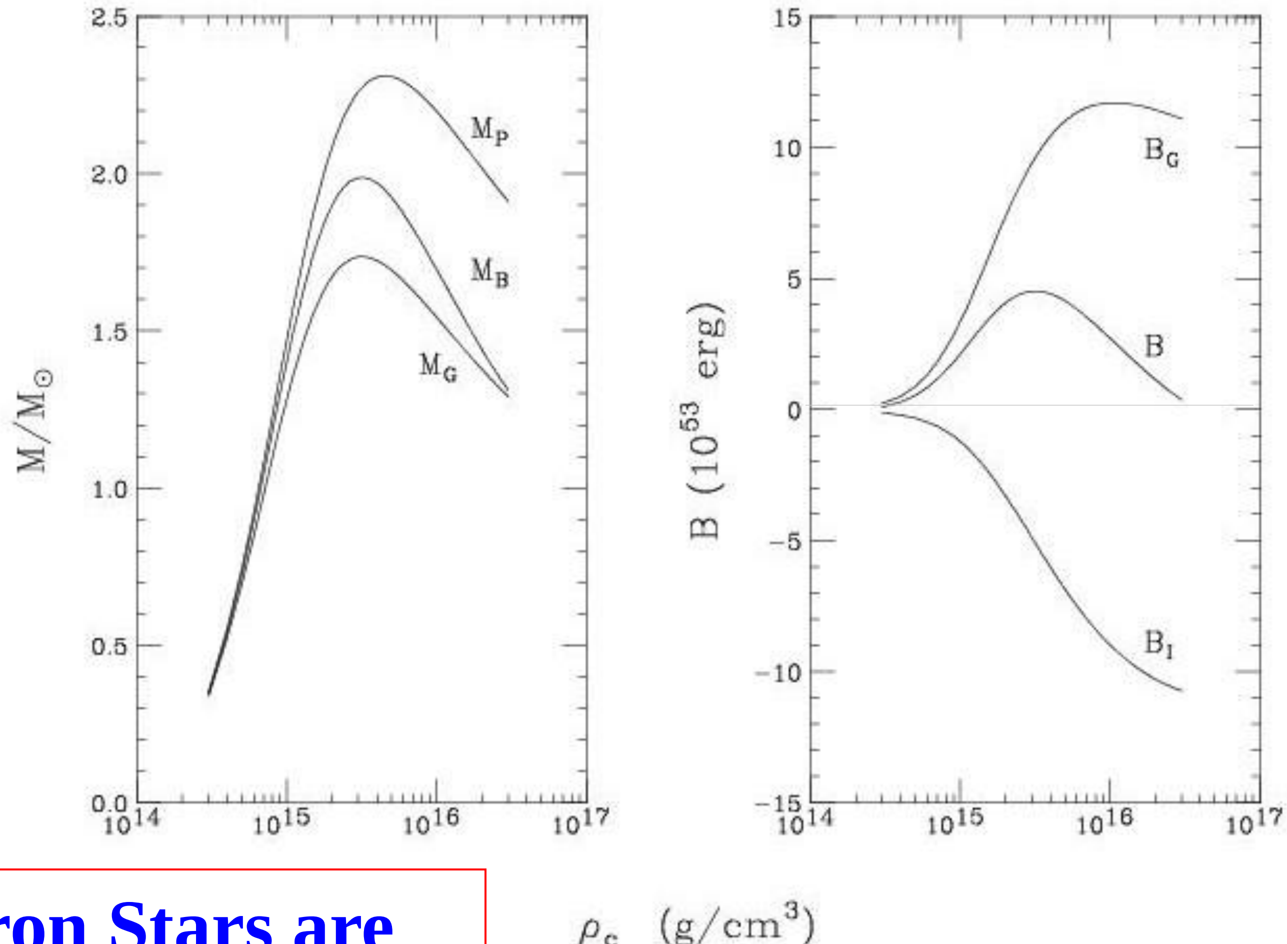
$\varepsilon' = (\rho - \rho_0)\, c^2$

- **Total energy:** $M_G\, c^2 = M_B\, c^2 + E_I + E_G = M_P\, c^2 + E_G$

  Total binding energy: $B = B_G + B_I = (M_B - M_G)\, c^2$

B **is the total energy released during the formation of a static neutron star from a rarified gas of** N_B **baryons**

Masses and binding energies of a Neutron Star



Bombaci (1995)

Neutron Stars are bound by gravity

Total binding energy: $\mathbf{B = B_G + B_I}$

Stability of the solutions of the TOV equations

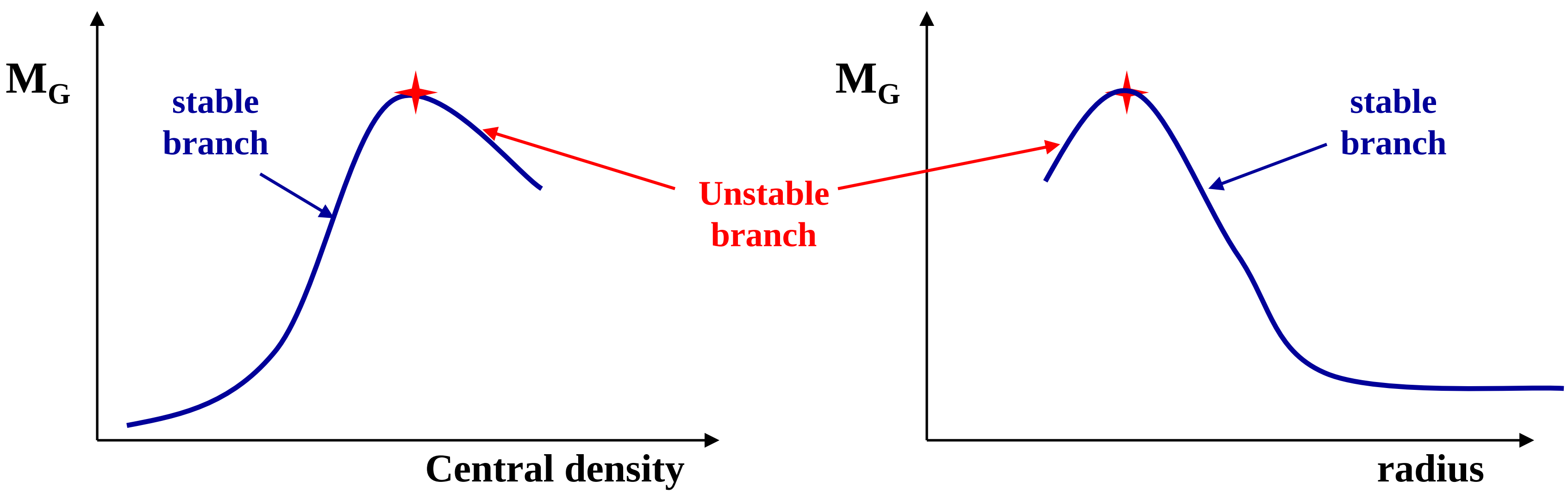
- ❑ **The solutions of the TOV eq.s represent static equilibrium configurations**
- ❑ **Stability of the solutions of TOV eq.s with respect to small perturbations**

Assumption: the time-dependent stellar configuration, which undergoes small radial perturbations, could be described by the EOS of a perfect fluid in "chemical" equilibrium (catalyzed matter)

Stable configurations must have

$$dM_G/d\rho_c > 0$$

This is a necessary but not sufficient condition for stability



The first calculation of the Neutron Stars structure

- **Neutron ideal relativistic Fermi gas**

  (Oppenheimer, Volkoff, 1939).

$M_{max} = 0.71\ M_{\odot}$, $R = 9.5$ km , $n_c/n_0 = 13.75$

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$M_{max} < M_{PSR1913+16} = 1.4408 \pm 0.0003\ M_{\odot}$

Too soft EOS : needs repulsions from nn strong interaction !

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Too soft EOS : needs repulsions from nn strong interaction !

- **Role of the weak interaction**

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

Some protons must be present in dense matter to balance this reaction.

The core of a Neutron Star can not be made of pure neutron matter

Before we start a systematic study of neutron star properties using different models for the EOS of dense matter, we want to answer the following question:

Is it possible to establish an upper bound for the maximum mass of a Neutron Star which does not depend on the deatils of the high density equation of state?

Upper bound on M_{max}

Assumpions:

(a) General Relativity is the correct theory of gravitation.

(b) The stellar matter is a perfect fluid described by a one-parameter EOS,
$P = P(\rho)$.

(c) $\rho \geq 0$ (gravity is attractive)

(d) "microscopic stability" condition: $dP/d\rho \geq 0$

(e) The EOS is known below some fiducial density ρ^*

(f) Causality condition

$$s = (dP/d\rho)^{1/2} \leq c$$

s = speed of sound in dense matter

Under the assumpions (a)—(f) is has been shown
by **Rhoades and Ruffini, (PRL 32, 1974)** that:

- **The upper bound M^{upper} is independent on the details of the EOS below the fiducial density ρ^***

- **M^{upper} scales with ρ^* as:**

$$M^{upper} = 6.8 \left(\frac{10^{14}\,\mathrm{g/cm}^3}{\rho^*} \right)^{1/2} M_{sun}$$

ρ^*	M^{upper}/M_{sun}
ρ_0	4.06
$2\rho_0$	2.87

if $M > M^{upper}$

The compact star is a Black Hole

$\rho_0 = 2.8 \times 10^{14}\,\mathrm{g/cm}^3$ = saturation density of nuclear matter

General features of a "realistic" EOS

Any "realistic" EOS must satisfy the following basic requirements:

(a) saturation properties of symmetric nuclear

$n_0 = 0.16 — 0.18\ \mathrm{fm}^{-3}$ $\quad (E/A)_0 = -16 \pm 1\ \mathrm{MeV}$

(b) Nuclear Symmetry Energy $\quad E_{sym}(n_0) = 28 — 32\ \mathrm{MeV}$,

$E_{sym}(n)$ **"well behaved" at high density**

(c) Nuclear incompressibility $\quad K_0 = 220 \pm 20\ \mathrm{MeV}$

(e) Causality condition:

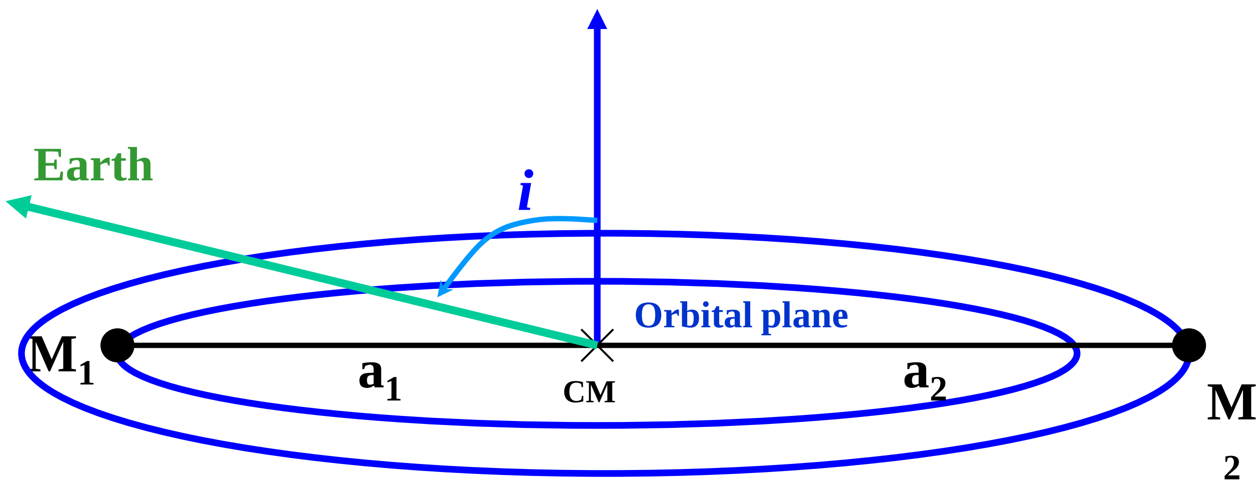
speed of sound $\quad s = (dP/d\rho)^{1/2} \leq c$

Observational determination of the mass of Neutron Stars

Determination of the masses of neutron stars

1) X-ray binaries

The method makes use of the **Kepler's Third Law**.
Consider two spherical masses M_1 and M_2 in circular orbit around their center of mass (the method is valid in the general case of elliptic orbits).



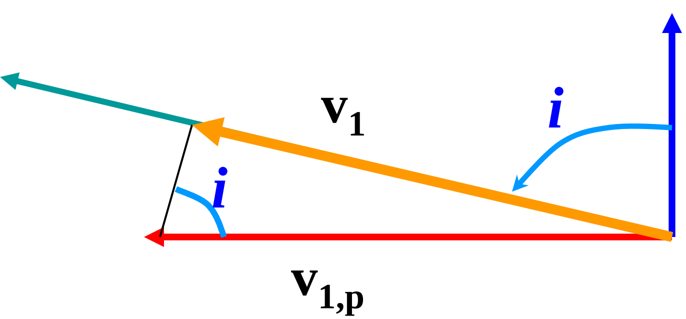
$$a = a_1 + a_2$$

In the CM frame:

$$M_1 a_1 = M_2 a_2$$

$v_{1,p} = 2\pi a_1 / P_b$ = **velocity of M_1 in the orb. plane**

P_b = **orbital period**



$$v_1 = v_{1,p} \sin i = \frac{2\pi}{P_b} a_1 \sin i$$

Any **spectral feature** emitted by the star M_1 will be **Doppler shifted.**

measurig **P_b , v_1** $\rightarrow$ **$a_1 \sin i$**

Kepler's Third Law:

$$G\frac{M_1+M_2}{a^3}=\frac{(2\pi)^2}{P_b^2}$$

$$a=\frac{M_1+M_2}{M_2}a_1$$

Mass function for the star M_1

$$f_1(M_1,M_2,\sin i)\equiv\frac{(M_2\sin i)^3}{(M_1+M_2)^2}=\frac{P_b v_1^3}{2\pi\ G}$$

For some X-ray binaries its has been possible to measure **both the mass functions for the optical companion star as well as the X-ray (NS)**

$$\begin{cases} f_X \equiv \dfrac{(M_{op}\sin i)^3}{(M_X+M_{op})^2} \\ f_{op} \equiv \dfrac{(M_X\sin i)^3}{(M_X+M_{op})^2} \end{cases} \Longrightarrow \begin{cases} q \equiv \left(\dfrac{f_{op}}{f_X}\right)^{1/3} = \dfrac{M_X}{M_{op}} \\ M_X = \dfrac{f_X q (1+q^2)}{\sin^3 i} \end{cases}$$

The determination of the stellar masses depends on the value of **sin *i*.**

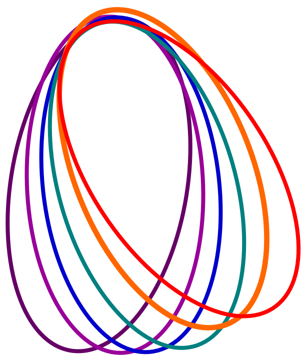
Geometrical constraints can be given on the possible values of **sin *i*:**
in some case **the X-ray component is eclipsed by the companion star →** ***i*** **~ 90°** , **sin *i* ~ 1**

2) Radio binary pulsar

Tight binary systems: **P_b = a few hours.**

General Relativistic effects are crucial to describe the **orbital motion**

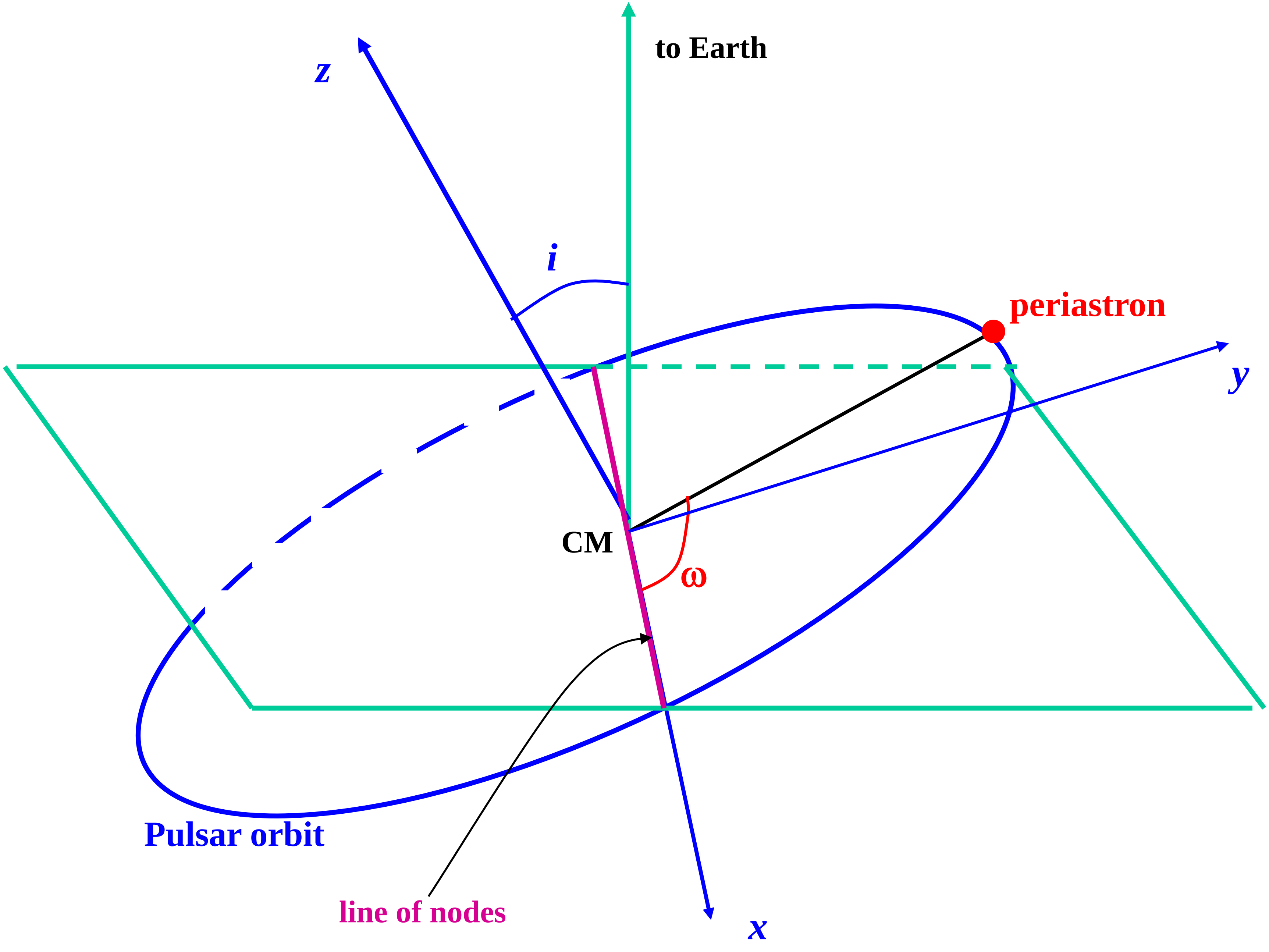
measure of post Keplerian parameters	⇒	high precision NS mass determination



Periastron advance : $\dot{\omega} \neq 0$

e.g. **Perielium advance for mercury,** $\dot{\omega}$ **= 43 *arcsec*/100 *yr***

Orbital decay: $\dot{P}_b \neq 0$ ⇒ **evidence for gravitational waves**



to Earth
z
i
periastron
y
CM
ω
Pulsar orbit
line of nodes
x

Post-Keplerian Parameters

The expressions for post-Keplerian parameters depend on theory of gravity. In the case of **General Relativity**:

$$\dot\omega = 3T_\odot^{2/3} \left(\frac{P_b}{2\pi}\right)^{-5/3} \frac{1}{1-e^2} (m_p + m_c)^{2/3}$$

$$\gamma = T_\odot^{2/3} \left(\frac{P_b}{2\pi}\right)^{1/3} e \frac{m_c(m_p+2m_c)}{(m_p+m_c)^{4/3}}$$

$$r = T_\odot m_c$$

$$s = \sin i = T_\odot^{-1/3} \left(\frac{P_b}{2\pi}\right)^{-2/3} x \frac{(mp+m_c)^{2/3}}{m_c}$$

$$\dot P_b = -\frac{192\pi}{5} T_\odot^{5/3} \left(\frac{P_b}{2\pi}\right)^{-5/3} f(e) \frac{m_p m_c}{(m_p+m_c)^{1/3}}$$

$$\Omega_{\rm geod} = \left(\frac{2\pi}{P_b}\right)^{5/3} T_\odot^{2/3} \frac{m_c(4m_p+3m_c)}{2(m_p+m_c)^{4/3}} \frac{1}{1-e^2}$$

$$T_\odot = GM_\odot/c^3 = 4.9254909\mu s$$

$\dot\omega$: Periastron precession

γ: Time dilation and grav. redshift

r: Shapiro delay “range”

s: Shapiro delay “shape”

$\dot P_b$: Orbit decay due to GW emission

Ω_{geod}: Frequency of geodetic precession resulting from spin-orbit coupling

$m_p = M_p/M_\odot$ pulsar mass

$m_c = M_c/ M_\odot$ companion star mass

$$x = \frac{a_1 \sin i}{c}$$

$$f(e) = \left(1-e^2\right)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

PSR 1913+16

(Hulse and Taylor 1974)

NS (radio PSR) + NS("silent")

P_{PSR} = 59 ms

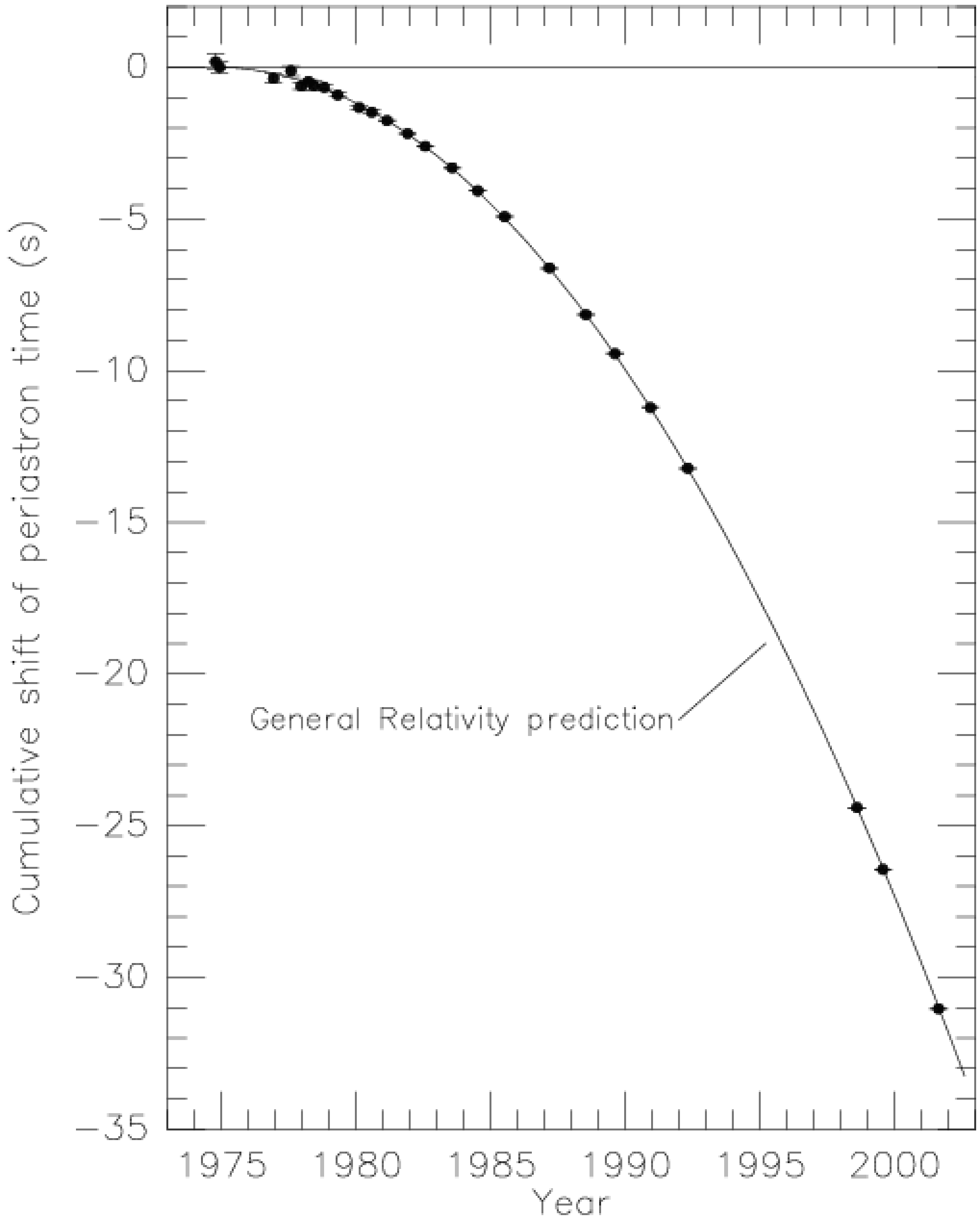
P_b = 7 h 45 min

$\dot{\omega} = 4.22^0 / yr$

Parameter	Value
Orbital period P_b (d)	0.322997462727(5)
Projected semi-major axis x (s)	2.341774(1)
Eccentricity e	0.6171338(4)
Longitude of periastron ω (deg)	226.57518(4)
Epoch of periastron T_0 (MJD)	46443.99588317(3)
Advance of periastron $\dot{\omega}$ (deg yr^{-1})	4.226607(7)
Gravitational redshift γ (ms)	4.294(1)
Orbital period derivative $(\dot{P}_b)^{obs}$ (10^{-12})	$-2.4211(14)$

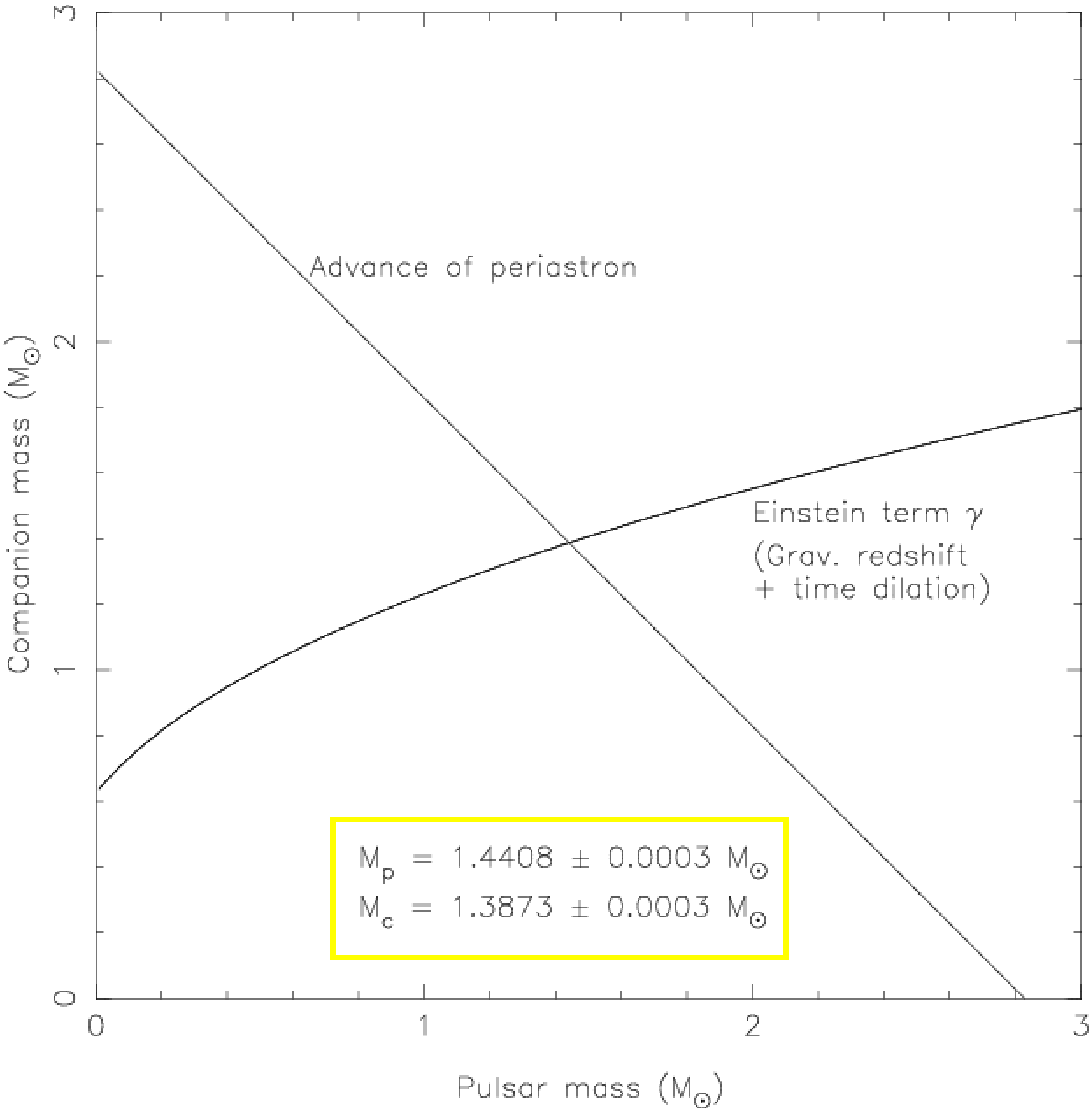
PSR 1913+16

Test of General Relativity and indirect evidence for gravitational radiation



The parabola indicates the predicted accumulated shift in the time of periastron caused by the **decay of the orbit**. The measured value at the epoch of periastron are indicated by the data points

PSR 1913+16


Advance of periastron
Einstein term γ
(Grav. redshift
+ time dilation)
M_p = 1.4408 ± 0.0003 M_⊙
M_c = 1.3873 ± 0.0003 M_⊙
Companion mass (M_⊙)
Pulsar mass (M_⊙)
0
1
2
3
0
1
2
3

PSR J0737-3039

(Burgay, D'Amico, Possenti, et al., Nature, 2003)

NS(PSR) + NS(PSR)

first **double pulsar**

$P_{PSR1} = 22.7$ ms

$P_{PSR2} = 2.77$ s

$P_b = 2$ h 24 min

$e \sim 0.088$

$$\dot{\omega} = 16.88^0 / yr$$

EVIDENCE FOR GRAVITATIONAL WAVE EMISSION

$dP_b/dt = -1.24 \times 10^{-12}$

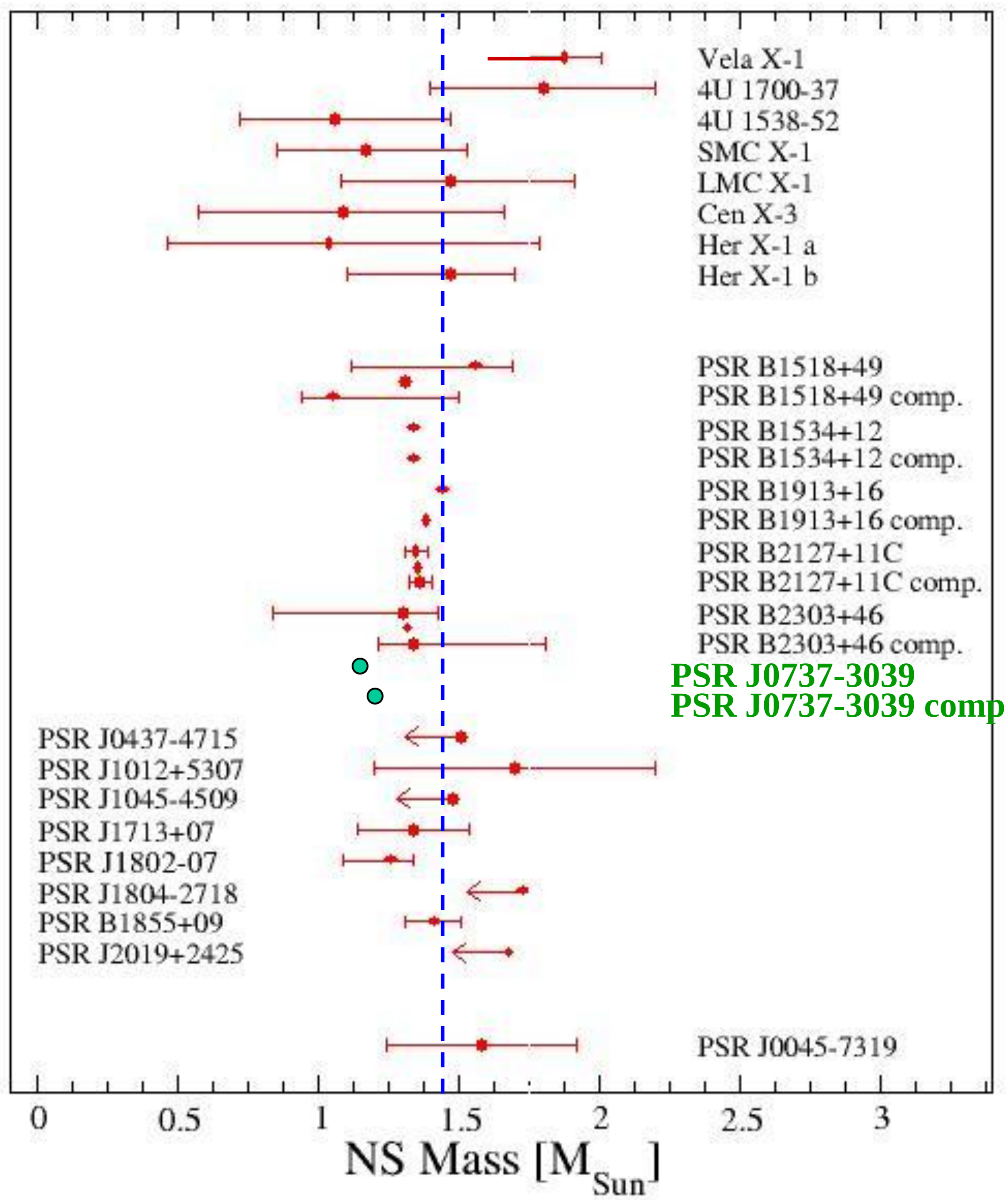
$T_{merg} \sim 85$ Myr

$M_1 = 1.34\ M_\odot$

$M_2 = 1.25\ M_\odot$

The VIRGO gravitational waves antenna - Cascina (Pisa)

Measured Neutron Star Masses

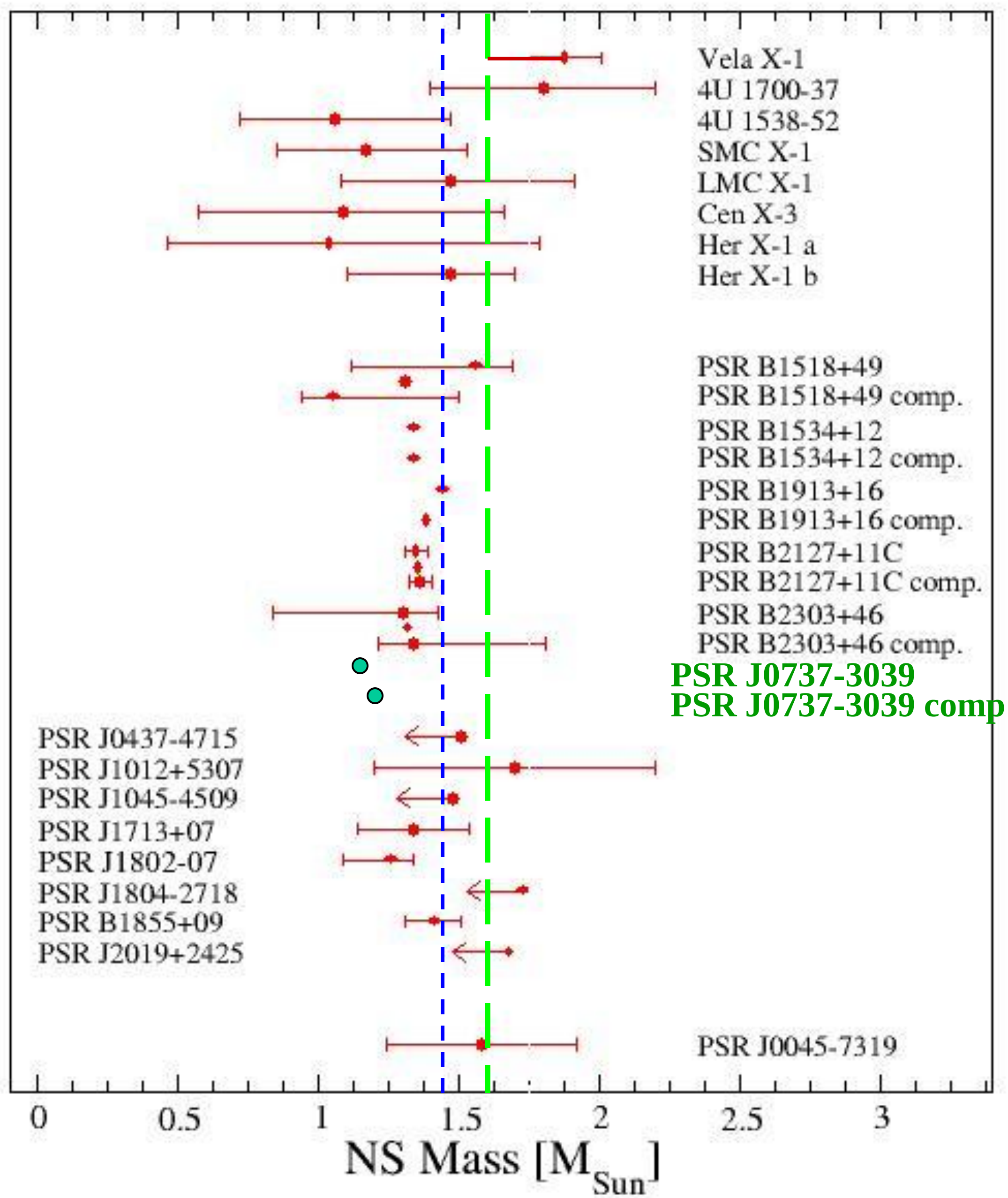


$M_{max} \geq M_{measured}$

$M_{max} \geq 1.44\ M_{\odot}$

"very soft" EOS are ruled out

Measured Neutron Star Masses



$M_{max} \geq M_{measured}$

$M_{max} \geq 1.44\ M_{\odot}$

"very soft" EOS are ruled out

$M_{max} \geq 1.57\ M_{\odot}$

$= M_{low}(\text{Vela X-1})$

Quaintrell et al., 2003,
Astron. & Astrophys., 401, 313

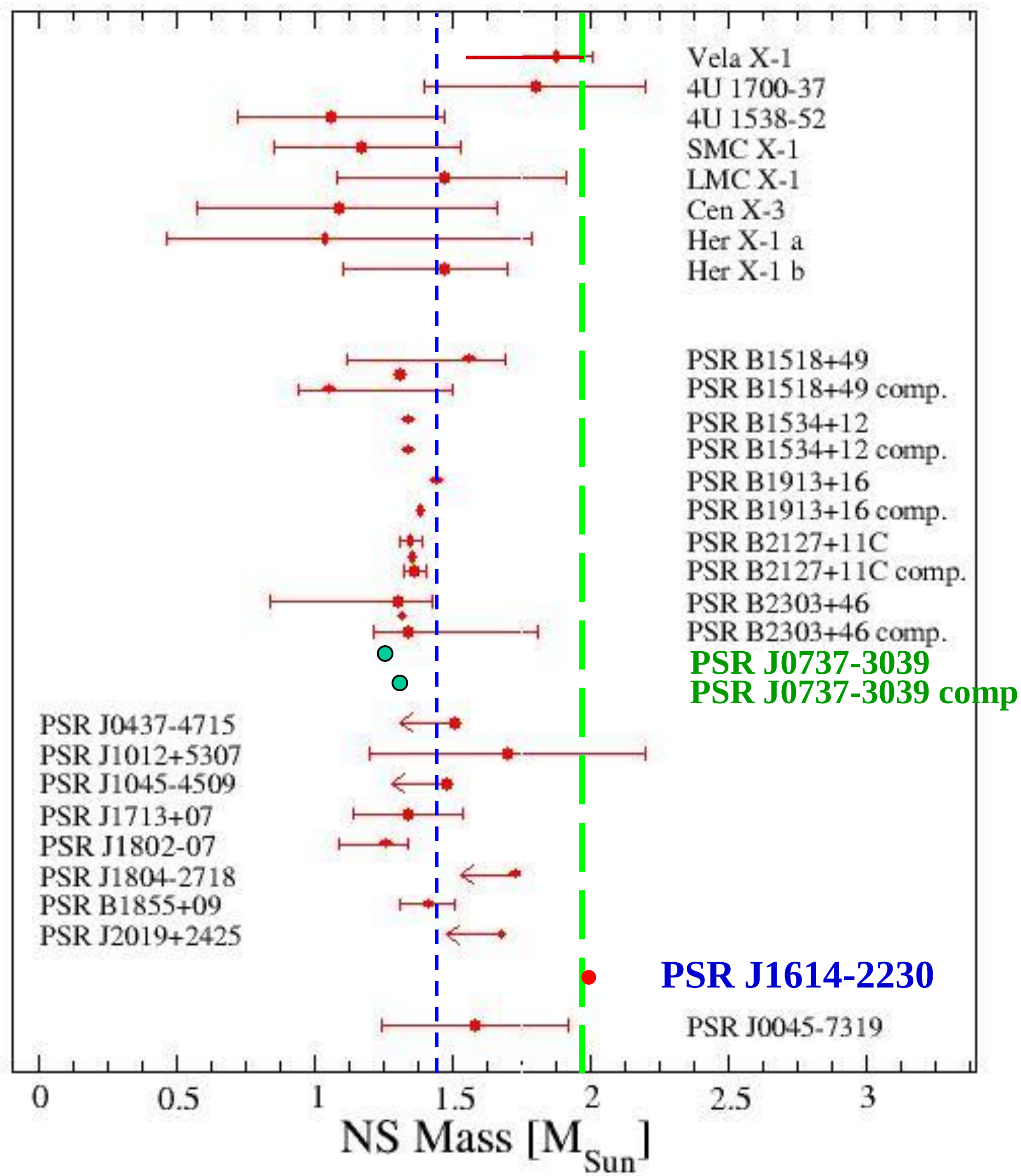
PRS J1614–2230 a "heavy" Neutron Star

NS – WD binary system	(He WD)
$\mathbf{M_{WD} = 0.5\ M_\odot}$	(companion mass)
$\mathbf{P_b = 8.69\ hr}$	(orbital period)
P = 3.15 ms	(PRS spin period)
$\boldsymbol{i = 89.17° \pm 0.02°}$	(inclination angle)

$$\mathbf{M_{NS} = 1.97 \pm 0.04\ M_\odot}$$

P. Demorest et al., Nature 467 (2010) 1081

Measured Neutron Star Masses



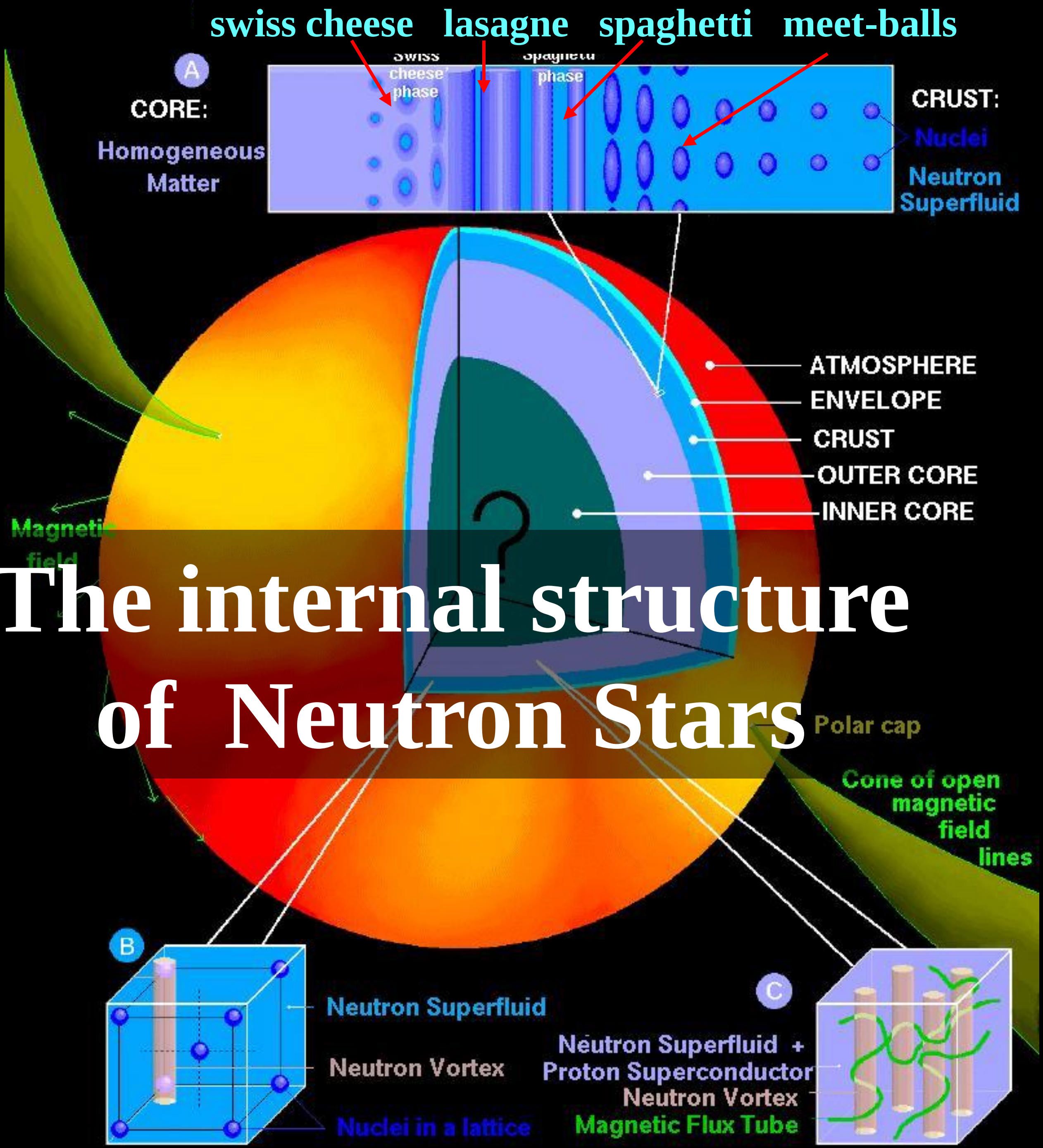
$$M_{max} \geq M_{measured}$$

$M_{max} \geq 1.97\ M_{\odot}$

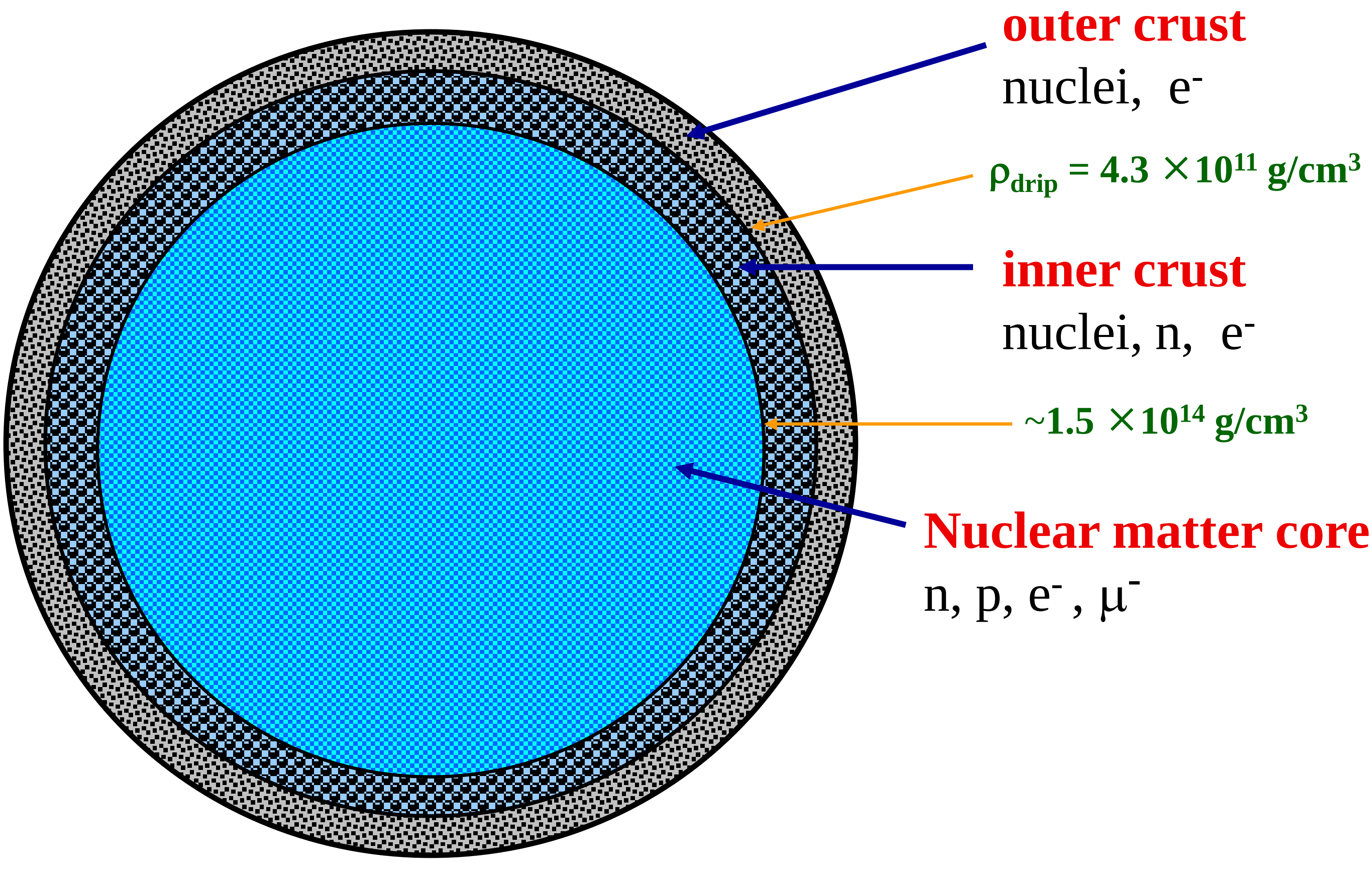
Demorest et al., 2010

Very stringent constrain on the EOS

The internal structure of Neutron Stars

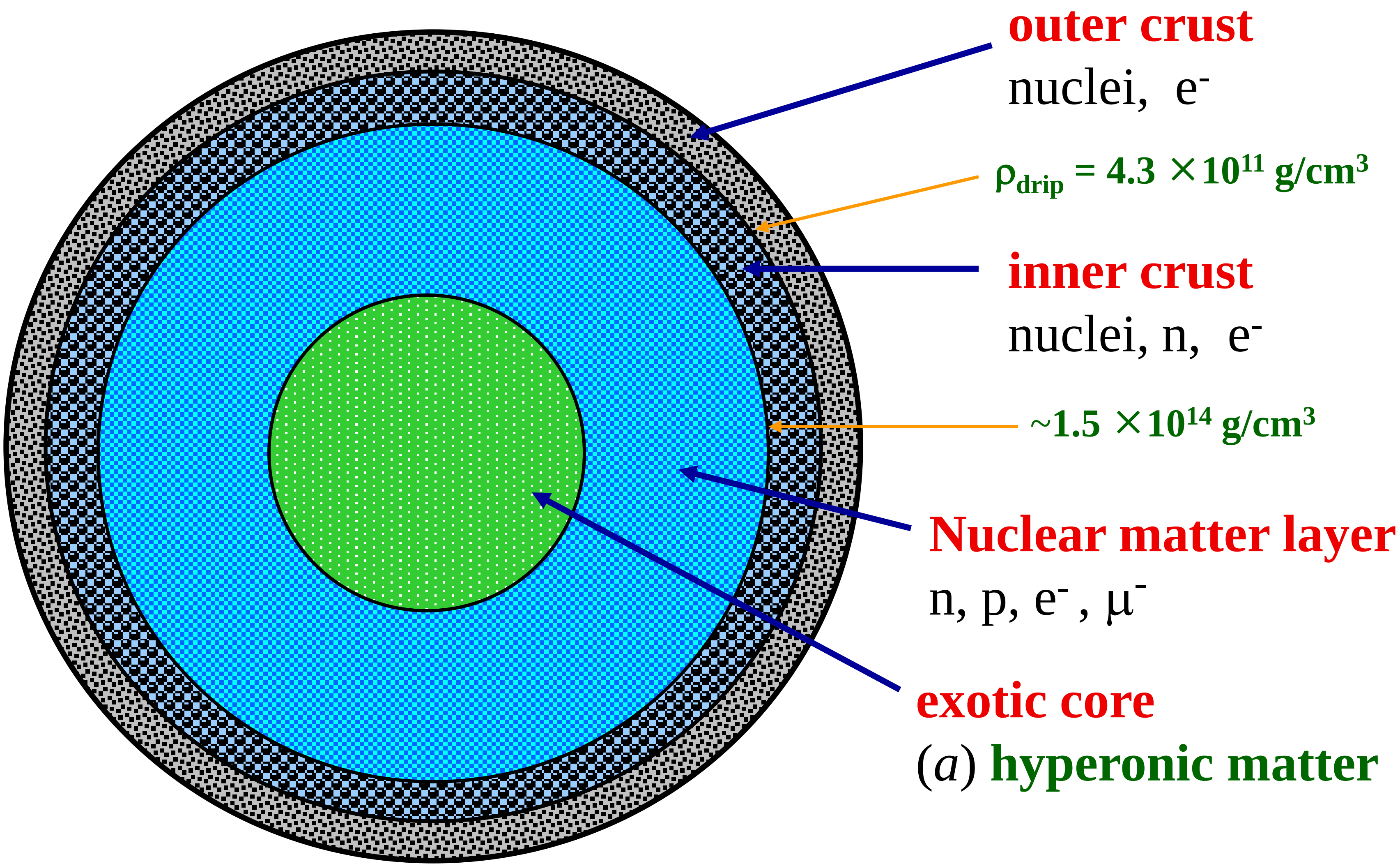


Schematic cross section of a Neutron Star



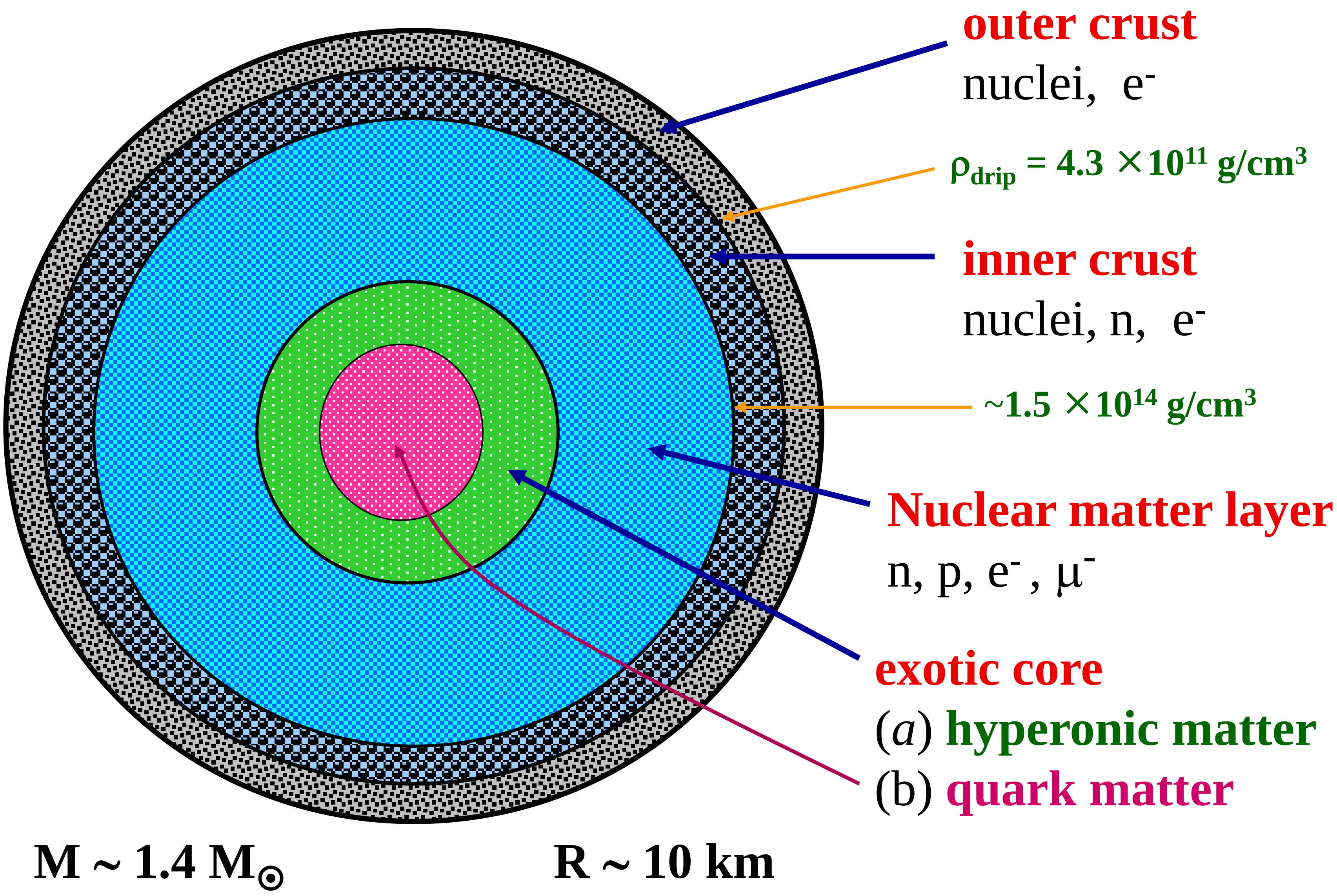
$M \sim 1.4\, M_\odot$ **$R \sim 10$ km**

Schematic cross section of a Neutron Star



$M \sim 1.4\ M_\odot$ $R \sim 10$ km

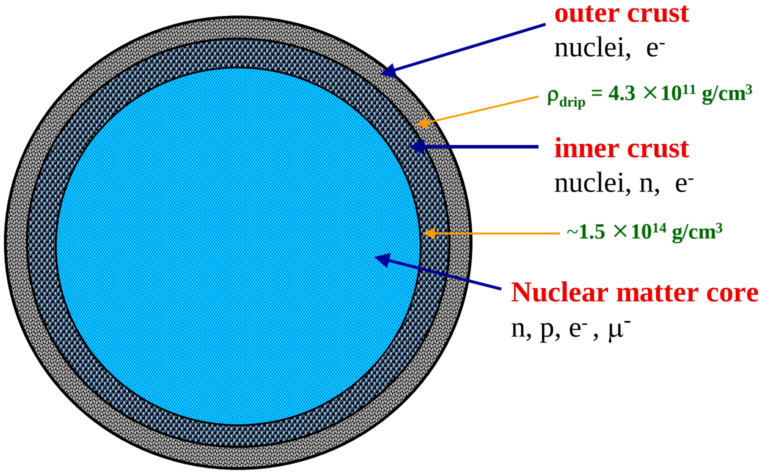
Schematic cross section of a Neutron Star



Neutron Stars with a nuclear matter core

As we have already seen due to the weak interaction,
the core of a Neutron Star can not be made of pure neutron matter.

Core constituents: n, p, e^-, μ^-



β-stable nuclear matter

$$p + e^- \leftrightarrow n + \nu_e$$

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

if $\mu_e \geq m_\mu = 105.6 MeV$

$$e^- \leftrightarrow \mu^- + \nu_e + \bar{\nu}_\mu$$

$$p + \mu^- \leftrightarrow n + \nu_\mu$$

$$\mu_\nu = \mu_{\bar{\nu}} = 0$$

neutrino-free matter

- **Equilibrium with respect to the weak interaction processes**

$$\mu_n - \mu_p = \mu_e$$

$$\mu_\mu = \mu_e$$

- **Charge neutrality**

$$n_p = n_e + n_\mu$$

To be solved for any given value of the total baryon number density n_B

Proton fraction in β-stable nuclear matter and role of the nuclear symmetry energy

$$\hat{\mu} \equiv \mu_n - \mu_p = -\frac{\partial(E/A)}{\partial x} = 2\frac{\partial(E/A)}{\partial \beta}$$

$$\begin{cases} \beta = (n_n - n_p)/n = 1 - 2x \\ n = n_n + n_p \end{cases}$$

$\beta = (n_n - n_p)/n = 1 - 2x$ asymmetry paramter $x = n_p/n$ proton fraction

$n = n_n + n_p$ total baryon density

Proton fraction in β-stable nuclear matter and role of the nuclear symmetry energy

$$\hat{\mu} \equiv \mu_n - \mu_p = -\frac{\partial(E/A)}{\partial x} = 2\frac{\partial(E/A)}{\partial \beta}$$

$$E_{sym}(n) \equiv \frac{1}{2}\frac{\partial^2(E/A)}{\partial \beta^2}\bigg|_{\beta=0}$$

$\beta = (n_n - n_p)/n = 1 - 2x$ asymmetry paramter

$n = n_n + n_p$ total baryon density

$x = n_p/n$ proton fraction

Energy per nucleon for asymmetric nuclear matter(*)

$$E(n,\beta)/A = E(n,\beta=0)/A + E_{sym}(n)\,\beta^2$$

$\beta = 0$ **symm nucl matter**

$\beta = 1$ **pure neutron matter**

$$E_{sym}(n) = E(n,\beta=1)/A - E(n,\beta=0)/A$$

$$\Rightarrow \quad \hat{\mu} = 4\, E_{sym}(n)\,[1 - 2x]$$

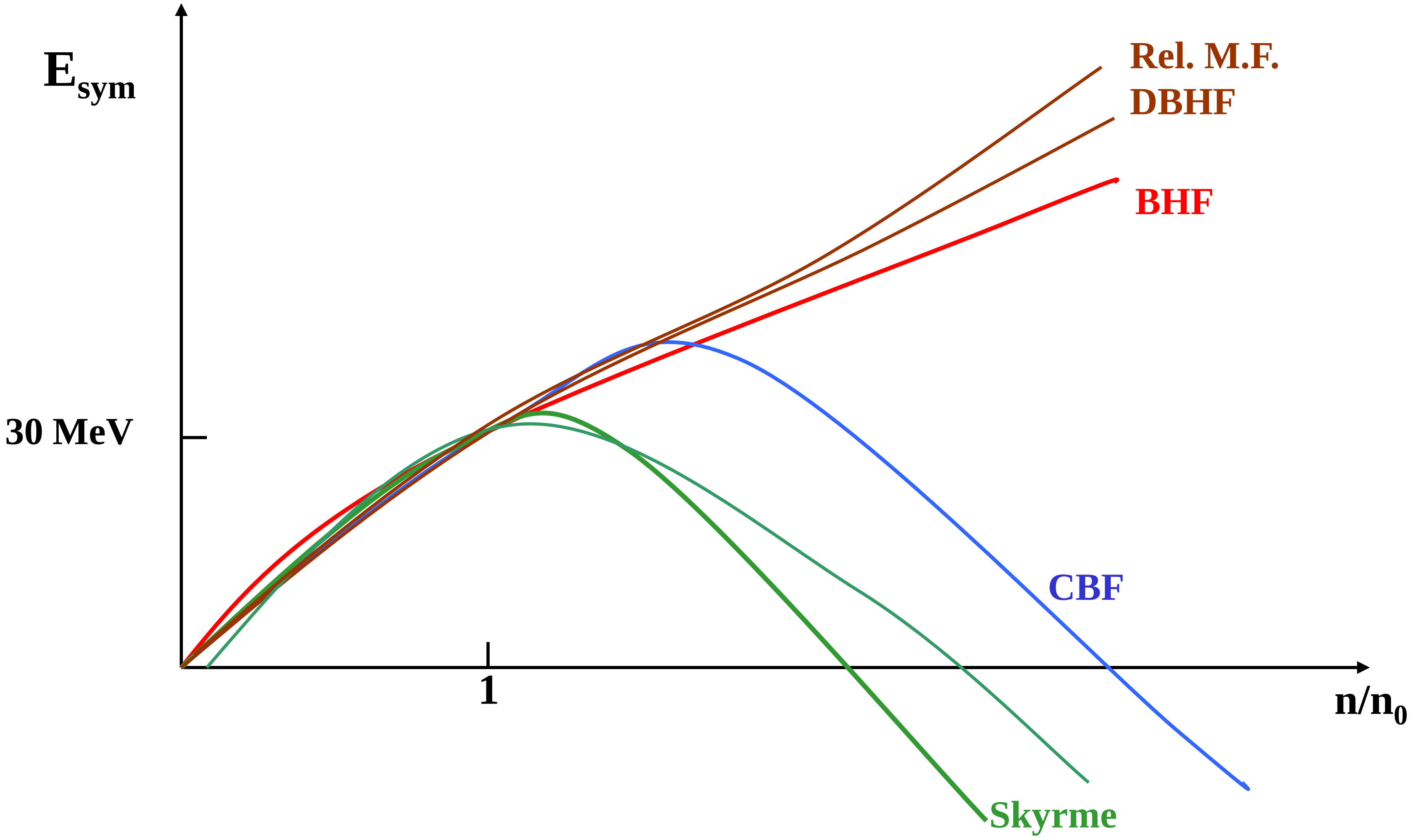
Chemical equil. + charge neutrality (no muons)

$$3\pi^2(\hbar c)^3\, n\, x(n) - [4\, E_{sym}(n)\,(1 - 2\, x(n))]^3 = 0$$

The composition of β-stable nuclear matter is strongly dependent on the nuclear symmetry energy.

(*) Bombaci, Lombardo, Phys. Rev: C44 (1991)

Schematic behaviour of the nuclear symmetry energy



Microscopic EOS for nuclear matter: Brueckner-Bethe-Goldstone theory

$$G_{\tau\tau'}(\omega) = V + V \sum_{k_a k_b} \frac{|k_a k_b\rangle Q_{\tau\tau'} \langle k_a k_b|}{\omega - e_\tau(k_a) - e_{\tau'}(k_b)} G_{\tau\tau'}(\omega)$$

$$e_\tau(k) = \frac{\hbar^2 k^2}{2M} + U_\tau(k)$$

$$U_\tau(k) = \sum_{\tau'} \sum_{k'} \langle \vec{k}\vec{k}' | G_{\tau\tau'}(e_\tau + e_{\tau'}) | \vec{k}\vec{k}' \rangle$$

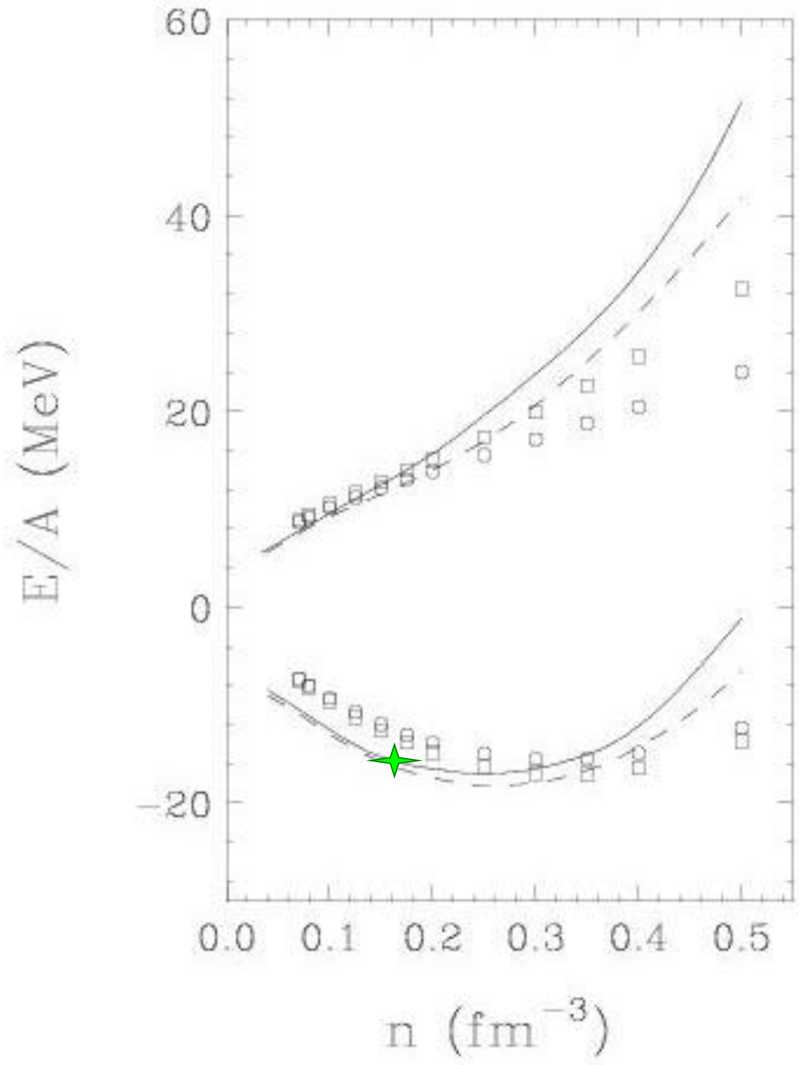
V is the **nucleon-nucleon interaction** (*e.g.* **the Argonne v14, Paris, Bonn potential**) **plus a density depependent Three-Body Force (TBF) necessary to reproduce the empirical saturation on nuclear matter**

- **Energy per baryon in the Brueckner-Hartree-Fock (BHF) approximation**

$$\frac{E}{A} = \frac{1}{A} \sum_{\tau} \sum_{k} \frac{\hbar^2 k^2}{2M} + \frac{1}{2A} \sum_{\tau} \sum_{k} U_\tau(k)$$

Energy per baryon

(two body forces only)



Upper curves: neutron matter
lower curves: symmetric nuclear matter

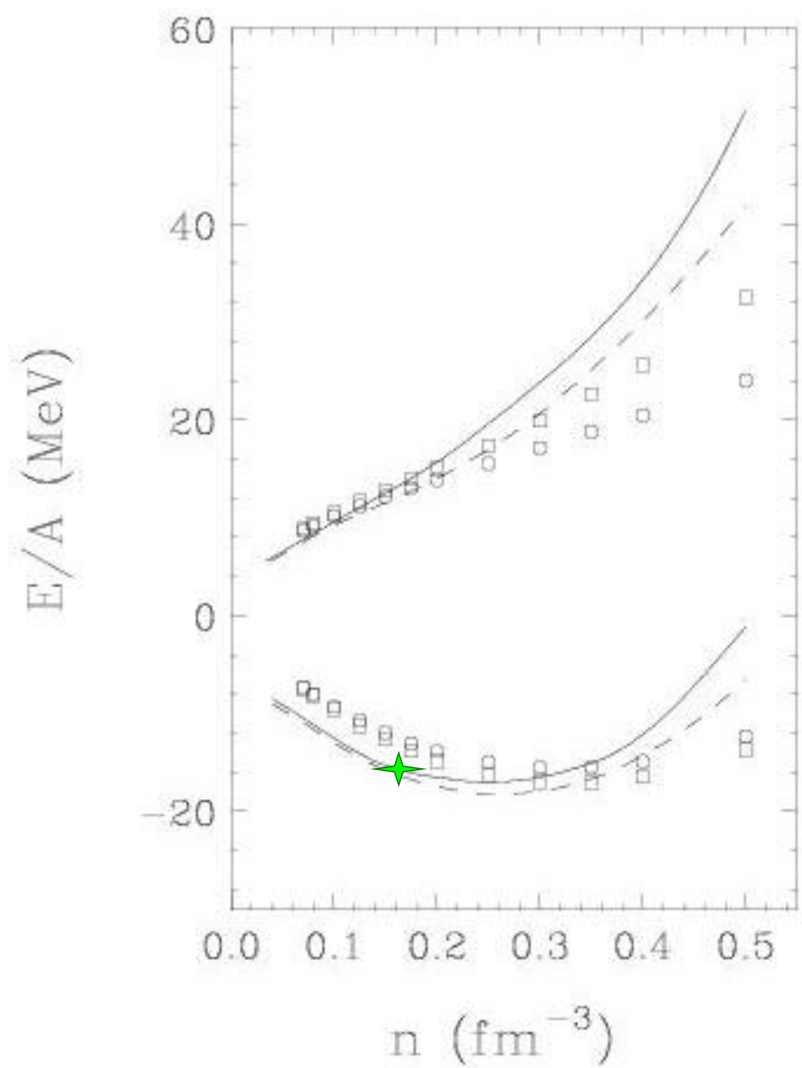
Empirical saturation point ✦

BHF with A14 - - - - - - - - -
BHF with Paris ———————
WFF: CBF with U14 □□□□□□□□□
WFF: CBF with A14 ○○○○○○○○○

Baldo, Bombaci, Burgio, Astr. & Astrophys. 328, (1997)

Energy per baryon

(two body forces only)



Three Body Forces (TBF) are necessary to get the correct saturation point of nuclear matter in non-relativistic many-body calculations

Empirical saturation point

BHF with A14

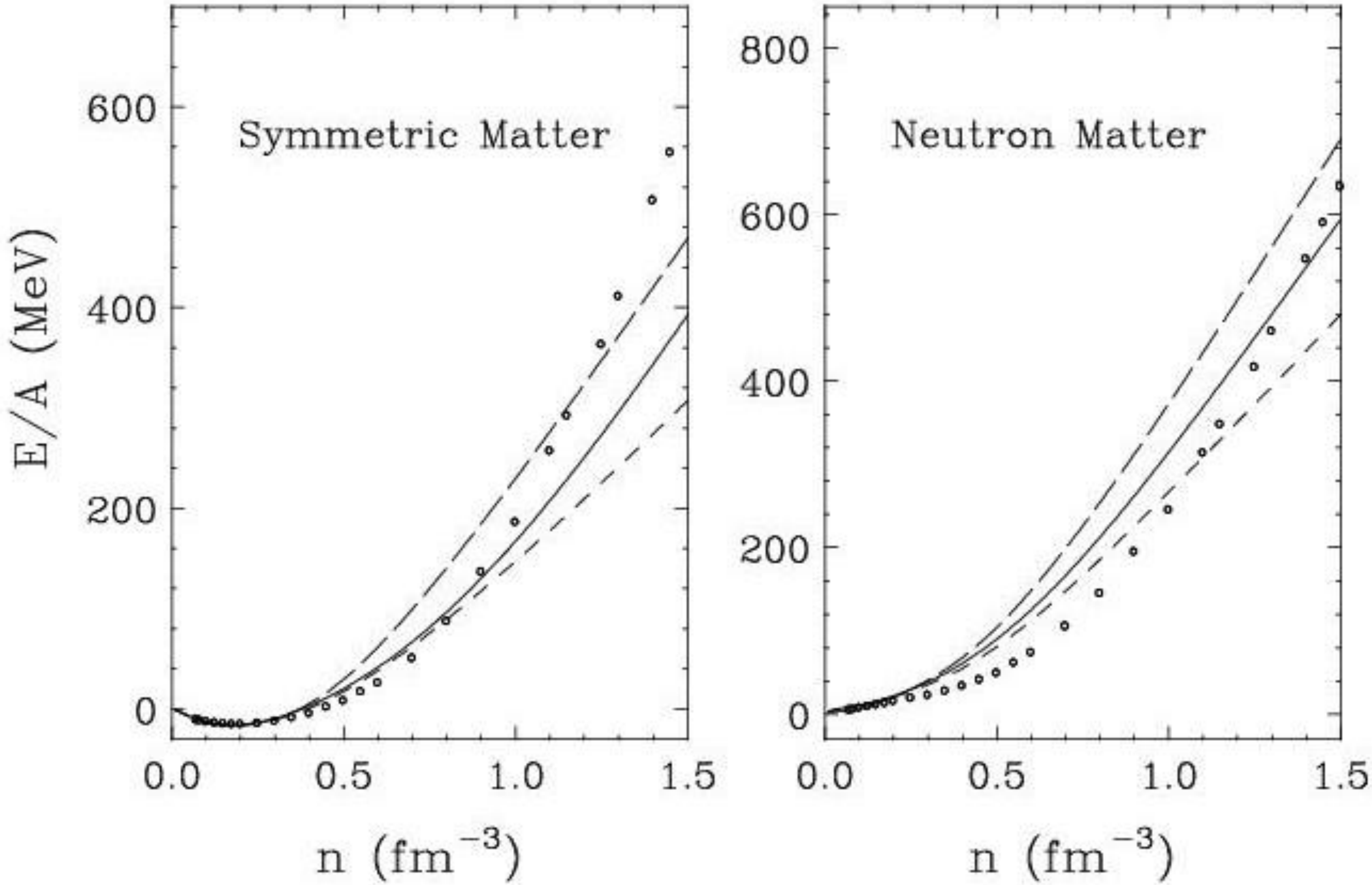
BHF with Paris

WFF: CBF with U14

WFF: CBF with A14

Baldo, Bombaci, Burgio, Astr. & Astrophys. 328, (1997)

Energy per baryon



BBB1: BHF with A14+TBF - - - - - - - -

BBB2: BHF with Paris+TBF ————

DBHF: Bonn A — — — —

WFF: CBF with A14+TBF o o o o o o o o o o

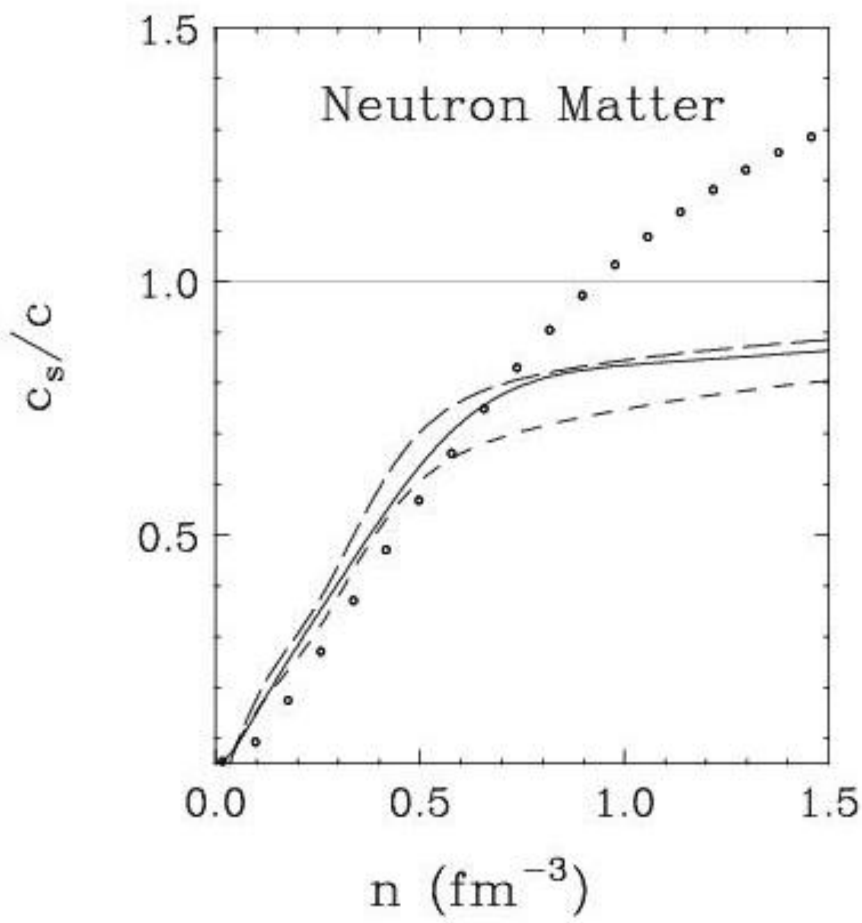
Baldo, Bombaci, Burgio, Astr. & Astrophys. 328, (1997)

Saturation properties BHF EOS (with TBF)

EOS	n_0 (fm^{-3})	E_0/A (MeV)	K (MeV)
A14+TBF	0.178	-16.46	253
Paris+TBF	0.176	-16.01	281
empirical saturation	0.17 ± 0.1	-16 ± 1	220 ± 20

The parameters of this TBF are chosen to reproduce the empirical sauration point, nevertheless the values of these parameters are almost the same of the Urbana VII TBF model, where the fit was done on the energy and radii of few body nuclei (^3H, ^3He).

Speed of sound



BBB1: BHF with A14+TBF - - - - - - - - -
BBB2: BHF with Paris+TBF ———————
DBHF: Bonn A — — — —
WFF: CBF with A14+TBF ○○○○○○○○○○

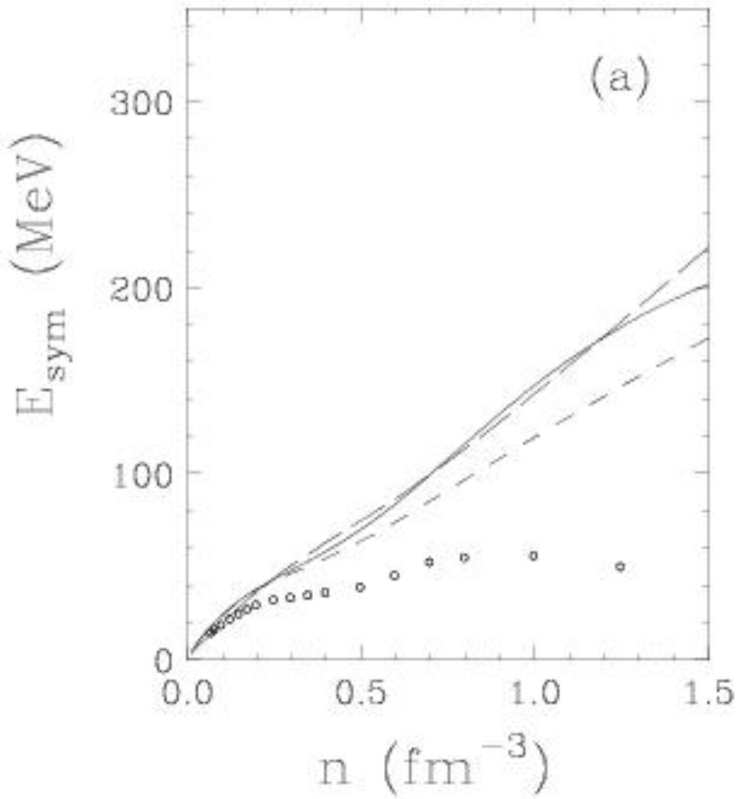
At high density extrapolation using

$E/A = Q^{\lambda}(n) / (1 + b\, n^{\lambda})$

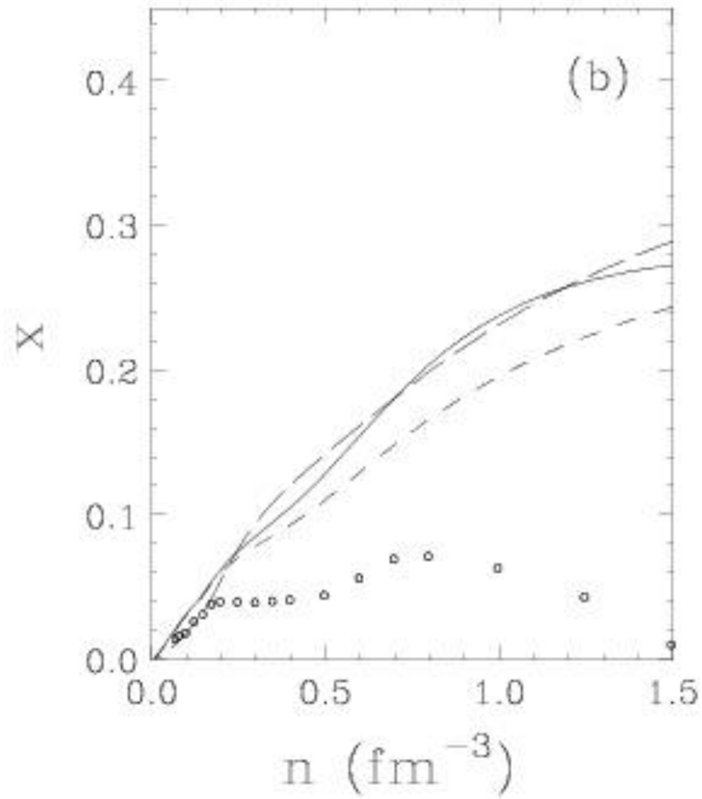
$Q^{\lambda}(n)$ = polinomial of degree λ

Baldo, Bombaci, Burgio, A&A 328, (1997)

Symmetry energy



Proton fraction in β-stable nucl. matter



BBB1: BHF with A14+TBF

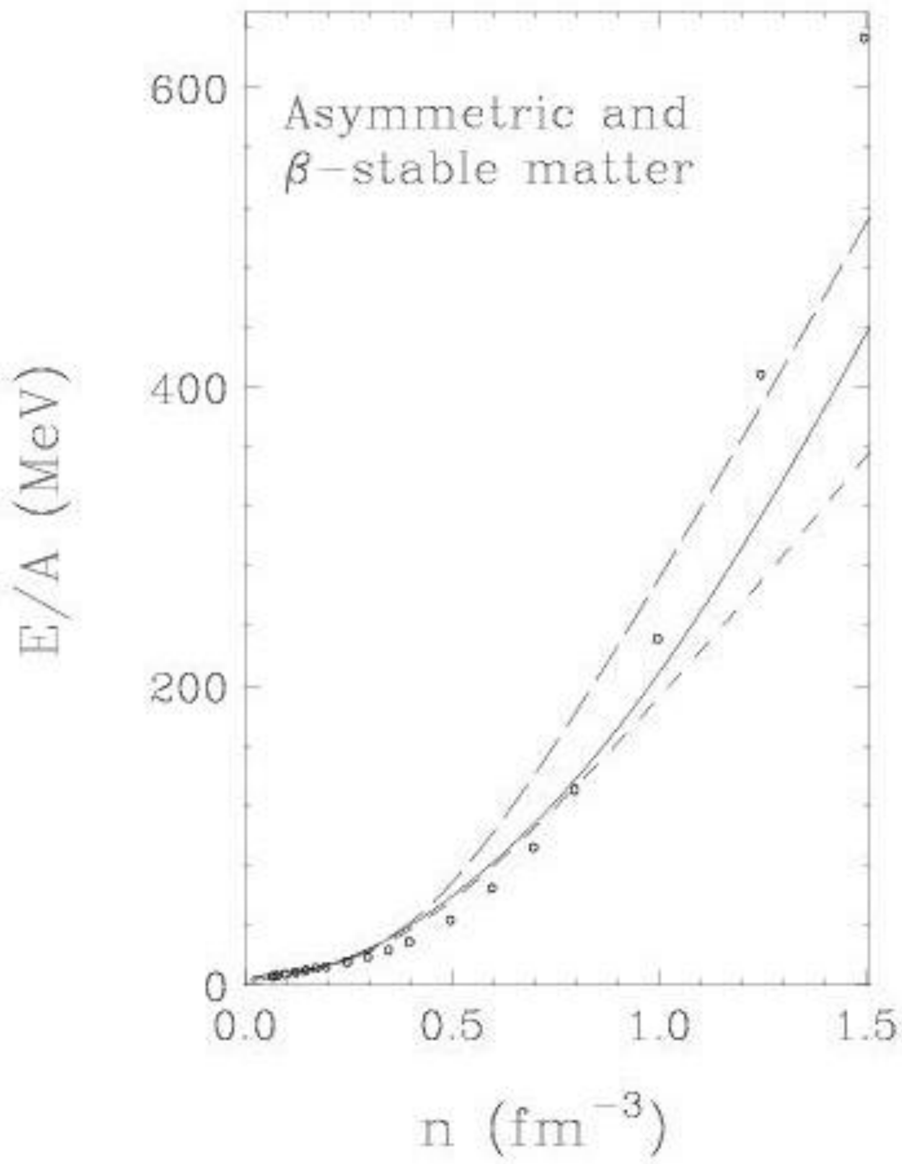
BBB2: BHF with Paris+TBF

DBHF: Bonn A

WFF: CBF with A14+TBF

Baldo, Bombaci, Burgio, Astr. & Astrophys. 328, (1997)

E/A in β-stable nuclear matter



BBB1: BHF with A14+TBF - - - - - -
BBB2: BHF with Paris+TBF
DBHF: Bonn A ———————
WFF: CBF with A14+TBF — — —
o o o o o o o o o o

Baldo, Bombaci, Burgio, Astr. & Astrophys. 328, (1997)

The EOS for β-stable matter

Pressure:

$$P_{nucl}(n) = n^2 \frac{d(E/A)}{dn}$$

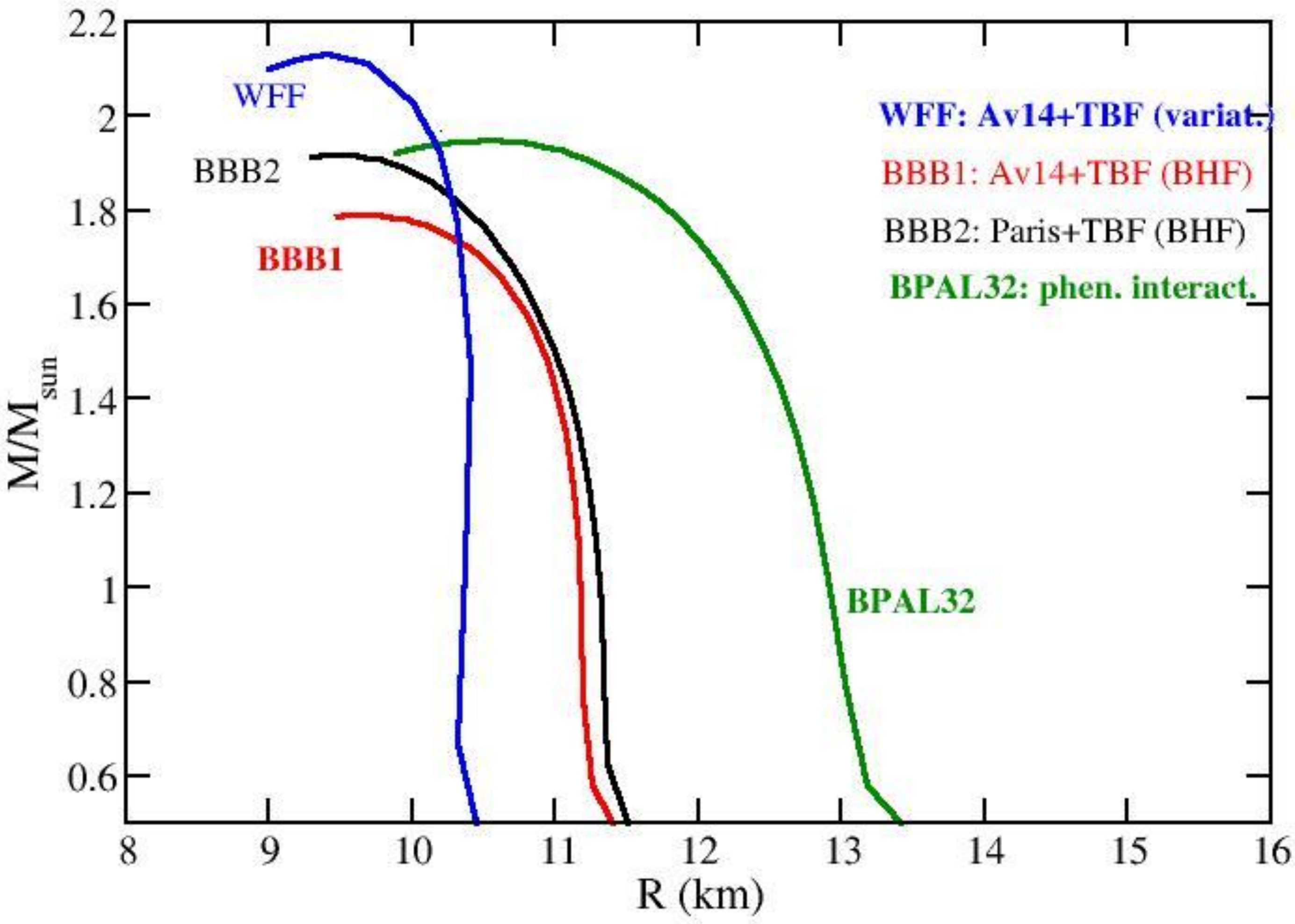
$$P = P_{nucl} + P_{lep}$$

Mass density:

$$\rho = \frac{1}{c^2}\left(\varepsilon_{nucl} + \varepsilon_{lep}\right) = \frac{1}{c^2}\left(n\frac{E}{A} + m_N c^2 n + \varepsilon_{lep}\right)$$

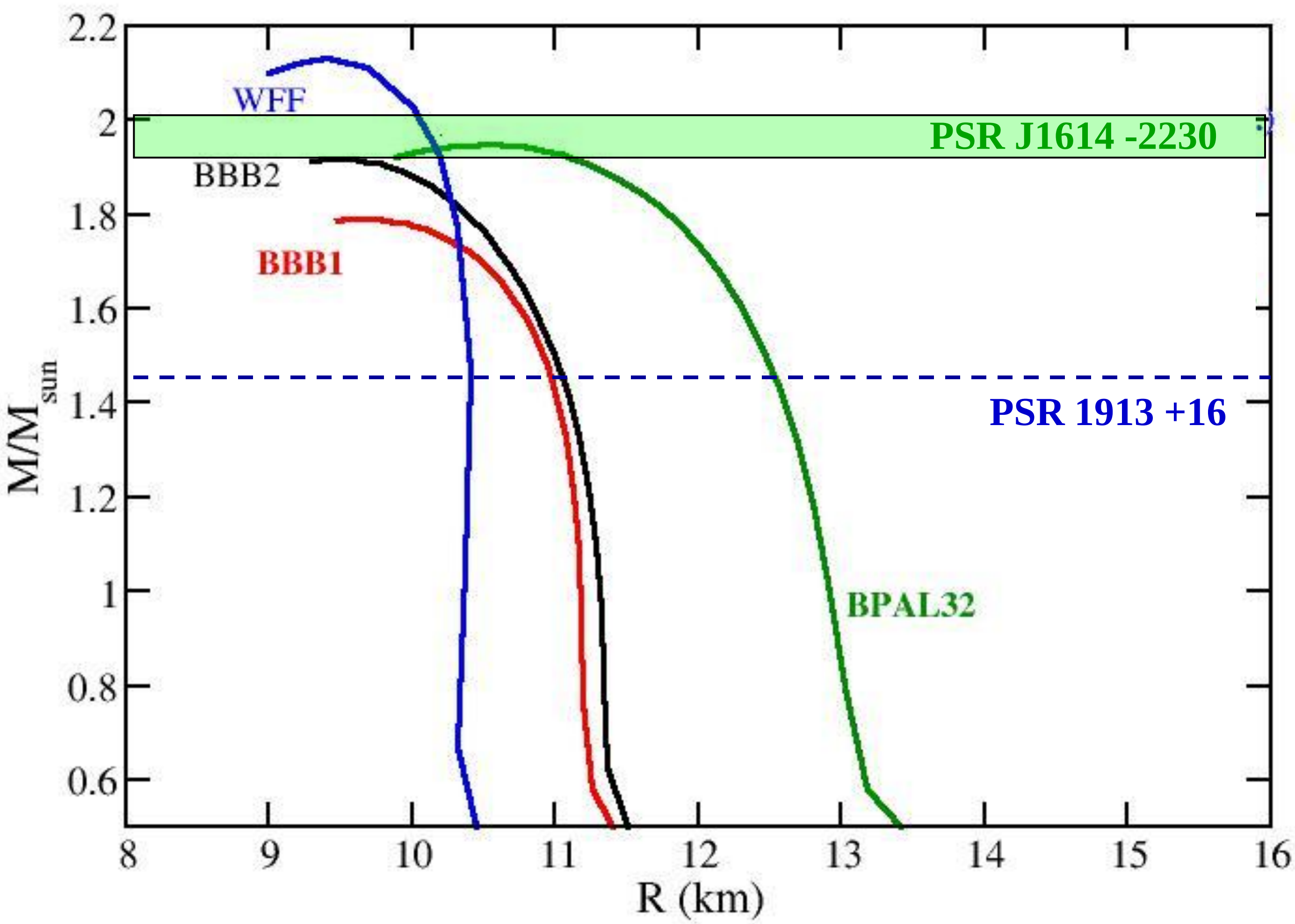
Leptons are treated as **non-interacting relativistic fermionic gases**

Mass-Radius relation for *nucleonic* Neutron Stars



WFF: Wiringa-Ficks-Fabrocini, 1988.
BPAL: Bombaci, 1995.
BBB: Baldo-Bombaci-Burgio, 1997.

Mass-Radius relation for *nucleonic* Neutron Stars



WFF: Wiringa-Ficks-Fabrocini, 1988.
BPAL: Bombaci, 1995.
BBB: Baldo-Bombaci-Burgio, 1997.

Maximum mass configuration of pure nucleonic Neutron Stars for different EOS

EOS	$M_G/M_\odot$	R(km)	n_c / n_0
BBB1	1.79	9.66	8.53
BBB2	1.92	9.49	8.45
WFF	2.13	9.40	7.81
BPAL12	1.46	9.04	10.99
BPAL22	1.74	9.83	9.00
BPAL32	1.95	10.54	7.58
KS	2.24	10.79	6.30

KS: Krastev, Sammarruca, 2006, Phys. Rev. C74, (2006) 025808. DBHF with Bonn – B potential

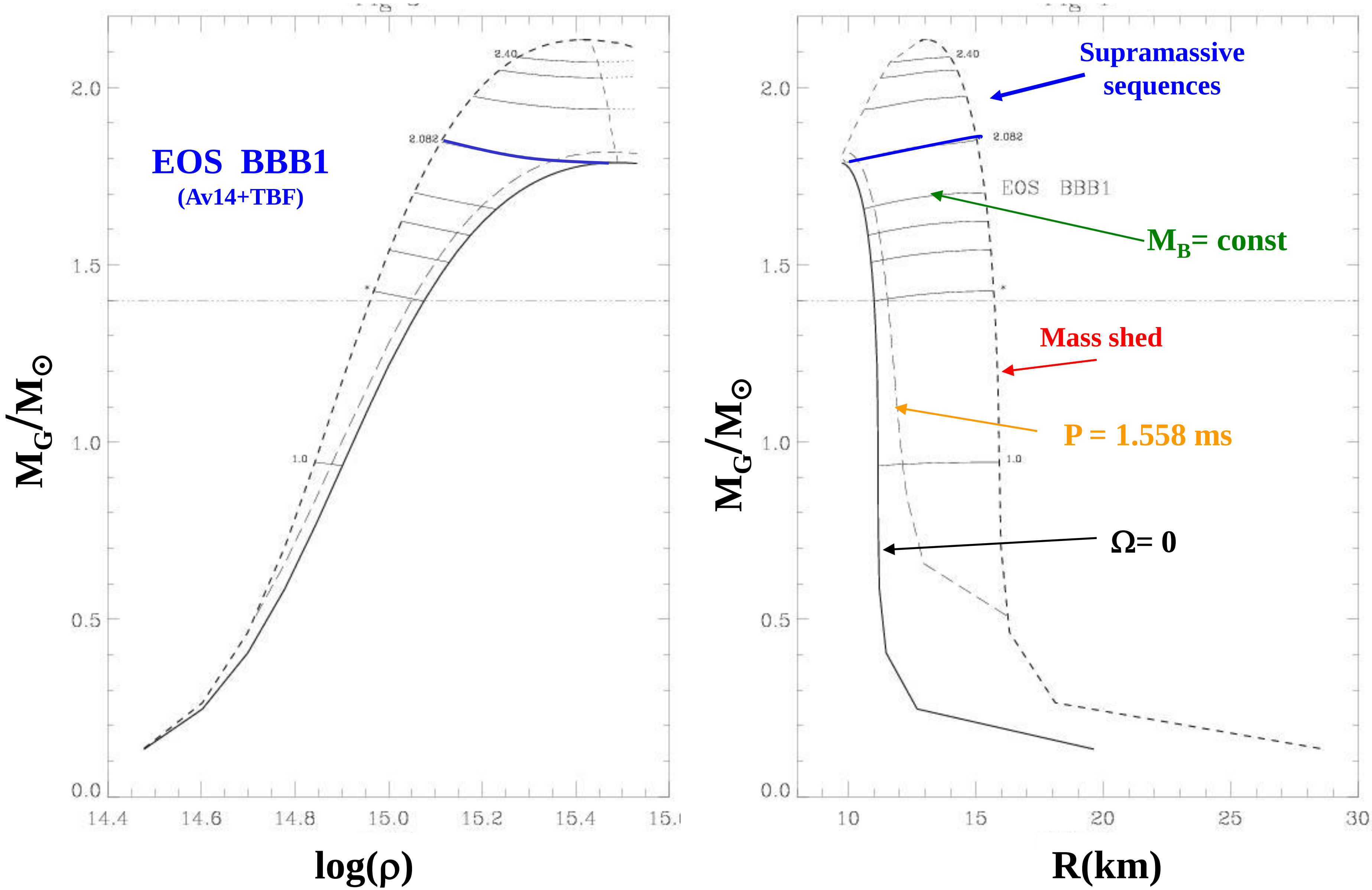
Properties of neutron stars with $M_G = 1.4\ M_\odot$

EOS	R(km)	n_c / n_0	x_c
BBB1	11.0	4.06	0.139
BBB2	11.1	4.00	0.165
WFF	10.41	4.13	0.066

Crustal properties of neutron stars with $M_G = 1.4\ M_\odot$

EOS	ρ_c (10^{15} g/cm^3)	R(km)	R_{core}	ΔR_{inner}	ΔR_{outer}	ΔR_{crust}
BPAL12	2.5	9.98	8.56	1.15	0.27	1.42
BPAL22	1.2	11.81	9.63	1.75	0.43	2.18
BPAL32	0.9	12.60	10.06	2.05	0.49	2.54

Rotating Neutron Stars



Datta, Thampan, Bombaci, Astron. and Astrophys. 334 (1998)

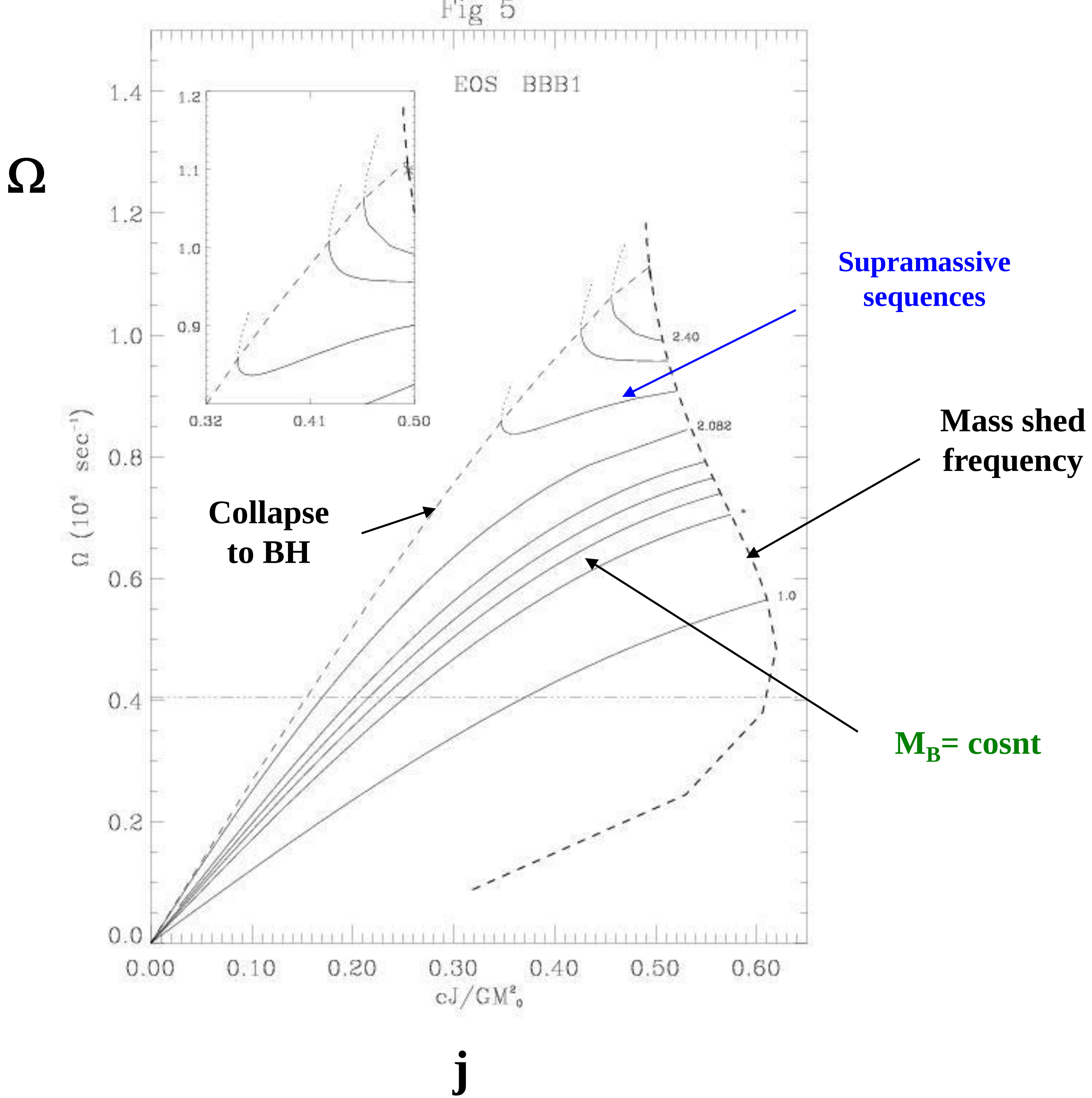


Fig 5
EOS BBB1
Ω
Ω (10⁴ sec⁻¹)
cJ/GM²₀
j
2.40
2.082
1.0
Supramassive sequences
Mass shed frequency
Collapse to BH
M_B= cosnt

Neutron Stars or Hyperon Stars

Why is it very likely to have hyperons in the core of a Neutron Star?

(1) The central density of a Neutron Star is "high"

$\boldsymbol{\rho_c \approx (4 - 10)\, \rho_0}$ $(\rho_0 = 0.17\ \text{fm}^{-3})$

(2) The nucleon chemical potentials increase very rapidly as function of density.

→ Above a threshold density ($\boldsymbol{\rho_c \approx (2 - 3)\, \rho_0}$) hyperons are created in the stellar interior.

A. Ambarsumyan, G.S. Saakyan, (1960)
V.R. Pandharipande (1971)

Threshold density for hyperons in neutron matter

- **Non-relativistic free Fermi neutron gas**

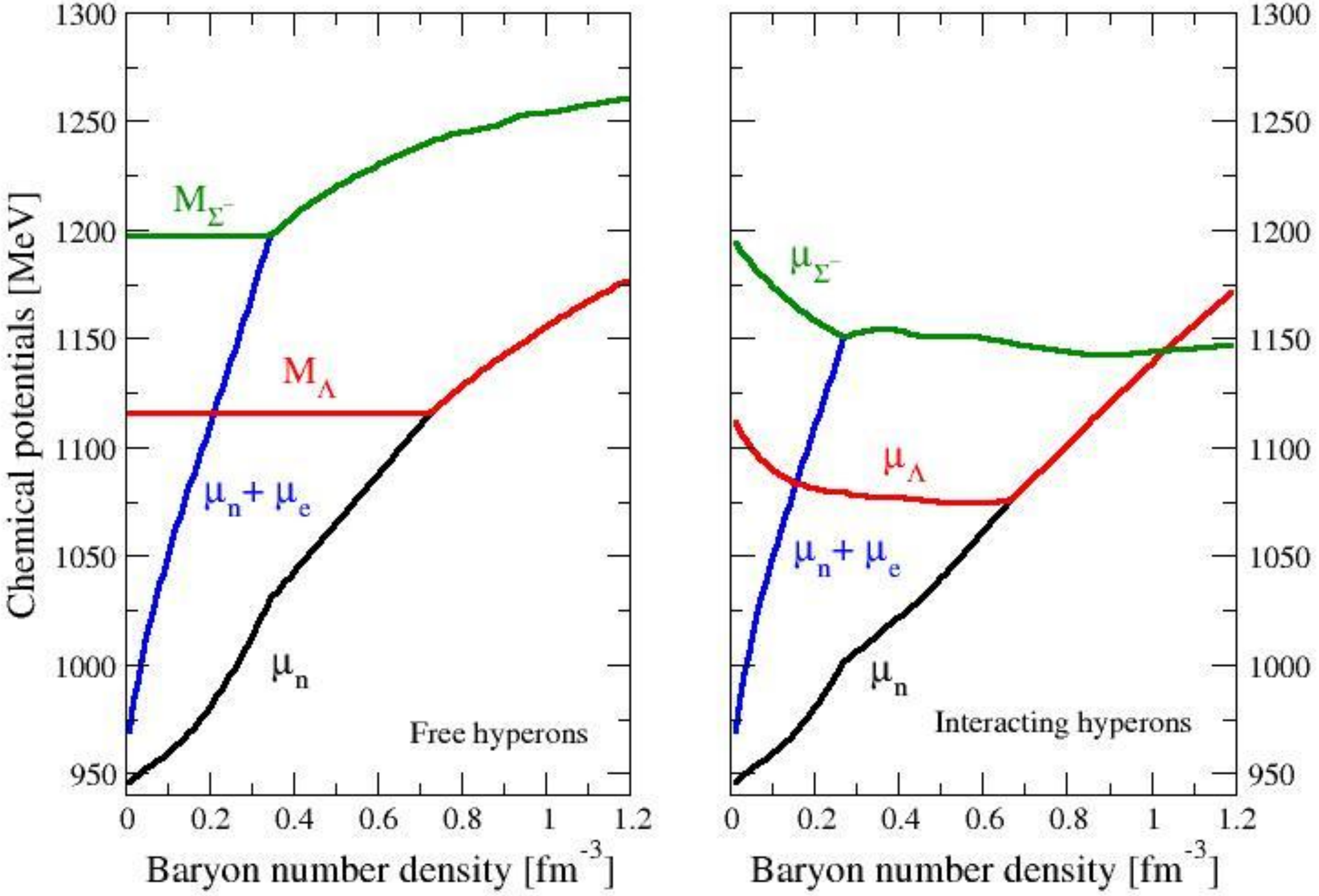
$$\frac{\hbar^2 k_{F_n}^2}{2m_n} + m_n c^2 \geq m_\Lambda c^2 \qquad n_n = \frac{k_{F_n}^3}{3\pi^2}$$

$$n_{cr} = \frac{1}{3\pi^2}\left\{\frac{\left[2m_n c^2 (m_\Lambda - m_n) c^2\right]^{1/2}}{\hbar c}\right\}^3$$

m_Λ= 1115.68 MeV/c^2 **m_n= 939.56 MeV/c^2**

n_{cr}= 0.837 fm^{-3} **n_{cr}/n_0 = 5.23** **n_0= 0.16 fm^{-3}**

Baryon chemical potentials in dense hyperonic matter



$$\mu_n = \mu_\Lambda$$

n + e$^-$ $\rightarrow$ Σ^- + ν_e

$$\mu_n + \mu_e = \mu_{\Sigma^-}$$

I. Vidaña, Ph.D. thesis (2001)

Microscopic EOS for hyperonic matter: extended Brueckner theory

$$G(\omega)_{B_1B_2B_3B_4} = V_{B_1B_2B_3B_4} + \sum_{B_5B_6} V_{B_1B_2B_5B_6} \frac{Q_{B_5B_6}}{\omega - e_{B_5} - e_{B_6}} G(\omega)_{B_5B_6B_3B_4}$$

$$e_{B_i}(k) = M_{B_i}c^2 + \frac{\hbar^2 k^2}{2M_{B_i}} + U_{B_i}(k)$$

$$U_{B_i}(k) = \sum_{B_j} \sum_{k' \leq k_{F B_j}} \langle \vec{k}\vec{k}' | G_{B_iB_jB_iB_j}(\omega = e_{B_i} + e_{B_j}) | \vec{k}\vec{k}' \rangle$$

V is the **baryon--baryon interaction for the baryon octet (n, p, Λ, Σ^-, Σ^0, Σ^+, Ξ^-, Ξ^0) (e.g. the Nijmegen potential).**

- **Energy per baryon in the BHF approximation**

$$E/N_B = 2\sum_{B_i}\int_0^{k_F[B_i]} \frac{d^3k}{(2\pi)^3} \left\{ M_{B_i}c^2 + \frac{\hbar^2 k^2}{2M_{B_i}} + \frac{1}{2}U_{B_i}^N(k) + \frac{1}{2}U_{B_i}^Y(k) \right\}$$

Baldo, Burgio, Schulze, Phys.Rev. C61 (2000) 055801;
Vidaña, Polls, Ramos, Engvik, Hjorth-Jensen, Phys.Rev. C62 (2000) 035801;
Vidaña, Bombaci, Polls, Ramos, Astron. Astrophys. 399, (2003) 687.

Isospin and Strangeness channels

	S = 0	S = -1	S = -2	S = -3	S = -4
I = 0	$(NN \to NN)$		$\begin{pmatrix} \Lambda\Lambda \to \Lambda\Lambda & \Lambda\Lambda \to \Xi N & \Lambda\Lambda \to \Sigma\Sigma \\ \Xi N \to \Lambda\Lambda & \Xi N \to \Xi N & \Xi N \to \Sigma\Sigma \\ \Sigma\Sigma \to \Lambda\Lambda & \Sigma\Sigma \to \Xi N & \Sigma\Sigma \to \Sigma\Sigma \end{pmatrix}$		$(\Xi\Xi \to \Xi\Xi)$
I = 1/2		$\begin{pmatrix} \Lambda N \to \Lambda N & \Lambda N \to \Sigma N \\ \Sigma N \to \Lambda N & \Sigma N \to \Sigma N \end{pmatrix}$		$\begin{pmatrix} \Lambda\Xi \to \Lambda\Xi & \Lambda\Xi \to \Sigma\Xi \\ \Sigma\Xi \to \Lambda\Xi & \Sigma\Xi \to \Sigma\Xi \end{pmatrix}$	
I = 1	$(NN \to NN)$		$\begin{pmatrix} \Xi N \to \Xi N & \Xi N \to \Lambda\Sigma & \Xi N \to \Sigma\Sigma \\ \Lambda\Sigma \to \Xi N & \Lambda\Sigma \to \Lambda\Sigma & \Lambda\Sigma \to \Sigma\Sigma \\ \Sigma\Sigma \to \Xi N & \Sigma\Sigma \to \Lambda\Sigma & \Sigma\Sigma \to \Sigma\Sigma \end{pmatrix}$		$(\Xi\Xi \to \Xi\Xi)$
I = 3/2		$(\Sigma N \to \Sigma N)$		$(\Sigma\Xi \to \Sigma\Xi)$	
I = 2			$(\Sigma\Sigma \to \Sigma\Sigma)$		

β-stable hadronic matter

- **Equilibrium with respect to the weak interaction processes**

$$\mu_p = \mu_n - \mu_e = \mu_{\Sigma^+}$$

$$\mu_n = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_\Lambda$$

$$\mu_n + \mu_e = \mu_{\Sigma^-} = \mu_{\Xi^-}$$

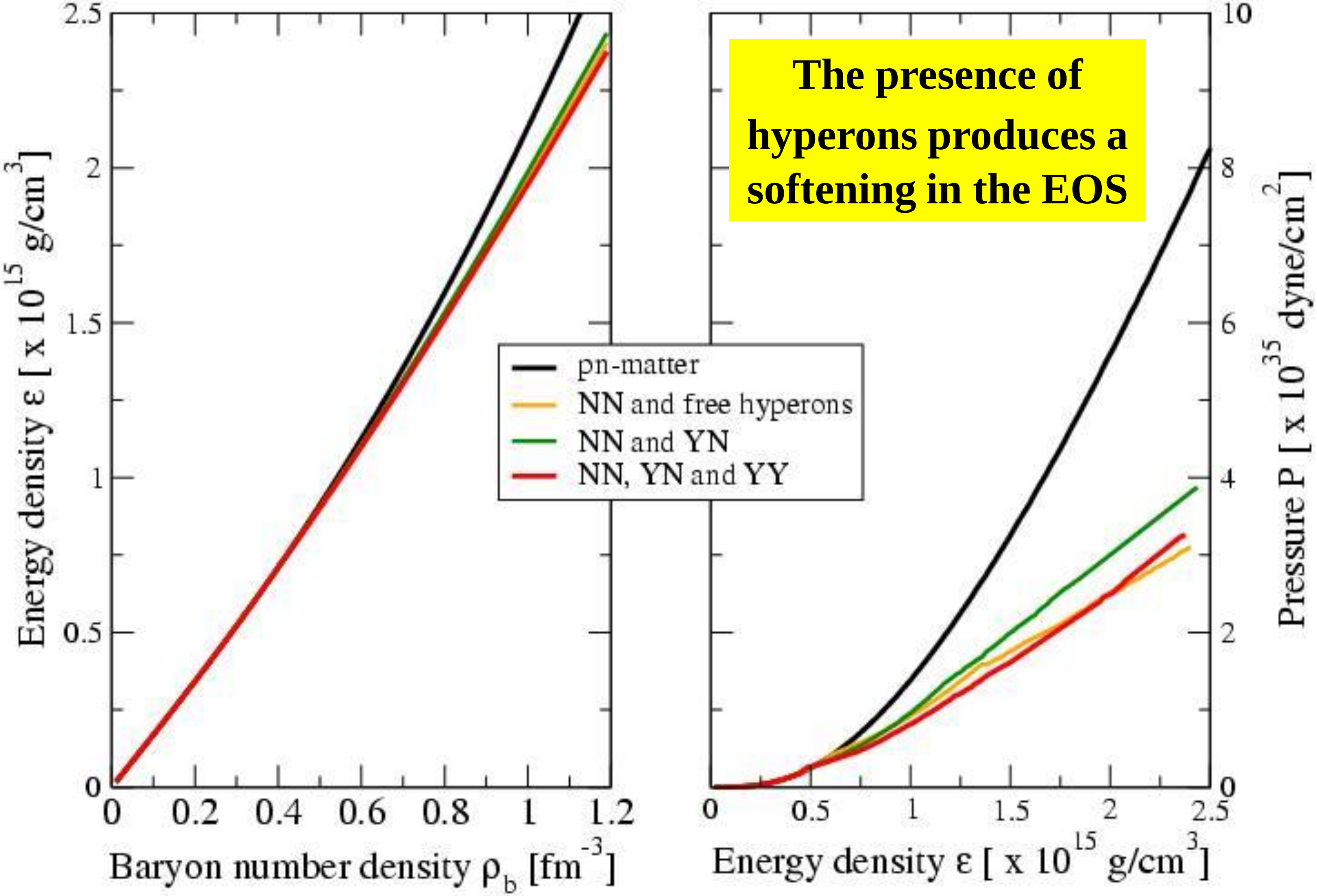
$$\mu_\mu = \mu_e$$

- **Charge neutrality**

$$n_p + n_{\Sigma^+} = n_e + n_\mu + n_{\Sigma^-} + n_{\Xi^-}$$

For any given value of the total baryon number density n_B

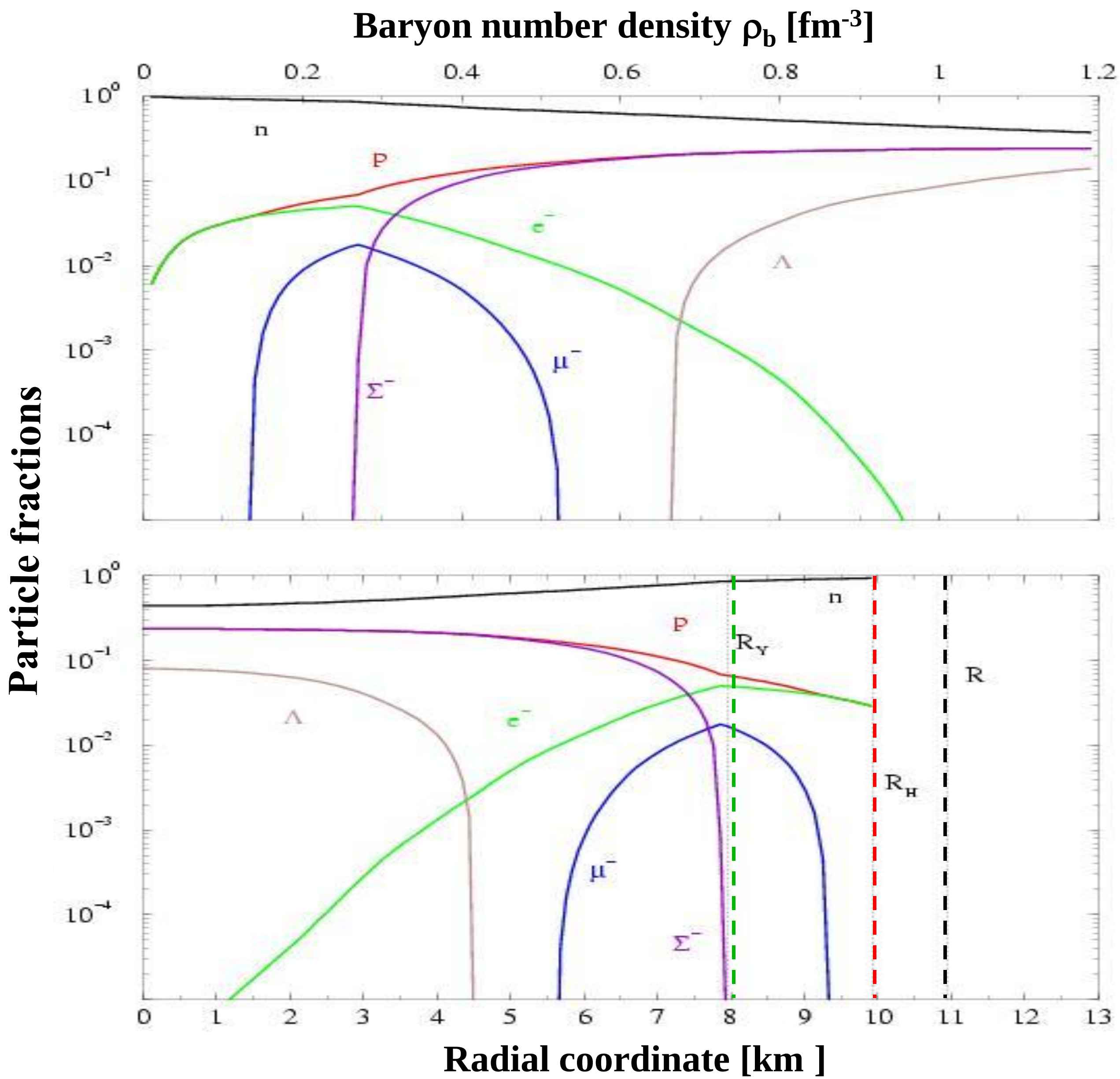
The Equation of State of Hyperonic Matter



NSC97e

I. Vidaña et al., Phys. Rev: C62 (2000) 035801

Composition of hyperonic beta-stable matter

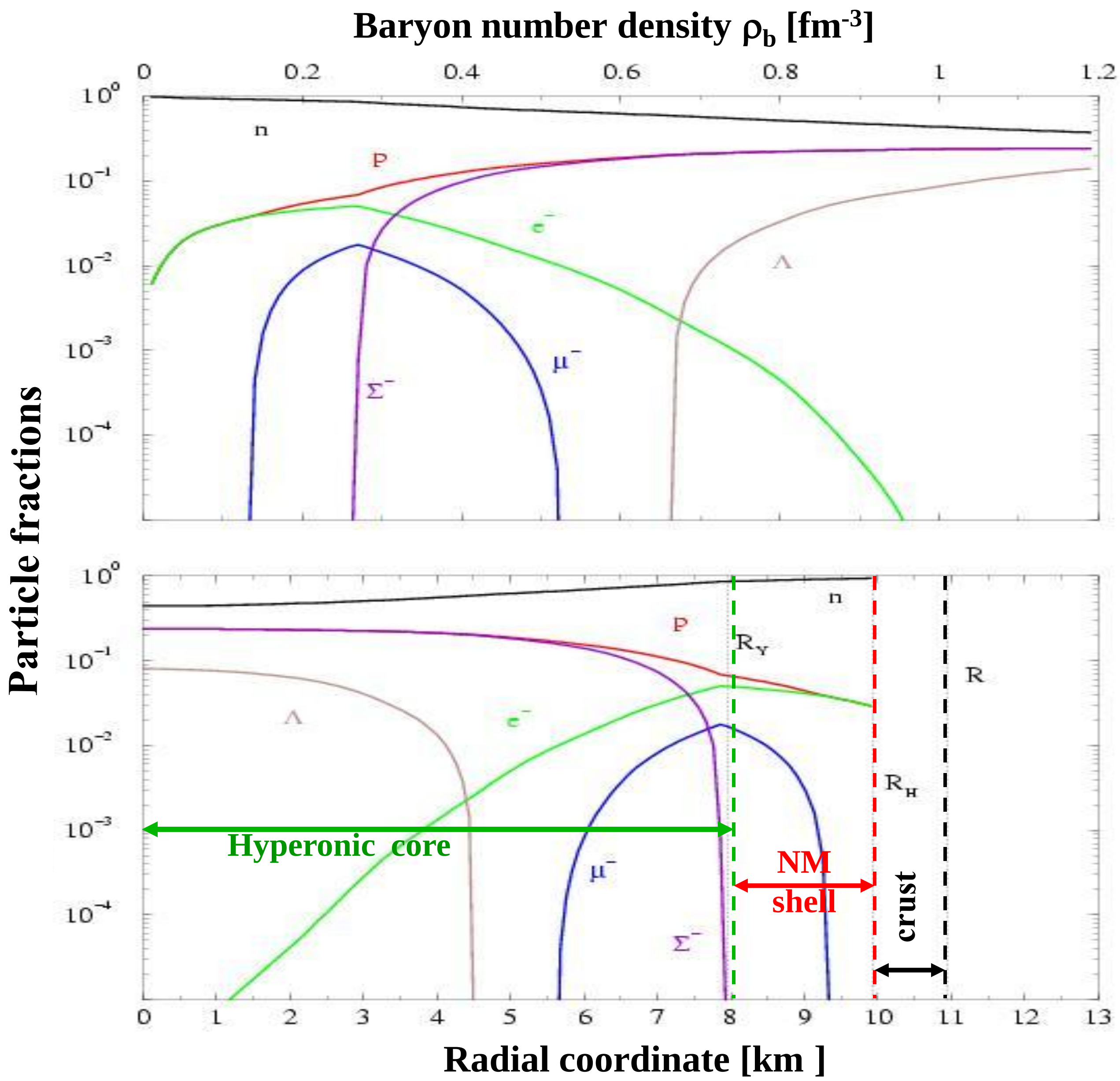


Hyperonic Star

$M_B = 1.34\ M_\odot$

I. Vidaña, I. Bombaci, A. Polls, A. Ramos, Astron. and Astrophys. 399 (2003) 687

Composition of hyperonic beta-stable matter

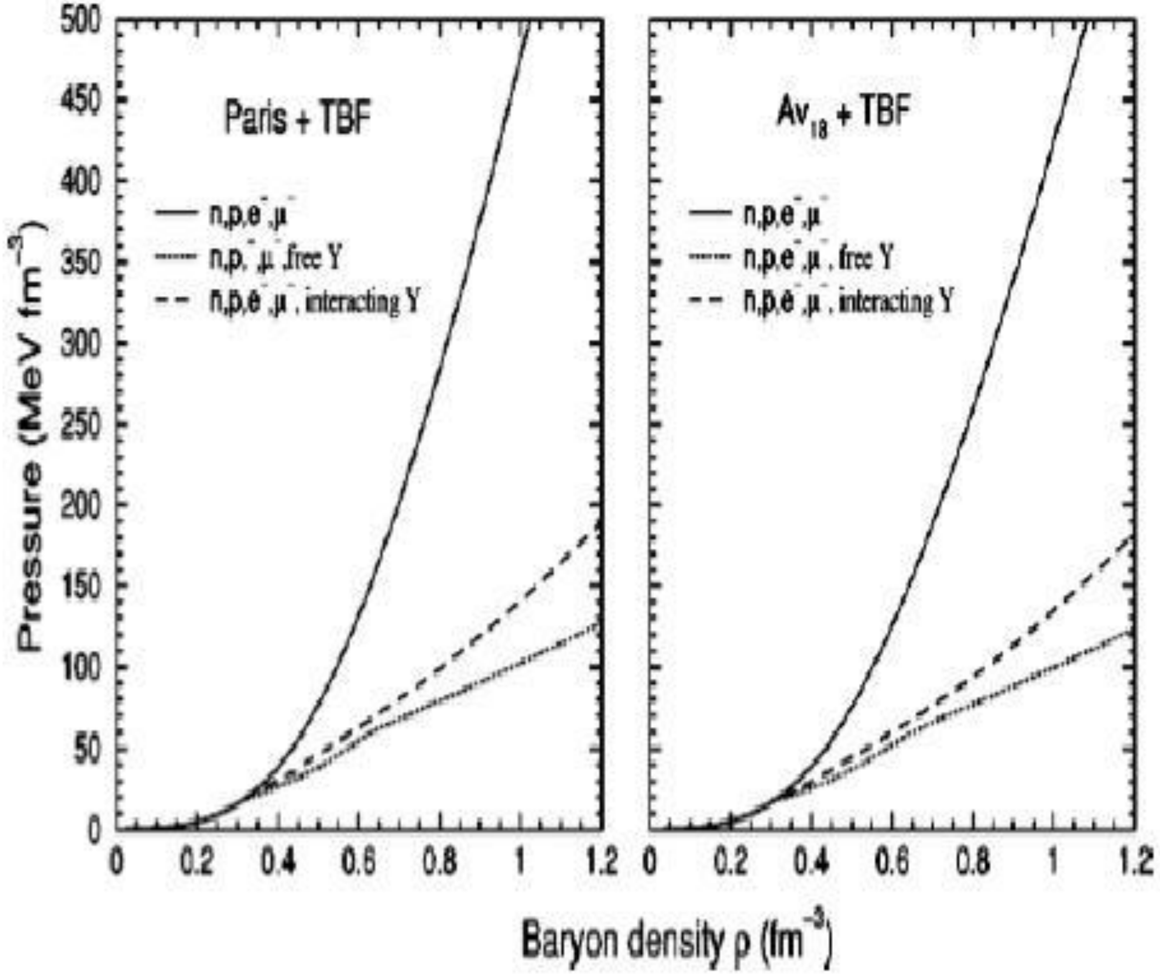


Hyperonic Star

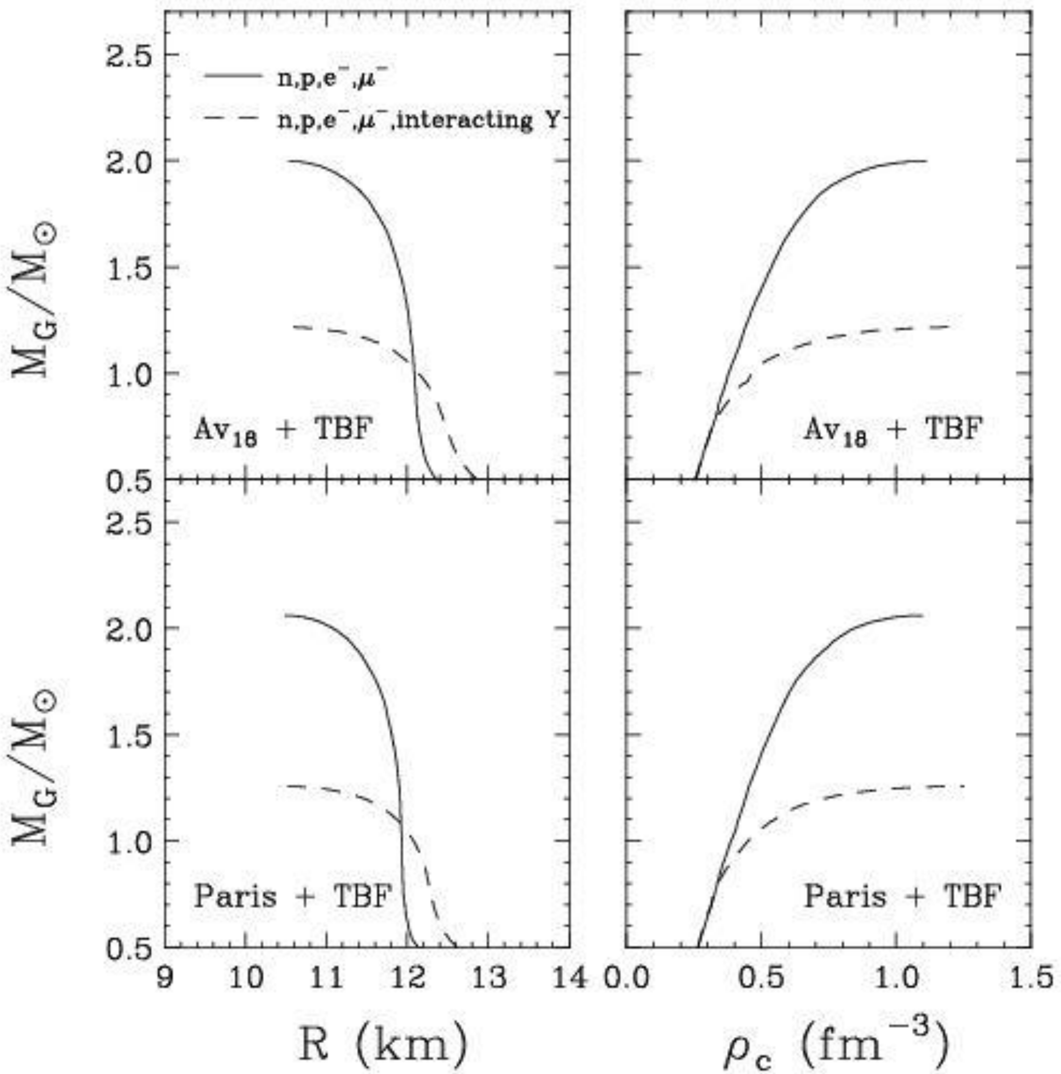
$M_B = 1.34\ M_\odot$

I. Vidaña, I. Bombaci, A. Polls, A. Ramos, Astron. and Astrophys. 399 (2003) 687

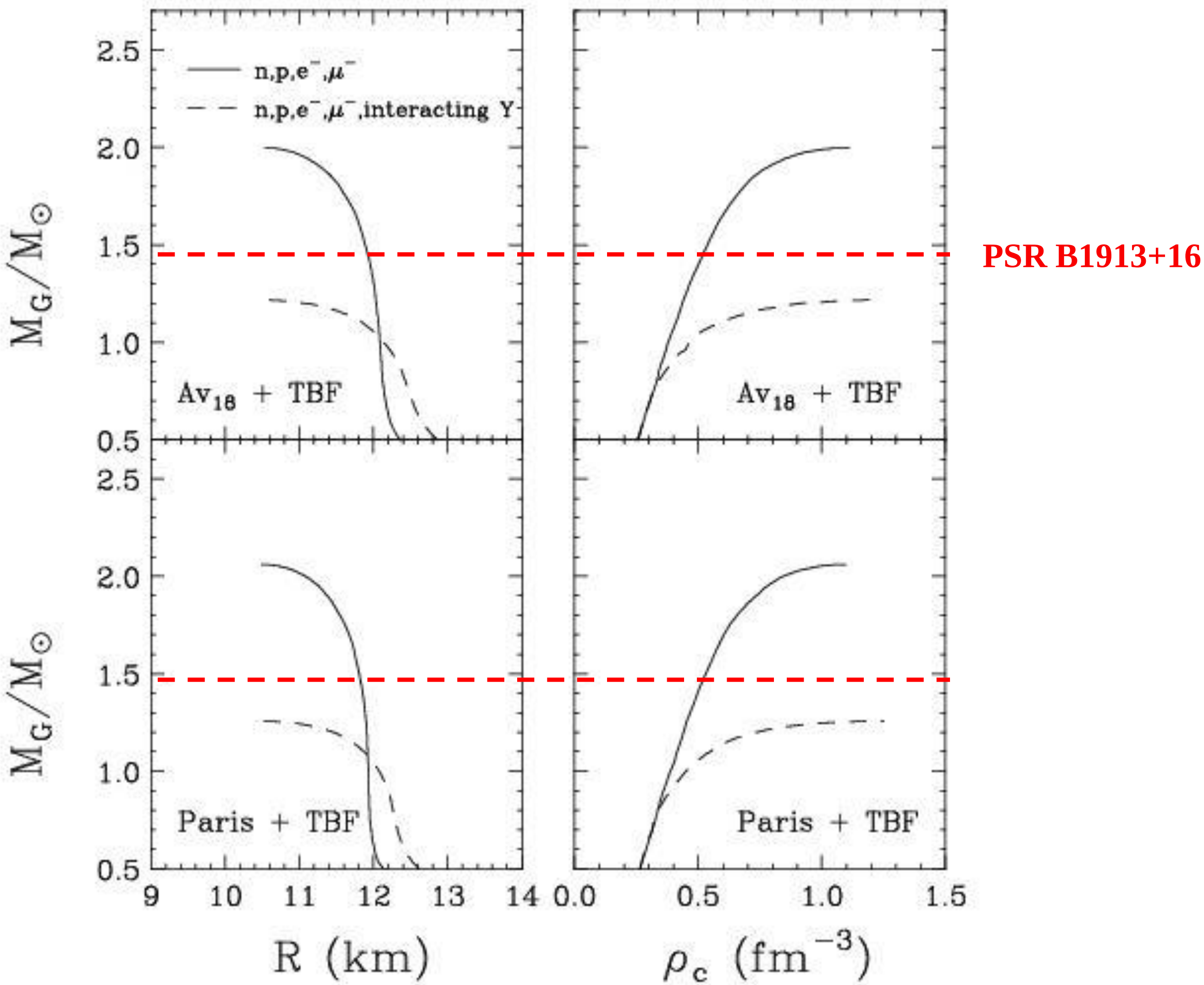
EOS of Hyperonic Matter: Paris (Av18) + Nijm_SC89 + TBF



M. Baldo, G.F. Burgio, H.-J. Schulze, Phys.Rev. C61 (2000)



M. Baldo, G.F. Burgio, H.-J. Schulze, Phys.Rev. C61 (2000)



M. Baldo, G.F. Burgio, H.-J. Schulze, Phys.Rev. C61 (2000)

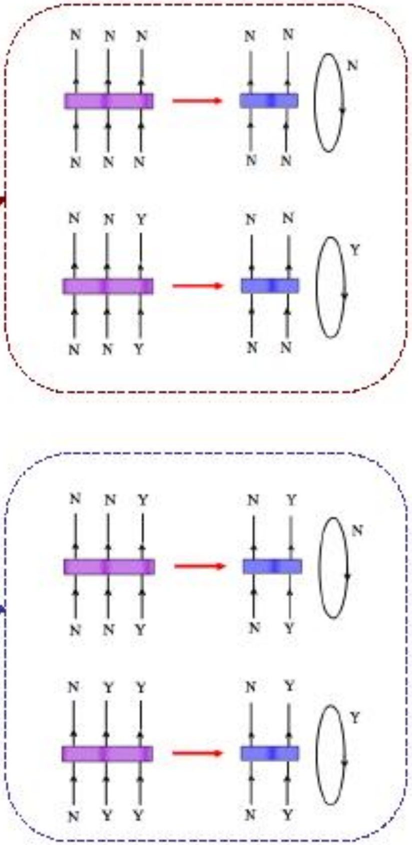
Estimation of the effect of hyperonic three-body forces on the maximum mass of neutron stars

phenomenological density dependent contact terms that mimic the effects of three-body forces

$$\varepsilon_{CT} = a_{NN}\rho_N^2 + b_{NN}\rho_N^{\gamma_{NN}}$$

$$+a_{\Lambda N}\rho_\Lambda\rho_N + b_{\Lambda N}\rho_\Lambda\rho_N\left(\frac{\rho_\Lambda^{\gamma_{\Lambda N}} + \rho_N^{\gamma_{\Lambda N}}}{\rho_\Lambda + \rho_N}\right)$$

$$+a_{\Sigma N}\rho_\Sigma\rho_N + b_{\Sigma N}\rho_\Sigma\rho_N\left(\frac{\rho_\Sigma^{\gamma_{\Sigma N}} + \rho_N^{\gamma_{\Sigma N}}}{\rho_\Sigma + \rho_N}\right)$$



$$\rho_N = \rho_n + \rho_p, \quad \rho_\Sigma = \rho_{\Sigma^-} + \rho_{\Sigma^0} + \rho_{\Sigma^+}$$

NYY → YY and YYY → YY
not included for consistency

Vidaña, Logoteta, Providencia, Polls, Bombaci, EPL 94 (2011) 11002

The parameters a_{NN} b_{NN} γ_{NN} are fixed to reproduce the empirical saturation point of nuclear matter

0.16 fm^{-3}, -16 MeV

γ_{NN}	a_{NN} [MeV fm^3]	b_{NN} [MeV fm$^{3\gamma_{NN}}$]	K_∞ [MeV]
2	−33.44	213.02	211
2.5	−22.08	355.03	236
3	−16.40	665.68	260
3.5	−12.99	1331.36	285

Assume that TBF involving Λ and Σ are the same, i.e.:

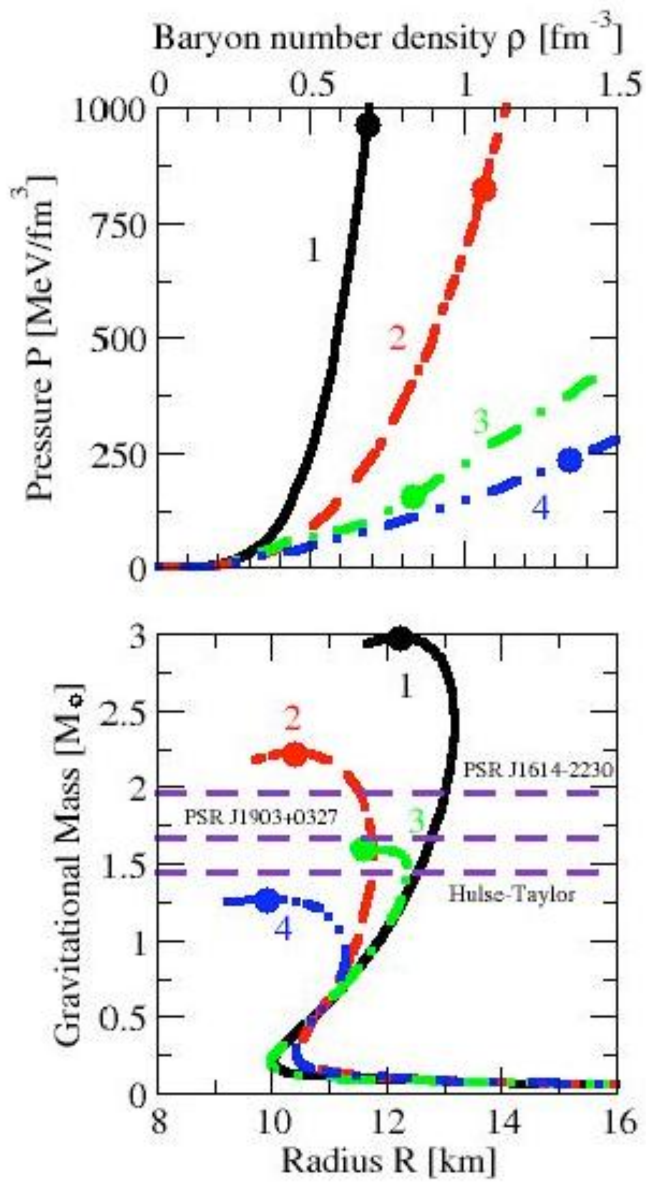
$$a_{\Lambda N} = a_{\Sigma N} \equiv a_{YN} \qquad b_{\Lambda N} = b_{\Sigma N} \equiv b_{YN} \qquad \gamma_{\Lambda N} = \gamma_{\Sigma N} \equiv \gamma_{YN}$$

$$a_{YN} = x\ a_{NN} \qquad b_{YN} = x\ b_{NN} \qquad \gamma_{YN} = x\ \gamma_{\Sigma N} \qquad x = 0,\ 1/3,\ 2/3,\ 1$$

$$\left(\frac{B}{A}\right)_\Lambda = -28\,\mathrm{MeV} = U_\Lambda(k=0) + a_{YN}\rho_0 + b_{YN}\rho_0^{\gamma_{YN}}$$

$$U_\Lambda(k=0) = -30.8\,\mathrm{MeV}$$

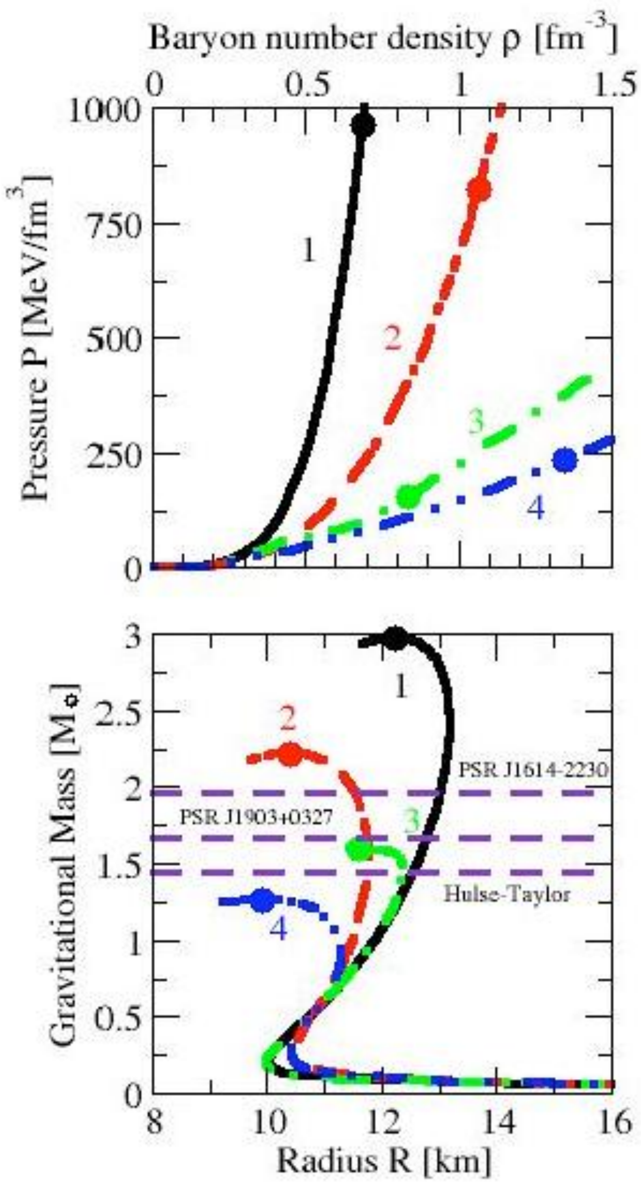
effect of hyperonic TBF on the maximum mass of neutron stars



γ_{NN}	x	γ_{YN}	Maximum Mass
	0	-	1.27 (2.22)
	1/3	1.49	1.33
2	2/3	1.69	1.38
	1	1.77	1.41
	0	-	1.29 (2.46)
	1/3	1.84	1.38
2.5	2/3	2.08	1.44
	1	2.19	1.48
	0	-	1.34 (2.72)
	1/3	2.23	1.45
3	2/3	2.49	1.50
	1	2.62	1.54
	0	-	1.38 (2.97)
	1/3	2.63	1.51
3.5	2/3	2.91	1.56
	1	3.05	1.60

Vidaña, Logoteta, Providencia, Polls, Bombaci, EPL 94 (2011) 11002

effect of hyperonic TBF on the maximum mass of neutron stars



γ_{NN}	x	γ_{YN}	Maximum Mass
	0	-	1.27 (2.22)
	1/3	1.49	1.33
2	2/3	1.69	1.38
	1	1.77	1.41
	0	-	1.29 (2.46)
	1/3	1.84	1.38
2.5	2/3	2.08	1.44
	1	2.19	1.48
	0	-	1.34 (2.72)
	1/3	2.23	1.45
3	2/3	2.49	1.50
	1	2.62	1.54
	0	-	1.38 (2.97)
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Vidaña, Logoteta, Providencia, Polls, Bombaci, EPL 94 (2011) 11002

Relativistic Quantum Field Theory in the mean field approximation for Hyperonic Matter and Hyperon Stars

Parameters fixed to:

Empirical saturation point of symmetric nuclear matter

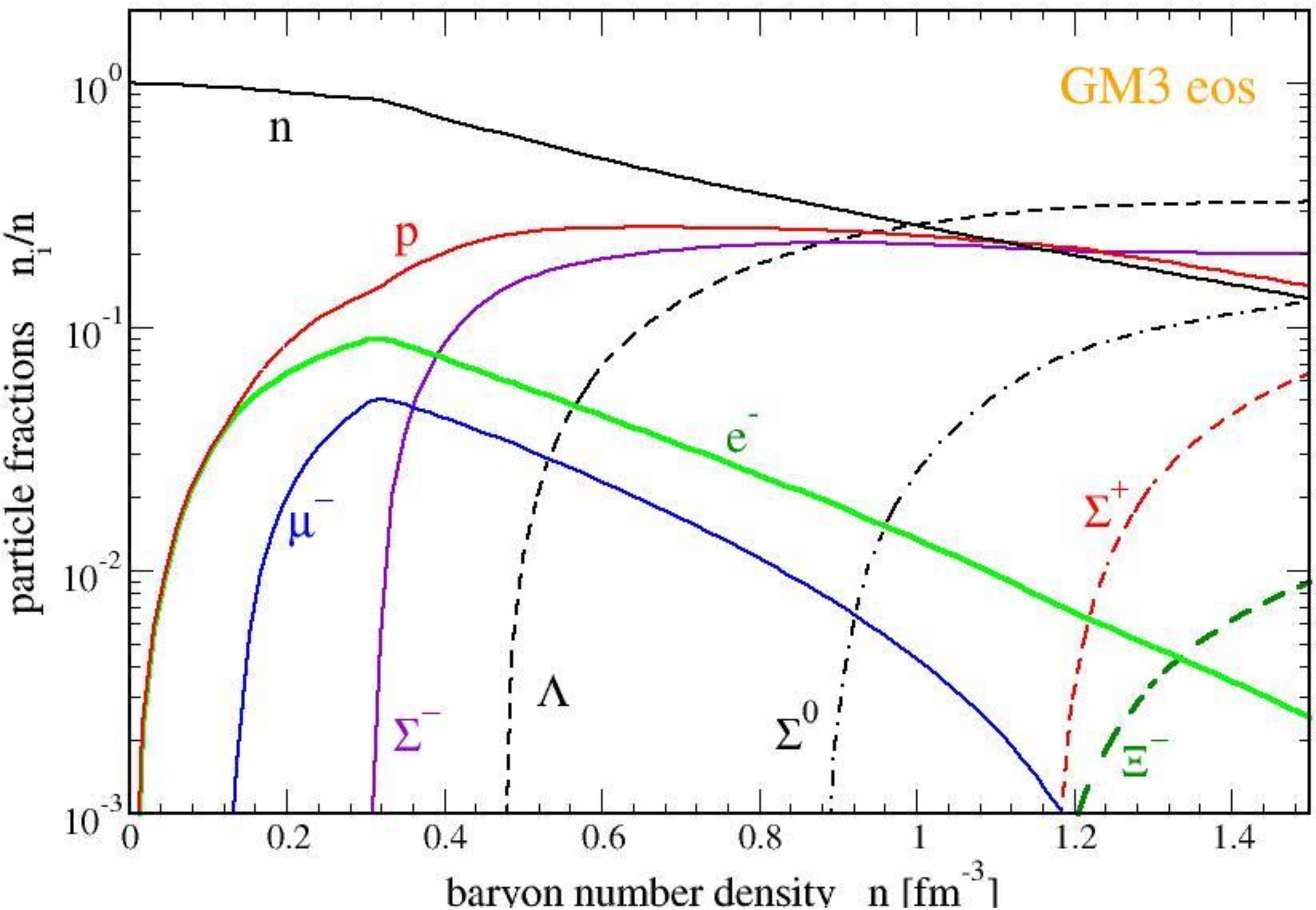
Nuclear incompressibility : K = 210 – 300 MeV

Nuclear symmetry energy at saturation density

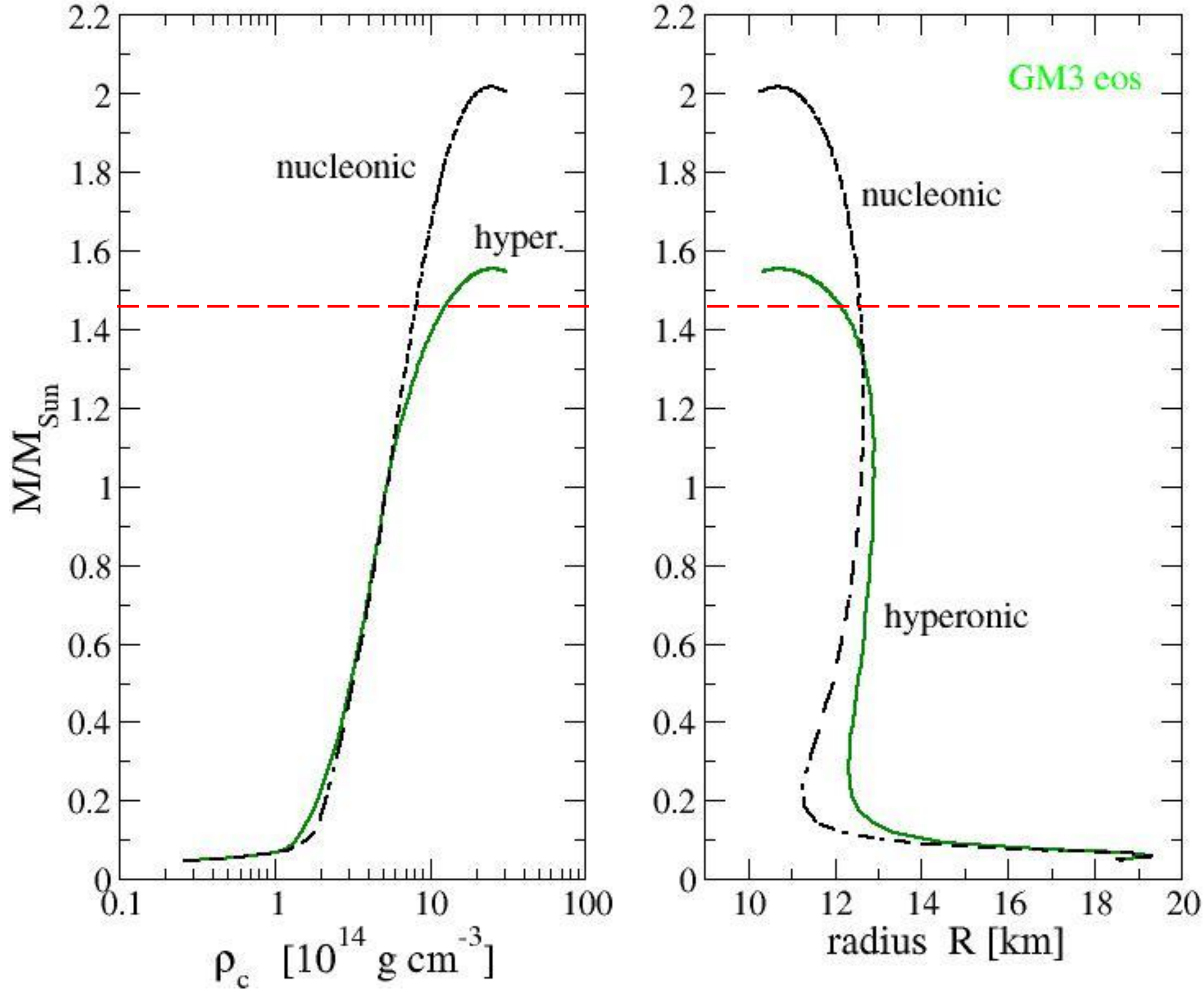
Binding energy of Λ in nuclear matter (B_Λ = - 28 MeV)

Measured masses of neutron stars: $M_{max} \geq 1.50\, M_\odot$

Glendenning, Astrophys. Jour. 293 (1985)
Glendenning and Moszkowski, Phys. Rev. Lett. 67, (1991) (**GM EOS**)

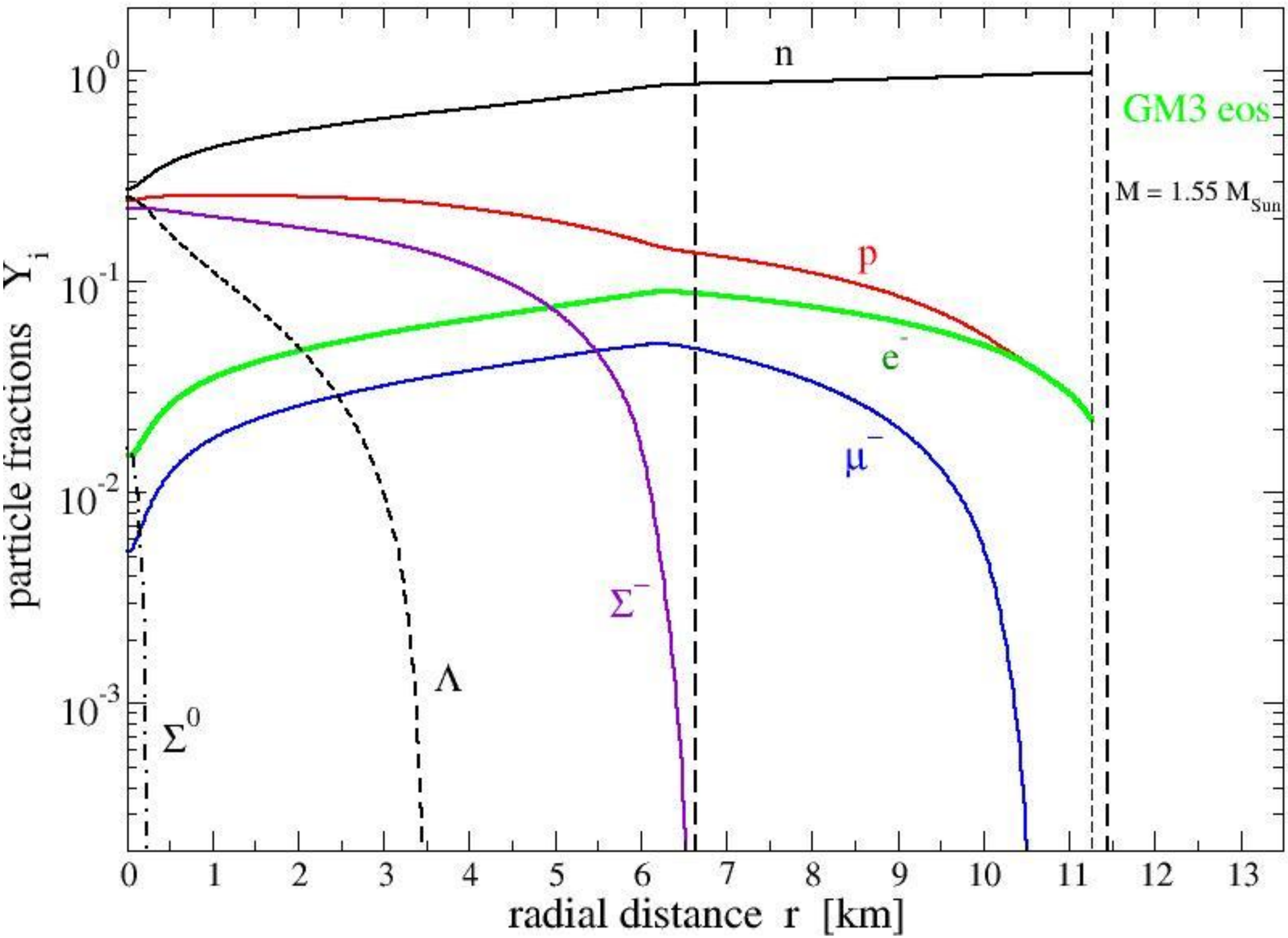


GM3 eos
n
p
e-
μ-
Σ-
Λ
Σ0
Σ+
Ξ-
particle fractions n_i/n
baryon number density n [fm-3]
10^0
10^-1
10^-2
10^-3
0
0.2
0.4
0.6
0.8
1
1.2
1.4



GM3 EOS: Glendenning, Moszkowsky, PRL 67(1991)

Relativistic Mean Field Theory of hadrons interacting via meson exchange



n
GM3 eos
M = 1.55 M_Sun
p
e⁻
μ⁻
Σ⁻
Λ
Σ0
particle fractions Y_i
radial distance r [km]
0
1
2
3
4
5
6
7
8
9
10
11
12
13

Hyperons in Neutron Stars: implications for the stellar structure

The presence of hyperons reduces the maximum mass of neutron stars: $\Delta M_{max} \approx (0.5 - 0.8)\, M_{\odot}$

Therefore, **to neglect hyperons always leads to an overestimate of the maximum mass of neutron stars**

Microscopic EOS for hyperonic matter:

"very soft" EOS non compatible with measured NS masses.

→ **Need for extra pressure at high density**

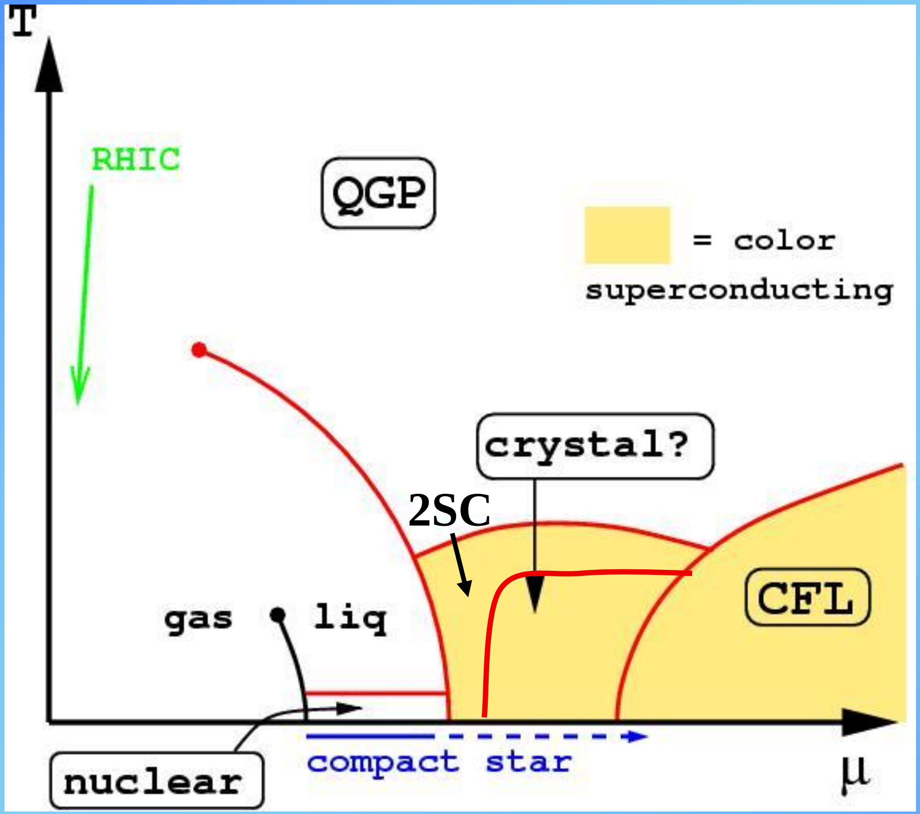
- **Improved NY, YY two-body interaction**
- **Three-body forces: NNY, NYY, YYY**

Quark Matter in Neutron Stars

QCD

Ultra-Relativistic Heavy Ion Collisions

→ **Quark-deconfinement phase transition** expected at $\rho_c \approx (3-5)\,\rho_0$



The core of the most massive Neutron Stars is one of the best candidates in the Universe where such a deconfined phase of quark matter can be found

What quark flavors are expected in a Neutron Star?

flavor	Mass	Q/|e|
u	5 ±3 MeV	2/3
d	10 ±5 MeV	-1/3
s	200 ±100 MeV	-1/3
c	1.3 ± 0.3 GeV	2/3
b	4.3 ±0.2 GeV	-1/3
t	175 ± 6 GeV	2/3

Suppose: $m_u = m_d = m_s = 0$ (✱)

u,d,s non-interacting
(ideal ultrarelativ. Fermi gas)

- **Threshold density for the *c* quark**

$$s \rightarrow c + e^- + \bar{\nu}_e$$

u,d,s in beta-equil.
$Q_{tot} = 0$

(✱) → $n_B = n_u = n_d = n_s$

$$E_{Fq} = \hbar c\, k_{Fq} = \hbar c\, (\pi^2 n_q)^{1/3} = \hbar c\, (\pi^2 n_B)^{1/3} \geq m_c = 1.3 \text{ GeV} \Longrightarrow n_B \sim 29 \text{ fm}^{-3} \sim 180\, n_0$$

Only *u, d, s* quark flavors are expected in Neutron Stars.

A simple model for the EOS of Strange Quark Matter

Grand canonical potential (per unit volume)

$$\Omega^{(0)} = \Omega_u^{(0)} + \Omega_d^{(0)} + \Omega_s^{(0)}$$

In the following we assume: $m_u = m_d = 0, \;\; m_s \neq 0$

$$\Omega_q^{(0)} = -\frac{1}{(\hbar c)^3}\frac{1}{4\pi^2}\mu_q^4 \qquad\qquad (q = u, d)$$

$$\Omega_s^{(0)} = -\frac{1}{(\hbar c)^3}\frac{1}{4\pi^2}\left\{\mu_s\mu_s^*\left(\mu_s^2 - \frac{5}{2}m_s^2\right) + \frac{3}{2}m_s^4 \ln\left(\frac{\mu_s + \mu_s^*}{m_s}\right)\right\}$$

$$\mu_s^* \equiv \left(\mu_s^2 - m_s^2\right)^{1/2} = \hbar c \; k_{Fs}$$

μ_u μ_d μ_s : chemical potentials for quarks

The expression for the **linear (in α_c) perturbative contribution** $\Omega^{(1)}$ to the grand canonical potential can be found in **Farhi and Jaffe, Phys. Rev. D30 (1984) 2379**

Equation of State (T = 0)

$$\begin{cases} P(\mu_u,\mu_d,\mu_s) = -\Omega \;\cong\; -\Omega^{(0)} - \Omega^{(1)} - B \\ \rho(\mu_u,\mu_d,\mu_s) \cong \dfrac{1}{c^2}\left\{ \Omega^{(0)} + \Omega^{(1)} + \sum_{f=u,d,s} \mu_f n_f + B \right\} \end{cases}$$

$$n_f = -\left(\frac{\partial \Omega_f}{\partial \mu_f}\right)_{T,V}$$

B = bag constant

$$n = \frac{1}{3}\left(n_u + n_d + n_s\right)$$

total baryon number density

β-stable Strange Quark Matter

$$u + e^- \leftrightarrow d + \nu_e$$

$$u + e^- \leftrightarrow s + \nu_e$$

$$d \rightarrow u + e^- + \bar{\nu}_e$$

$$s \rightarrow u + e^- + \bar{\nu}_e$$

$$s + u \leftrightarrow d + u$$

$$e^- \leftrightarrow \mu^- + \nu_e + \bar{\nu}_\mu$$

$$\ldots\ldots, \ etc.$$

$$\mu_\nu = \mu_{\bar{\nu}} = 0$$

neutrino-free matter

β-stable Strange Quark Matter

- **Equilibrium with respect to the weak interaction processes**

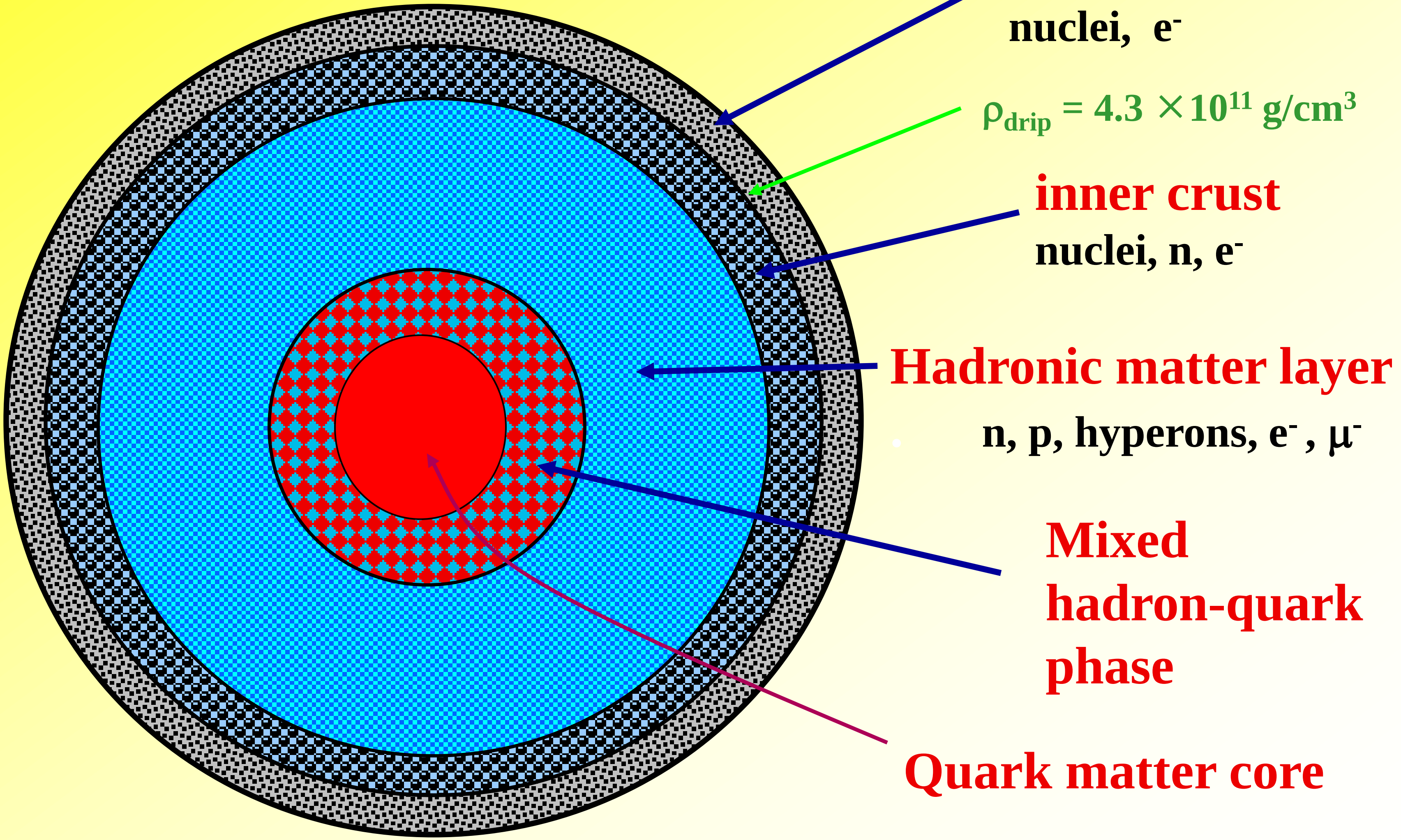
$$\mu_d = \mu_u + \mu_e$$
$$\mu_d = \mu_s$$
$$\mu_\mu = \mu_e$$

- **Charge neutrality**

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e - n_\mu = 0$$

To be solved for any given value of the total baryon number density n_B

Hybrid Stars (neutron stars with a quark matter core)



The EOS for Hybrid Stars

⁎ Hadronic phase :

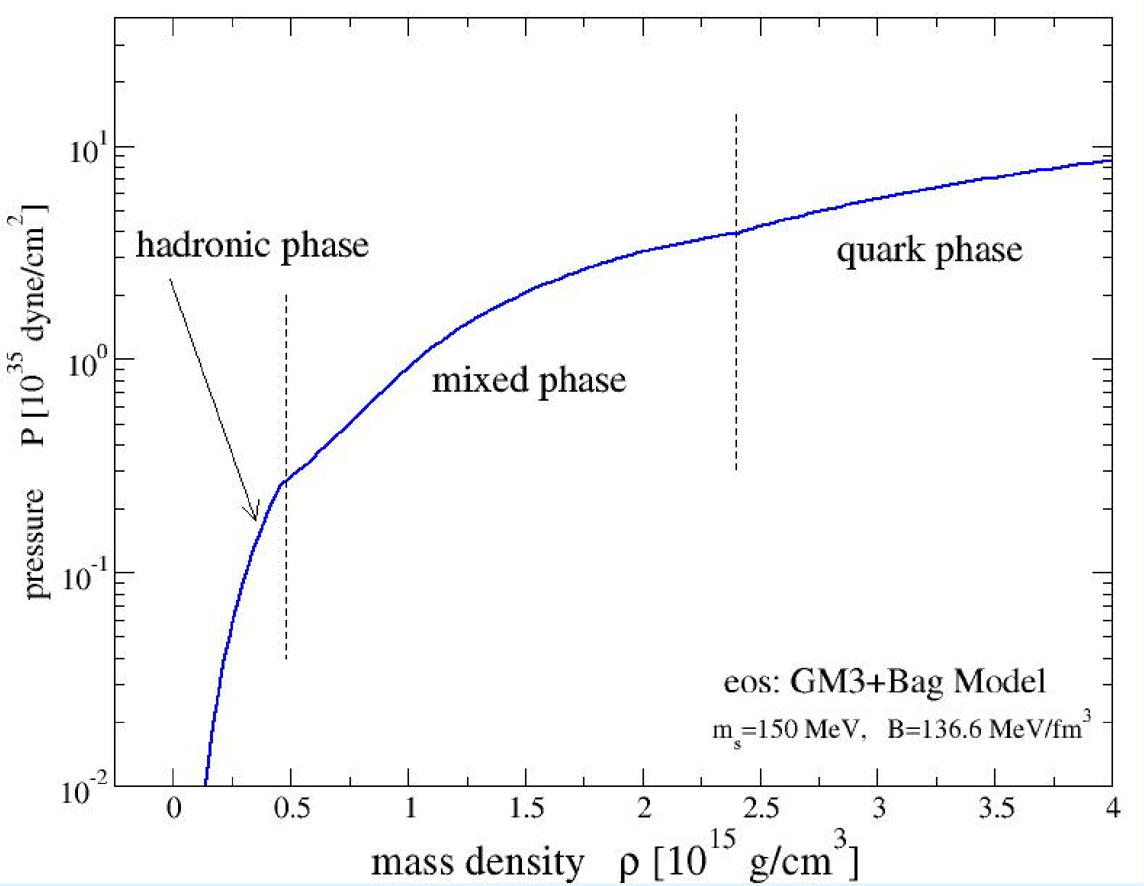
Relativistic Mean Field Theory of hadrons interacting via meson exch. [e.g. Glendenning, Moszkowsky, PRL 67(1991)]

⁎ Quark phase :

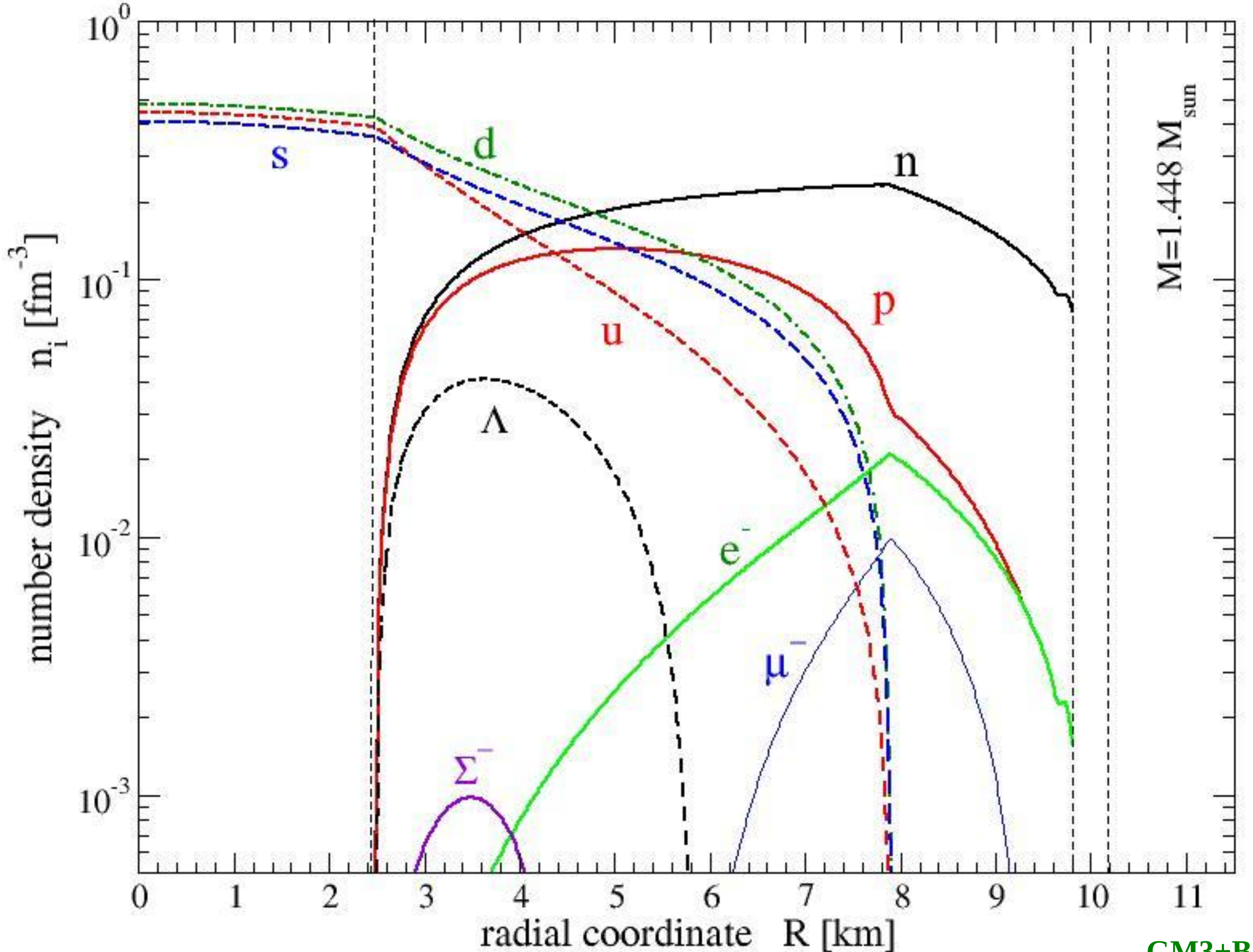
EOS based on the MIT bag model for hadrons. [Farhi, Jaffe, Phys. Rev. D46(1992)]

⁎ Mixed phase :

Gibbs construction for a multicomponent system with two conserved “charges”. [Glendenning, Phys. Rev. D46 (1992)]



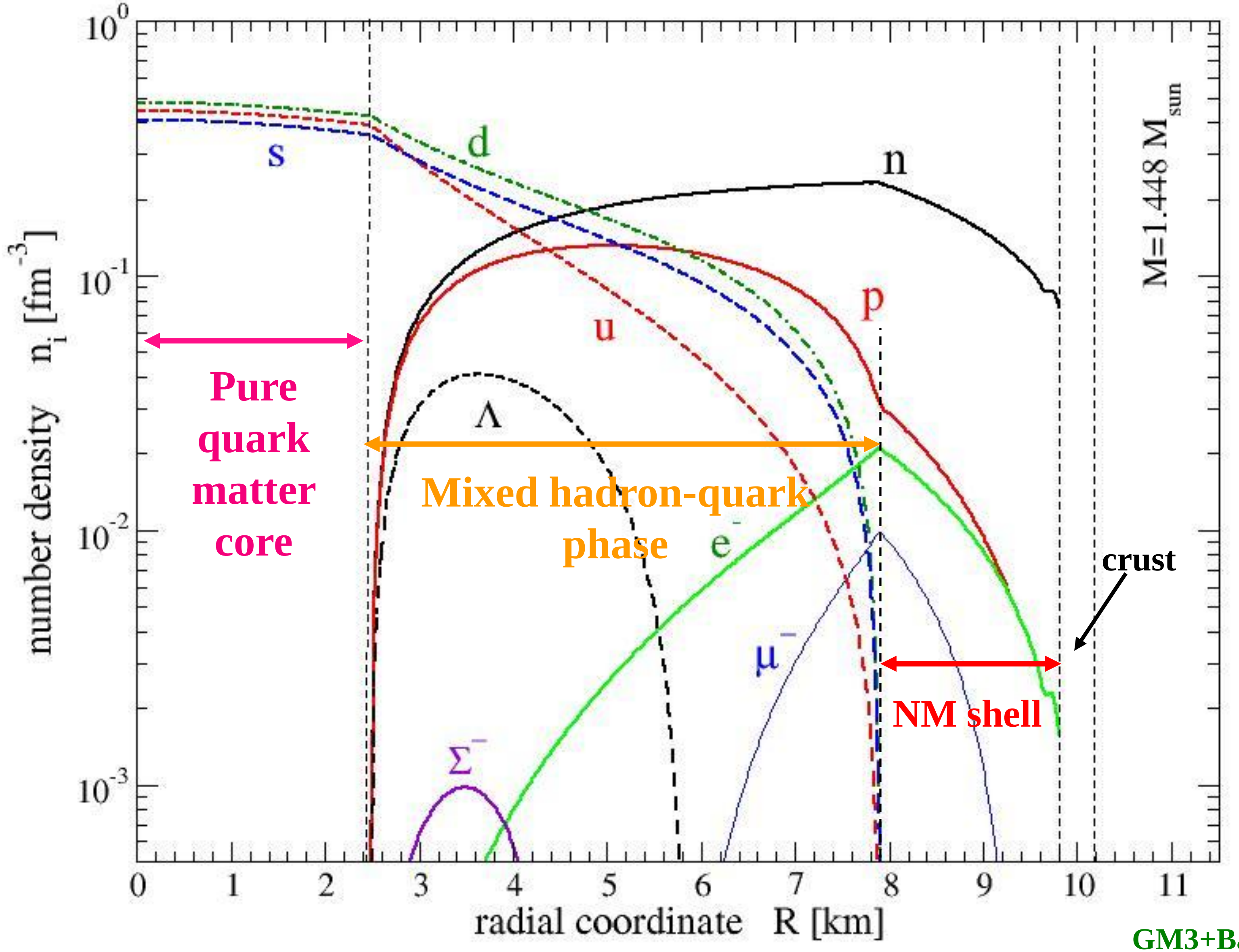
Hybrid Star



I. Bombaci, I. Parenti, I. Vidaña (2004)

GM3+Bag model
m_s=150 MeV, B=13.6.6Mev/fm^3

Hybrid Star



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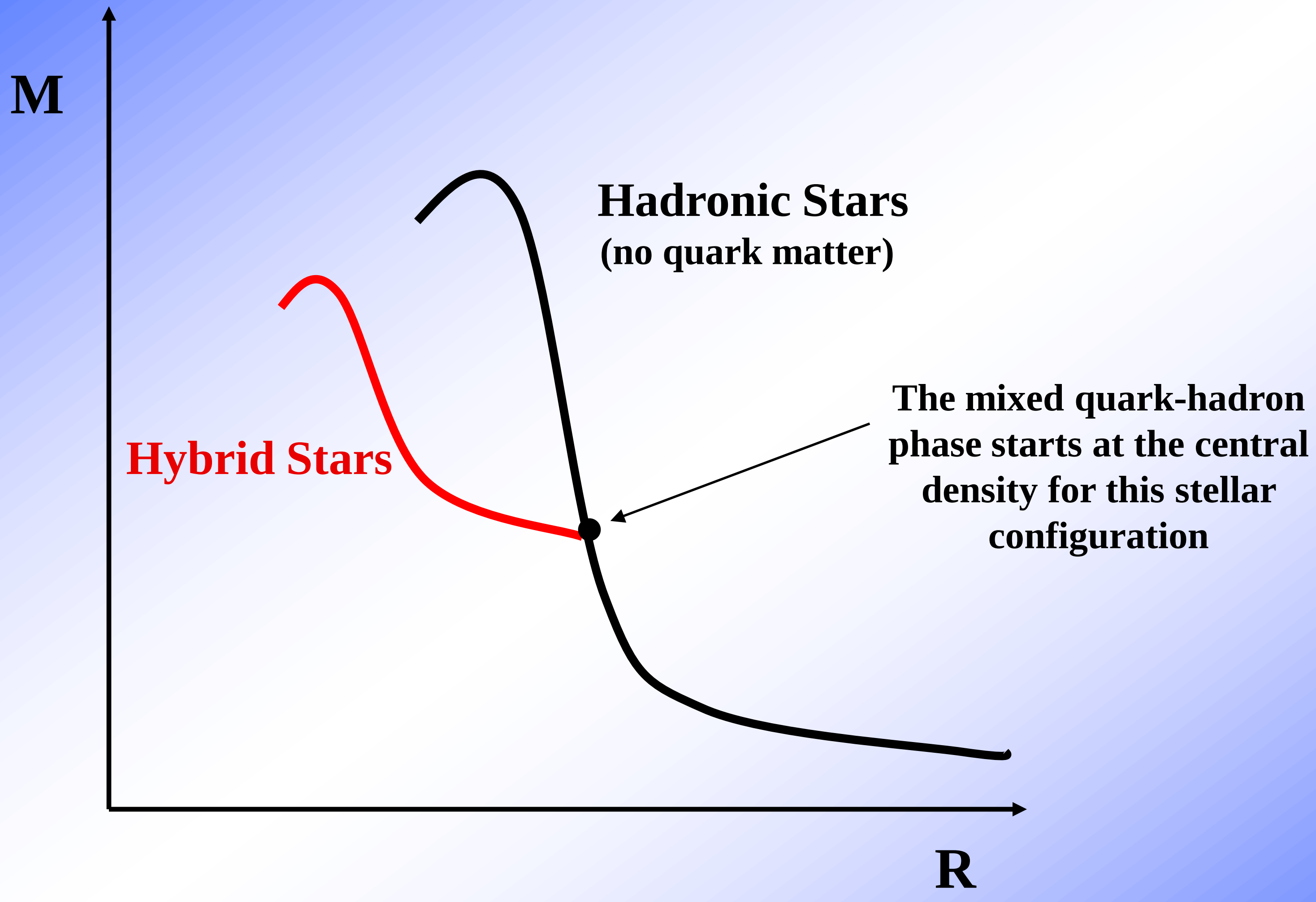
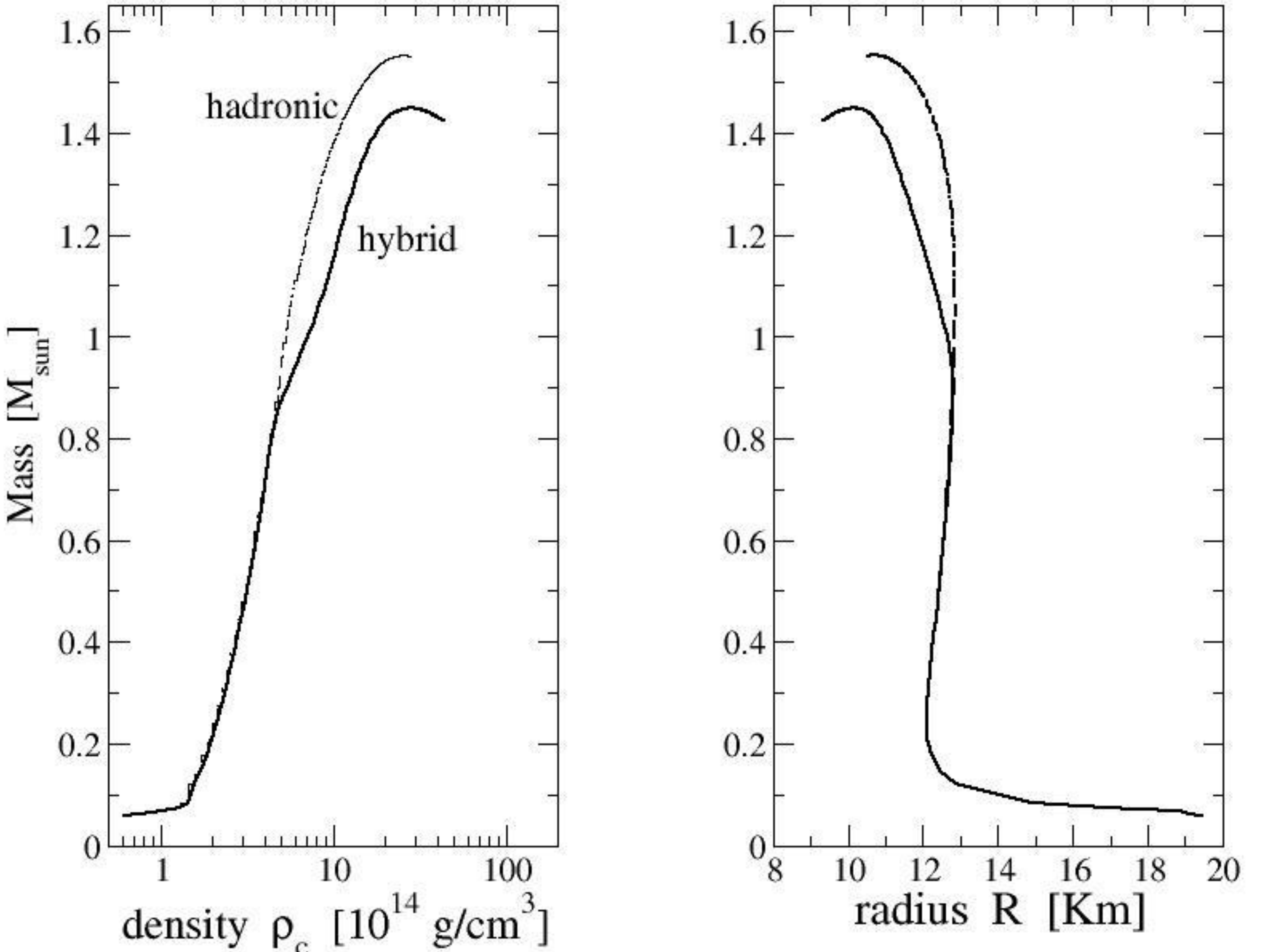
The mass-radius relation



Hybrid Stars

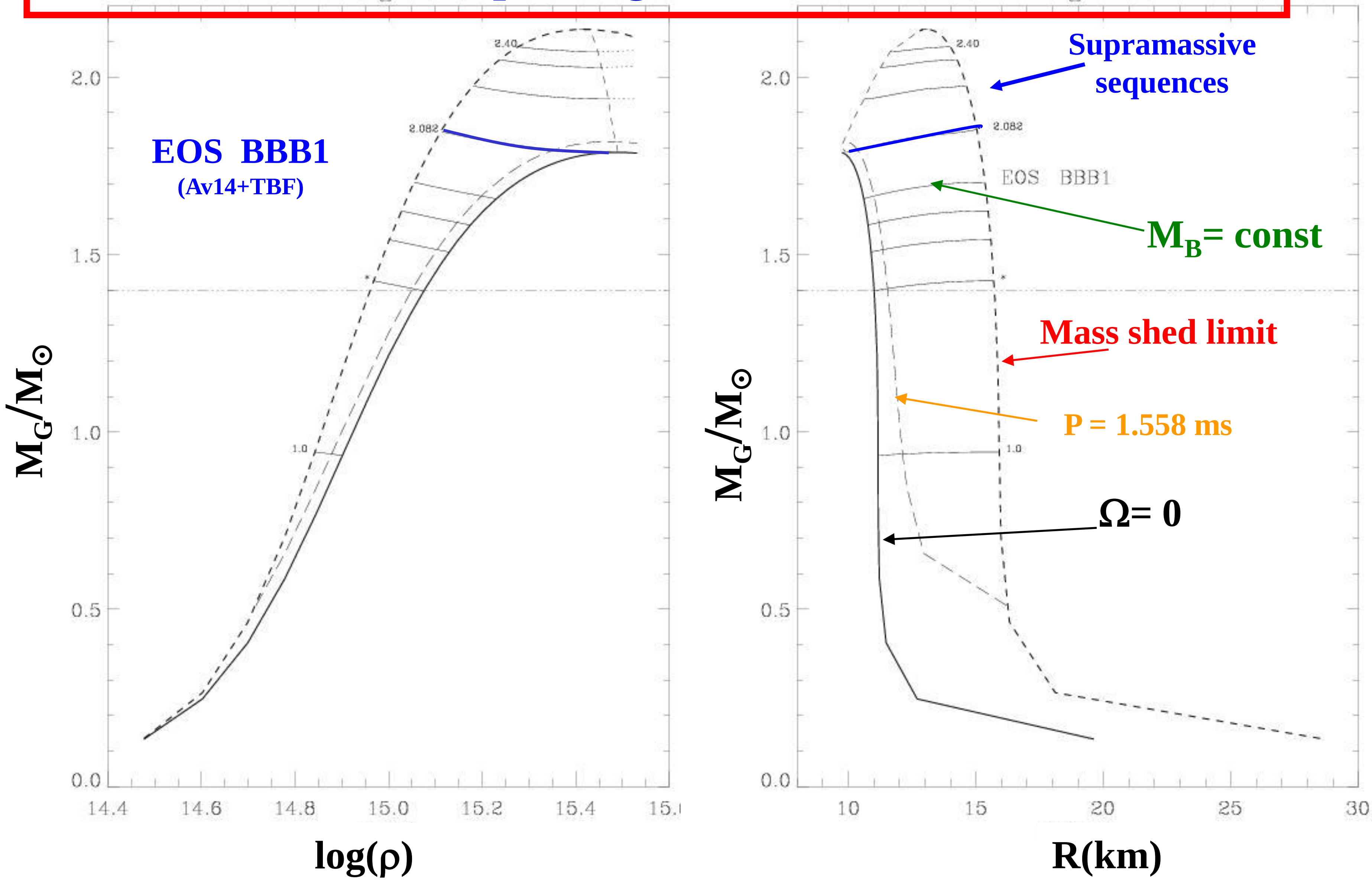


EOS: GM3 + Bag model

(B=136 MeV/fm^3, m_s=150 MeV

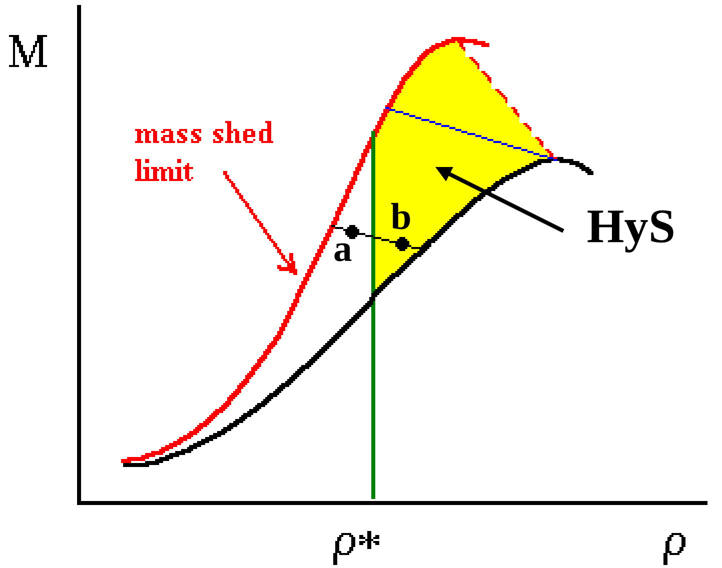
I. Bombaci, I. Parenti, I. Vidaña (2004)

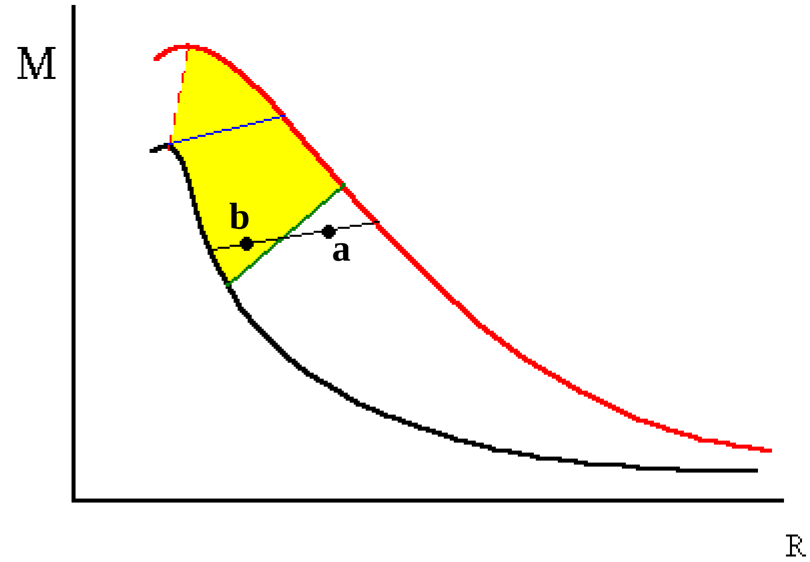
Possible signature for the deconfinement phase transition in isolated spinning-down neutron stars



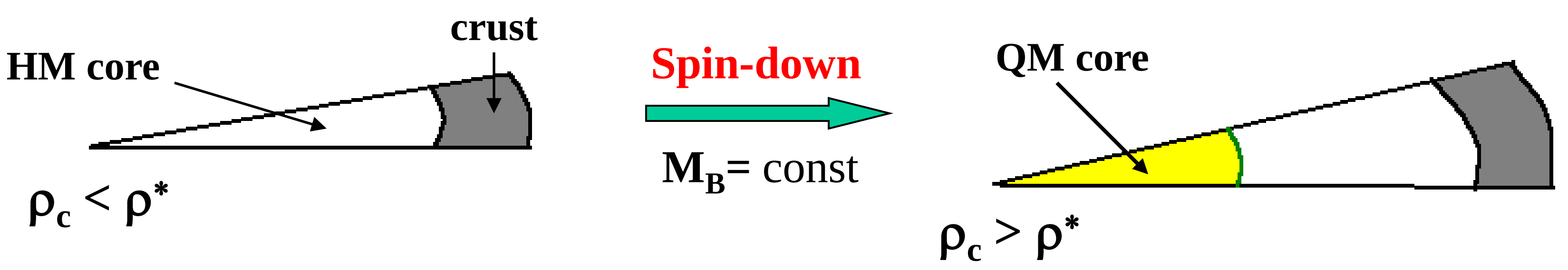
Datta, Thampan, Bombaci, Astron. and Astrophys. 334 (1998)

Possible signature for the deconfinement phase transition in isolated spinning-down neutron stars





ρ* = critical density for quark deconfinement

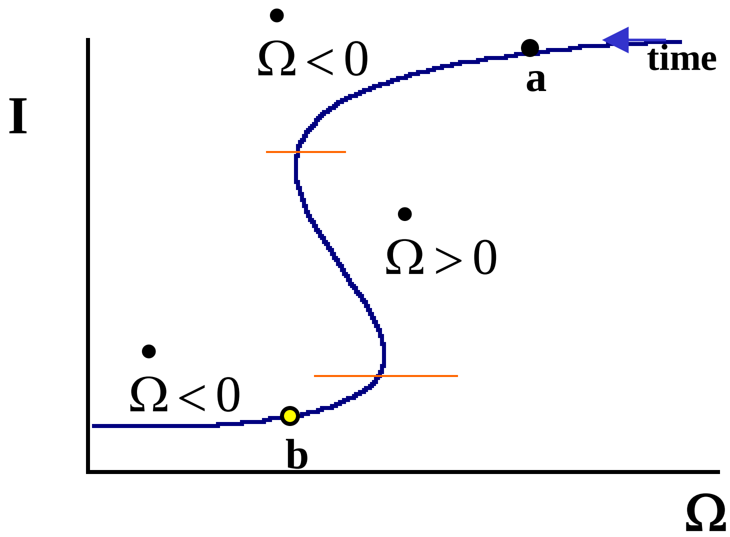


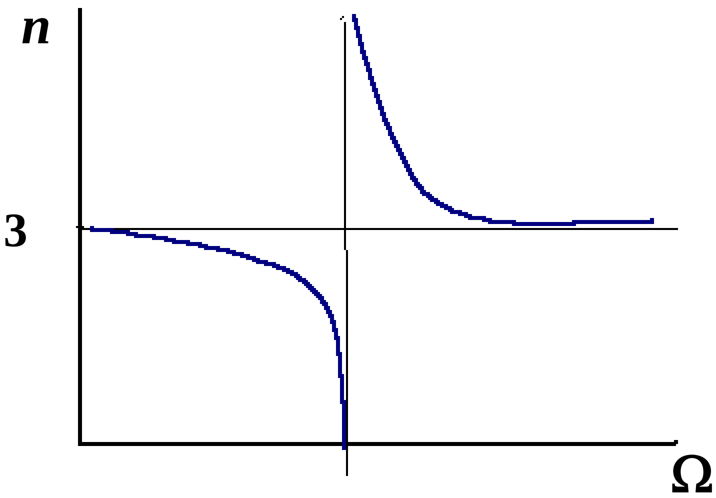
Spin-down: **J decreases**, Ω decreases

ρ_c increases, I decreases

apparent braking index

$$\tilde{n}(\Omega) \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2 = n - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}$$





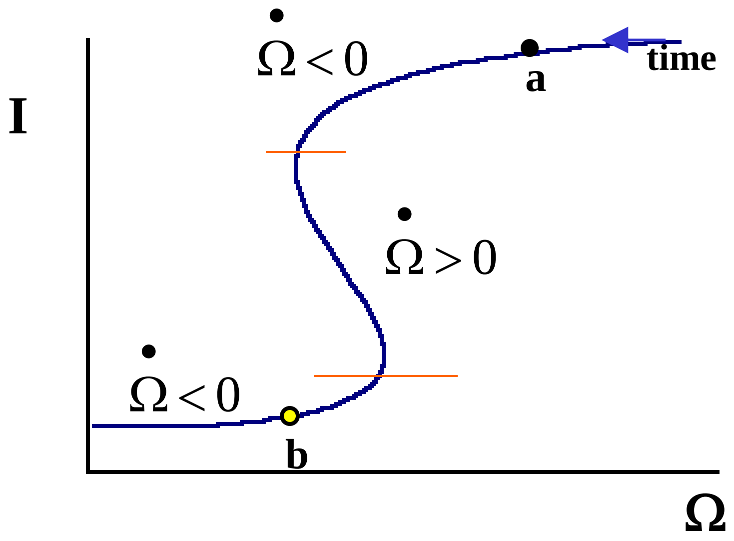
measured large value of the braking index $|n| >> 3$

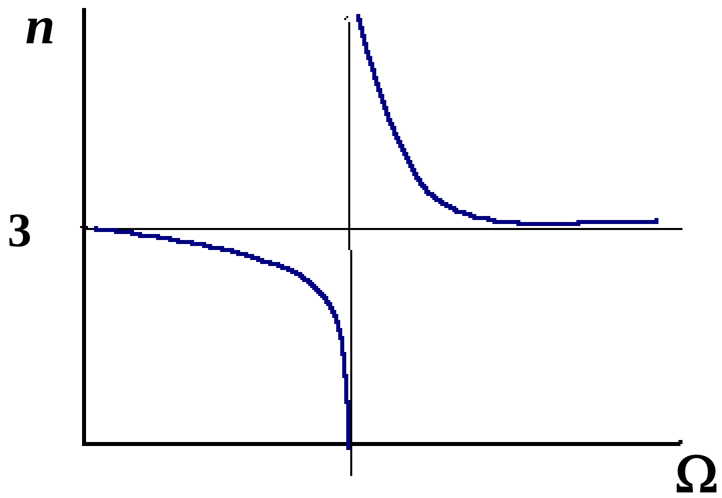
Observational signature for quark deconfinement phase transition in compact stars

Glendenning, Pei, Weber, 1997

apparent braking index

$$\tilde{n}(\Omega) \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2 = n - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}$$



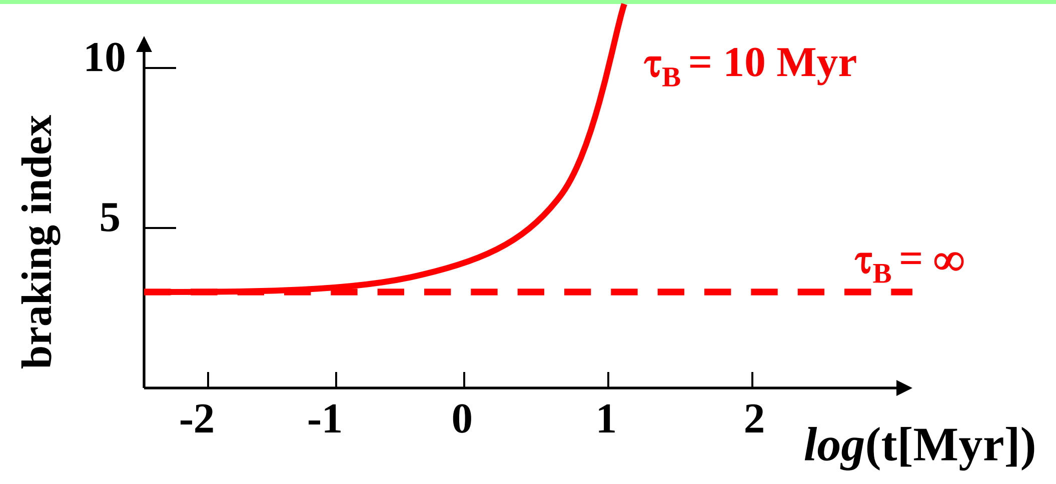


Effects of magnetic field decay on the braking index

(see the first lecture)

braking index

$$n(t) \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2 = 3 - \frac{3c^3 I \dot{B}}{R^6 B^3 \sin^2\alpha\ \Omega^2}$$



Tauris and Konar,
Astron. and Astrophys. 376
(2001)

The Strange Matter hypothesis

↓

Strange Stars

new family of compact stars made of strange quark matter (*u,d,s* quark matter)

The Strange Matter hypothesis

Bodmer (1971), Terazawa (1979), Witten (1984): BTW hypothesis

Three-flavor *u,d,s* quark matter, in equilibrium with respect to the weak interactions, could be the true ground state of strongly interacting matter, rather than ^{56}Fe

$$E/A|_{SQM} \leq E(^{56}Fe)/56 \sim 930.4 \text{ MeV}$$

The Strange Matter hypothesis

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$$E/A|_{SQM} \leq E(^{56}Fe)/56 \sim 930.4 \text{ MeV}$$

Stability of Nuclei with respect to u,d quark matter

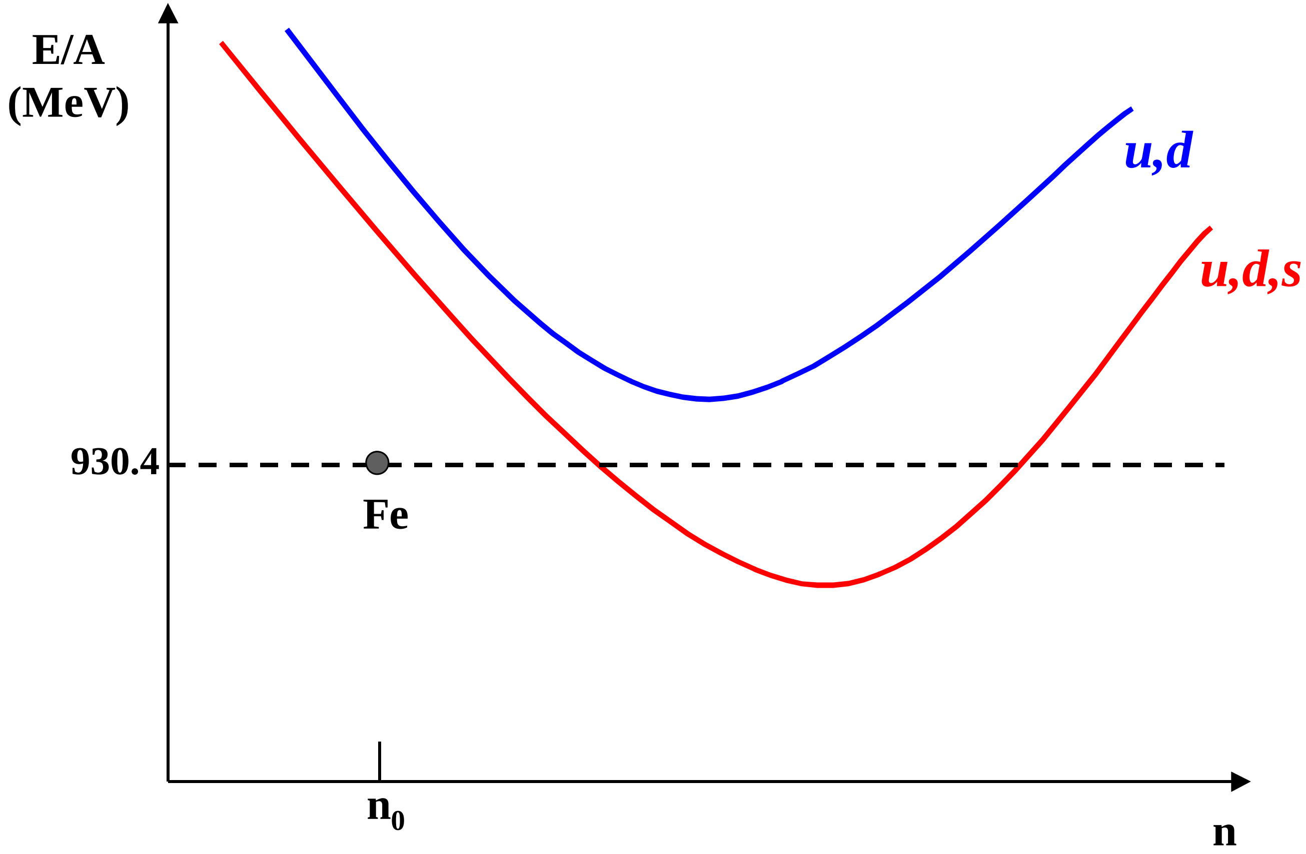
The success of traditional nuclear physics provides a clear indication that **quarks in the atomic Nucleus are confined within protons and neutrons**

$$E/A|_{ud} \geq E(^{56}Fe)/56$$

The Strange Matter hypothesis

Bodmer (1971), Terazawa (1979), Witten (1984): BTW hypothesis

ect
of



d

EOS for SQM: massless quarks

(ultra-relativistic ideal gas +bag constant)

$$\varepsilon = K\, n^{4/3} + B$$
$$P = (1/3) K\, n^{4/3} - B$$

$$P = (1/3)\,(\varepsilon - 4B)$$

$$E/A = K\, n^{1/3} + B/n$$

$$K = \frac{9}{4}\hbar c\ \pi^{2/3}$$

u,d,s QM : deg.fact.= 2×3×3 ($n_u = n_d = n_s$)

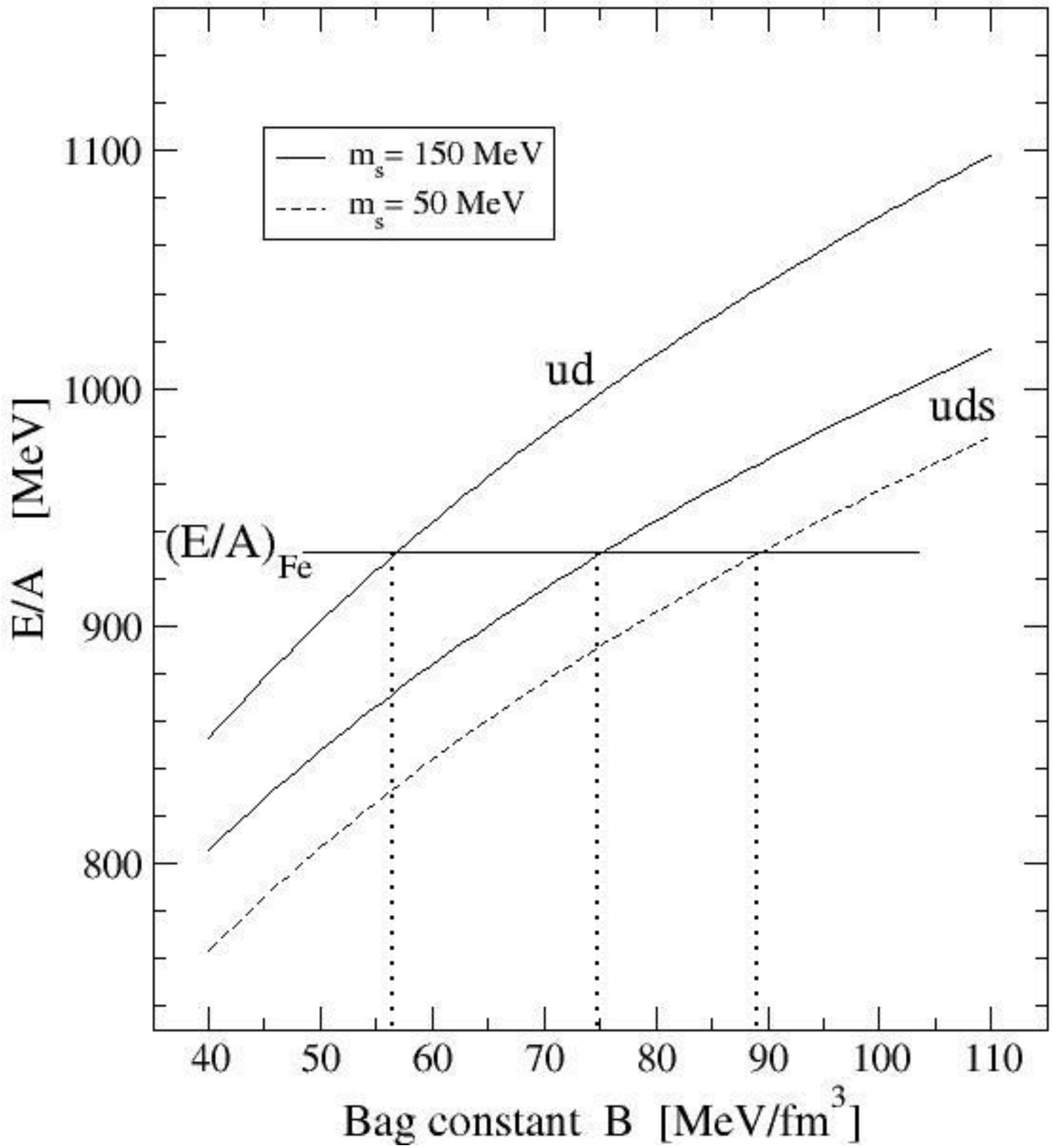
$$K = \frac{9}{4}\hbar c \left(\frac{3}{2}\pi^2\right)^{1/3}$$

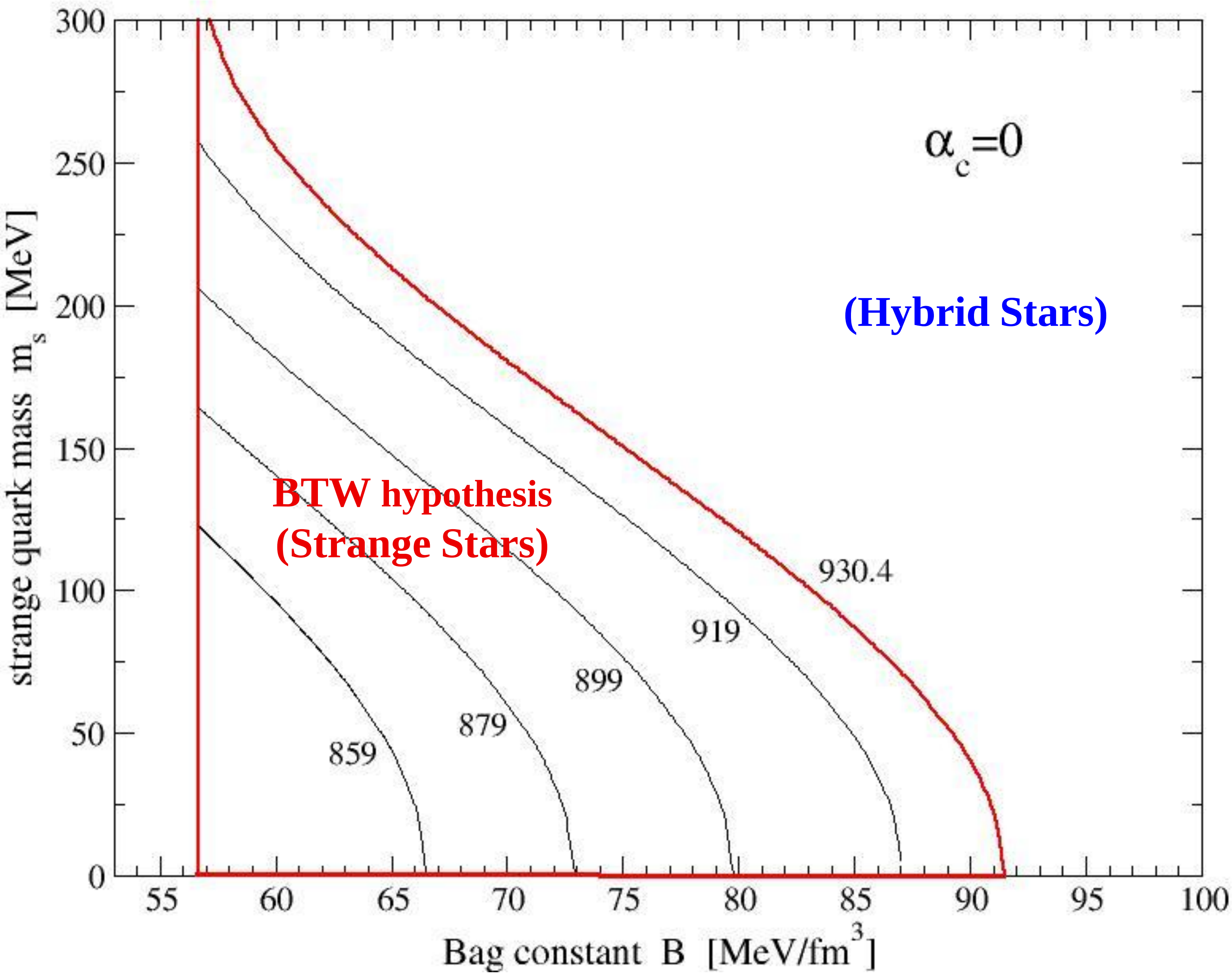
u,d (isospin-symm.)QM : deg.fact.= 2×2×3

Saturation point of QM

$$\begin{cases} n_s = \left(\dfrac{3B}{K}\right)^{3/4} \\ \left.\dfrac{E}{A}\right|_s = \dfrac{4B}{n_s} = 4B\left(\dfrac{K}{3B}\right)^{3/4} \end{cases}$$

Saturation energy of quark matter





300
250
200
150
100
50
0
55
60
65
70
75
80
85
90
95
100
strange quark mass m_s [MeV]
Bag constant B [MeV/fm^3]
α_c=0
(Hybrid Stars)
BTW hypothesis
(Strange Stars)
930.4
919
899
879
859

Stability of atomic nuclei against decay to SQM droplets

- **If the SQM hypothesis is true, why nuclei do not decay into SQM droplets (strangelets) ?**

- **One should explain the existence of atomic nuclei in Nature.**

a) Direct decay to a SQM droplet

$$^{56}Fe \rightarrow {}^{56}(SQM) \qquad (1)$$

weak process $u \rightarrow s + e^{+} + \nu_e$

$$d + u \rightarrow s + u \qquad (2)$$

To have the direct decay to ^{56}(SQM) one needs ~ 56 simultaneous strangeness changing weak processes (2).

The probability for the direct decay (1) is : $P \sim (G_F^{\,2})^A \sim 0$

The *mean-life time* of ^{56}Fe with respect to the direct decay to a drop of SQM is

τ >> age of the Universe

b) Step by step decay to a SQM droplet

$$^{56}Fe \rightarrow {}^{56}X^{1\Lambda} \rightarrow {}^{56}Y^{2\Lambda} \; \ldots\ldots \rightarrow {}^{56}(SQM)$$

$$^{56}Fe \rightarrow \begin{cases} {}^{56}Fe^{\Lambda} \\ {}^{56}Mn^{\Lambda} \end{cases}$$

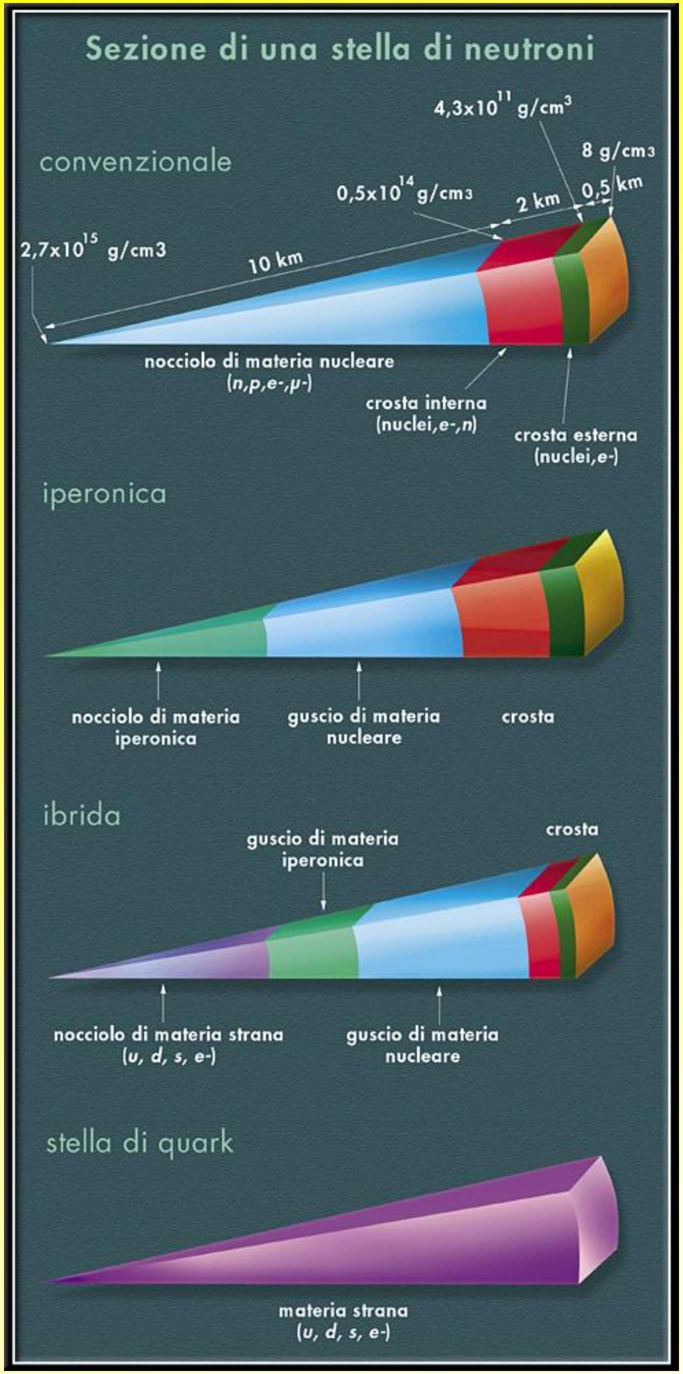
These processes are not energetically possible since

$$Q = M({}^{56}Fe) - M({}^{56}X^{1\Lambda}) < 0$$

Thus, **according to the BTW hypothesis**, **nuclei are metastable states of strong interacting matter with a *mean-life time***

τ >> age of the Universe

"Neutron Stars"



"traditional"
Neutron Stars

Hyperon Stars

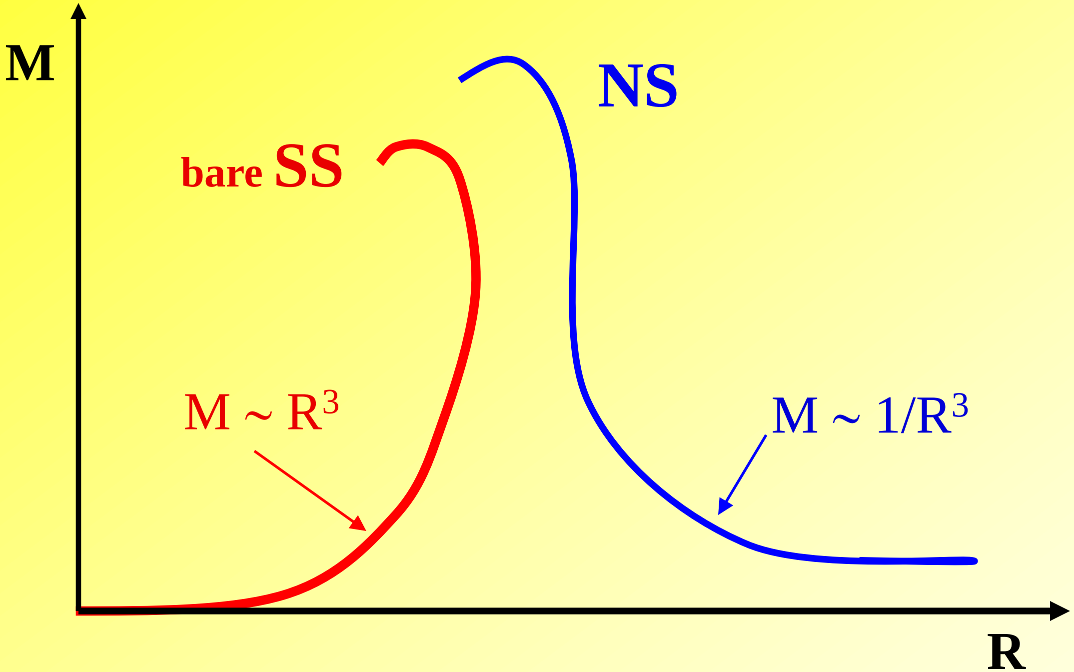
} **Hadronic Stars**

Hybrid Stars

Strange Stars

} **Quark Stars**

The Mass-Radius relation for Strange Stars



- “low” mass **Strange stars** are **self-bound** bodies
  i.e. they are bound by the strong interactions.
- Neutron Stars (Hadronic Stars) are **bound by gravity**.

- **Gravitational energy:** $E_G = (M_G - M_P)\, c^2 \leq 0$

  Gravitational binding energy: $B_G = -E_G$

B_G is the gravitational energy released moving the infinitesimal mass elements ρdV from infinity to form the star.

In the Newtonian limit

$$E_G = -G \int_0^R 4\pi\, r^2 \frac{m(r)\rho(r)}{r} dr \equiv E_G^{Newt}$$

- **Internal energy:** $E_I = (M_P - M_B)\, c^2 = \int_0^R \varepsilon'(r) dV$

  Internal binding energy: $B_I = -E_I$

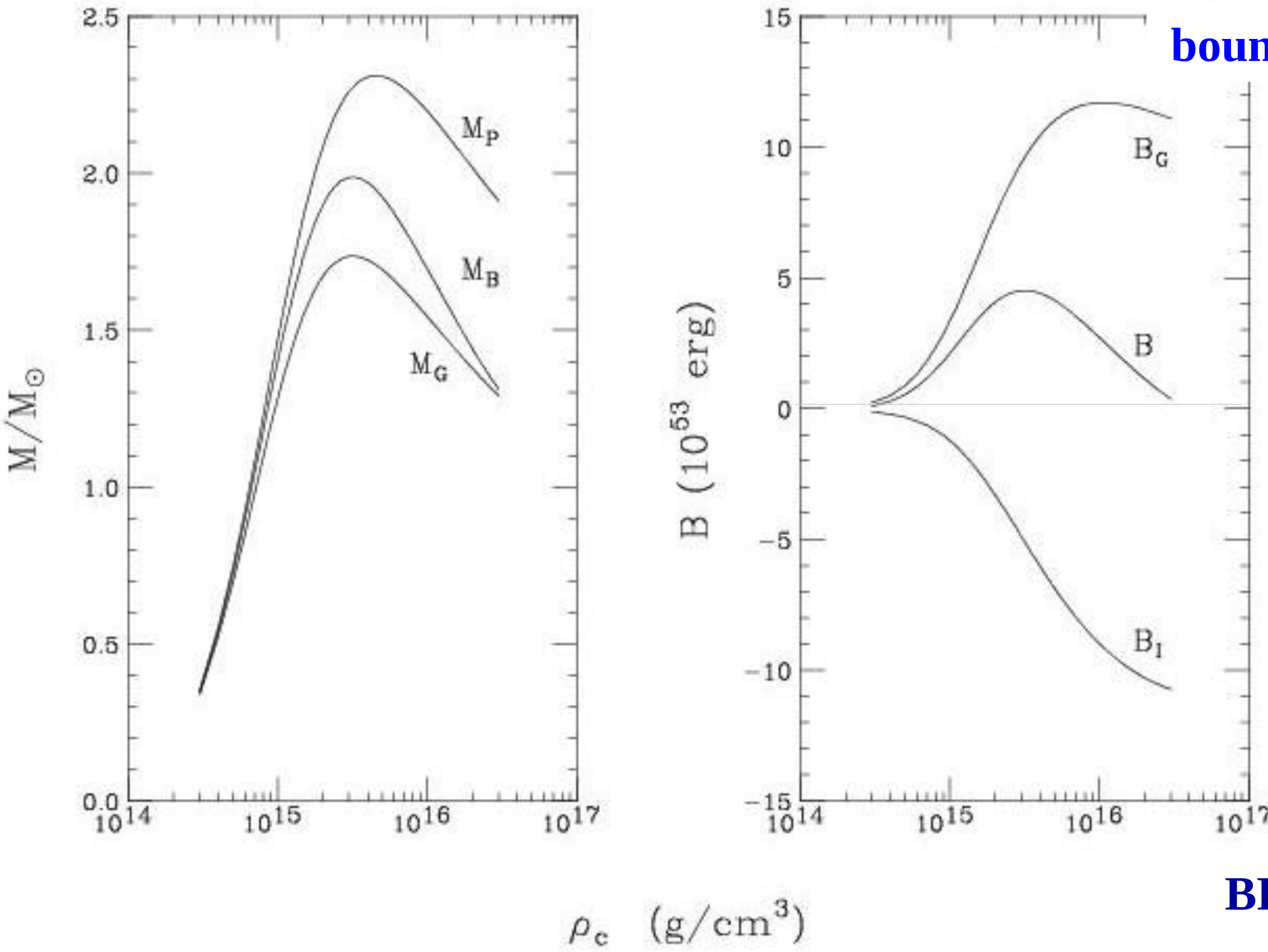
$\varepsilon' = (\rho - \rho_0)\, c^2$

- **Total energy:** $M_G c^2 = M_B c^2 + E_I + E_G = M_P c^2 + E_G$

  Total binding energy: $B = B_G + B_I = (M_B - M_G)\, c^2$

B is the total energy released during the formation of a static neutron star from a rarified gas of N_B baryons

Masses and binding energies of Neutron Stars



Bombaci (1995)

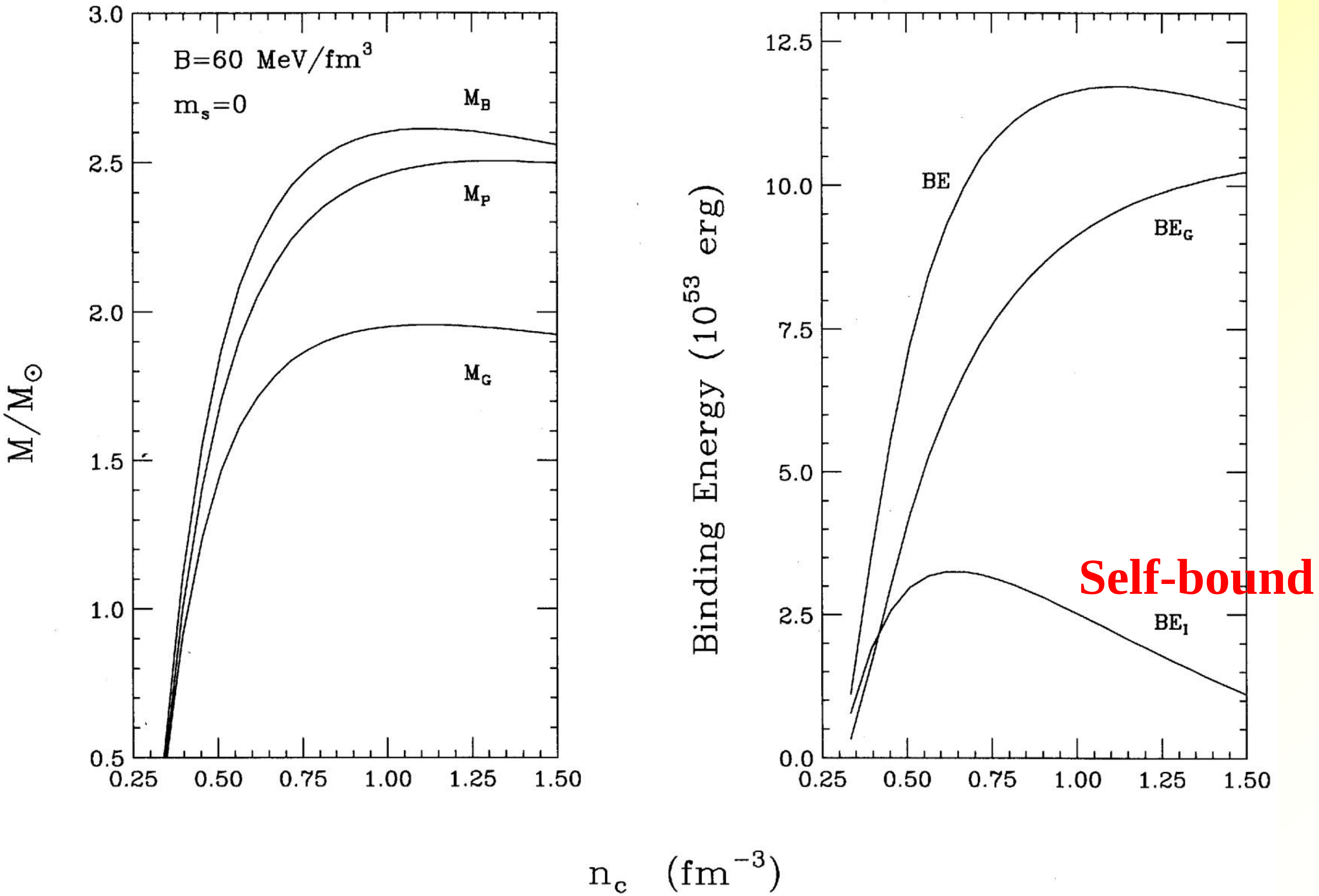
BPAL22 EOS

B_G = gravit. binding energy

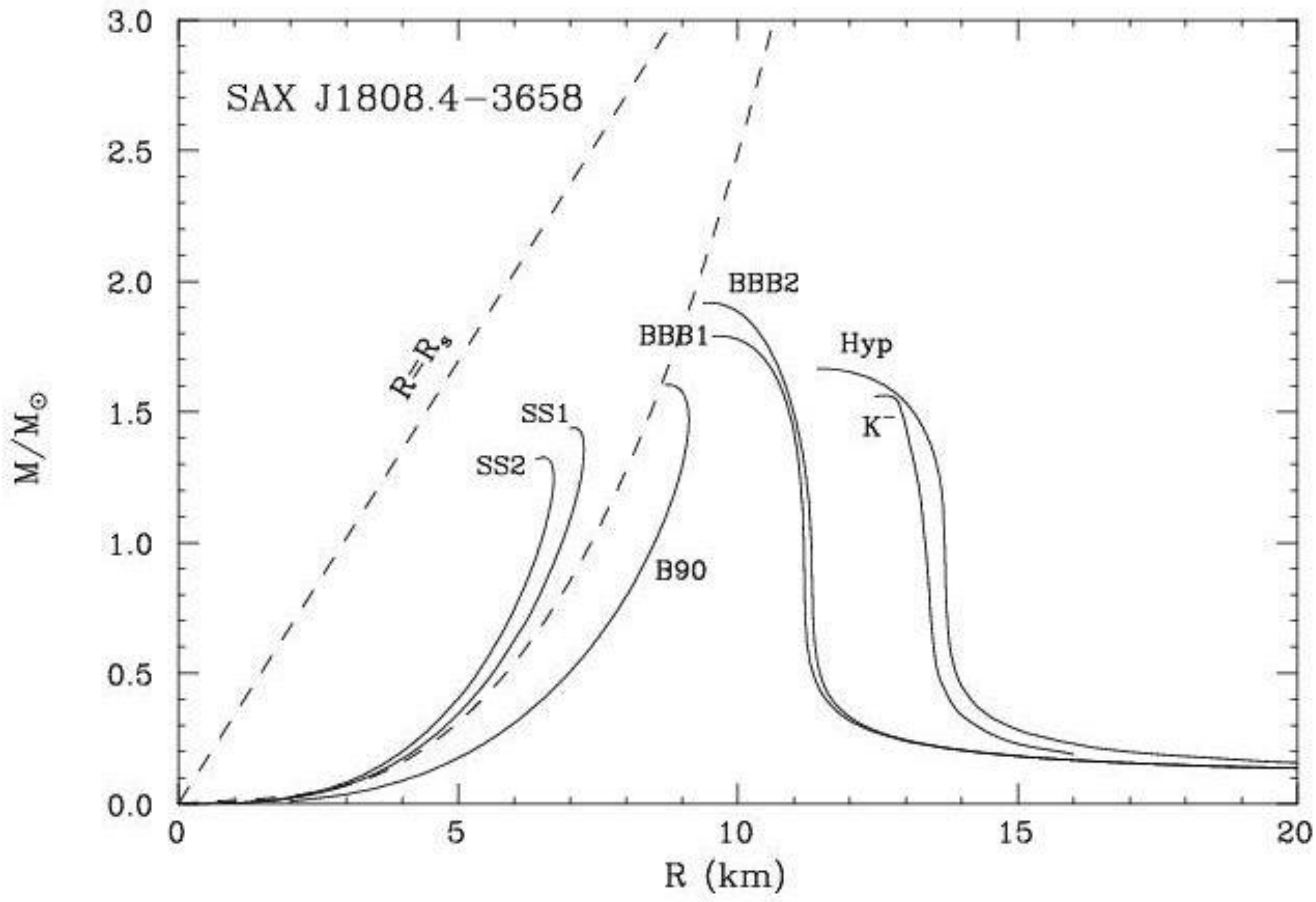
B_I = internal. binding energy

$B = B_G + B_I$ = total binding energy

Masses and binding energies of Strange Stars



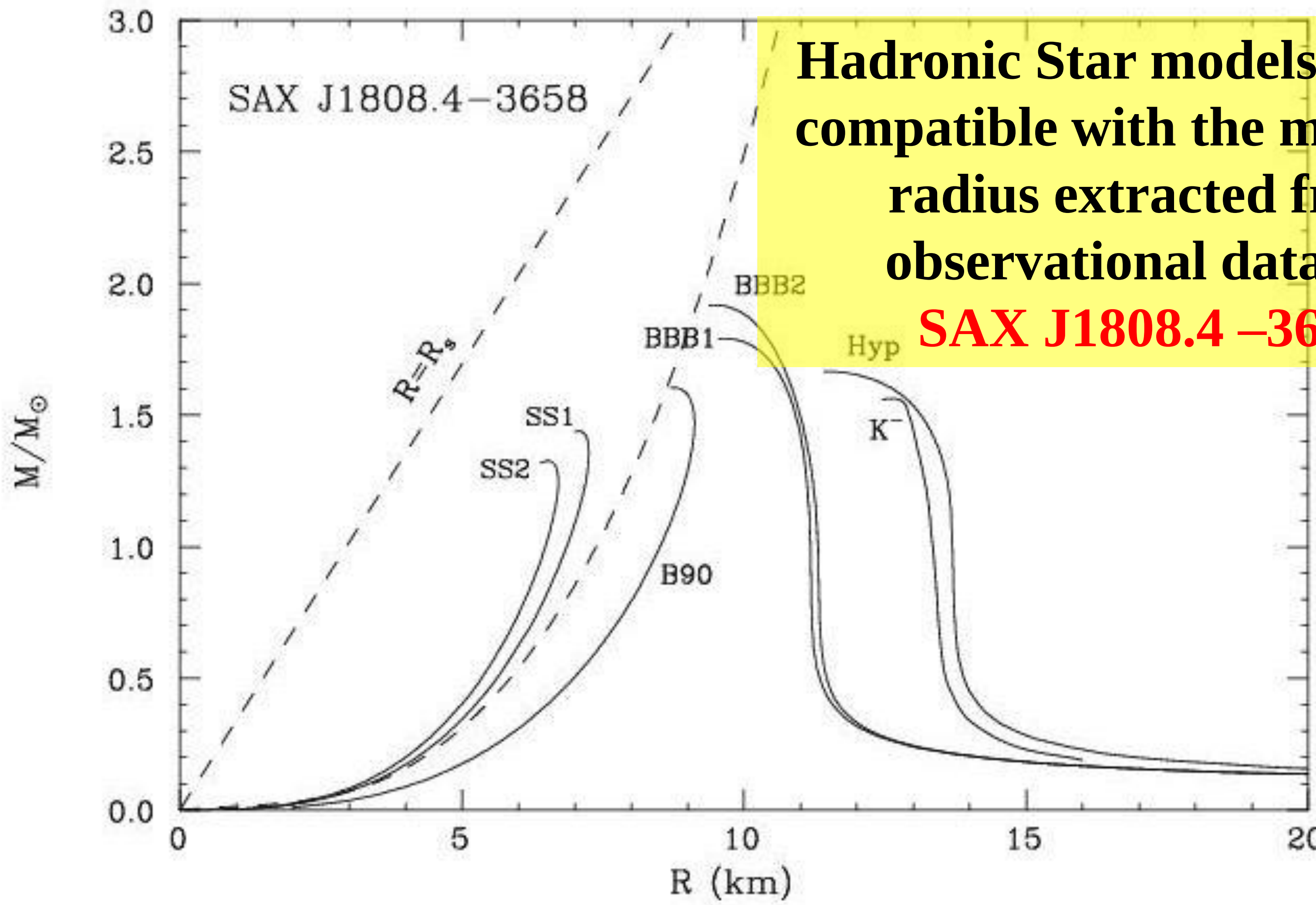
A strange star candidate: SAX J1808.4 –3658



X.D. Li, I. Bombaci, M. Dey, J. Dey, E.P.J. Van den Heuvel, Phys. Rev. Lett. 83 (1999) 3776

SS1, SS2: M. Dey, I. Bombaci, J. Dey, S. Ray, B.C. Samanta, Phys. Lett. B438 (1998) 123

A strange star candidate: SAX J1808.4 –3658



Hadronic Star models are not compatible with the mass and radius extracted from observational data for SAX J1808.4 –3658

X.D. Li, I. Bombaci, M. Dey, J. Dey, E.P.J. Van den Heuvel, Phys. Rev. Lett. 83 (1999) 3776

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