Transport properties of β -stable nuclear matter

Arianna Carbone[1] & Omar Benhar [2]

[1]Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona
[2]INFN & Dipartimento di Fisica, "Sapienza" Università di Roma

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Outline

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   - Nuclear Matter & Neutron stars structure
   - Landau theory for normal Fermi liquids
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3. Results
   - Results
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Motivations & Objectives

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- Oscillating modes of rotating stars are driven unstable through the emission of gravitational waves (CFS instability)
- Dissipation effects interior to the stars, as viscosity, can damp, or even suppress, these unstable oscillating modes
- The knowledge of the viscosity is an essential element to predict the stability of a rotating star

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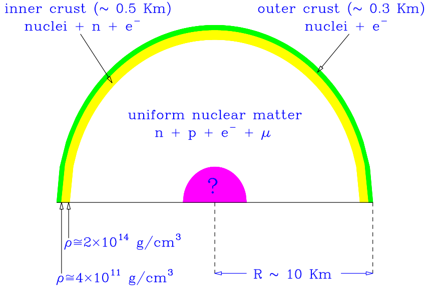
Our effort has been put in calculating both equilibrium and transport properties with the same dynamical model:

Landau-AK formalism using the effective potential derived in the Correlated Basis Function (CBF) theory

Neutron stars structure

External core: homogeneous fluid of neutrons, protons and electrons stable with respect to β-decay and electronic capture



Landau theory for normal Fermi liquids

- Interacting systems of fermions at low temperature

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Quasiparticle distribution

$$n_{\mathbf{p}} = \frac{1}{e^{\beta(\epsilon_{\mathbf{p}}[n_{\mathbf{p}}]-\mu)} + 1}$$

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- The Boltzmann-Landau transport equation

$$\frac{\partial n_{\mathbf{p}}(\mathbf{r},t)}{\partial t} + \frac{d\mathbf{r}}{dt}\frac{\partial n_{\mathbf{p}}(\mathbf{r},t)}{\partial \mathbf{r}} + \frac{d\mathbf{p}}{dt}\frac{\partial n_{\mathbf{p}}(\mathbf{r},t)}{\partial \mathbf{p}} = I[n_{\mathbf{p}}(\mathbf{r},t)]$$

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- Considering a small deviation from equilibrium of the state of the system, $n_{\mathbf{p}} = n^0_{\mathbf{p}} + \delta n_{\mathbf{p}}$, the transport equation reads

$$\mathbf{v}_{\mathbf{p}} \cdot \nabla_{\mathbf{r}} \delta n_{\mathbf{p}} = I[\delta n_{\mathbf{p}}]$$

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- The collisions integral

$$\begin{aligned} I[n_1] &= \frac{1}{V^2}\sum_{2}\sum_{34} \delta_{\mathbf{p_1}+\mathbf{p_2},\mathbf{p_3}+\mathbf{p_4}} \delta_{\sigma_1+\sigma_2,\sigma_3+\sigma_4} \delta(\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4) \\ &\times \quad W(12;34)[n_3 n_4(1-n_1)(1-n_2) - n_1 n_2 (1-n_3)(1-n_4)] \end{aligned}$$

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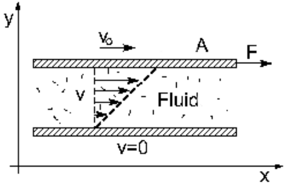
$$\sigma'_{xy} = \eta\frac{\partial u_x}{\partial y} = -\sum_\sigma \int \frac{d^3p}{(2\pi\hbar)^3} p_x (v_{\mathbf{p}})_y \delta n_{\mathbf{p}}$$



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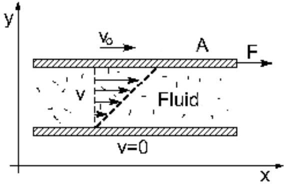

- Solve the transport equation

Landau theory for normal Fermi liquids

Abrikosov-Khalatnikov solution (1959):

$$\eta_{AK} = \frac{1}{5}\rho\, m^* v_F^2 \tau \frac{2}{\pi^2(1-\lambda_\eta)}$$

$$\tau \equiv \frac{8\pi^4}{m^{*3}\langle W\rangle T^2}, \quad \langle W\rangle \equiv \int \frac{d\Omega}{2\pi}\frac{W(\theta,\phi)}{\cos(\theta/2)}, \quad \lambda_\eta = \frac{\langle W(\theta,\phi)[1-3\sin^4(\theta/2)\sin^2\phi]\rangle}{\langle W(\theta,\phi)\rangle}$$

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Brooker-Sykes solution (1968):

$$\eta = \frac{1}{5}\rho\, m^* v_F^2 \tau \frac{2}{\pi^2(1-\lambda_\eta)} C(\lambda_\eta)$$

$$C(\lambda_\eta) = \frac{1-\lambda_\eta}{4}\sum_{k=0}^{\infty}\frac{4k+3}{(k+1)(2k+1)[(k+1)(2k+1)-\lambda_\eta]}$$

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Comparison between solutions:

$$0.75 < \frac{\eta}{\eta_{AK}} < 0.92$$

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External core of neutron stars as a multicomponent Fermi liquid:

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External core of neutron stars as a multicomponent Fermi liquid:

- Three components Boltzmann-Landau transport equation

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial n_\alpha}{\partial \mathbf{r}} \cdot \frac{\partial \epsilon_{\mathbf{p}\alpha}}{\partial \mathbf{p}} - \frac{\partial n_\alpha}{\partial \mathbf{p}} \cdot \frac{\partial \epsilon_{\mathbf{p}\alpha}}{\partial \mathbf{r}} = I_\alpha$$

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- Three components shear viscosity

$$\eta = \eta_n + \eta_p + \eta_e$$

$$\eta_\alpha = \frac{1}{5}\rho_\alpha m^*_\alpha v^2_{F\alpha}\tau_\alpha \frac{2}{\pi^2(1-\lambda_{\alpha\alpha})}C(\lambda_{\alpha\alpha})$$

$$\tau_\alpha = \frac{8\pi^4}{\sum_\beta m^*_\alpha (m^{*2}_\beta < W_{\alpha\beta} l_{\alpha\beta} >) T^2}, \quad \lambda_{\alpha\alpha} = \frac{\sum_\beta < W_{\alpha\beta}(\theta,\phi) l^\alpha_{\alpha\beta} >}{\sum_\beta < W_{\alpha\beta}(\theta,\phi) l_{\alpha\beta} >}$$

The effective potential

- Two body cluster contribution in the CBF theory:

$$v_{\text{eff}}(ij) = f_{ij}\left(-\frac{1}{m}\nabla^2 + v(ij)\right) f_{ij} = \sum_{n=1}^{6} v_{\text{eff}}^{n}(r_{ij}) O_{ij}^{n}$$

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- Three-nucleon contributions are taken into account considering the approach proposed by Lagaris-Pandharipande

$$v(ij) \rightarrow \widetilde{v}(ij) = \sum [v_\pi^n(r_{ij}) + v_I^n(r_{ij}) e^{-\gamma_1 \rho} + v_S^n(r_{ij})] O_{ij}^n$$

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- Calculate the scattering cross section and obtain the scattering probability to evaluate the mean lifetime of the quasiparticle state

$$W = W(\theta, \phi) = \left(\frac{2\pi}{\mu}\right)^2 \left(\frac{d\sigma}{d\Omega}\right)_{cm} [E_{cm} = f(\theta), \theta_{cm} = \phi]$$

The effective neutron mass

- Calculate the single particle energy using the effective potential

$$e_\lambda(p) = \frac{p^2}{2m} + \frac{\rho}{2} \sum_\mu \sum_n x_\mu \int d^3r \, v^n_{\text{eff}}(r) \left[A^n_{\lambda\mu} - B^n_{\lambda\mu} j_0(pr) \ell(p_F r) \right]$$

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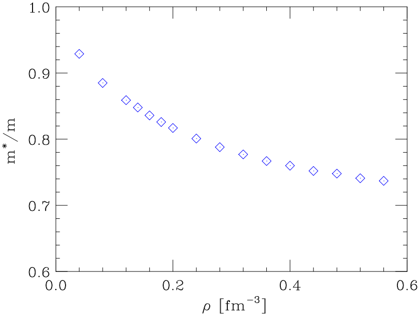
- Obtain the effective neutron mass

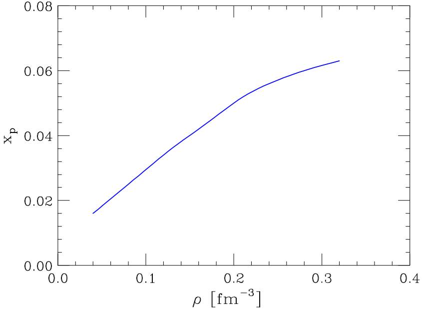
$$\frac{1}{m^*} = \frac{1}{p}\frac{\mathrm{d}e}{\mathrm{d}p}$$



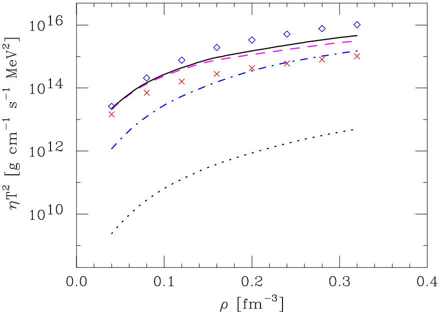
Proton fraction

Solve the coupled equations which define the equilibrium of the system:

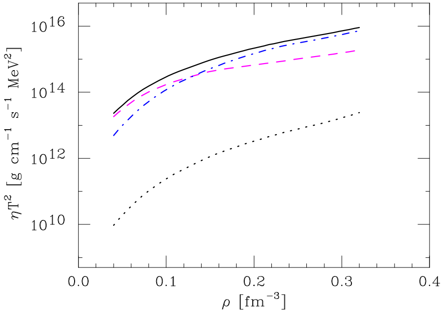
- β-stability: $\mu_n = \mu_p + \mu_e$
- charge neutrality: $\rho_p = x_p \rho = \rho_e$



Results I:



Results II: double proton fraction



- solid line=β-stable matter; dashed=n; dot-dashed=e; dotted=p; diamonds=PNM; cross=Argonne v8'

Conclusions & Outlooks

- We calculated the viscosity of the external core of a neutron star, using a realistic model of nuclear matter and of the dynamic to its interior
- The percentage of electrons play a fundamental role in determining the weight of the viscosity of each particle family
- This work can be improved including the presence of muons and successively hyperons (if that's what really neutron stars are made of!)
- Calculate consistently the shear viscosity of the superfluid / superconducting phase

Thank you! [1]

[1]Arianna Carbone, PhD student with Dr. Artur Polls & Dr. Arnau Rios on "Three body forces with self-consistent Green's functions".
ariannac@ecm.ub.es