# The nuclear shell model: a brief tutorial

Luigi Coraggio

Istituto Nazionale di Fisica Nucleare - Sezione di Napoli

May, 30th 2011

# A tutorial to the nuclear shell model

- A brief introduction to the nuclear shell-model
- Problematics
- Shell-model codes
- Applications

# A tutorial to the nuclear shell model

- A brief introduction to the nuclear shell-model
- Problematics
- Shell-model codes
- Applications

# A tutorial to the nuclear shell model

- A brief introduction to the nuclear shell-model
- Problematics
- Shell-model codes
- Applications

# A tutorial to the nuclear shell model

- A brief introduction to the nuclear shell-model
- Problematics
- Shell-model codes
- Applications

M. Goeppert-Mayer and H. D. Jensen

1963 Nobel Prizes in Physics

# The nuclear shell model

The concept of the nuclear shell model is anologous to the atomic one: the many-nucleons wavefunction is approximated by an antisymmetrized product of single-particle wavefunctions

$$\Phi(A) = \begin{vmatrix} \phi_1(r_1) & \phi_1(r_2) & \dots & \phi_1(r_A) \\ \phi_2(r_1) & \phi_2(r_2) & \dots & \phi_2(r_A) \\ \dots & \dots & \dots & \dots \\ \phi_A(r_1) & \phi_A(r_2) & \dots & \phi_A(r_A) \end{vmatrix}$$

The single-particle wavefunctions $\phi_i$ are eigenfunctions of a spherical potential well, i.e. eigenfunctions of the hamiltonian $h_0 = \frac{p^2}{2M} + u(r)$

# The nuclear shell model

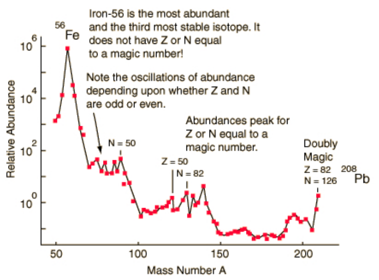
The concept of the nuclear shell model is anologous to the atomic one: the many-nucleons wavefunction is approximated by an antisymmetrized product of single-particle wavefunctions

$$\Phi(A) = \begin{vmatrix} \phi_1(r_1) & \phi_1(r_2) & \dots & \phi_1(r_A) \\ \phi_2(r_1) & \phi_2(r_2) & \dots & \phi_2(r_A) \\ \dots & \dots & \dots & \dots \\ \phi_A(r_1) & \phi_A(r_2) & \dots & \phi_A(r_A) \end{vmatrix}$$

The single-particle wavefunctions $\phi_i$ are eigenfunctions of a spherical potential well, i.e. eigenfunctions of the hamiltonian $h_0 = \frac{p^2}{2M} + u(r)$

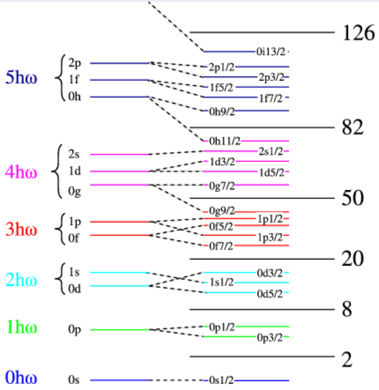
# The nuclear shell model

With a few physical assumptions the hamiltonian $H_0 = \sum_{i=1}^{A} h_0^i$ is able to explain nuclear properties such as the ground-state $J^\pi$ and the "magic numbers"



# The nuclear shell model

In order to reproduce the correct sequence 2, 8, 20, 28, 50, ... it is needed to couple the spherical potential well (harmonic oscillator, Woods-Saxon potential, ...) with a spin-orbit potential $f(r)\mathbf{l} \cdot \mathbf{s}$



# The nuclear shell model

The independent-particle hamiltonian $H_0$ is not able, however, to predict satisfactorily the excited spectra of the nuclei, so a residual two-body interaction $V^{res}$ needs to be taken into account, that breaks the degeneracy of states with different $J^{\pi}$ and the same single-particle configuration, so introducing a configuration mixing

$$H = H_0 + H_I = \sum_{i=1}^{A}\left(\frac{p_i^2}{2M} + u_i\right) + \sum_{i<j} V_{ij}^{res}$$

More details about $V^{res}$ will be given later on

# The nuclear shell model

The independent-particle hamiltonian $H_0$ is not able, however, to predict satisfactorily the excited spectra of the nuclei, so a residual two-body interaction $V^{res}$ needs to be taken into account, that breaks the degeneracy of states with different $J^{\pi}$ and the same single-particle configuration, so introducing a configuration mixing

$$H = H_0 + H_I = \sum_{i=1}^{A} \left( \frac{p_i^2}{2M} + u_i \right) + \sum_{i<j} V_{ij}^{res}$$

More details about $V^{res}$ will be given later on

# The nuclear shell model

The above hamiltonian cannot be diagonalized without reducing the number of degrees of freedom.

So, the first step is to identify a "core" nucleus, whose degrees of freedom will be considered "frozen" within the shell-model hamiltonian

The best choice, made on physical grounds, is a nucleus with a number of protons and neutrons equal to a "magic number", i.e. a doubly closed-shell nucleus

$^{4}$He, $^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{56}$Ni, $^{100}$Sn, $^{132}$Sn, $^{208}$Pb, ...

# The nuclear shell model

The above hamiltonian cannot be diagonalized without reducing the number of degrees of freedom.

So, the first step is to identify a "core" nucleus, whose degrees of freedom will be considered "frozen" within the shell-model hamiltonian

The best choice, made on physical grounds, is a nucleus with a number of protons and neutrons equal to a "magic number", i.e. a doubly closed-shell nucleus

$^{4}$He, $^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{56}$Ni, $^{100}$Sn, $^{132}$Sn, $^{208}$Pb, ...

# The nuclear shell model

The above hamiltonian cannot be diagonalized without reducing the number of degrees of freedom.

So, the first step is to identify a "core" nucleus, whose degrees of freedom will be considered "frozen" within the shell-model hamiltonian

The best choice, made on physical grounds, is a nucleus with a number of protons and neutrons equal to a "magic number", i.e. a doubly closed-shell nucleus

$^{4}$He, $^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{56}$Ni, $^{100}$Sn, $^{132}$Sn, $^{208}$Pb, ...

This constraint allows to study only nuclei with $Z$ and $N$ larger than $Z_c$ and $N_c$, and the physics of those nuclei will be described only in terms of the nucleons exceeding the $A_c$ nucleons of the core, the so-called "valence nucleons"

This is simplification of the computational problem, but something more has to be done.

Note that the proton- and neutron-major shells are well separated in energy by $H_0$.

This means that our model space can be limited to the two proton- and neutron-major shells located in energy just above the core

# The nuclear shell model

This constraint allows to study only nuclei with $Z$ and $N$ larger than $Z_c$ and $N_c$, and the physics of those nuclei will be described only in terms of the nucleons exceeding the $A_c$ nucleons of the core, the so-called "valence nucleons"

This is simplification of the computational problem, but something more has to be done.

Note that the proton- and neutron-major shells are well separated in energy by $H_0$.

This means that our model space can be limited to the two proton- and neutron-major shells located in energy just above the core

# The nuclear shell model

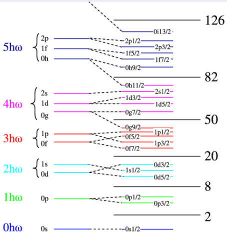
This constraint allows to study only nuclei with $Z$ and $N$ larger than $Z_c$ and $N_c$, and the physics of those nuclei will be described only in terms of the nucleons exceeding the $A_c$ nucleons of the core, the so-called “valence nucleons”

This is simplification of the computational problem, but something more has to be done.

Note that the proton- and neutron-major shells are well separated in energy by $H_0$.

This means that our model space can be limited to the two proton- and neutron-major shells located in energy just above the core

# The nuclear shell model

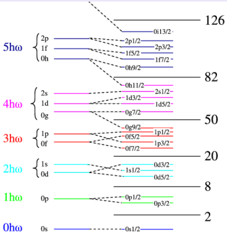



This choice limits the number of the nuclei that can be studied to those with $Z_c, N_c < Z, N < Z_c', N_c'$, where $Z_c', N_c'$ are the number of protons and neutrons of the next doubly closed-shell nucleus

For example: $^{16}\mathrm{O} \rightarrow {}^{40}\mathrm{Ca}$

Model space: $0d_{5/2}$, $0d_{3/2}$, $1s_{1/2}$

# The nuclear shell model

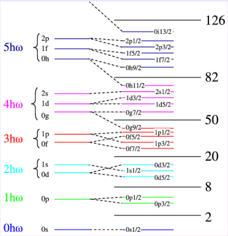



This choice limits the number of the nuclei that can be studied to those with $Z_c, N_c < Z, N < Z'_c, N'_c$, where $Z'_c, N'_c$ are the number of protons and neutrons of the next doubly closed-shell nucleus

For example: $^{16}O \rightarrow ^{40}Ca$

Model space: $0d_{5/2},\ 0d_{3/2},\ 1s_{1/2}$

# The nuclear shell model

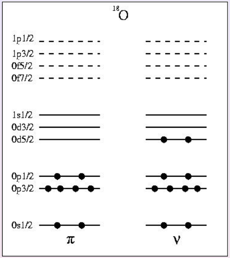



This choice limits the number of the nuclei that can be studied to those with $Z_c, N_c < Z, N < Z'_c, N'_c$, where $Z'_c, N'_c$ are the number of protons and neutrons of the next doubly closed-shell nucleus

For example: $^{16}O \rightarrow ^{40}Ca$

Model space: $0d_{5/2},\ 0d_{3/2},\ 1s_{1/2}$

# The nuclear shell model




The shell-model hamiltonian to be diagonalized in the second quantization formalism is

$$H = \sum_{i=1}^{n} \epsilon_i a_i^\dagger a_i + \sum_{ijkl} V_{ijkl}^{res} a_i^\dagger a_j^\dagger a_l a_k$$

The $\epsilon_j$s are the eigenvalues of the single-particle hamiltonian $H_0$, the $V_{ijkl}^{res}$ are the two-body matrix elements of the residual potential

# The nuclear shell model

The eigenfunctions $\Psi_\alpha$ are linear combinations of antisymmetrized product of single-particle wavefunctions

$$\Psi_\alpha = \sum_\beta c_\alpha^\beta \Phi_\alpha^\beta \quad ,$$

where

$$\Phi_\alpha^\beta = \left[ (a_1^\dagger)^{k_1} (a_2^\dagger)^{k_2} \ldots (a_n^\dagger)^{k_n} \right]_{\alpha\beta} \Psi_c$$

$n$ is the number of single-particle levels in the model space and $A_{val} = \sum_i k_i$ is the number of valence nucleons

# Problematics

Now we know what we need to study a nucleus or a class of nuclei within the shell model

- To identify the best model space
- To fix the $\epsilon_j$ and the $V^{res}_{ijkl}$
- Diagonalize the shell-model hamiltonian

# Problematics

Now we know what we need to study a nucleus or a class of nuclei within the shell model

- To identify the best model space
- To fix the $\epsilon_j$ and the $V^{res}_{ijkl}$
- Diagonalize the shell-model hamiltonian

# Problematics

Now we know what we need to study a nucleus or a class of nuclei within the shell model

- To identify the best model space
- To fix the $\epsilon_j$ and the $V^{res}_{ijkl}$
- Diagonalize the shell-model hamiltonian

# Problematics

Now we know what we need to study a nucleus or a class of nuclei within the shell model

- To identify the best model space
- To fix the $\epsilon_j$ and the $V_{ijkl}^{res}$
- Diagonalize the shell-model hamiltonian

# The model space

The model space is the set of single-particle orbitals that are accessible by the valence nucleons

As has been told before, usually it is made up by the proton- and neutron-major shells energetically located just above the “core” filled ones

For example: for nuclei ranging from $^{16}$O to $^{40}$Ca, we consider the $0s$ and the $0p$ shells filled by 8 protons and 8 neutrons ($^{16}$O core), and the valence nucleons interact in the 6 $1s\ 0d$ proton and neutron orbitals

# The model space

The model space is the set of single-particle orbitals that are accessible by the valence nucleons

As has been told before, usually it is made up by the proton- and neutron-major shells energetically located just above the "core" filled ones

For example: for nuclei ranging from $^{16}$O to $^{40}$Ca, we consider the $0s$ and the $0p$ shells filled by 8 protons and 8 neutrons ($^{16}$O core), and the valence nucleons interact in the 6 $1s\,0d$ proton and neutron orbitals

# The model space

The model space is the set of single-particle orbitals that are accessible by the valence nucleons

As has been told before, usually it is made up by the proton- and neutron-major shells energetically located just above the “core” filled ones

For example: for nuclei ranging from $^{16}$O to $^{40}$Ca, we consider the $0s$ and the $0p$ shells filled by 8 protons and 8 neutrons ($^{16}$O core), and the valence nucleons interact in the 6 $1s\,0d$ proton and neutron orbitals

# The model space

In such a frame, one can describe only some positive-parity states, if one needs to perform a more detailed study more degrees of freedom are needed to be defrozen

This means to include more single-particle orbitals, the choice grounded on physical assumptions

It could happen also the opposite, the physics of the systems under investigation could suggest that a further reduction of the model space can be done, so to simplify the computational problem (we will see an example later)

$$H = \sum_{i=1}^{n} \epsilon_i a_i^\dagger a_i + \sum_{ijkl} V_{ijkl}^{res} a_i^\dagger a_j^\dagger a_l a_k$$

# The model space

In such a frame, one can describe only some positive-parity states, if one needs to perform a more detailed study more degrees of freedom are needed to be defrozen

This means to include more single-particle orbitals, the choice grounded on physical assumptions

It could happen also the opposite, the physics of the systems under investigation could suggest that a further reduction of the model space can be done, so to simplify the computational problem (we will see an example later)

$$H = \sum_{i=1}^{n} \epsilon_i a_i^\dagger a_i + \sum_{ijkl} V_{ijkl}^{res} a_i^\dagger a_j^\dagger a_l a_k$$

# The model space

In such a frame, one can describe only some positive-parity states, if one needs to perform a more detailed study more degrees of freedom are needed to be defrozen

This means to include more single-particle orbitals, the choice grounded on physical assumptions

It could happen also the opposite, the physics of the systems under investigation could suggest that a further reduction of the model space can be done, so to simplify the computational problem (we will see an example later)

$$H = \sum_{i=1}^{n} \epsilon_i a_i^\dagger a_i + \sum_{ijkl} V_{ijkl}^{res} a_i^\dagger a_j^\dagger a_l a_k$$

# The single-particle energies

The choice of the "best" set of single-particle energies $\epsilon_j$ is crucial

We can:

- use the experimental energy spectra of nuclei with $A = A_c + 1$
- fit them so to reproduce the experimental energy spectra of seniority $v = 1$ states
- calculate them as eigenvaues of a theoretical $H_0$ (Woods-Saxon potential, Hartree-Fock potential from Skyrme or Gongny forces, ...)

# The single-particle energies

The choice of the “best” set of single-particle energies $\epsilon_j$ is crucial

We can:

- use the experimental energy spectra of nuclei with $A = A_c + 1$
- fit them so to reproduce the experimental energy spectra of seniority $v = 1$ states
- calculate them as eigenvaues of a theoretical $H_0$ (Woods-Saxon potential, Hartree-Fock potential from Skyrme or Gongny forces, ...)

# The residual two-body interaction

The residual two-body interaction $V^{res}$ has to be "effective"

i.e., $V^{res}$ has to take implicitly into account of the degrees of freedom that are explicitly considered frozen

This means that $V^{res}$ should contain the core-polarization and particle-particle excitations , so that the eigenvalues of the shell-model hamiltonian are the same (or at least very close) to those of the nuclear many-body hamiltonian diagonalized in the full Hilbert space

# The residual two-body interaction

The residual two-body interaction $V^{res}$ has to be "effective"

i.e., $V^{res}$ has to take implicitly into account of the degrees of freedom that are explicitly considered frozen

This means that $V^{res}$ should contain the core-polarization and particle-particle excitations , so that the eigenvalues of the shell-model hamiltonian are the same (or at least very close) to those of the nuclear many-body hamiltonian diagonalized in the full Hilbert space

# The residual two-body interaction

The residual two-body interaction $V^{res}$ has to be “effective”

i.e., $V^{res}$ has to take implicitly into account of the degrees of freedom that are explicitly considered frozen

This means that $V^{res}$ should contain the core-polarization and particle-particle excitations , so that the eigenvalues of the shell-model hamiltonian are the same (or at least very close) to those of the nuclear many-body hamiltonian diagonalized in the full Hilbert space

# The residual two-body interaction

The possible ways to derive a shell-model residual interaction $V^{res}$ can be grouped into three main approaches

- Empirical $V^{res}$ fitted on experimental data
- Empirical $V^{res}$ with a simple analytical expression
- Realistic effective $V^{res}$ derived microscopically from the free nucleon-nucleon potential $V_{NN}$ (the house specialty)

# The residual two-body interaction

The possible ways to derive a shell-model residual interaction $V^{res}$ can be grouped into three main approaches

- Empirical $V^{res}$ fitted on experimental data
- Empirical $V^{res}$ with a simple analytical expression
- Realistic effective $V^{res}$ derived microscopically from the free nucleon-nucleon potential $V_{NN}$ (the house specialty)

# The residual two-body interaction

The possible ways to derive a shell-model residual interaction $V^{res}$ can be grouped into three main approaches

- Empirical $V^{res}$ fitted on experimental data
- Empirical $V^{res}$ with a simple analytical expression
- Realistic effective $V^{res}$ derived microscopically from the free nucleon-nucleon potential $V_{NN}$ (the house specialty)

# The residual two-body interaction

The possible ways to derive a shell-model residual interaction $V^{res}$ can be grouped into three main approaches

- Empirical $V^{res}$ fitted on experimental data
- Empirical $V^{res}$ with a simple analytical expression
- Realistic effective $V^{res}$ derived microscopically from the free nucleon-nucleon potential $V_{NN}$ (the house specialty)

# $V^{res}$ fitted to the experiment

The $V^{res}$ two-body matrix elements are treated as free parameters

They are derived by way of a best-fit procedure to a selected set of experimental data

- USDA, USDB (*sd*-shell region, $^{16}$O core)
- KB3G, FPD6, GXPF1A (*pf*-shell region, $^{40}$Ca core)
- Warburton-Brown ($^{132}$Sn and $^{208}$Pb cores)

# $V^{res}$ fitted to the experiment

The $V^{res}$ two-body matrix elements are treated as free parameters

They are derived by way of a best-fit procedure to a selected set of experimental data

- USDA, USDB (*sd*-shell region, $^{16}$O core)
- KB3G, FPD6, GXPF1A (*pf*-shell region, $^{40}$Ca core)
- Warburton-Brown ($^{132}$Sn and $^{208}$Pb cores)

# $V^{res}$ fitted to the experiment

The $V^{res}$ two-body matrix elements are treated as free parameters

They are derived by way of a best-fit procedure to a selected set of experimental data

- USDA, USDB (*sd*-shell region, $^{16}$O core)
- KB3G, FPD6, GXPF1A (*pf*-shell region, $^{40}$Ca core)
- Warburton-Brown ($^{132}$Sn and $^{208}$Pb cores)

# $V^{res}$ fitted to the experiment

The $V^{res}$ two-body matrix elements are treated as free parameters

They are derived by way of a best-fit procedure to a selected set of experimental data

- USDA, USDB (*sd*-shell region, $^{16}$O core)
- KB3G, FPD6, GXPF1A (*pf*-shell region, $^{40}$Ca core)
- Warburton-Brown ($^{132}$Sn and $^{208}$Pb cores)

## $V^{res}$ fitted to the experiment

The $V^{res}$ two-body matrix elements are treated as free parameters

They are derived by way of a best-fit procedure to a selected set of experimental data

- USDA, USDB (*sd*-shell region, $^{16}$O core)
- KB3G, FPD6, GXPF1A (*pf*-shell region, $^{40}$Ca core)
- Warburton-Brown ($^{132}$Sn and $^{208}$Pb cores)

## $V^{res}$ fitted to the experiment

Pros: these shell-model interactions are avery refined tool, very successful, and nowadays the most widely employed ones

Cons: the predictions of the physics that characterize unexplored features of the spectroscopy of the nuclei could be biased by the choice of the experimental databases

# $V^{res}$ fitted to the experiment

Pros: these shell-model interactions are avery refined tool, very successful, and nowadays the most widely employed ones

Cons: the predictions of the physics that characterize unexplored features of the spectroscopy of the nuclei could be biased by the choice of the experimental databases

# Schematic interactions

They have a simple analytical expression, whose few parameters have to be fitted to experimental data

For example, they could be expressed as Gaussian of Yukawian form functions coupled to exchange operators consistent with those of the free $V_{NN}$

$$V^{res} = V_0(r) + V_\sigma \sigma_1 \cdot \sigma_2 + V_\tau \tau_1 \cdot \tau_2 + V_{\sigma\tau}(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) + V_T \frac{(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r}) - (\sigma_1 \cdot \sigma_2)}{r^2}$$

Another class are those interactions that contain only few relevant component of the nucleon-nucleon potential and a very small number of free parameters:

- pairing or pairing plus quadrupole interactions
- surface delta interaction
- spin and isospin dependent Migdal interaction

# Schematic interactions

They have a simple analytical expression, whose few parameters have to be fitted to experimental data

For example, they could be expressed as Gaussian of Yukawian form functions coupled to exchange operators consistent with those of the free $V_{NN}$

$$V^{res} = V_0(r) + V_\sigma \sigma_1 \cdot \sigma_2 + V_\tau \tau_1 \cdot \tau_2 + V_{\sigma\tau}(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) + V_T \frac{(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r}) - (\sigma_1 \cdot \sigma_2)}{r^2}$$

Another class are those interactions that contain only few relevant component of the nucleon-nucleon potential and a very small number of free parameters:

- pairing or pairing plus quadrupole interactions
- surface delta interaction
- spin and isospin dependent Migdal interaction

# Schematic interactions

They have a simple analytical expression, whose few parameters have to be fitted to experimental data

For example, they could be expressed as Gaussian of Yukawian form functions coupled to exchange operators consistent with those of the free $V_{NN}$

$$V^{res} = V_0(r) + V_\sigma \sigma_1 \cdot \sigma_2 + V_\tau \tau_1 \cdot \tau_2 + V_{\sigma\tau} (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) + V_T \frac{(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r}) - (\sigma_1 \cdot \sigma_2)}{r^2}$$

Another class are those interactions that contain only few relevant component of the nucleon-nucleon potential and a very small number of free parameters:

- pairing or pairing plus quadrupole interactions
- surface delta interaction
- spin and isospin dependent Migdal interaction

# Schematic interactions

They have a simple analytical expression, whose few parameters have to be fitted to experimental data

For example, they could be expressed as Gaussian of Yukawian form functions coupled to exchange operators consistent with those of the free $V_{NN}$

$$V^{res} = V_0(r) + V_\sigma \sigma_1 \cdot \sigma_2 + V_\tau \tau_1 \cdot \tau_2 + V_{\sigma\tau}(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) + V_T \frac{(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r}) - (\sigma_1 \cdot \sigma_2)}{r^2}$$

Another class are those interactions that contain only few relevant component of the nucleon-nucleon potential and a very small number of free parameters:

- pairing or pairing plus quadrupole interactions
- surface delta interaction
- spin and isospin dependent Migdal interaction

# Schematic interactions

They have a simple analytical expression, whose few parameters have to be fitted to experimental data

For example, they could be expressed as Gaussian of Yukawian form functions coupled to exchange operators consistent with those of the free $V_{NN}$

$$V^{res} = V_0(r) + V_\sigma \sigma_1 \cdot \sigma_2 + V_\tau \tau_1 \cdot \tau_2 + V_{\sigma\tau} (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) + V_T \frac{(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r}) - (\sigma_1 \cdot \sigma_2)}{r^2}$$

Another class are those interactions that contain only few relevant component of the nucleon-nucleon potential and a very small number of free parameters:

- pairing or pairing plus quadrupole interactions
- surface delta interaction
- spin and isospin dependent Migdal interaction

# Schematic interactions

They have a simple analytical expression, whose few parameters have to be fitted to experimental data

For example, they could be expressed as Gaussian of Yukawian form functions coupled to exchange operators consistent with those of the free $V_{NN}$

$$V^{res} = V_0(r) + V_\sigma \sigma_1 \cdot \sigma_2 + V_\tau \tau_1 \cdot \tau_2 + V_{\sigma\tau} (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) + V_T \frac{(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r}) - (\sigma_1 \cdot \sigma_2)}{r^2}$$

Another class are those interactions that contain only few relevant component of the nucleon-nucleon potential and a very small number of free parameters:

- pairing or pairing plus quadrupole interactions
- surface delta interaction
- spin and isospin dependent Migdal interaction

# Schematic interactions

Pros: they are very useful in order to understand what is the relevant physics underlying the spectroscopic structure of the nuclei

Cons: low-resolution in the reproduction of experimental data, nowadays they are considered out-of-date

# Schematic interactions

Pros: they are very useful in order to understand what is the relevant physics underlying the spectroscopic structure of the nuclei

Cons: low-resolution in the reproduction of experimental data, nowadays they are considered out-of-date

# Realistic effective interactions

The shell-model $V^{res}$ is derived directly from the free nucleon-nucleon potential by way of theoretical approaches

We will see some details tomorrow

# Realistic effective interactions

The shell-model $V^{res}$ is derived directly from the free nucleon-nucleon potential by way of theoretical approaches

We will see some details tomorrow

# Realistic effective interactions

Pros: no parameters are involved in the shell-model calculation, except from the single-particle energies

Cons: the theory to derive these interactions is very complicated and still under investigation, the results could depend upon the performances of the input $V_{NN}$ (but this could be also an advantage ...)

# Realistic effective interactions

Pros: no parameters are involved in the shell-model calculation, except from the single-particle energies

Cons: the theory to derive these interactions is very complicated and still under investigation, the results could depend upon the performances of the input $V_{NN}$ (but this could be also an advantage ...)

# The diagonalization of the hamiltonian

The difficulties to diagonalize a shell-model hamiltonian can range from matrix whose dimension is $10^0$ to one with $10^9$

For example: the number of basis states in the region of $^{100}$Sn core can be:

| $^{102}$Sn | (2 valence neutrons) | $^{112}$Sn | (12 valence neutrons) |
|---|---|---|---|
| $J^\pi = 0^+$ | $J^\pi = 2^+$ | $J^\pi = 0^+$ | $J^\pi = 2^+$ |
| 5 | 9 | $6 * 10^4$ | $6 * 10^5$ |

while in the region of $^{132}$Sn core can be:

| $^{134}$Sn | (2 valence neutrons) | $^{154}$Sm | (12 valence protons 10 valence neutrons) |
|---|---|---|---|
| $J^\pi = 0^+$ | $J^\pi = 2^+$ | $J^\pi = 0^+$ | $J^\pi = 2^+$ |
| 6 | 12 | $4 * 10^{13}$ | $3 * 10^{14}$ |

# The diagonalization of the hamiltonian

The difficulties to diagonalize a shell-model hamiltonian can range from matrix whose dimension is $10^0$ to one with $10^9$

For example: the number of basis states in the region of $^{100}$Sn core can be:

| $^{102}$Sn | (2 valence neutrons) | $^{112}$Sn | (12 valence neutrons) |
|---|---|---|---|
| $J^\pi = 0^+$ | $J^\pi = 2^+$ | $J^\pi = 0^+$ | $J^\pi = 2^+$ |
| 5 | 9 | $6 * 10^4$ | $6 * 10^5$ |

while in the region of $^{132}$Sn core can be:

| $^{134}$Sn | (2 valence neutrons) | $^{154}$Sm | (12 valence protons 10 valence neutrons) |
|---|---|---|---|
| $J^\pi = 0^+$ | $J^\pi = 2^+$ | $J^\pi = 0^+$ | $J^\pi = 2^+$ |
| 6 | 12 | $4 * 10^{13}$ | $3 * 10^{14}$ |

# The diagonalization of the hamiltonian

The difficulties to diagonalize a shell-model hamiltonian can range from matrix whose dimension is $10^0$ to one with $10^9$

For example: the number of basis states in the region of $^{100}$Sn core can be:

| $^{102}$Sn | (2 valence neutrons) | $^{112}$Sn | (12 valence neutrons) |
|---|---|---|---|
| $J^\pi = 0^+$ | $J^\pi = 2^+$ | $J^\pi = 0^+$ | $J^\pi = 2^+$ |
| 5 | 9 | $6 * 10^4$ | $6 * 10^5$ |

while in the region of $^{132}$Sn core can be:

| $^{134}$Sn | (2 valence neutrons) | $^{154}$Sm | (12 valence protons 10 valence neutrons) |
|---|---|---|---|
| $J^\pi = 0^+$ | $J^\pi = 2^+$ | $J^\pi = 0^+$ | $J^\pi = 2^+$ |
| 6 | 12 | $4 * 10^{13}$ | $3 * 10^{14}$ |

# Shell-model codes

Nowadays there several codes for large-scale shell-model calculations on the market:

- OXBASH
- NuShell
- ANTOINE, NATHAN
- Oslo (this is the one I will briefly introduce to)

# Shell-model codes

Nowadays there several codes for large-scale shell-model calculations on the market:

- OXBASH
- NuShell
- ANTOINE, NATHAN
- Oslo (this is the one I will briefly introduce to)

# Shell-model codes

Nowadays there several codes for large-scale shell-model calculations on the market:

- OXBASH
- NuShell
- ANTOINE, NATHAN
- Oslo (this is the one I will briefly introduce to)

# Shell-model codes

Nowadays there several codes for large-scale shell-model calculations on the market:

- OXBASH
- NuShell
- ANTOINE, NATHAN
- Oslo (this is the one I will briefly introduce to)

# Shell-model codes

Nowadays there several codes for large-scale shell-model calculations on the market:

- OXBASH
- NuShell
- ANTOINE, NATHAN
- Oslo (this is the one I will briefly introduce to)

# The Oslo shell-model code

This code has been written by Torgeir Engeland in C++ language, and recently improved by Morten Hjorth-Jensen and co-workers

It is easy to be provided, fast, versatile

It can be downloaded for Linux machines at the address : http://folk.uio.no/mhjensen/cp/software.html

# The Oslo shell-model code

This code has been written by Torgeir Engeland in C++ language, and recently improved by Morten Hjorth-Jensen and co-workers

It is easy to be provided, fast, versatile

It can be downloaded for Linux machines at the address : http://folk.uio.no/mhjensen/cp/software.html

# The Oslo shell-model code

This code has been written by Torgeir Engeland in C++ language, and recently improved by Morten Hjorth-Jensen and co-workers

It is easy to be provided, fast, versatile

It can be downloaded for Linux machines at the address :
http://folk.uio.no/mhjensen/cp/software.html

# The Oslo shell-model code

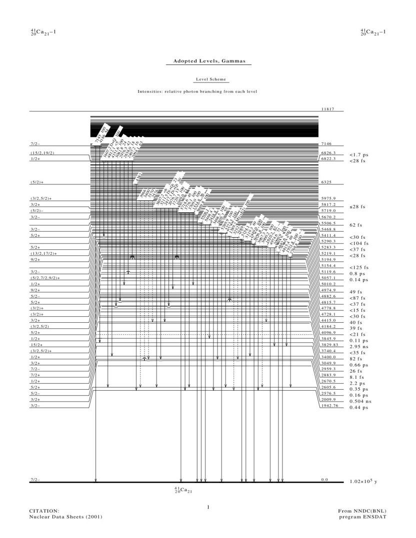
The two-body matrix elements have to be given in proton-neutron formalism, antisymmetrized and normalized

Let us now consider a simple case: nuclei beyond $^{40}$Ca doubly-closed core

# The Oslo shell-model code

The two-body matrix elements have to be given in proton-neutron formalism, antisymmetrized and normalized

Let us now consider a simple case: nuclei beyond $^{40}$Ca doubly-closed core


Adopted Levels, Gammas
Level Scheme
Intensities: relative photon branching from each level
CITATION:
Nuclear Data Sheets (2001)
From NNDC(BNL)
program ENSDAT

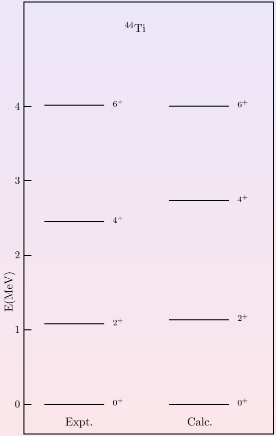
# An application

In such a case, the single-particle energies are given by the proton and neutron ground-state energies of $^{41}$Sc and $^{41}$Ca respect to $^{40}$Ca

- Proton $\epsilon_{7/2} = -1.1$ MeV
- Neutron $\epsilon_{7/2} = -8.4$ MeV

The two-body matrix elements of $V^{res}$ can be fitted on the experimental spectra of $^{42}$Ti, $^{42}$Ca, $^{42}$Sc

| $n_a l_a j_a$ $n_b l_b j_b$ $n_c l_c j_c$ $n_d l_d j_d$ | $J^\pi$ | $T_z$ | TBME |
|---|---|---|---|
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $0^+$ | 1 | -2.50 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $2^+$ | 1 | -0.95 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $4^+$ | 1 | 0.25 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $6^+$ | 1 | 0.70 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $0^+$ | -1 | -3.00 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $2^+$ | -1 | -1.45 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $4^+$ | -1 | -0.25 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $6^+$ | -1 | 0.20 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $0^+$ | 0 | -3.00 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $1^+$ | 0 | -2.40 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $2^+$ | 0 | -1.45 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $3^+$ | 0 | -1.50 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $4^+$ | 0 | -0.25 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $5^+$ | 0 | -1.50 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $6^+$ | 0 | 0.20 |
| $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$ | $7^+$ | 0 | -2.34 |

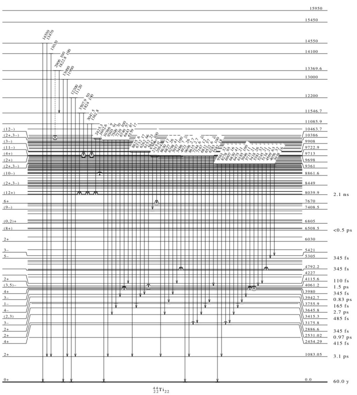


$^{44}$Ti
E(MeV)
0
1
2
3
4
6$^+$
4$^+$
2$^+$
0$^+$
6$^+$
4$^+$
2$^+$
0$^+$
Expt.
Calc.

Adopted Levels, Gammas

Level Scheme

Intensities: relative photon branching from each level



CITATION:
Nuclear Data Sheets (1999)

1

From NNDC(BNL)
program ENSDAT

# The end

Instead of a conclusion, I would be happy that this could be a starting point for all of you

Explore the possibilities of the shell model and of the Oslo code, try to do some calculations

Thank you for your attention and patience

# The end

Instead of a conclusion, I would be happy that this could be a starting point for all of you

Explore the possibilities of the shell model and of the Oslo code, try to do some calculations

Thank you for your attention and patience

# The end

Instead of a conclusion, I would be happy that this could be a starting point for all of you

Explore the possibilities of the shell model and of the Oslo code, try to do some calculations

Thank you for your attention and patience