Nuclear Astrophysics

I. Hydrostatic stellar burning

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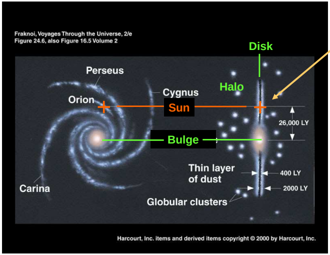
Otranto, may 30-june 3, 2011

What is nuclear astrophysics?

Nuclear astrophysics aims at understanding the nuclear processes that take place in the universe. These nuclear processes generate energy in stars and contribute to the nucleosynthesis of the elements.

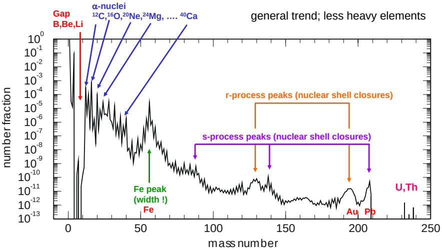
3. The solar abundance distribution



solar abundances:

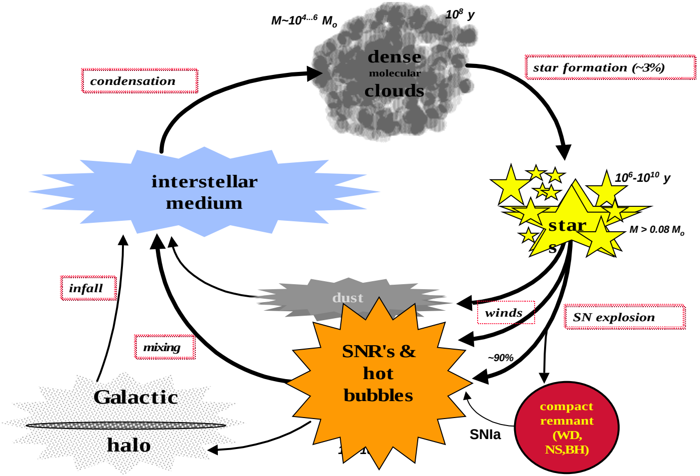
Elemental (and isotopic) composition of Galaxy at location of solar system at the time of it's formation

Hydrogen mass fraction	X = 0.71
Helium mass fraction	Y = 0.28
Metallicity (mass fraction of everything else)	Z = 0.019
Heavy Elements (beyond Nickel) mass fraction	4E-6



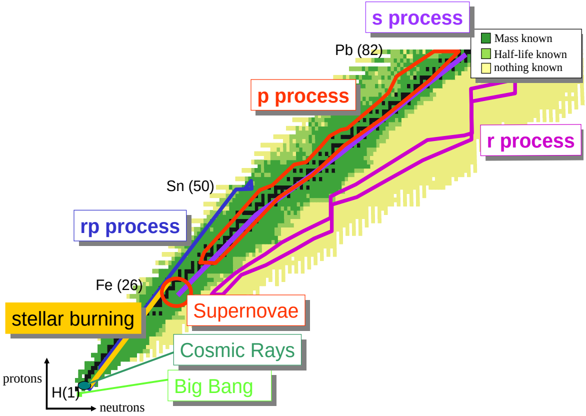
N. Grevesse and A. J. Sauval, Space Science Reviews **85**, 161

Hoyle's cosmic cycle

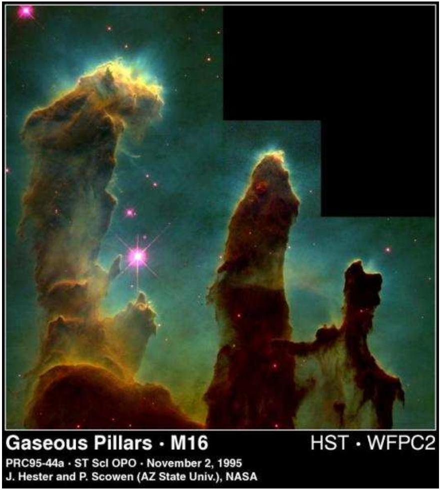


Nucleosynthesis processes

In 1957: Burbidge, Burbidge, Fowler, Hoyle, [Rev. Mod. Phys. **29**, 547 (1957)] suggested the synthesis of the elements in stars.



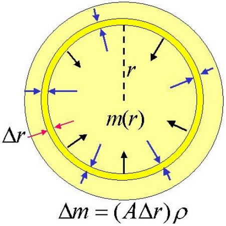
Star formation



- Stars are formed from the contraction of molecular clouds due to their own gravity.
- Contraction increases temperature and eventually nuclear fusion reactions begin. A star is born.
- Contraction time depends on mass: 10 millions years for a star with the mass of the Sun; 100,000 years for a star 11 times the mass of the Sun.

The evolution of a Star is governed by gravity

What is a Star?



in equilibrium: gravity $\leftrightarrow$ pressure

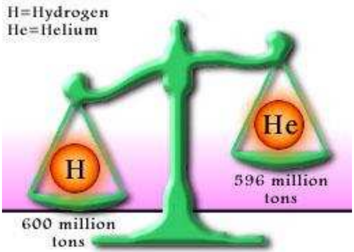
- A star is a self-luminous gaseous sphere.
- Stars produce energy by nuclear fusion reactions. A star is a self-regulated nuclear reactor.
- Gravitational collapse is balanced by pressure generated from nuclear reactions:
  $dF_{grav} = -G\frac{m(r)dm}{r^2} = dF_{press} = [(P(r+dr) - P(r))dA$
- Further, equation needed to describe the pressure as function of density, composition (nuclear reactions), temperature (heat transport) $\rightarrow$ Equation of State (EOS)
- Star evolution, lifetime and death depends on mass. Two groups:
  - Stars with masses less than 8 solar masses (white dwarfs)
  - Stars with masses greater than 8 solar masses (supernova explosions)

Where does the energy come from?

Energy comes from nuclear reactions in the core.

$$4\,{}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + \mathrm{neutrinos} + 26.7\mathrm{MeV}$$

$$E = mc^2$$



The Sun converts 600 million tons of hydrogen into 596 million tons of helium every second. The difference in mass is converted into energy. The Sun will continue burning hydrogen during 5 billions years.
Energy relieased by H-burning:
6.45×10^{18} erg g^{-1}
Solar Luminosity: 3.846×10^{33} erg s^{-1}

Types of processes

Transfer (strong interaction)

$$^{15}\mathrm{N}(p,\alpha)^{12}\mathrm{C}, \qquad \sigma \simeq 0.5 \text{ b at } E = 2.0 \text{ MeV}$$

Capture (electromagnetic interaction)

$$^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}, \qquad \sigma \simeq 10^{-6} \text{ b at } E = 2.0 \text{ MeV}$$

Weak (weak interaction)

$$p(p,e^{+}\nu)d, \qquad \sigma \simeq 10^{-20} \text{ b at } E = 2.0 \text{ MeV}$$

Stellar reaction rate

Consider N_a and N_b particles per cubic centimeter of particle types a and b. The rate of nuclear reactions is given by:

$$r = N_a N_b \sigma(v) v$$

In stellar environment the velocity (energy) of particles follows a thermal distribution that depends on the type of particles.

- Nuclei (Maxwell-Boltzmann): $\phi(v) = N 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right)$

The product σv has to be averaged over the velocity distribution $\phi(v)$

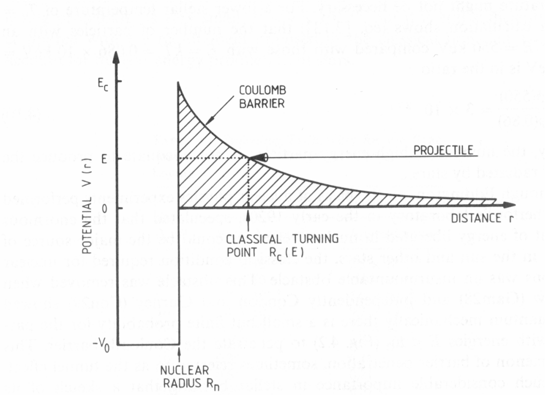
$$\langle \sigma v \rangle = \int_0^\infty \int_0^\infty \phi(v_a)\phi(v_b)\sigma(v) v dv_a dv_b$$

Changing to center-of-mass coordinates, integrating over the cm-velocity and using $E = \mu v^2/2$

$$\langle \sigma v \rangle = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{kT}\right) dE$$

Charged-particle cross section

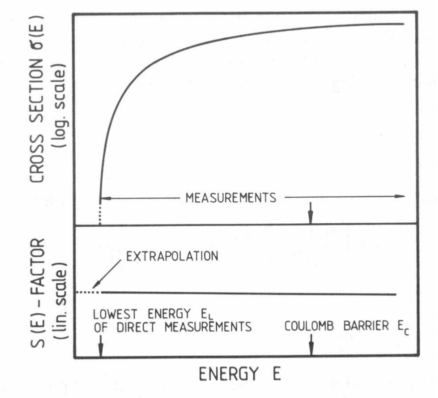
Stars' interior is a plasma made of charged particles (nuclei, electron). Nuclear reactions proceed by tunnel effect. For $p+p$ reaction Coulomb barrier 550 keV, but the typical energy in the sun is only 1.35 keV.

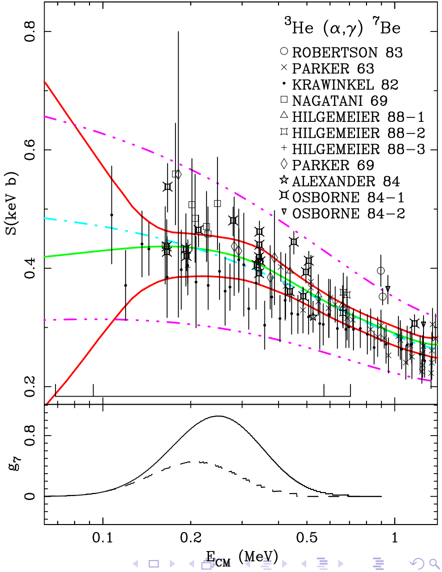


cross section: $\sigma(E) = \frac{1}{E} S(E) e^{-2\pi\eta}$; $\quad \eta = \frac{Z_1 Z_2 e^2}{\hbar}\sqrt{\frac{\mu}{2E}} = \frac{b}{E^{1/2}}$

Astrophysical S factor

S factor allows accurate extrapolations to low energy.

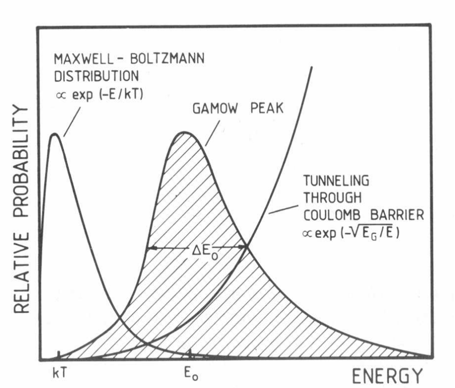




Gamow window

Using definition of S factor:

$$\langle \sigma v \rangle = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp\left[-\frac{E}{kT} - \frac{b}{E^{1/2}}\right] dE$$



Gamow window

Assuming that S factor is constant over the Gamow window and approximating the integrand by a Gaussian one gets:

$$\langle \sigma v \rangle = \left(\frac{2}{\mu}\right)^{1/2} \frac{\Delta}{(kT)^{3/2}} S(E_0) \exp\left(-\frac{3E_0}{kT}\right)$$

$$\begin{aligned} E_0 &= 1.22[\mathrm{keV}](Z_1^2 Z_2^2 \mu T_6^2)^{1/3} \\ \Delta &= 0.749[\mathrm{keV}](Z_1^2 Z_2^2 \mu T_6^5)^{1/6} \end{aligned}$$

(T_x measures the temperature in 10^x K.)
Examples for solar conditions:

reaction	E_0 [keV]	$\Delta/2$ [keV]	$I_{\rm max}$	T dependence of $\langle \sigma v \rangle$
p+p	5.9	3.2	1.1×10^{-6}	$T^{3.9}$
p+^{14}N	26.5	6.8	1.8×10^{-27}	T^{20}
α+^{12}C	56.0	9.8	3.0×10^{-57}	T^{42}
^{16}O+^{16}O	237.0	20.2	6.2×10^{-239}	T^{182}

It depends very sensitively on temperature!

The ppI chain

Step 1: $p + p \rightarrow {}^{2}\mathrm{He}$ (not possible)

$p + p \rightarrow d + e^{+} + \nu_e$

Step 2: $d + p \rightarrow {}^{3}\mathrm{He}$

$d + d \rightarrow {}^{4}\mathrm{He}$ (d abundance too low)

Step 3: ${}^{3}He + p \rightarrow {}^{4}\mathrm{Li}$ (${}^{4}Li$ is unbound)

${}^{3}He + d \rightarrow {}^{4}\ He + n$ (d abundance too low)

${}^{3}He + {}^{3}\ He \rightarrow {}^{4}\ He + 2p$

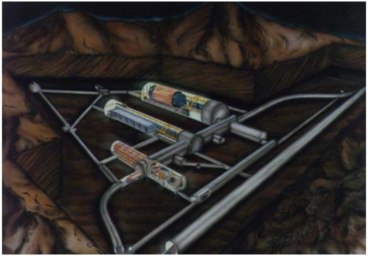
$d + d$ is not going, because Y_d is extremely small and $d + p$ leads to rapid destruction.
${}^{3}He + {}^{3}\ He$ works, because Y_{3He} increases as nothing destroys it.

The relevant S-factors

$p(p, e^+\nu_e)d$: $S_{11}(0) = (4.00 \pm 0.05) \times 10^{-25}$ MeVb
calculated

$p(d, \gamma)^3$He: $S_{12}(0) = 2.5 \times 10^{-7}$ MeVb
measured at LUNA

^{3}He(^{3}He,2p)^{4}He: $S_{33}(0) = 5.4$ MeVb
measured at LUNA



Laboratory Underground for Nuclear Astrophysics (Gran Sasso)

Burning of Deuterium

Deuterons are burnt by the reaction $d(p,\gamma)^3$He:

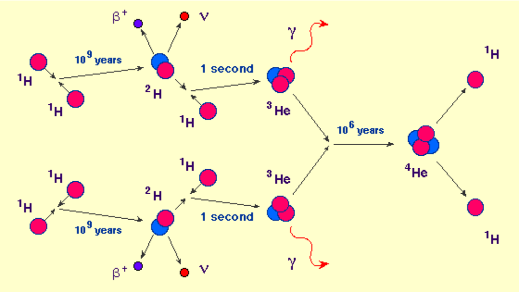
$$\frac{dD}{dt} = r_{11} - r_{12}$$
$$= \frac{H^2}{2}\langle\sigma v\rangle_{11} - HD\langle\sigma v\rangle_{12}$$

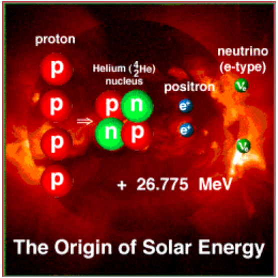
In equilibrium ($\frac{dD}{dt} = 0$) one has

$$\left(\frac{D}{H}\right)_e = \frac{\langle\sigma v\rangle_{11}}{2\langle\sigma v\rangle_{12}}$$

$$(D/H)_e = 5.6 \times 10^{-18} \text{ for } \mathrm{T}_6 = 5$$

The ppI chain





^{4}He as catalyst

^{4}He can act as catalyst initializing the ppII and ppIII chains.
With which nucleus will ^{4}He fuse?

- protons:
  the fusion of ^{4}He and protons lead to ^{5}Li which is unbound.
- deuterons:
  the fusion of deuterons with ^{4}He can make stable ^{6}Li; however, the deuteron abundance is too low for this reaction to be significant
- ^{3}He:
  ^{3}He and ^{4}He can fuse to ^{7}Be. This is indeed the break-out reaction from the ppI chain.

Once ^{7}Be is produced, it can either decay by electron capture or fuse with a proton. Thus, the reaction sequence branches at ^{7}Be into the ppII and ppIII chains.

The solar pp chains

$$ {}^1_1\mathrm{H} + {}^1_1\mathrm{H} \longrightarrow {}^2_1\mathrm{H} + e^+ + \nu_e $$

$$ {}^2_1\mathrm{H} + {}^1_1\mathrm{H} \longrightarrow {}^3_2\mathrm{He} + \gamma $$

85% | 15%

$$ {}^3_2\mathrm{He} + {}^3_2\mathrm{He} \longrightarrow {}^4_2\mathrm{He} + 2\,{}^1_1\mathrm{H} $$

(PP I)

$$ {}^3_2\mathrm{He} + {}^4_2\mathrm{He} \longrightarrow {}^7_4\mathrm{Be} + \gamma $$

15% | 0.02%

$$ {}^7_4\mathrm{Be} + e^- \longrightarrow {}^7_3\mathrm{Li} + \nu_e $$

$$ {}^7_3\mathrm{Li} + {}^1_1\mathrm{H} \longrightarrow 2\,{}^4_2\mathrm{He} $$

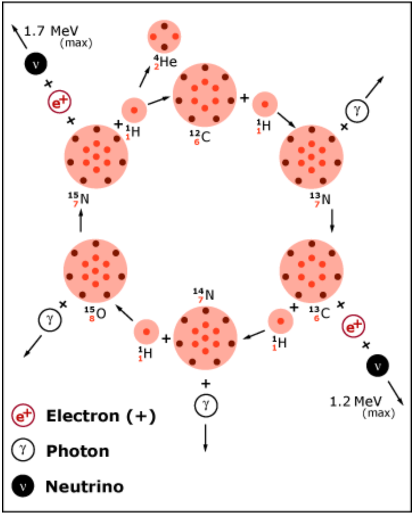
(PP II)

$$ {}^7_4\mathrm{Be} + {}^1_1\mathrm{H} \longrightarrow {}^8_5\mathrm{B} + \gamma $$

$$ {}^8_5\mathrm{B} \longrightarrow {}^8_4\mathrm{Be} + e^+ + \nu_e $$

$$ {}^8_4\mathrm{Be} \longrightarrow 2\,{}^4_2\mathrm{He} $$

(PP III)



requires presence of ^{12}C as catalyst

Hydrogen burning: pp-chains vs CNO cycle

Slowest reaction determines efficiency (energy production) of chain:

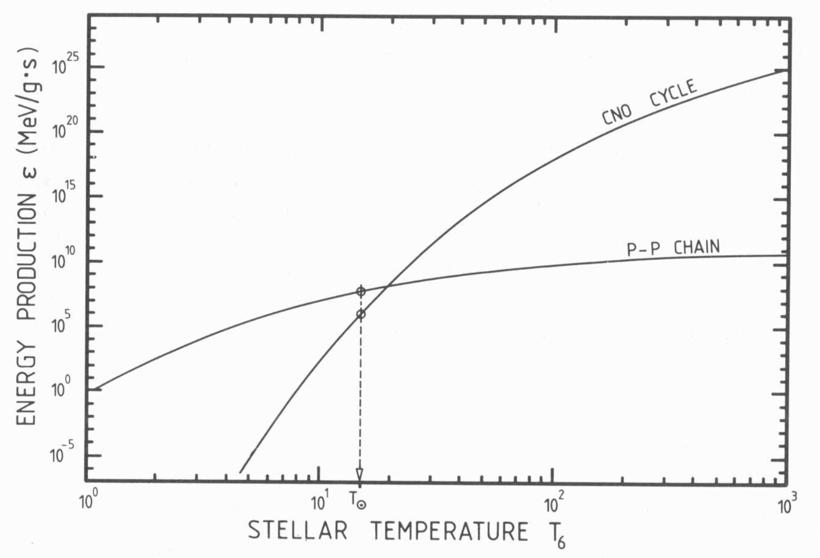
pp-chains:
p+p fusion, mediated by weak interaction

CNO cycle:
^{14}N+p, largest Coulomb barrier, mediated by electromagnetic interaction (in contrast to strong interaction in ^{15}N+p)

Temperature dependence quite different:
$\langle \sigma v \rangle \sim T^{(\tau-2)/3}$
with $\tau = \frac{3E_0}{kT}$; $E_0 = 1.22[keV](Z_1^2 Z_2^2 \mu T_6^2)^{1/3}$

At $T_6 = 15$ (solar core): $\langle \sigma v \rangle \sim T^{3.9}$ (pp); $\langle \sigma v \rangle \sim T^{20}$ (CNO)

Energy generation: CNO cycle vs pp-chains



Consequences

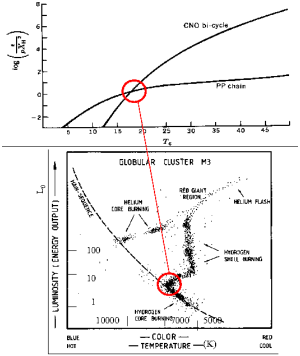
- stars slightly heavier than the Sun burn hydrogen via CNO cycle
- this goes significantly faster; such stars have much shorter lifetimes

mass [$M_\odot$]	timescale [y]
0.4	2×10^{11}
0.8	1.4×10^{10}
1.0	1×10^{10}
1.1	9×10^{9}
1.7	2.7×10^{9}
3.0	2.2×10^{8}
5.0	6×10^{7}
9.0	2×10^{7}
16.0	1×10^{7}
25.0	7×10^{6}
40.0	1×10^{6}

hydrogen burning timescales depend strongly on mass. Stars slightly heavier than the Sun burn hydrogen by CNO cycle.

Future Sun will burn hydrogen by CNO cycle

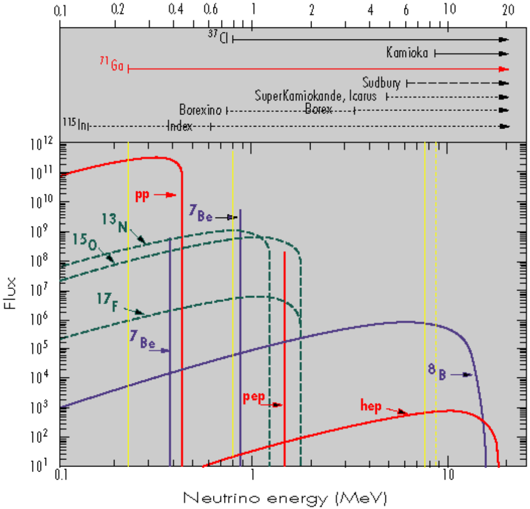
- by continuous hydrogen burning, the Sun reduces its hydrogen reservoir in the core
- at some point in the future energy production by the pp-chains will not suffice to balance the solar energy household
- to gain sufficient energy the solar core will gravitationally contract and thereby increase the temperature
- hydrogen core burning in the Sun then switches from pp-chains to CNO cycle



when a star changes from core pp-burning to CNO cycle, its evolutionary track leaves main sequence in HR-diagram

Solar neutrino fluxes and detector thresholds

Solar hydrogen burning produces neutrinos (Bahcall)

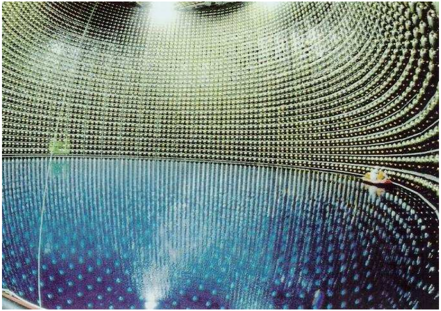


Depending on material, the detectors are blind for neutrinos with energies smaller than a threshold.

Neutrino astronomy

- In the 1950s, Ray Davis (2002 Nobel Prize Laureate) decided to measure the solar neutrinos. (Every second, 10 billion neutrinos pass through every square cm on Earth).
- In 1967, the detector (615 tons of C_2Cl_4) was installed at Homestake Gold Mine, South Dakota (1,500 m depth).
- In 1968, the first measurement was a factor 3 smaller than predictions. Similar results by other experiments.

Super-Kamiokande, Japan (50,000 tons pure water)

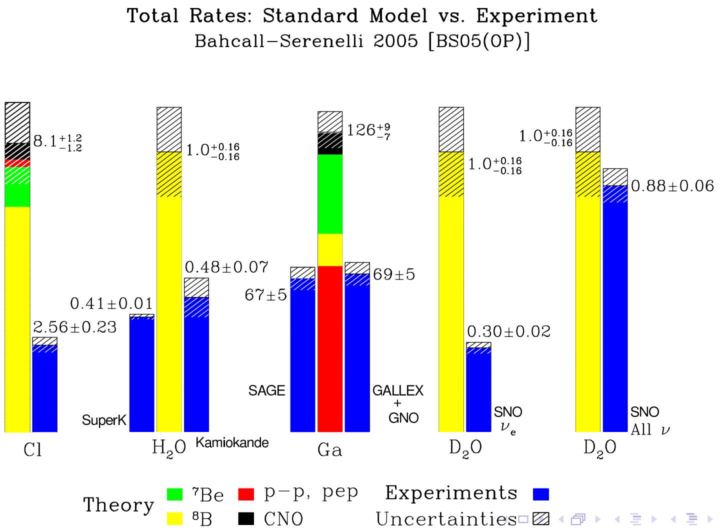


Sudbury Neutrino Observatory, Canada (1,000 tons heavy water)

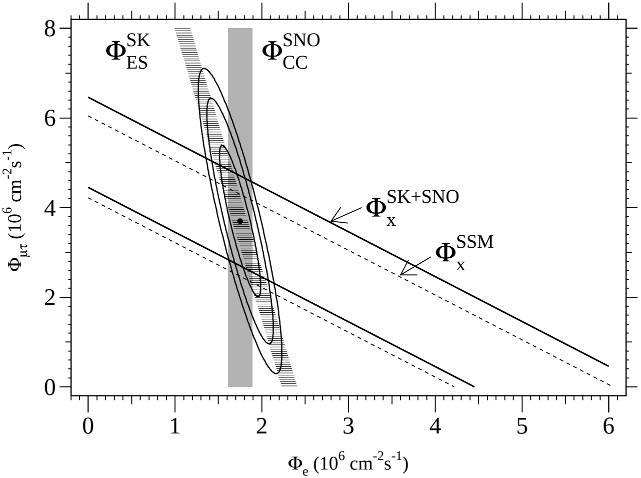
Detecting solar neutrinos

- Homestake:
  - first observation of solar neutrinos
  - detection $\nu_e + {}^{37}\text{Cl} \rightarrow e^- + {}^{37}\text{Ar}$
  - blind for $E_\nu < 814$ keV
- Kamiokande, Super-Kamiokande:
  - proof that neutrinos are from Sun
  - detection $\nu_e + e^- \rightarrow \nu_e + e^{-'}$ (Cerenkov)
  - blind for $E_\nu < 5000$ keV
- GALLEX:
  - observation of pp neutrinos, in agreement with luminosity
  - detection $\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge}$
  - blind for $E_\nu < 233$ keV
- Sudbury SNO:
  - proof of solar neutrino oscillations
  - detection $\nu_e + \text{d} \rightarrow \text{p} + \text{p} + e^-$ (charged current)
  - detection $\nu_x + \text{d} \rightarrow \text{p} + \text{n} + \nu_x$ (neutral current)
  - neutral current reaction detects all neutrino flavors
  - blind for $E_\nu < 2224$ keV

Observed solar neutrino deficit



The SNO proof of neutrino oscillations



Observed TOTAL neutrino flux agrees with solar model predictions!

End of hydrogen core burning

When the hydrogen fuel in the core gets exhausted, an isothermal core of about 8% of the stellar mass can develop in the center. Continous hydrogen burning adds to the core mass which eventually rises over the Schönberg-Chandrasekhar mass limit. Then the core's temperature (and density) rise. Finally the central temperature is high enough ($T_c \approx 10^8$ K) to ignite helium core burning.

Hydrogen burning continues in a shell outside the helium core. This (hydrogen shell burning) occurs at higher temperatures than hydrogen core burning.

Which reaction can start helium burning?

Consider a supply of protons and ^{4}He.

We first note again that ^{5}Li is unbound. Although this nucleus is continuously formed by p+^{4}He reactions, the scattering is elastic and the formed ^{5}Li nuclei decay within 10^{-22} s.

As a consequence ^{4}He 'survives' in the core until sufficiently large temperatures are achieved to overcome the larger Coulomb barrier between ^{4}He nuclei. Unfortunately the ^{8}Be ground state, formed by elastic ^{4}He+^{4}He scattering, is a resonance too and decays within 10^{-16} s back to two ^{4}He nuclei.

The Salpeter-Hoyle suggestion

In 1952 Salpeter pointed out that the ^{8}Be lifetime might be sufficiently large that there is a chance that it captures another ^{4}He nucleus. This *triple-alpha reaction*

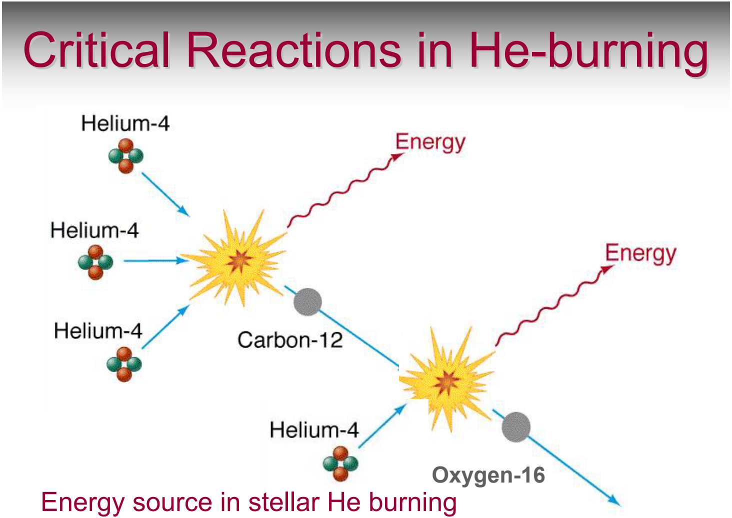
$$3\ ^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma$$

can then form ^{12}C and supply energy. However, the simultaneous collision of 3 ^{4}He (α-particles) is too rare to give the burning rate necessary in stellar models. So Hoyle predicted a resonance in ^{12}C to speed up the collision. And indeed, this *Hoyle state* was experimentally observed shortly after its prediction.
^{12}C can then react with another ^{4}He nucleus forming ^{16}O via

$$^{12}\mathrm{C} + \alpha \rightarrow {}^{16}\mathrm{O} + \gamma$$

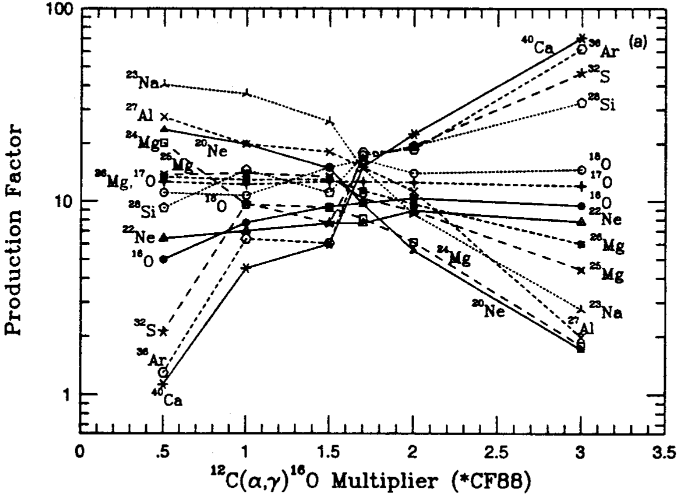
These two reactions make up helium burning.

Helium burning reactions



Energy source in stellar He burning
Energy release determined by associated reaction rates

Influence of alpha+^{12}C on nucleosynthesis



At the end of helium burning

Nucleosynthesis yields from stars may be divided into production by stars above or below $9M_\odot$.

- **stars with $M \lesssim 9M_\odot$**
  the stars are expected to shed their envelopes during helium burning and become white dwarfs. Most of the matter returned to the ISM is unprocessed.
- **stars with $M > 9M_\odot$**
  these stars will ignite carbon burning under non-degenerate conditions. The subsequent evolution proceeds in most cases to core collapse. These stars make the bulk of newly processed matter that is returned to the ISM.

Carbon Burning

Burning conditions:

for stars > 8 M_o (solar masses) (ZAMS)

T~ 600-700 Mio

ρ ~ 10^5-10^6 g/cm^3

Major reaction sequences:

$$^{12}\mathrm{C}+{}^{12}\mathrm{C}\rightarrow {}^{24}\mathrm{Mg}^*\rightarrow {}^{23}\mathrm{Mg}+n-2.62\ \mathrm{MeV}$$
$$\rightarrow {}^{20}\mathrm{Ne}+\alpha+4.62\ \mathrm{MeV}$$
$$\rightarrow {}^{23}\mathrm{Na}+p+2.24\ \mathrm{MeV}.$$

dominates by far

of course p's, n's, and a's are recaptured … ^{23}Mg can b-decay into ^{23}Na

Composition at the end of burning:

mainly ^{20}Ne, ^{24}Mg, with some ^{21,22}Ne, ^{23}Na, ^{24,25,26}Mg, ^{26,27}Al

of course ^{16}O is still present in quantities comparable with ^{20}Ne (not burning … yet) 21

Neon Burning

Neon burning is very similar to carbon burning.

Burning conditions:

for stars > 12 M_o (solar masses) (ZAMS)

T~ 1.3-1.7 Bio K
$\rho \sim 10^6$ g/cm^3

Why would neon burn before oxygen ???

Answer:

Temperatures are sufficiently high to initiate **photodisintegration** of ^{20}Ne

$$\left.\begin{array}{l} ^{20}\mathrm{Ne}+\gamma \rightarrow {}^{16}\mathrm{O} + \alpha \\ ^{16}\mathrm{O}+\alpha \rightarrow {}^{20}\mathrm{Ne} + \gamma \end{array}\right\}$$ equilibrium is established

this is followed by (using the liberated helium)

$$^{20}\mathrm{Ne}+\alpha \rightarrow {}^{24}\mathrm{Mg} + \gamma$$

so net effect: $$2\ {}^{20}\mathrm{Ne} \rightarrow {}^{16}\mathrm{O} + {}^{24}\mathrm{Mg} + 4.59\ \mathrm{MeV}.$$

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Oxygen Burning

Burning conditions:

T~ 2 Bio
$\rho \sim 10^7$ g/cm^3

Major reaction sequences:

$$
\begin{aligned}
{}^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{32}\mathrm{S}^* &\rightarrow {}^{31}\mathrm{S} + n + 1.45\ \mathrm{MeV} && (5\%) \\
&\rightarrow {}^{31}\mathrm{P} + p + 7.68\ \mathrm{MeV} && (56\%) \\
&\rightarrow {}^{30}\mathrm{P} + d - 2.41\ \mathrm{MeV} && (5\%) \\
&\rightarrow {}^{28}\mathrm{Si} + \alpha + 9.59\ \mathrm{MeV.} && (34\%)
\end{aligned}
$$

plus recapture of n,p,d,α

Main products:

^{28}Si,^{32}S (90%) and some ^{33,34}S,^{35,37}Cl,$^{36.38}$Ar, ^{39,41}K, ^{40,42}Ca

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Silicon Burning

Silicon burning is very similar to oxygen burning.

Burning conditions:

T~ 3-4 Bio
$\rho \sim 10^9$ g/cm^3

Reaction sequences:

- Silicon burning is fundamentally different to all other burning stages.
- **Complex network of fast (γ,n), (γ,p), (γ,a), (n,γ), (p,γ), and (a,γ) reactions**
- The net effect of Si burning is: 2 ^{28}Si --> ^{56}Ni,

need new concept to describe burning:

Nuclear Statistical Equilibrium (NSE)

Quasi Statistical Equilibrium (QSE)

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Nuclear burning stages

(e.g., 20 solar mass star)

Fuel	Main Product	Secondary Product	T (10^9 K)	Time (yr)	Main Reaction
H	He	^{14}N	0.02	10^7	$4\,H \xrightarrow{CNO} {}^4He$
He	O, C	^{18}O, ^{22}Ne s-process	0.2	10^6	$3\,He^4 \rightarrow {}^{12}C$ $^{12}C(\alpha,\gamma)^{16}O$
C	Ne, Mg	Na	0.8	10^3	$^{12}C + {}^{12}C$
Ne	O, Mg	Al, P	1.5	3	$^{20}Ne(\gamma,\alpha)^{16}O$ $^{20}Ne(\alpha,\gamma)^{24}Mg$
O	Si, S	Cl, Ar, K, Ca	2.0	0.8	$^{16}O + {}^{16}O$
Si	Fe	Ti, V, Cr, Mn, Co, Ni	3.5	0.02	$^{28}Si(\gamma,\alpha)\ldots$

Kippenhahn diagram for a 22 $M_\odot$ star

(A. Heger and S. Woosley)