

**CHIRAL INSPIRED THREE-NUCLEON  
INTERACTION IN NUCLEAR MATTER  
(Preliminary results)**

**Alessandro Lovato**

# OUTLINE

- Nuclear Hamiltonian
- UIX three-body potential
- Chiral NN potential
- Chiral NNN potential and its local form
- Comparative study of local three-body potential in nuclear matter
- Conclusions

# NUCLEAR HAMILTONIAN & N-N POTENTIAL

The non relativistic Hamiltonian describing nuclear matter is

$$H = \sum_{i=1}^A \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i=1}^A v_{ij} + \dots$$

Realistic nucleon-nucleon (NN) potentials are, for example:

- **Argonne  $v_{18}, v_8'$**

- Mainly phenomenological
- Local in coordinates

- **CD-BONN**

- Based on meson-exchange
- Nonlocal

- **Chiral N<sup>3</sup>LO**

- Based on **Chiral Lagrangians**
- Nonlocal

The parameters of these potentials have been obtained by fitting the ~ 4300 data below 350 MeV in the Nijmegen NN scattering database.



**They reproduce the experimental NN scattering data up to energies of 350 MeV with a  $\chi^2$  per datum close to 1.**

No many-body methods are needed for the fit: these potentials have high predictive power and can be used in “ab initio” calculations.

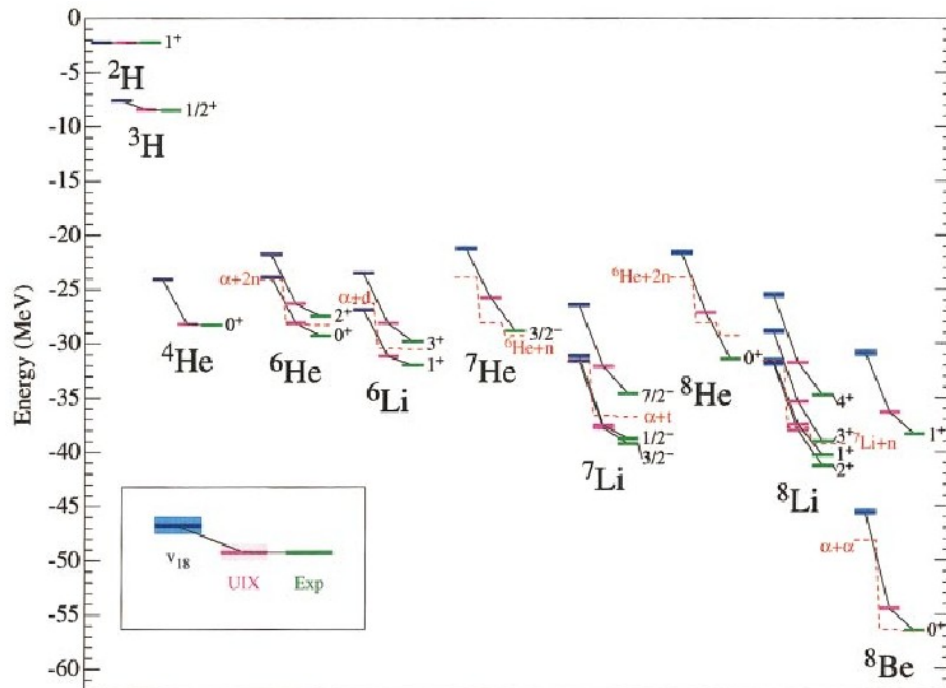
# N-N POTENTIAL IS NOT ENOUGH

When two body potential only is considered:

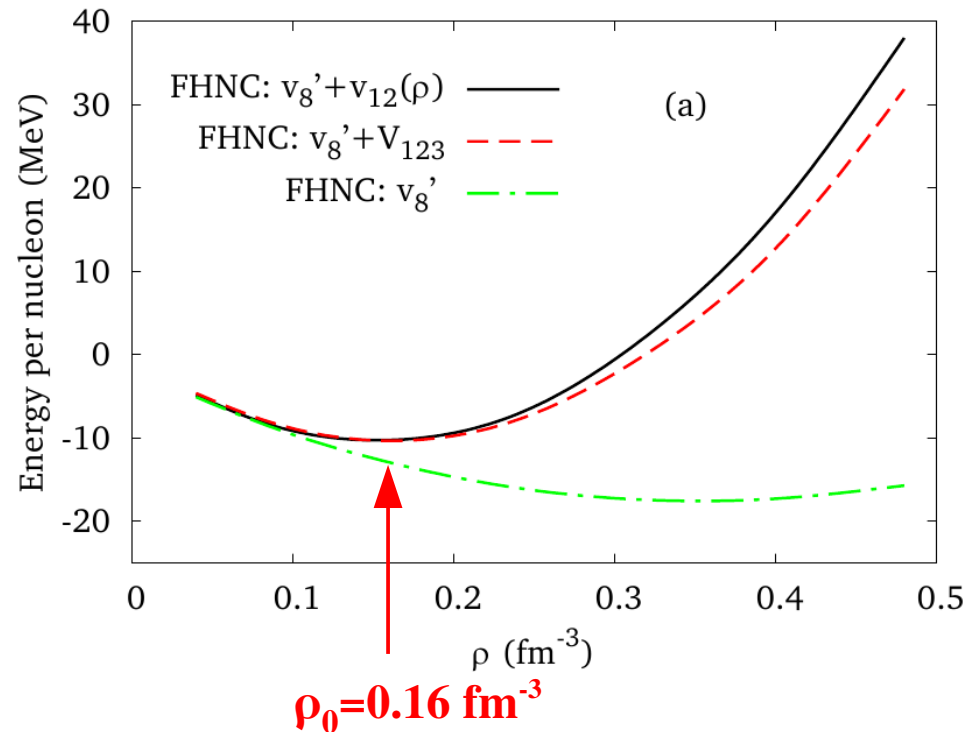
- The description of three- and four- nucleon bound and scattering states gives a  $\chi^2$  per datum much larger than 1.
- The equilibrium density  $\rho_0$  of Symmetric Nuclear Matter (SNM) is overestimated.

**Three-nucleon forces are needed !!!**

Light nuclei



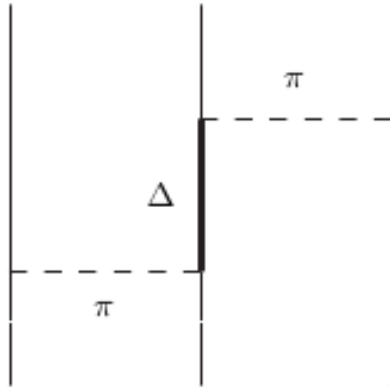
SNM



# UIX NNN POTENTIAL

One of the mostly used three nucleon potential is UIX. It consists of two contributions

- **Fujita Myiazawa  $V^{2\pi}$** : two pions are exchanged among nucleons and a  $\Delta$  resonance is excited in the intermediate state.



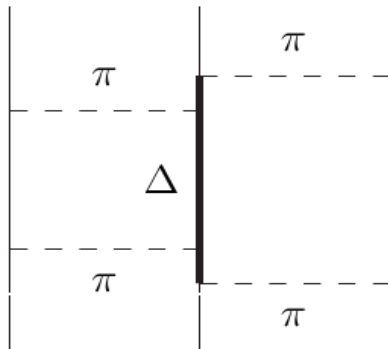
$$V^{2\pi} = A^{2\pi} (O_{123}^{2\pi} + O_{231}^{2\pi} + O_{312}^{2\pi}) \Rightarrow \text{Cyclic sum}$$

$$O_{123}^{2\pi} = \left( \{ \hat{X}_{12}, \hat{X}_{23} \} \{ \tau_{12}, \tau_{13} \} + \frac{1}{4} [ \hat{X}_{12}, \hat{X}_{23} ] [ \tau_{12}, \tau_{23} ] \right)$$

$$\hat{X}_{ij} = Y(m_\pi r) \sigma_{ij} + T(m_\pi r) S_{ij}$$

It solves the underbinding of light nuclei, but makes nuclear matter even more overbound.

- **Phenomenological scalar repulsive term  $V^R$** : introduced by Lagaris and Pandharipande in order FHNC/SOC calculation to reproduce the correct binding energy of SNM.



$$V^R = U_0 \sum_{cycl} T^2(m_\pi r_{12}) T^2(m_\pi r_{23})$$

# UIX NNN POTENTIAL

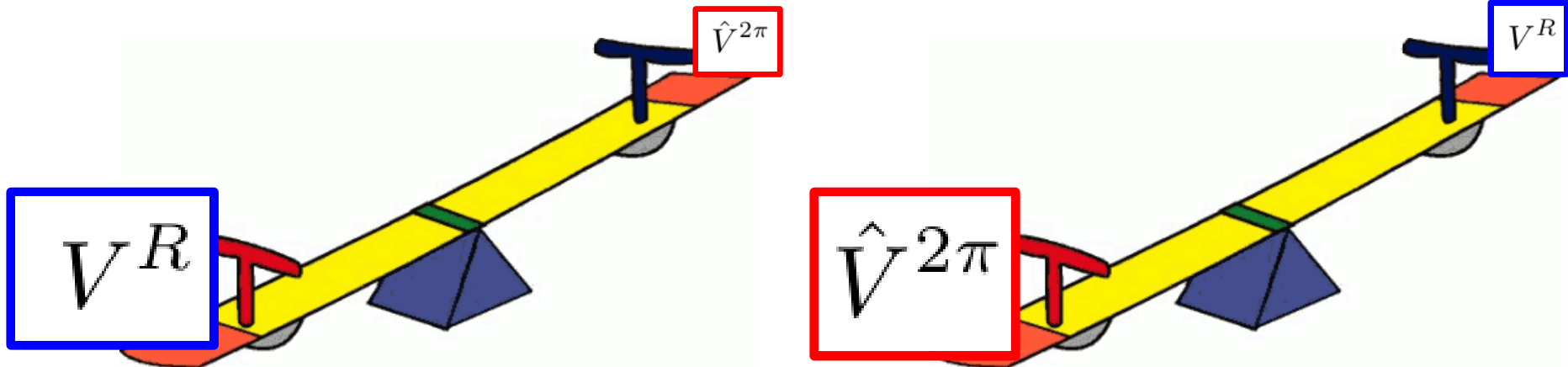
UIX potential has two free parameters to be fixed on experimental data

- $A^{2\pi}$  is chosen to reproduce the observed binding energies of  ${}^3\text{H}$  and of  ${}^4\text{He}$ .
- $U_0$  is adjusted in order for FHNC/SOC calculations to reproduce the empirical equilibrium density of SNM  $\rho_0=0.16 \text{ fm}^{-3}$ .

Lagaris and Pandharipande argued that, because of correlations, the relative weight of the contribution depends upon the density of the system:

High density

Low density



# UIX NNN POTENTIAL

UIX potential cannot be considered the final answer for what concerns the three-nucleon interaction issue.

## Theoretical problems



Both FM and scalar repulsive terms are mainly phenomenological: there are no a priori reasons to stop at the first order in the perturbative expansion in the coupling constant  $g_0 \sim 10$ .



Adjusting  $U_0$  to reproduce the correct value of  $\rho_0$ , that is calculated within the FHNC/SOC framework, makes the potential affected by the uncertainties of the many-body technique. Pure “ab initio” calculations are not possible anymore.

## Phenomenological problems



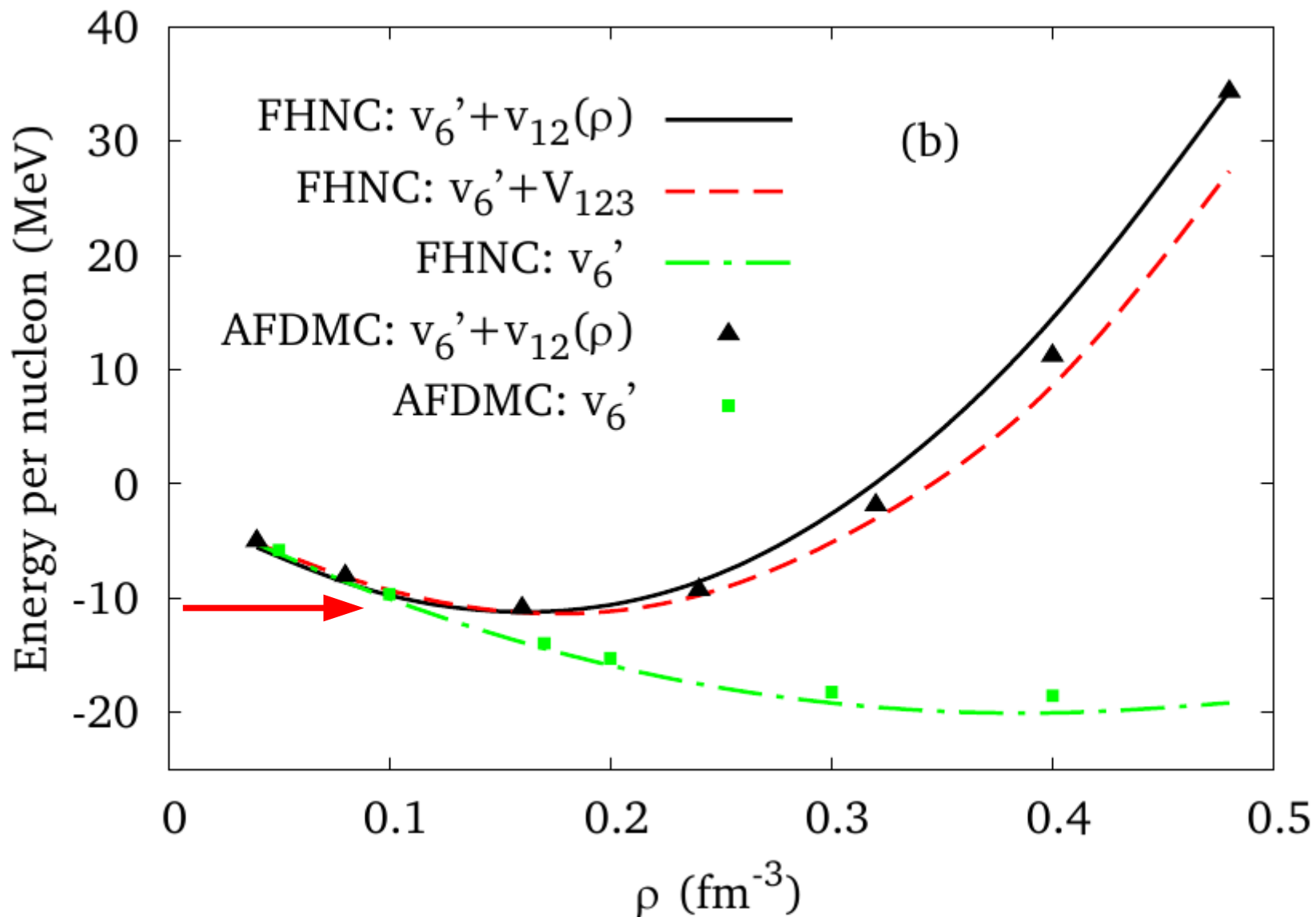
The value of the n-d scattering length obtained with Argonne  $v_{18} + UIX$  is not correctly reproduced

	$v_{18} + UIX$	Exp.
${}^2a_{nd}$ (fm)	0.578	$0.645 \pm 0.003 \pm 0.007$

# UIX NNN POTENTIAL



The binding energy of SNM at  $\rho=\rho_0$  is  $E(\rho_0)=-11$  MeV instead of -16 MeV

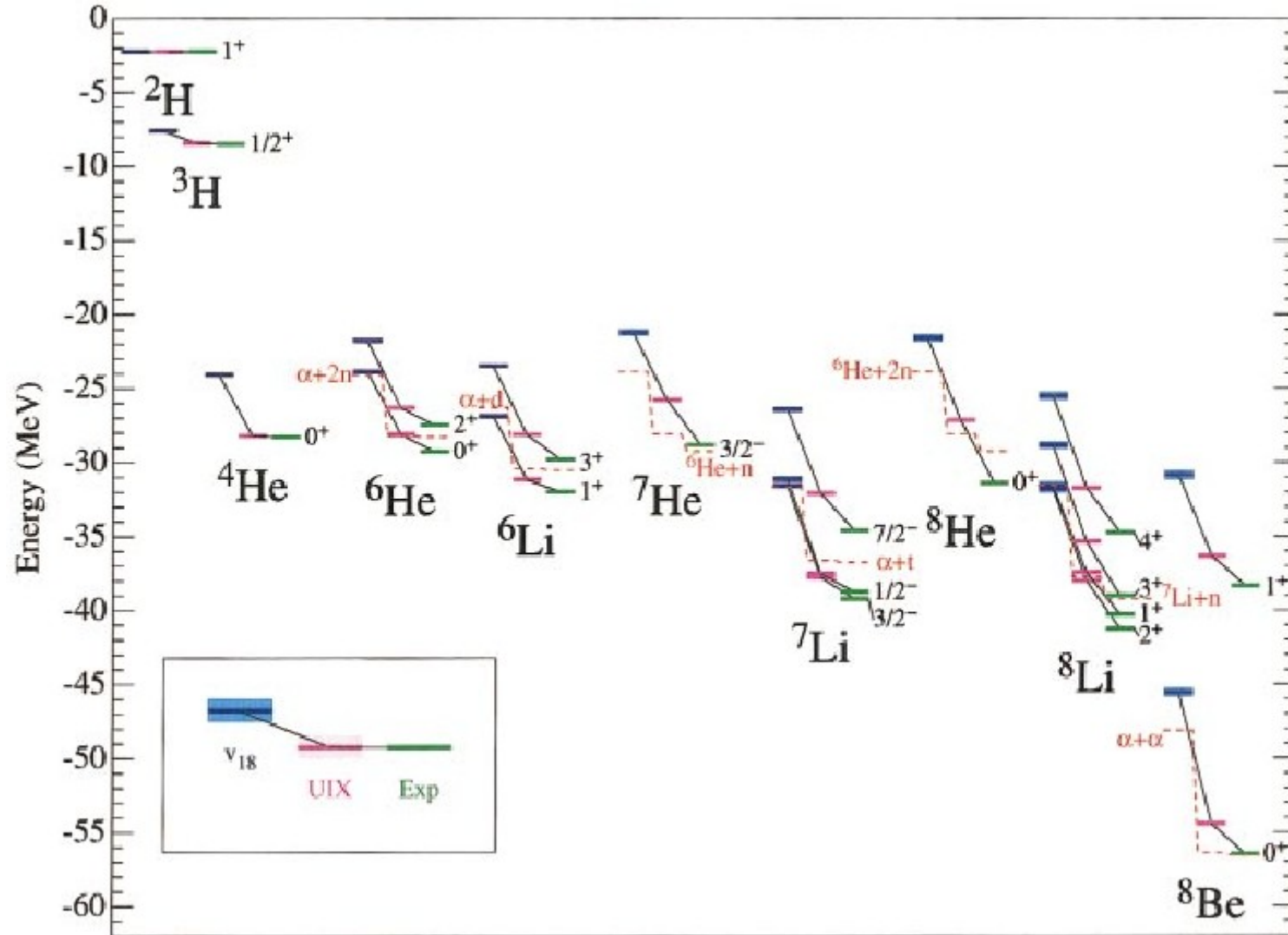




# UIX NNN POTENTIAL



The spectrum of (even) light nuclei is not very well reproduced



# BEYOND UIX

It is necessary to consider a more realistic three nucleon interaction, going beyond UIX

HOW ???

The two-nucleon potential can be decomposed in only few different spin-space structures respecting the symmetry of the interaction. For local  $v_{18}$ , or  $v_8$  potential for example

$$\hat{V}_{12} = \sum_{p=1}^n v^p(r_{12}) \hat{O}_{12}^p$$

Fitting the huge amount of NN scattering data provides the shape of the radial function  $v^p$ .

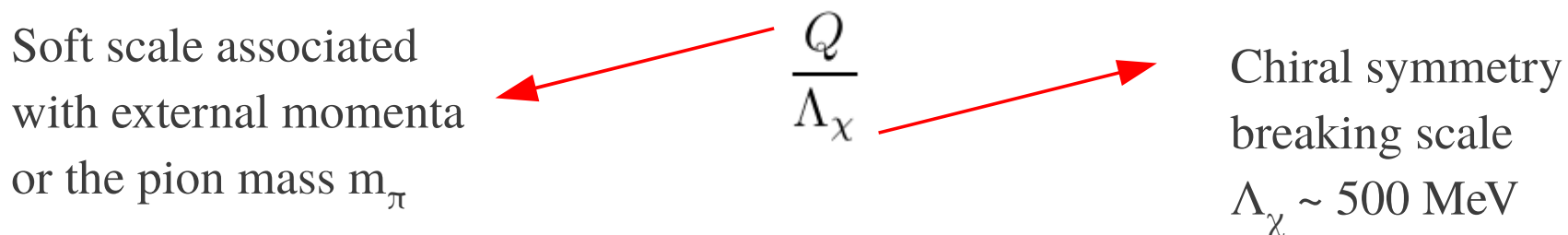
Because of the large variety of different possible structures in the three-nucleon force, **following the same phenomenological path as in the NN system and parametrizing its most general structure seems not to be feasible** without additional theoretical guidance.



**Chiral perturbation theory (ChPT)** is an effective field theory (EFT) of QCD which exploits its symmetries and symmetry-breaking pattern and allows to analyze the properties of hadronic systems at low energies in a systematic and model independent way.

# EFFECTIVE FIELD THEORIES

- Effective field theories have proved to be an important and very useful tool in nuclear and particle physics. One understands under an effective (field) theory an **approximate theory whose scope is to describe phenomena which occur at a chosen length (or energy) range**.
- The effective field theory we are interested in **describes reactions involving pions with external momenta of the order of  $m_\pi$  and (essentially) non-relativistic nucleons** whose three-momenta are of the order of  $m_\pi$ .
- The effective Lagrangian can be used to compute low-energy observables (such as scattering amplitudes) in a systematically improvable way via an expansion in powers of



- Usually effective field theories are not renormalizable, actually **ChPT is such that it forbids the renormalizable derivative-less interaction** of the type  $\pi^4$ .

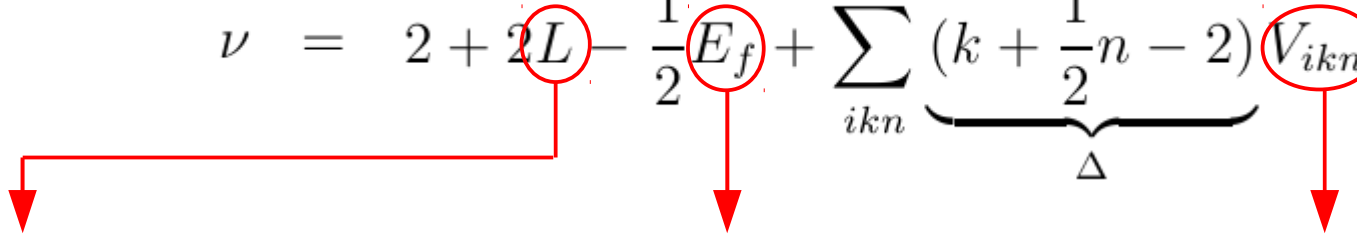
# WEINBERG POWER COUNTING

Using certain topological identities, Weinberg demonstrated that the order of a generic Feynman diagram, i.e. the power of the soft scale  $Q$ , is given by

$$\nu = 2 + 2L - \frac{1}{2}E_f + \sum_{ikn} \underbrace{\left(k + \frac{1}{2}n - 2\right)}_{\Delta} V_{ikn}$$

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Number of loops

Number of external  
fermionic lines

Vertex with  $i$  bosonic  
fields,  $k$  fermionic fields  
and  $n$  derivatives

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## CAVEAT: FEW NUCLEONS SYSTEMS



Contributions of reducible diagrams, i.e. those ones which contain purely nucleonic intermediate states are infrared divergent and do not respect the previous power counting: **infrared enhancement of the few-nucleon diagrams.**

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## WEINBERG SOLVED THE ISSUE



**Consider irreducible diagrams only and solve the Lippman Schwinger equation** which leads to a nonperturbative resummation of the contributions resulting from reducible diagrams.

# CHIRAL NN POTENTIAL

Chiral NN potential at  $N^3LO$  consists of one-, two- and three-pion exchanges and a set of contact interactions with zero, two and four derivatives. In addition, one has to take into account various isospin-breaking and relativistic corrections.

- Entem & Machleidt in 2003 performed the first fit of the Chiral  $N^3LO$  potential to Nijmegen scattering data

29 Fit  
Parameters



Bin (MeV)	No. of data	$N^3LO^a$	NNLO <sup>b</sup>	NLO <sup>b</sup>	AV18 <sup>c</sup>
0–100	795	1.05	6.66	57.8	0.96
100–190	411	1.50	28.3	62.0	1.31
190–290	851	1.93	66.8	111.6	1.82
0–290	2057	1.50	35.4	80.1	1.38

	$N^3LO^a$	CD-Bonn [10]	AV18 [22]	Empirical <sup>b</sup>
Deuteron				
$B_d(\text{MeV})$	2.224575	2.224575	2.224575	2.224575(9)
$A_S(\text{fm}^{-1/2})$	0.8843	0.8846	0.8850	0.8846(9)
$\eta$	0.0256	0.0256	0.0250	0.0256(4)
$r_d(\text{fm})$	1.978 <sup>c</sup>	1.970 <sup>c</sup>	1.971 <sup>c</sup>	1.97535(85)
$Q(\text{fm}^2)$	0.285 <sup>d</sup>	0.280 <sup>d</sup>	0.280 <sup>d</sup>	0.2859(3)
$P_D(\%)$	4.51	4.85	5.76	



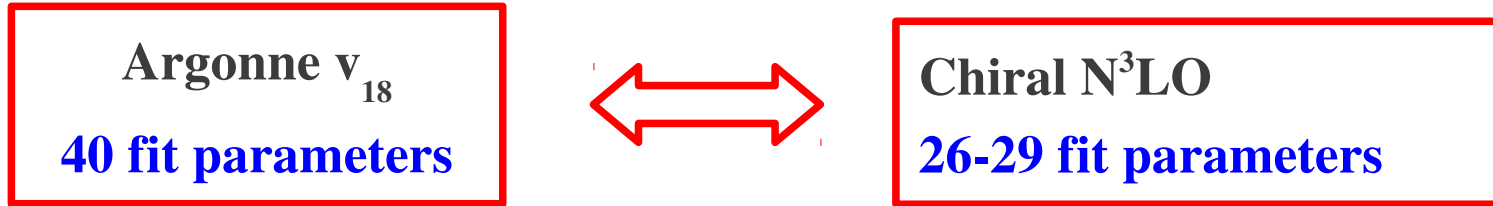
Predictions!

- Epelbaum et al. in 2004 did again the fit with only 26 parameters, performing a cutoff study using a different regularization scheme obtaining a similar  $\chi^2 \sim 1$ .



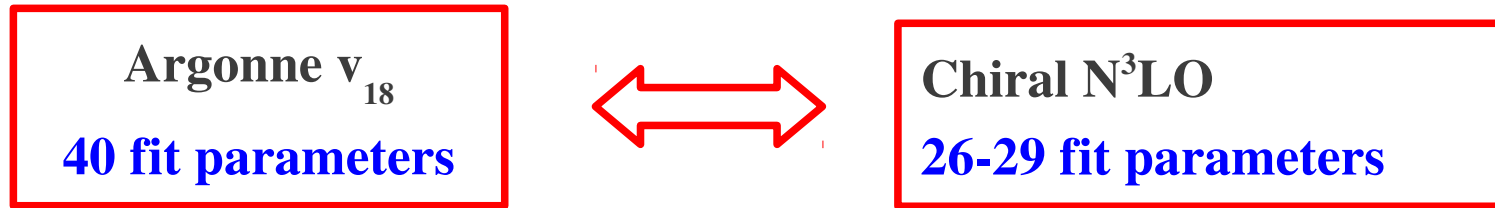
# CHIRAL NN POTENTIAL

Chiral  $N^3LO$  potential results to be as accurate as Argonne  $v_{18}$  potential in the description of experimental data, moreover



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Unfortunately no local version of Chiral  $N^3LO$  exists. **We can use this potential neither in FHNC/SOC nor in AFDMC calculations.**

This is not a crucial problem since Argonne  $v_{18}$  and  $v_8'$  describe amazingly well NN scattering data and deuteron properties.

If three-body local chiral potential exists we can use this potential combined with Argonne NN potential.



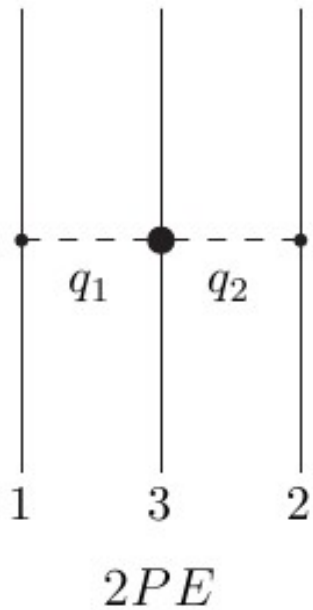
**MIXED APPROACH**

# NNN CHIRAL POTENTIAL IN CHIRAL $\Delta$ LESS THEORY

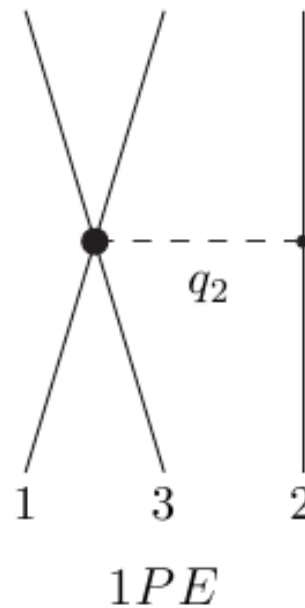
In a theory without explicit  $\Delta$  degrees of freedom, the first contribution to the **chiral 3NF** appears at  $N^2\text{LO}$  in the Weinberg counting scheme.

The contributions to this potential comes from three different physical processes

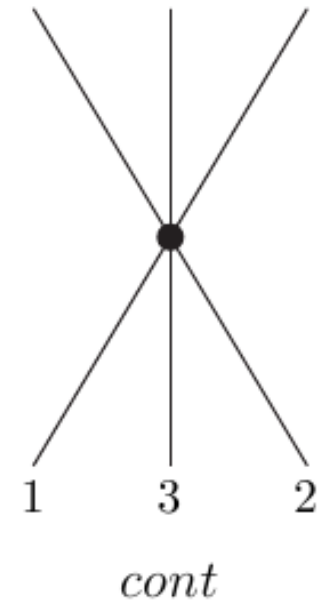
Two-pion exchange (**TPE**)



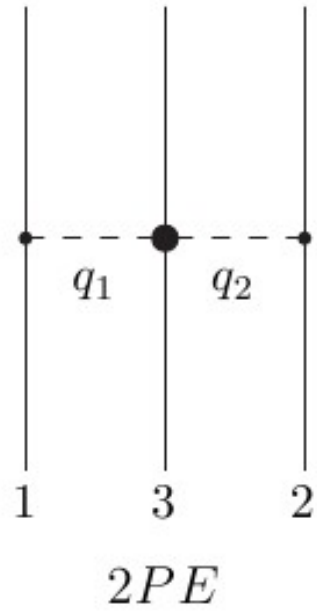
One-pion exchange (**OPE**)



Contact interaction (**cont**)



# NNN CHIRAL POTENTIAL: TPE TERM



$$\tilde{V}^{TPE}(3 : 12) = V_0 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{q}_1)}{(q_1^2 + m_\pi^2)} \frac{(\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2)}{(q_2^2 + m_\pi^2)} \times$$

$$\left[ \tau_{12} \underbrace{(-a + b\mathbf{q}_1 \cdot \mathbf{q}_2)}_{\mathbf{V}_a} - d\boldsymbol{\tau}_3 \cdot \underbrace{(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)}_{\mathbf{V}_b} \underbrace{\boldsymbol{\sigma}_3 \cdot (\mathbf{q}_1 \times \mathbf{q}_2)}_{\mathbf{V}_d} \right]$$

We are interested in the **local expression in coordinate space**

$$V^{TPE}(3 : 12) = \int \frac{d^3 q_1}{(2\pi)^3} \frac{d^3 q_2}{(2\pi)^3} \underbrace{F_\Lambda(q_1^2) F_\Lambda(q_2^2)}_{\text{Cutoff functions depending on transferred momenta}} e^{i\mathbf{q}_1 \cdot \vec{r}_{13}} e^{i\mathbf{q}_2 \cdot \vec{r}_{23}} \tilde{V}^{TPE}(3 : 12)$$

**Cutoff functions depending on transferred momenta**

Chiral N2LO Local potential in coordinate space is a sum of these terms

$$V_a(3 : 12) = aW_0 \tau_{12} (\boldsymbol{\sigma}_1 \cdot \vec{r}_{13}) (\boldsymbol{\sigma}_2 \cdot \vec{r}_{23}) y(r_{13}) y(r_{23})$$

$$V_b(3 : 12) = bW_0 \tau_{12} [\sigma_{12} y(r_{13}) y(r_{23}) + (\boldsymbol{\sigma}_1 \cdot \vec{r}_{23}) (\boldsymbol{\sigma}_2 \cdot \vec{r}_{23}) t(r_{23}) y(r_{13})$$

$$+ (\boldsymbol{\sigma}_1 \cdot \vec{r}_{13}) (\boldsymbol{\sigma}_2 \cdot \vec{r}_{13}) t(r_{13}) y(r_{23}) + (\vec{r}_{13} \cdot \vec{r}_{23}) (\boldsymbol{\sigma}_1 \cdot \vec{r}_{13}) (\boldsymbol{\sigma}_2 \cdot \vec{r}_{23}) t(r_{13}) t(r_{23})]$$

$$V_d(3 : 12) = dW_0 (\boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2) [(\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_1) y(r_{13}) y(r_{23}) + (\boldsymbol{\sigma}_3 \cdot \vec{r}_{23} \times \boldsymbol{\sigma}_1) (\boldsymbol{\sigma}_2 \cdot \vec{r}_{23}) t(r_{23}) y(r_{13})$$

$$+ (\boldsymbol{\sigma}_2 \cdot \vec{r}_{13} \times \boldsymbol{\sigma}_3) (\boldsymbol{\sigma}_1 \cdot \vec{r}_{13}) t(r_{13}) y(r_{23}) + (\boldsymbol{\sigma}_3 \cdot \vec{r}_{23} \times \vec{r}_{13}) (\boldsymbol{\sigma}_1 \cdot \vec{r}_{13}) (\boldsymbol{\sigma}_2 \cdot \vec{r}_{23}) t(r_{13}) t(r_{23})]$$

Where

$$W_0 = \frac{m_\pi^6}{(4\pi)^2} V_0 \quad z_0(r) = \frac{2}{\pi m_\pi^3} \int dq q^2 \frac{F_\Lambda(q^2)}{(q^2 + m_\pi^2)} j_0(qr) \quad y(r) = \frac{z_0'(r)}{r} \quad t(r) = \frac{y'(r)}{r}$$

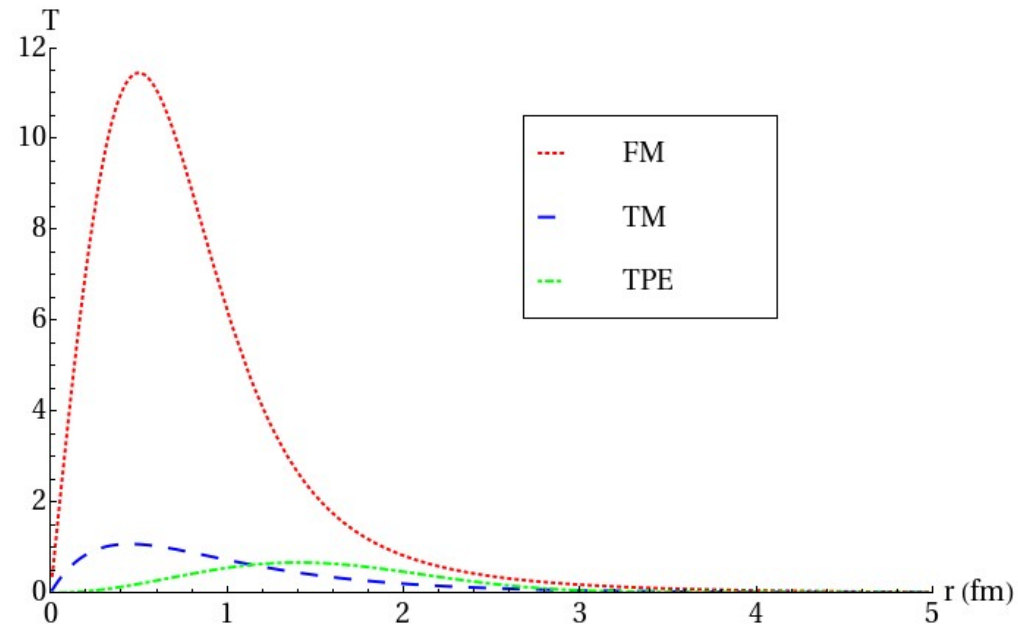
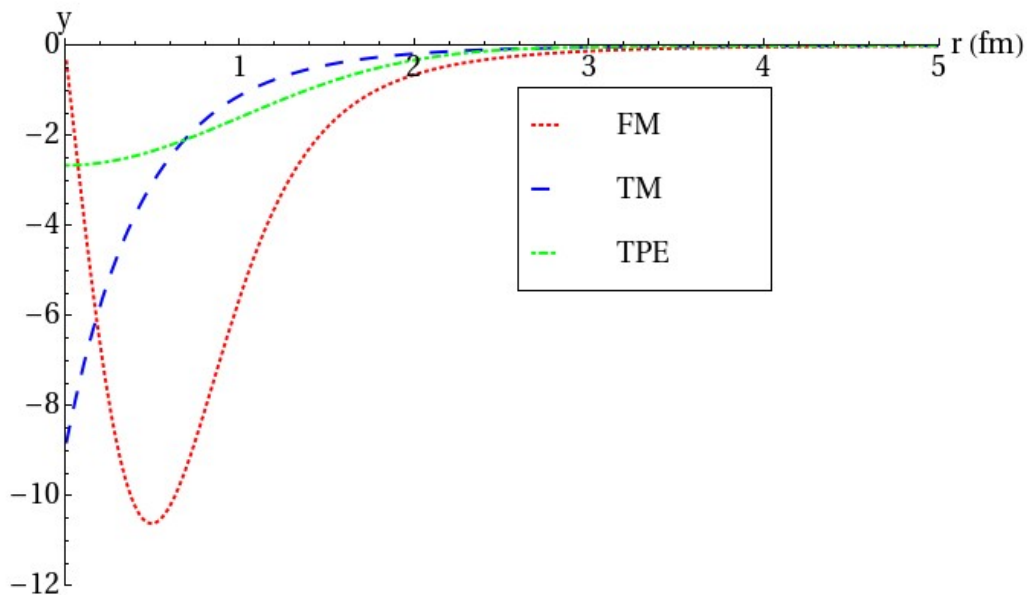
# TPE CHIRAL TERM, FM & TM

- It is easy to show that  $V_b$  and  $V_d$  correspond to the anticommutator and to the commutator term, respectively, of FM potential

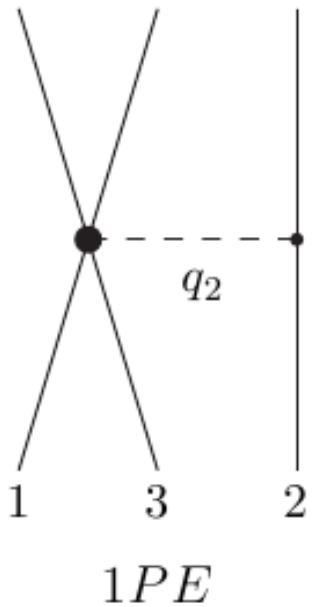
$$\left\{ \begin{array}{l} V_b(3 : 12) = \frac{bW_0}{4} \{\tau_{13}, \tau_{23}\} \{\sigma_{13}Y_{13} + S_{13}T_{13}, \sigma_{23}Y_{23} + S_{23}T_{13}\} \\ V_d(3 : 12) = \frac{dW_0}{4} [\tau_{13}, \tau_{23}] [\sigma_{13}Y_{13} + S_{13}T_{13}, \sigma_{23}Y_{23} + S_{23}T_{13}] \end{array} \right. \quad \left\{ \begin{array}{l} Y(r) = y(r) + \frac{r^2}{3}t(r) \\ T(r) = \frac{r^2}{3}t(r) \end{array} \right.$$

- $V_a$  term is not present in UIX, whereas the sum of  $V_a$ ,  $V_b$ , and  $V_d$  is precisely the so called Tucson Melbourne (TM) potential.

Despite the physical mechanism is different, TPE chiral term, FM and TM have the same expression. The only differences are in the radial functions and in the constants.



# NNN CHIRAL POTENTIAL: OPE TERM



This term appears only in the chiral potential

$$V^{OPE}(3 : 12) = -c_D V_0^D \frac{(\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2)}{(q_2^2 + m_\pi^2)} \left[ \alpha_1 (\boldsymbol{\sigma}_1 \cdot \mathbf{q}_2) \tau_{12} + \alpha_2 (\boldsymbol{\sigma}_1 \cdot \mathbf{q}_2) \tau_{13} + \alpha_3 \mathbf{q}_2 (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_3) \vec{\tau}_2 \cdot (\vec{\tau}_1 \times \vec{\tau}_3) + 1 \leftrightarrow 2 \right]$$

The force is symmetric with respect to an interchange of particles 1 and 2

$$V^{OPE}(3 : 12) \mathcal{A}_{12} |\Psi\rangle = \mathcal{A}_{12} V^{OPE}(3 : 12) |\Psi\rangle$$

$$\mathcal{A}_{12} = 1 - \frac{(1 + \sigma_{12})}{2} \frac{(1 + \tau_{12})}{2}$$

Once contracted with  $\mathcal{A}_{12}$ , all the different structures gives the same contribution.

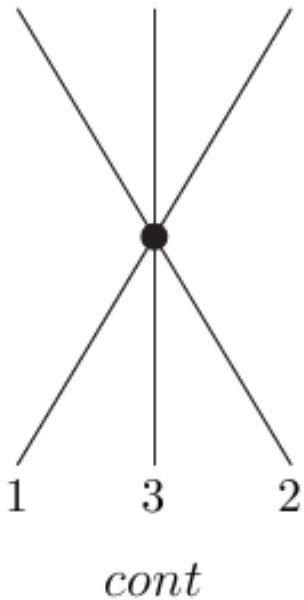
$$V^{OPE}(3 : 12) = -c_D V_0^D \frac{(\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2)}{(q_2^2 + m_\pi^2)} \left[ \alpha_1 (\boldsymbol{\sigma}_1 \cdot \mathbf{q}_2) \tau_{12} + 1 \leftrightarrow 2 \right] \quad \text{Epelbaum et al. (2002)}$$

The local expression reads

$$V_D(3 : 12) = c_D W_0^D \tau_{12} [\sigma_{12} y(r_{23}) z_0(r_{13}) + (\boldsymbol{\sigma}_1 \cdot \vec{r}_{23})(\boldsymbol{\sigma}_2 \cdot \vec{r}_{23}) t(r_{23}) z_0(r_{13}) + \sigma_{12} y(r_{13}) z_0(r_{23}) + (\boldsymbol{\sigma}_2 \cdot \vec{r}_{13})(\boldsymbol{\sigma}_1 \cdot \vec{r}_{13}) t(r_{13}) z_0(r_{23})]$$

**Because of the regulator there are no more contact terms  $z_0(\mathbf{r}) \neq \delta(\mathbf{r})$ , and the equivalence of the different contact terms is spoiled!**

# NNN CHIRAL POTENTIAL: cont TERM



This term also appears only in the chiral potential

$$V^{cont}(3 : 12) = c_E(\beta_1 + \beta_2\sigma_{12} + \beta_3\tau_{12} + \beta_4\sigma_{12}\tau_{12} + \beta_5\sigma_{12}\tau_{23} + \beta_6(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{\sigma}_3 + (\vec{\tau}_1 \times \vec{\tau}_2) \cdot \vec{\tau}_3 + 1 \leftrightarrow 2)$$

As before, it is possible to show that, once multiplied by the antisymmetrization operator, all the terms give the same contribution. For instance

$$(\tau_{12} + \tau_{13} + \tau_{23})\mathcal{A}_{123} = -3\mathcal{A}_{123}$$

Without any loss of generality, it is possible to choose one of the independent terms

$$V^{cont}(3 : 12) = c_E W_0^E \tau_{12}$$

The local expression reads

$$V_E(3 : 12) = W_0^E \tau_{12} z_0(r_{13}) z_0(r_{23})$$

Aside from the isospin factor this term resembles the UIX repulsive scalar term.

**Once again, because of the regulator this is no more a contact term !!!**

**In principle all the different terms have to be considered.**

# NNN POTENTIAL PARAMETERS

Very recently Kviesky et al. performed a comparative study of UIX, TM and chiral NNLOL three nucleon interactions combined with Argonne  $v_{18}$  NN potential.

They have found the best-fit values for the three-body potentials to reproduce simultaneously the **triton binding energy** and the **doublet n-d scattering length**.

$$\text{UIX: } V_b + V_d + V_E$$

$A_{2\pi}^{\text{PW}}$ (MeV)	$D_{2\pi}^{\text{PW}}$	$A_R$ (MeV)	$T$ (MeV)	$V(2N)$ (MeV)	$V_A(3N)$ (MeV)	$V_R(3N)$ (MeV)	$B(^4\text{He})$ (MeV)	$^2a_{nd}$ (fm)
-0.0293	0.25	0.0048	51.259	-58.606	-1.126	1.000	28.48	0.578
-0.0200	1.625	0.0176	47.472	-57.976	-0.923	2.950	28.33	0.644
-0.0250	1.25	0.0182	47.628	-57.967	-1.162	3.024	28.34	0.644
-0.0293	1.00	0.0181	47.876	-58.000	-1.369	3.015	28.33	0.643
-0.0350	0.8125	0.0191	47.998	-57.975	-1.649	3.147	28.33	0.645
-0.0400	0.6875	0.0198	48.133	-57.964	-1.897	3.249	28.38	0.645
-0.0450	0.5625	0.0198	48.414	-57.995	-2.148	3.248	28.38	0.643
-0.0500	0.50	0.0210	48.471	-57.952	-2.401	3.401	28.44	0.645
Exp.							28.30	$0.645 \pm 0.003 \pm 0.007$



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$$\text{TM': } V_a + V_b + V_d + V_E$$

$b (m_\pi^{-3})$	$d (m_\pi^{-3})$	$c_E$	$\Lambda (m_\pi)$	$T$ (MeV)	$V(2N)$ (MeV)	$V_A(3N)$ (MeV)	$V_R(3N)$ (MeV)	$B(^4\text{He})$ (MeV)	$^2a_{nd}$ (fm)
-2.580	-0.753	0.0	4.8	50.708	-58.144	-1.039	0.0	28.52	0.596
-8.256	-4.690	1.0	4.0	50.317	-57.366	-2.206	0.781	28.30	0.644
-3.870	-3.375	1.6	4.8	50.699	-57.641	-2.748	1.215	28.38	0.644
-2.064	-2.279	2.0	5.6	50.998	-57.940	-2.814	1.291	28.44	0.640
Exp.								28.30	$0.645 \pm 0.003 \pm 0.007$

# NNN POTENTIAL PARAMETERS

Very recently Kviesky et al. performed a comparative study of UIX, TM and chiral NNLO three nucleon interactions combined with Argonne  $v_{18}$  NN potential.

They have found the best-fit values for the three-body potentials to reproduce simultaneously the **triton binding energy** and the **doublet n-d scattering length**.

$$\text{N}^2\text{LO: } V_a + V_b + V_d + V_E$$

$c_3 (c_3^0)$	$c_4 (c_4^0)$	$c_D$	$c_E$	$T$ (MeV)	$V(2N)$ (MeV)	$V_A(3N)$ (MeV)	$V_R(3N)$ (MeV)	$B(^4\text{He})$ (MeV)	$^2a_{nd}$ (fm)
1.4	0.3636	-0.5	0.1	49.834	-57.278	-1.029	0.0	28.31	0.641
1.4	0.3786	-1	0.0	49.950	-57.401	-1.022	0.0	28.30	0.636
1.5	0.3735	-1	-0.03	49.839	-57.274	-1.076	0.036	28.29	0.644
1.7	0.9000	-2	-0.50	50.166	-57.181	-2.119	0.657	28.32	0.645
Exp.								28.30	$0.645 \pm 0.003 \pm 0.007$

# NNN POTENTIALS IN NUCLEAR MATTER

Consider two different contribution belonging to  $V_E$  of the chiral potential that give the same contribution in the limit of infinite cutoff

$$V_E^{\tau_{12}}(3 : 12) = V_0^E \tau_{12} Z_0(r_{13}) Z_0(r_{23})$$

$$V_E^I(3 : 12) = V_0^E Z_0(r_{13}) Z_0(r_{23}) .$$

In order  $V_E^{\tau_{12}, I}$  to give a positive contribution to the binding energy of light nuclei



$$\begin{cases} c_E^I > 0 \\ c_E^{\tau_{12}} < 0 \end{cases}$$

In PNM **the expectation value of a three-body contact term is zero**. Due to cutoff effect this is not true anymore. Moreover

$$\langle \tau_{12} \rangle_{PNM} = 1 \quad \longrightarrow \quad \begin{cases} \langle V_E^I \rangle_{PNM} > 0 \\ \langle V_E^{\tau_{12}} \rangle_{PNM} < 0 \end{cases}$$

**Inconsistency of the local formulation of N<sup>2</sup>LO potential**

We have computed the expectation value of  $V_E$  in Fermi gas of SNM and PNM

$\Lambda$ (MeV)	$\langle V_E^{\tau_{12}} \rangle_{SNM}^{FG}/A$ (MeV)	$\langle V_E^I \rangle_{SNM}^{FG}/A$ (MeV)	$\langle V_E^{I, \tau_{12}} \rangle_{PNM}^{FG}/A$ (MeV)
300	-2.61	10.21	9.15
400	-3.61	8.15	5.95
500	-4.37	6.93	3.60
600	-4.87	6.30	2.15
700	-5.15	5.98	1.30
800	-5.30	5.81	0.81
teorico	-5.55	5.55	0

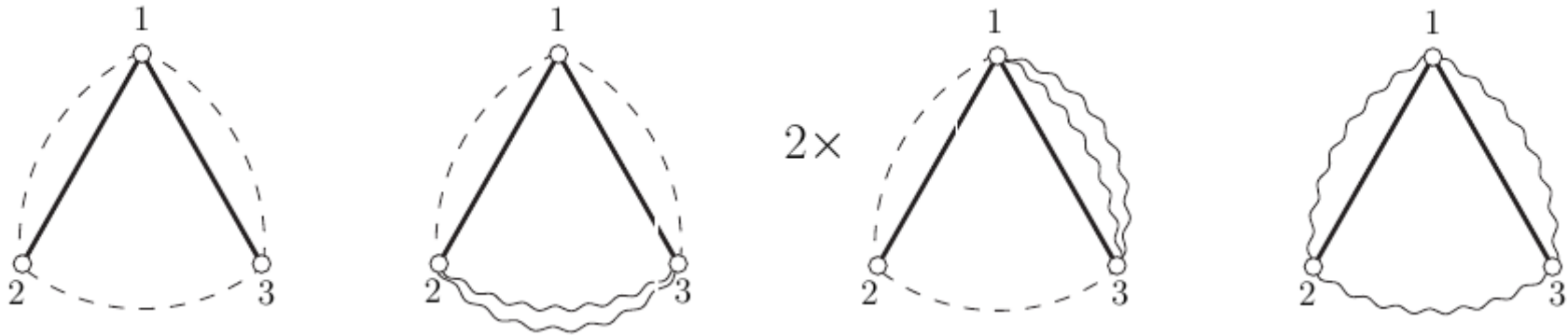
# NNN POTENTIALS IN NUCLEAR MATTER: FHNC/SOC

We are implementing UIX, TM' and chiral NNLO potential in nuclear matter using both FHNC/SOC and AFDMC technique.

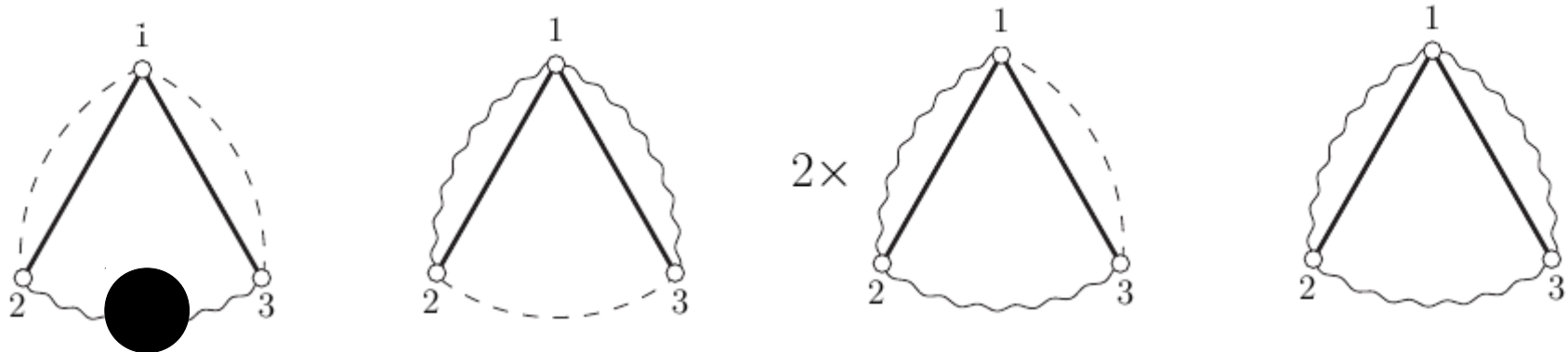
In **FHNC/SOC** (Omar lectures) the expectation value of the three body potential reads

$$\frac{\langle v_{123} \rangle}{A} = \frac{1}{3!} \rho^2 \sum_P \int dr_{12} dr_{13} v_{123}^P g_{123}^P \quad g_{123}^P = \frac{A!}{(A-3)!3!} \frac{\int d\sigma \tau_{123} dx_{4\dots A} \Phi^* F^\dagger O_{123}^P F \Phi}{\rho^3 \int dx_{1\dots A} \Phi^* F^\dagger F \Phi}$$

For the central part  $V_E$  of UIX and TM' potential we calculated the following diagrams



For the operatorial parts  $V_a$ ,  $V_b$ ,  $V_d$  and  $V_D$



# NNN POTENTIALS IN NUCLEAR MATTER: FHNC/SOC

To find the minimum of the energy

$$E_V = E_V(d_c, d_t, \beta_p, \alpha_p) \quad \longrightarrow \quad \text{SIMULATED ANNEALING METHOD}$$

The variational parameters are drawn from the Boltzmann distribution  $\exp(-E_V/T)$

We used a Metropolis algorithm with acceptance probability

$$P_{s,s'} = \exp \left[ - \frac{E(s') - E(s)}{T} \right] \quad s = \{d_c, d_t, \beta_p, \alpha_p\} \quad s' = \{d'_c, d'_t, \beta'_p, \alpha'_p\}$$

As  $T$  is lowered, the parameters stay closer to the minimum of  $E_V$ .

Variational principle can be violated because elementary diagrams are neglected and because of SOC approximation. To keep the violations under control we performed a constrained optimization by imposing:

$$|E_{PB} - E_{JF}| < 10\% T_F \quad \left| \rho \int d\vec{r}_{12} [g^c(r_{12}) - 1] - 1 \right| < 0.03$$

Up to now the optimizations have been carried out for the following three body potentials, while for the two body interaction we stick on Argonne  $v_8'$

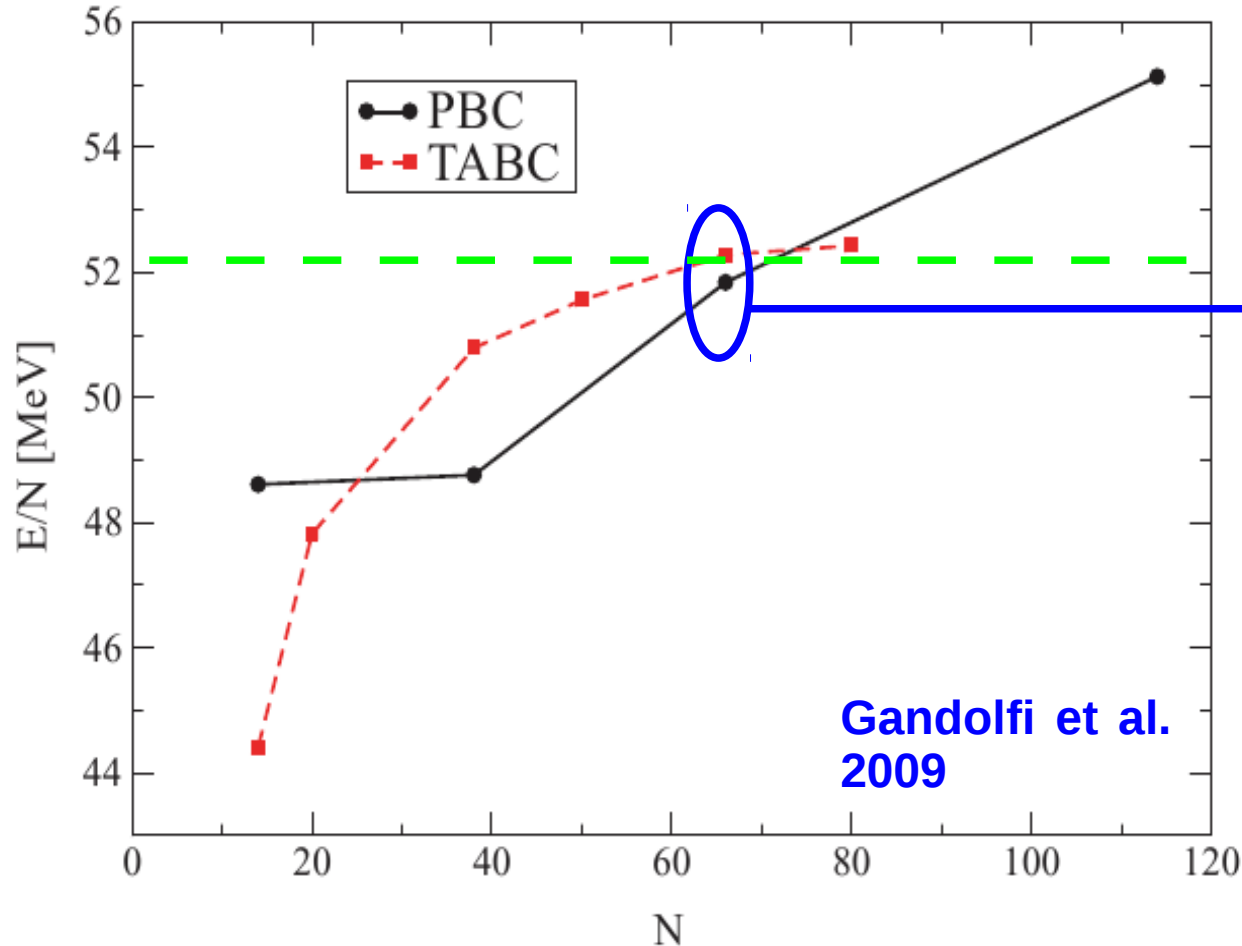
● UIX

● TM1 TM2 TM3

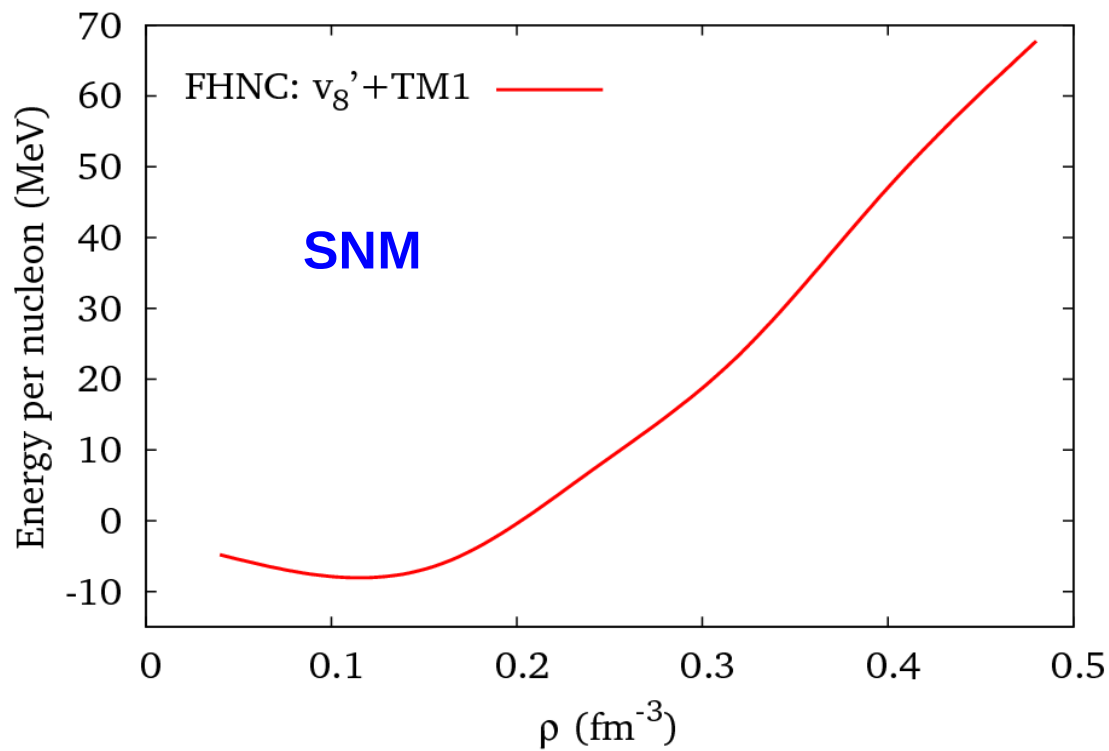
● N<sup>2</sup>LO 1  
(for PNM only)

# NNN POTENTIALS IN NUCLEAR MATTER: AFDMC

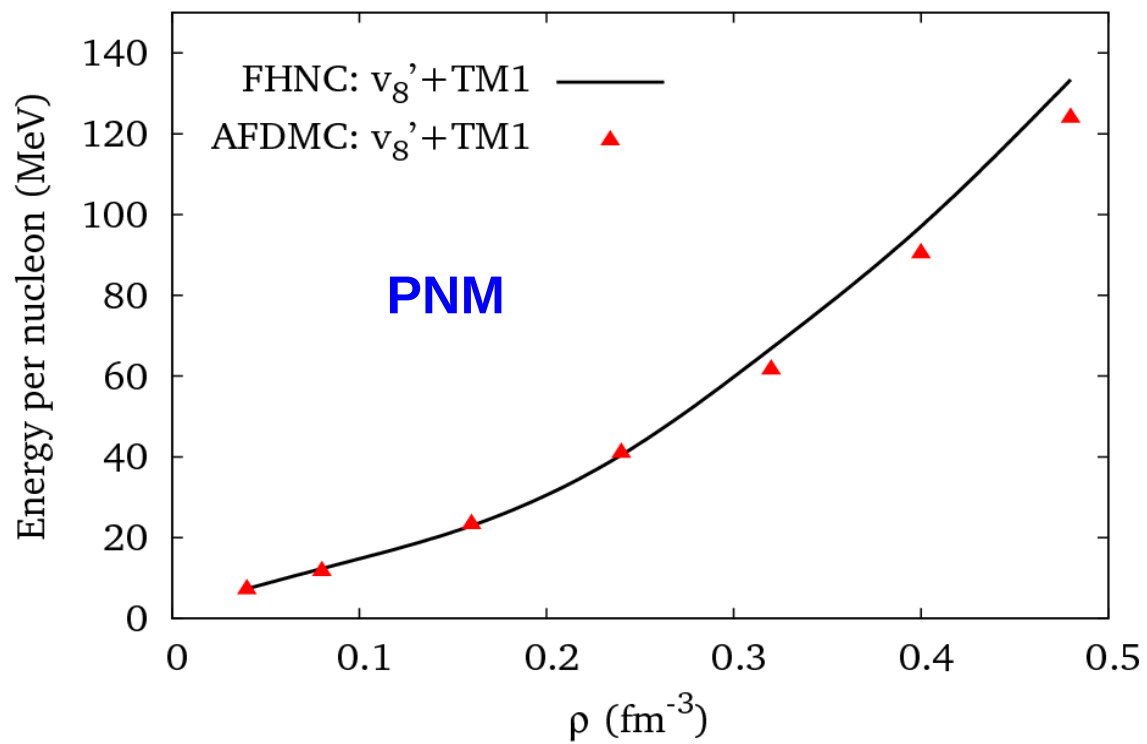
We carried out AFDMC simulations for PNM with **66 neutrons in periodic box system**.

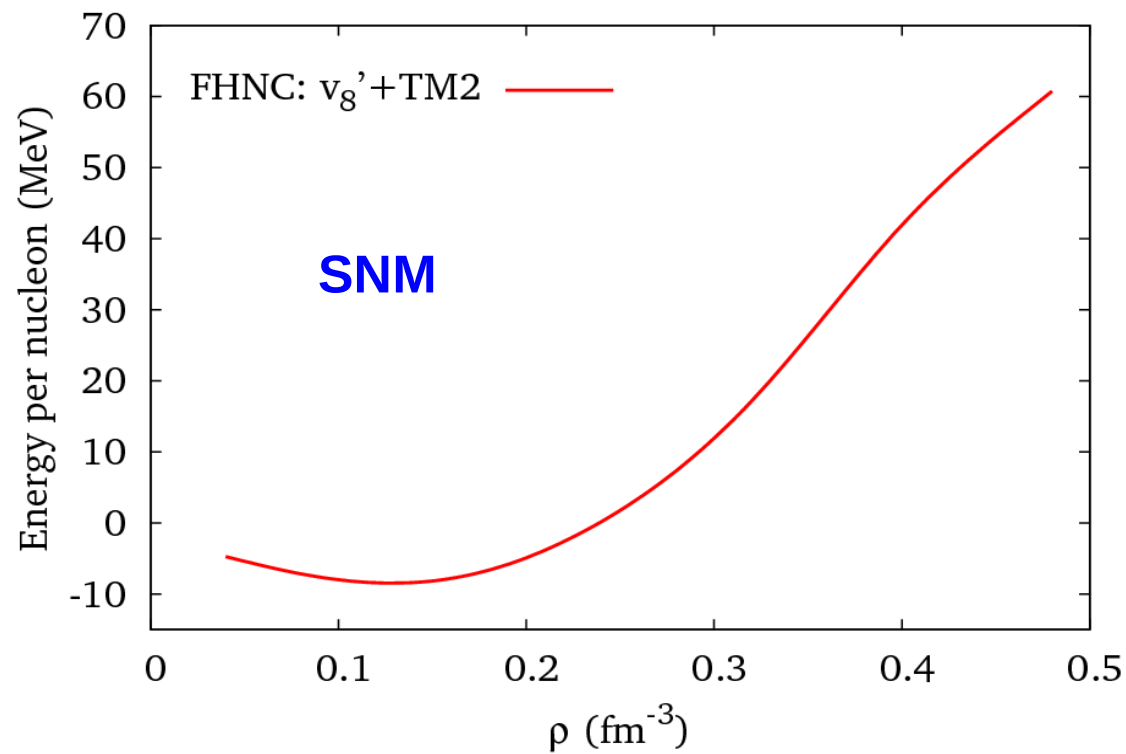


	$A = 14$	$A = 38$	$A = 66$	$A = 114$	$\infty$
$E/A(\text{MeV})$	56.51	53.50	55.43	56.58	55.71

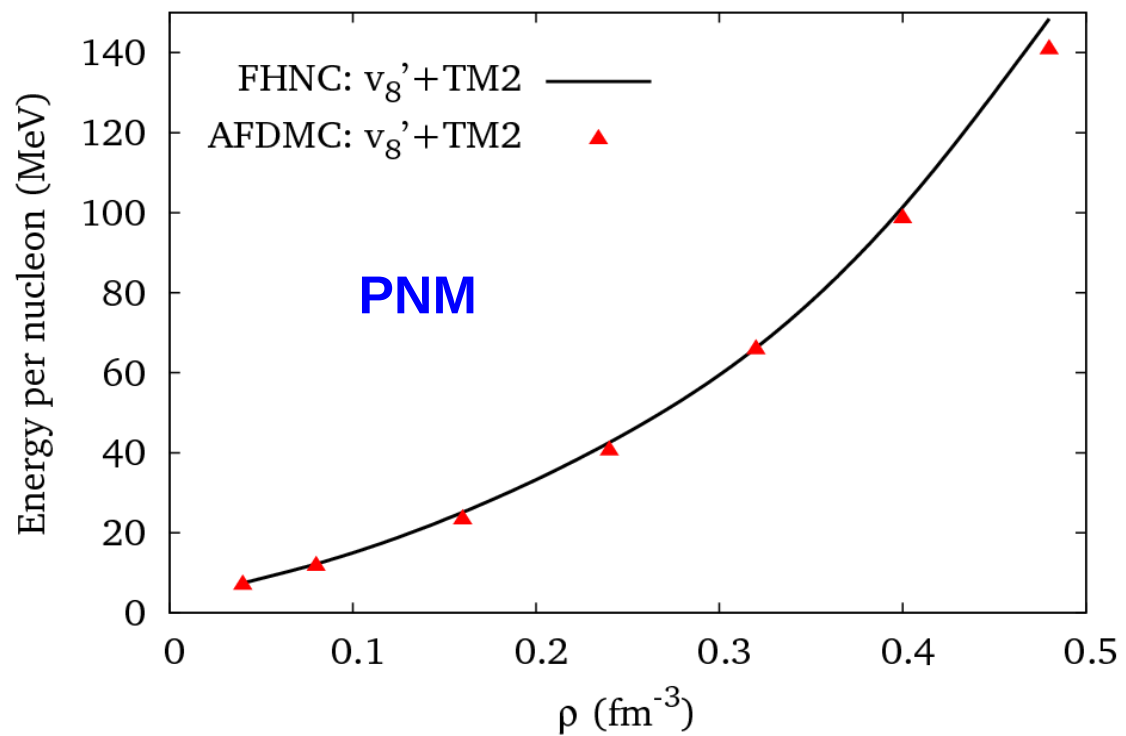


**TM1 results**

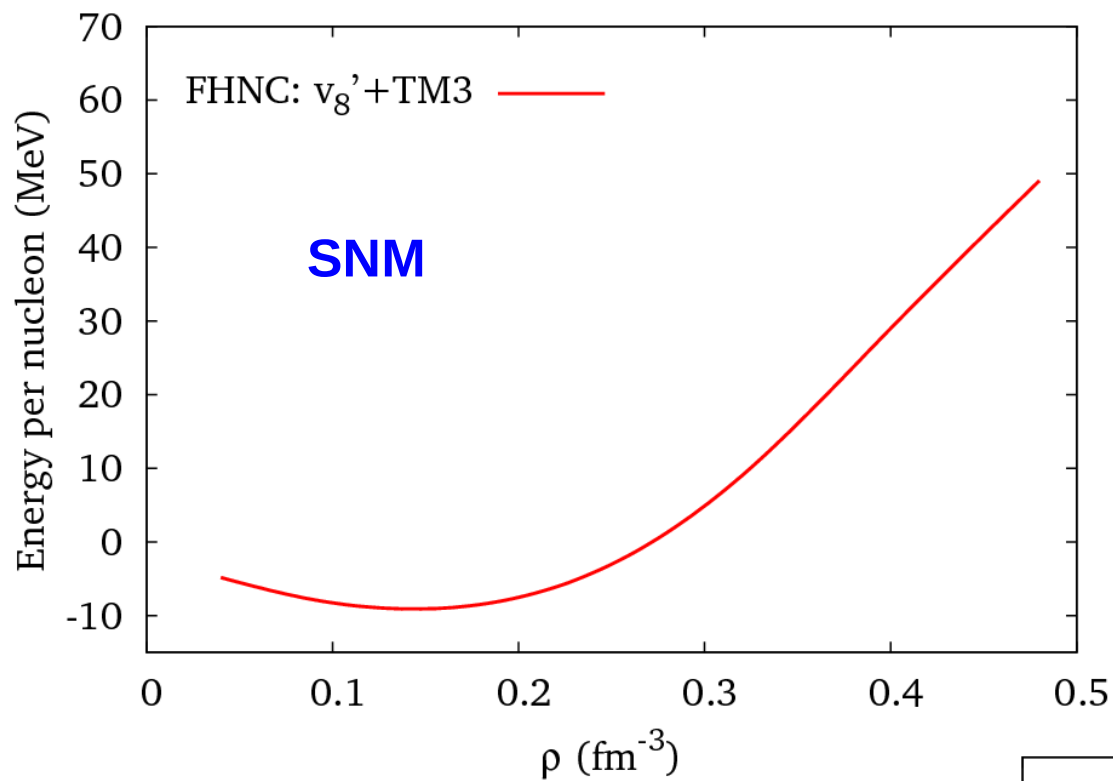




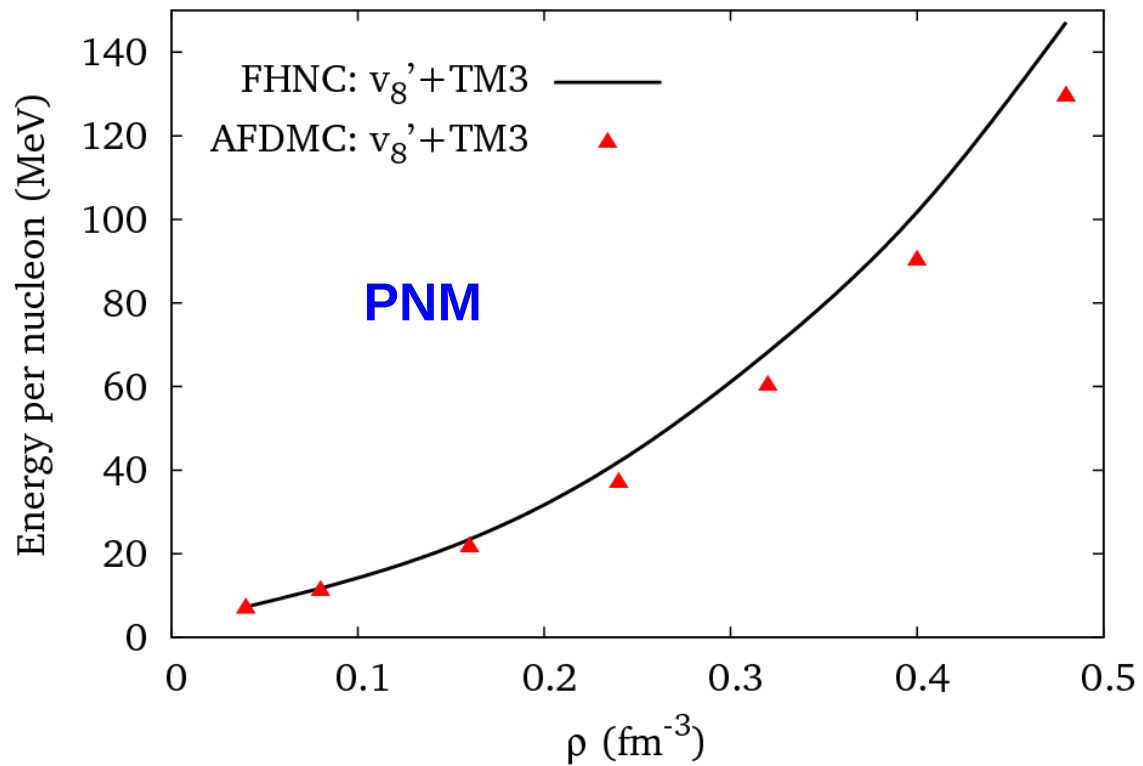
**TM2 results**



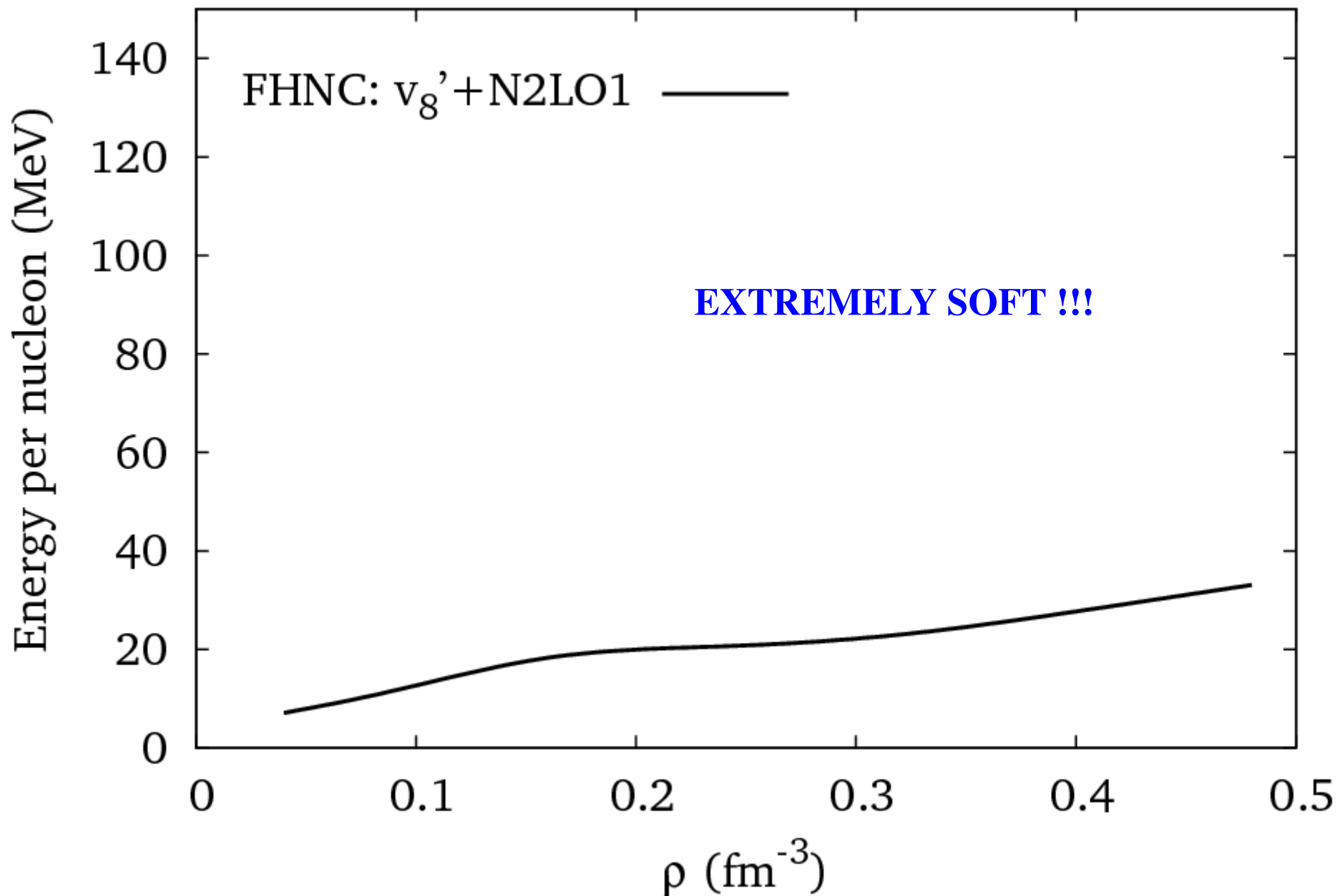




**TM3 results**



# N<sup>2</sup>LO results



# CONCLUSIONS

- No one of the new potentials seems to reproduce the correct binding energy of SNM
- Chiral NNLO seems to provide a very soft PNM EoS. Probably due to the wrong treatment of  $V_E$

