An introduction to the nuclear many-body problem

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Working hypotheses

- Nucleons, basic degrees of freedom.

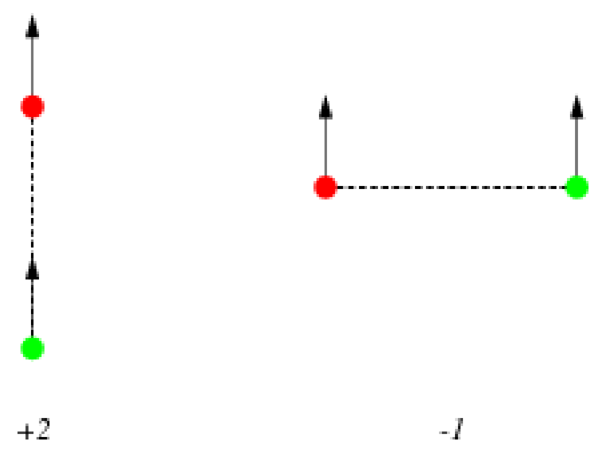
- Non relativistic system

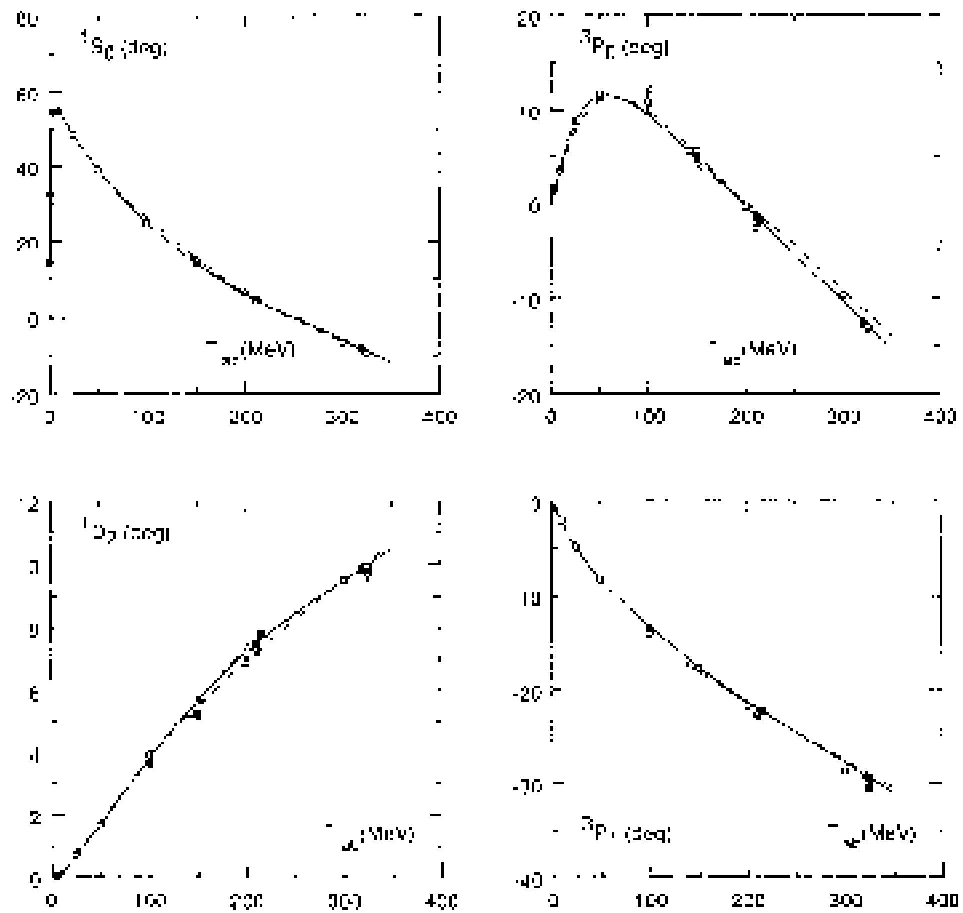
$$H|\Psi >= E|\Psi >$$

Deuteron

The only bound system of two nucleons

B = 2.22 MeV	
$\mu_d = \mu_p + \mu_n - 0.00222$	Non central forces
Q=2.82 mb	Tensor dependence





$^1S_0$ (deg)
T_{lab} (MeV)
$^3P_0$ (deg)
T_{lab} (MeV)
$^1D_2$ (deg)
T_{lab} (MeV)
$^3P_1$ (deg)
T_{lab} (MeV)

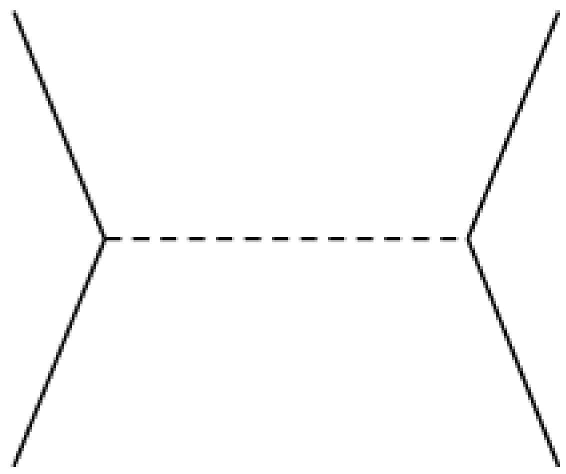
Penomenological interactions (Urbana and Argonne type)

$$V(i,j) = \sum_{p=1,18} v_p(r_{ij}) O^p_{ij}$$

$$\begin{aligned} O^{p=1,14}_{ij} &= \left[1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, (\mathbf{L}\cdot\mathbf{S}), \mathbf{L}^2, \mathbf{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), (\mathbf{L}\cdot\mathbf{S})^2\right] \\ &\otimes \left[1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\right] \end{aligned}$$

$$O^{p=15,18}_{ij} = \left[1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij},\right] \otimes \left[\tau_{zi} + \tau_{zj}\right]$$

Meson exchange potentials (Bonn)

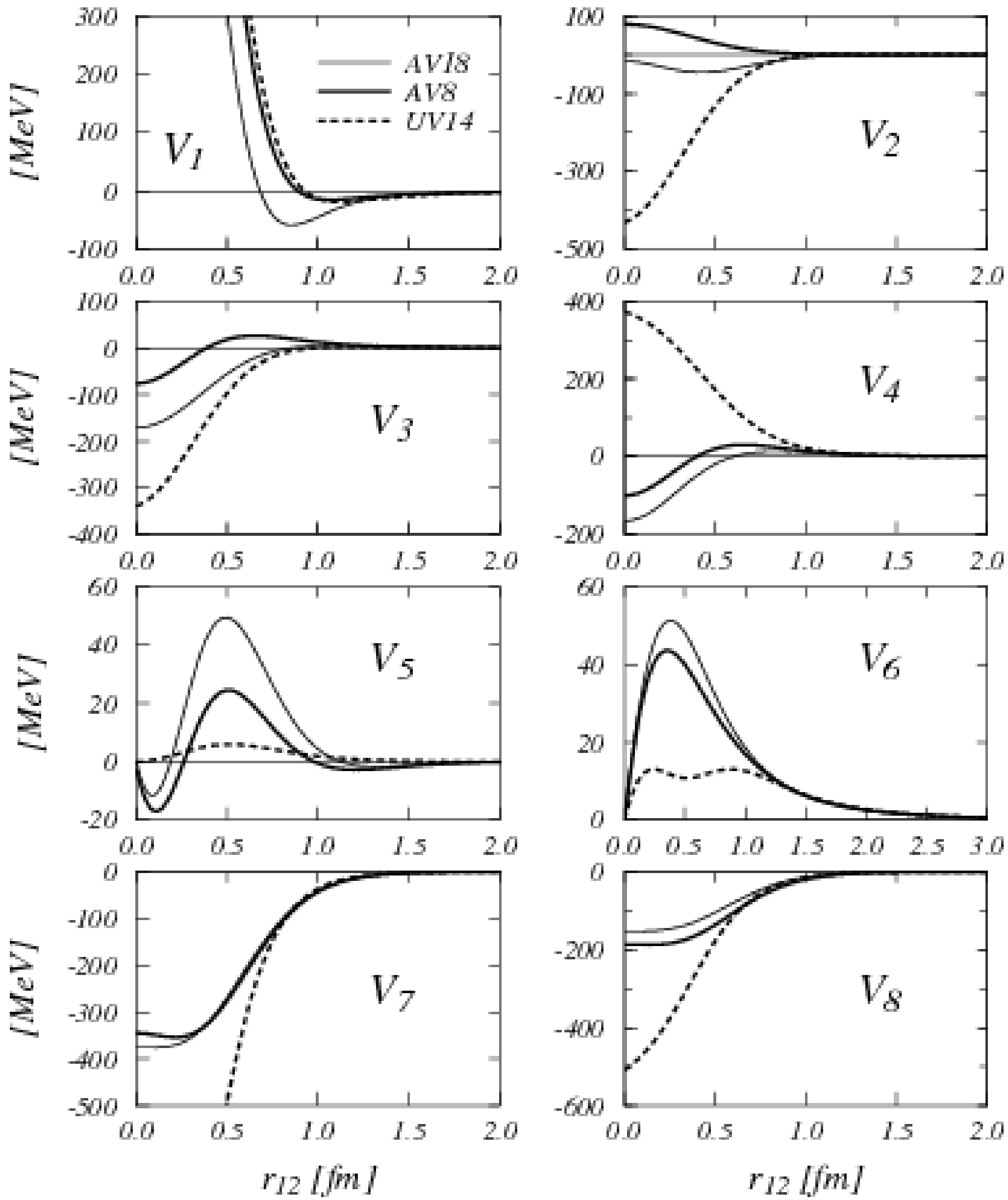


Mesons π, ρ, ω, σ

Effective field theories

- Perturbation expansion of a Lagrangian based on pion exchange which satisfies the symmetries of the QCD at MeV scale. Chiral symmetry.

- N^3LO


AV18
AV8
UV14
V_1
V_2
V_3
V_4
V_5
V_6
V_7
V_8
[MeV]
r_{12} [fm]

Green's function Monte Carlo

$$\Psi(\tau) = e^{-(H-E_0)\tau}\Psi_T = e^{-(H-E_0)\tau}\sum_N A_N \Psi_N = \sum_N e^{-(E_N-E_0)\tau} A_N \Psi_N$$

$$\Psi(\tau \to \infty) \to A_0 \Psi_0$$

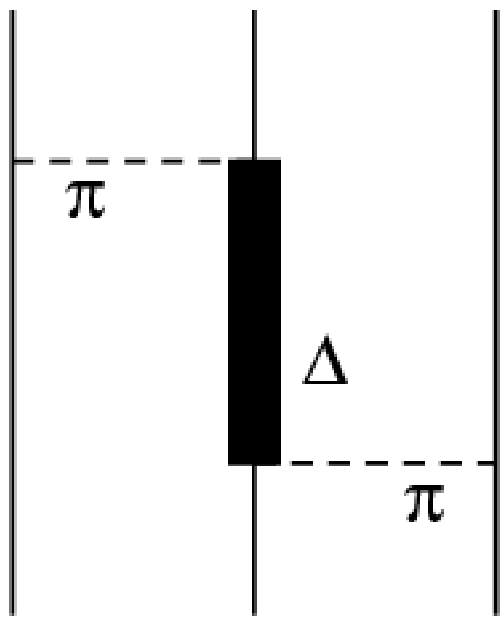
$$\frac{< \Psi_T | e^{-(H-E_0)\tau/2}\, H\, e^{-(H-E_0)\tau/2} | \Psi_T >}{< \Psi_T | e^{-(H-E_0)\tau/2} e^{-(H-E_0)\tau/2} | \Psi_T >} \geq E_0$$

^3H Binding energy

Exp 8.48 MeV

2 N potential	2N
CD Bonn	7.953
Nijm II	7.709
Nijm I	7.731
Nijm 93	7.664
Reid 93	7.648
AV14	7.683
AV18	7.576

3 body force



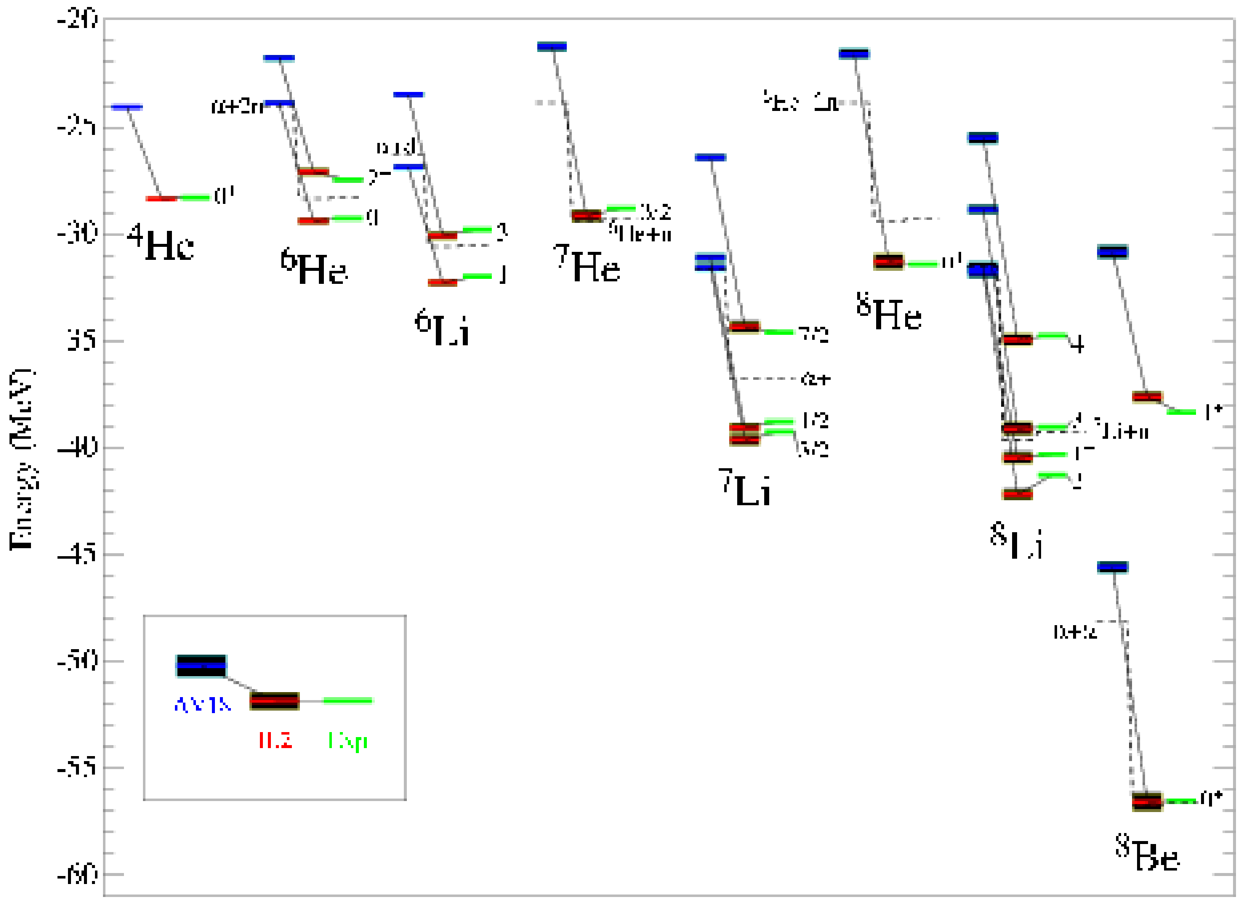
^3H Binding energy

Exp 8.48 MeV

2 N potential	2N	2N + 3N
CD Bonn	7.953	8.483
Nijm II	7.709	8.477
Nijm I	7.731	8.480
Nijm 93	7.664	8.480
Reid 93	7.648	8.480
AV14	7.683	8.480
AV18	7.576	8.479

Monte Carlo

S. C. Pieper and R. B. Wiringa, Ann. Rev. Nucl. Part. Sci. 51 (2001) 53.



Number of spin-isospin configurations

$$N_{conf} = 2^A \frac{A!}{Z!(A-Z)!}$$

Nucleo	Z	N=A-Z	N_{conf}
^3He	2	1	24
^4He	2	2	96
^6He	2	4	960
^6Li	3	3	1280
^8He	2	6	7168
^{12}C	6	6	3,784,704
^{16}O	8	8	$8.4 \cdot 10^8$
^{40}Ca	20	20	$1.5 \cdot 10^{23}$
^{48}Ca	20	28	$4.7 \cdot 10^{27}$

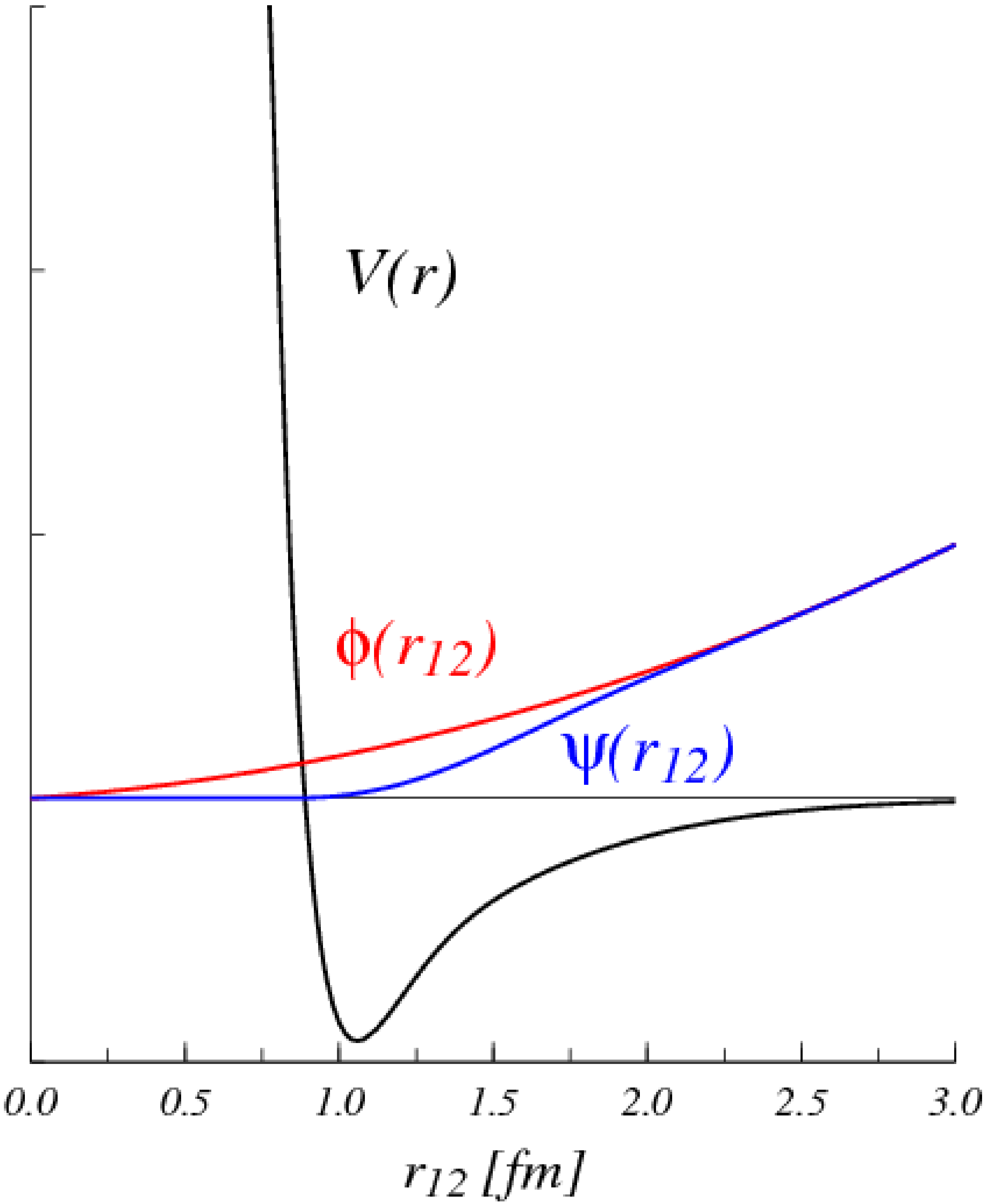
Perturbation theory

$$H = H_0 + H_1$$

$$H_0|\Phi_0\rangle = E_0|\Phi_0\rangle$$

$$H_0|\Phi_0\rangle = \sum_i h_i \prod_{k=1,A} |\phi_k\rangle = \sum_i \epsilon_i \prod_{k=1,A} |\phi_k\rangle$$

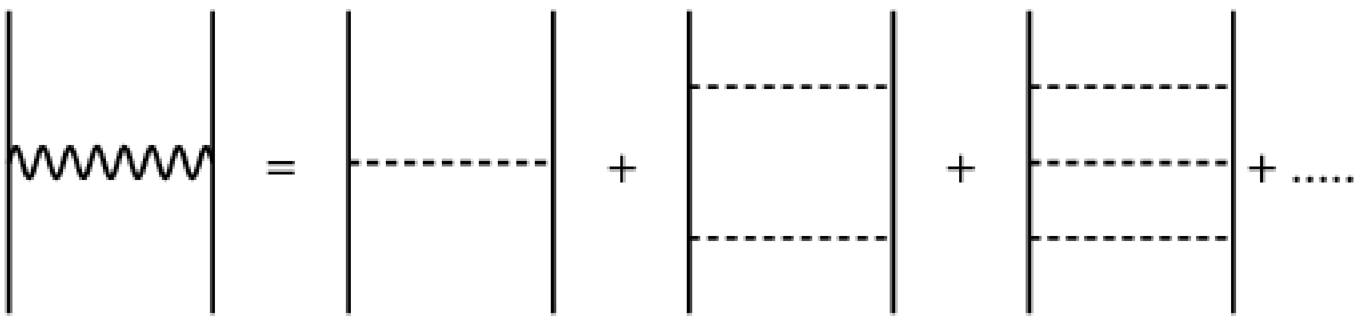
$$\begin{aligned} E &= \langle\Phi_0|H_0|\Phi_0\rangle + \langle\Phi_0|H_1 \sum_{n=0}^{\infty} \left(\frac{1}{E_0 - H_0} H_1\right)^n |\Phi_0\rangle_c \\ &= E_0 + \langle\Phi_0|H_1|\Phi_0\rangle + \langle\Phi_0|H_1 \frac{1}{E_0 - H_0} H_1|\Phi_0\rangle_c + \cdots \end{aligned}$$



V(r)
φ(r12)
ψ(r12)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
r12 [fm]

G matrix

$$\langle\Phi_0|G(\omega)|\Phi_0\rangle = \langle\Phi_0|H_1|\Phi_0\rangle + \sum_{p,q>F}\frac{\langle\Phi_0|H_1|\Phi_{pq}\rangle\langle\Phi_{pq}|G(\omega)|\Phi_0\rangle}{\omega-\epsilon_p-\epsilon_q}$$



G

Correlated Basis Function

$$\Psi(1,2,...,A) = F(1,2,...,A)\Phi(1,2,....,A)$$

$$\delta E[\Psi] = \delta \left[\frac{< \Psi|H|\Psi >}{< \Psi|\Psi >} \right] = \delta \left[\frac{< \Phi|F^{\dagger}HF|\Phi >}{< \Psi|\Psi >} \right] = 0$$