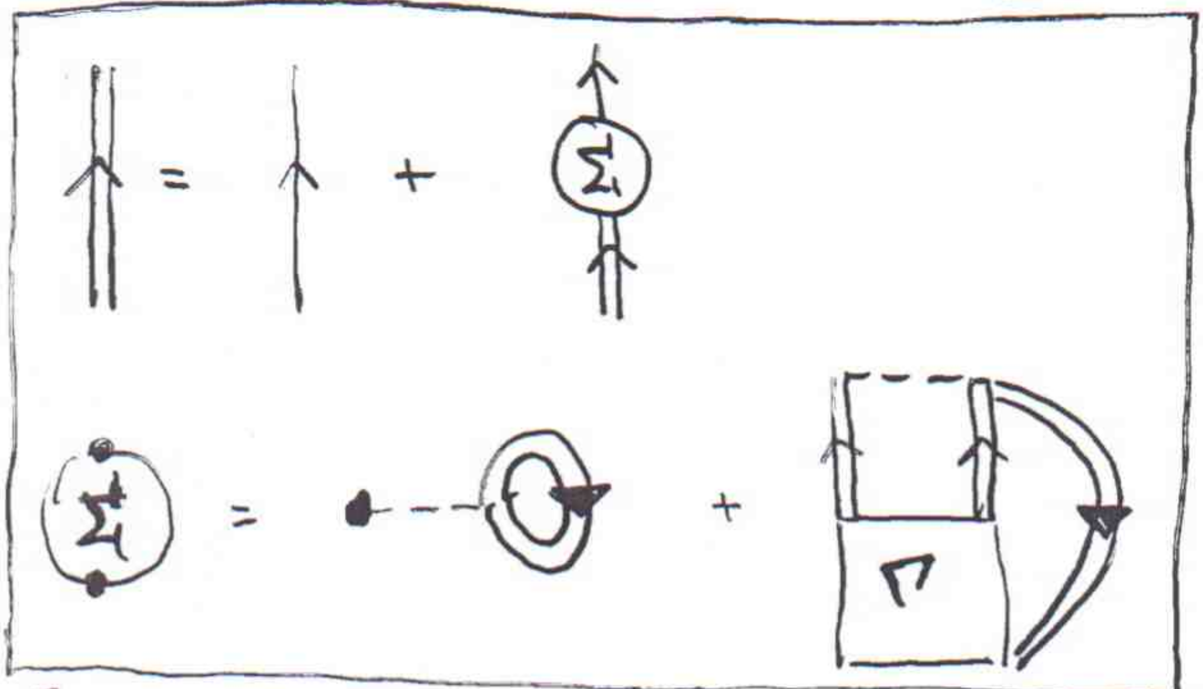
Due to the short-range repulsion we must propagate particles to all orders ⇒ ladder approach ⇒ we treat the holes at the same foot ⇒ Galitskii - Feynman G-matrix

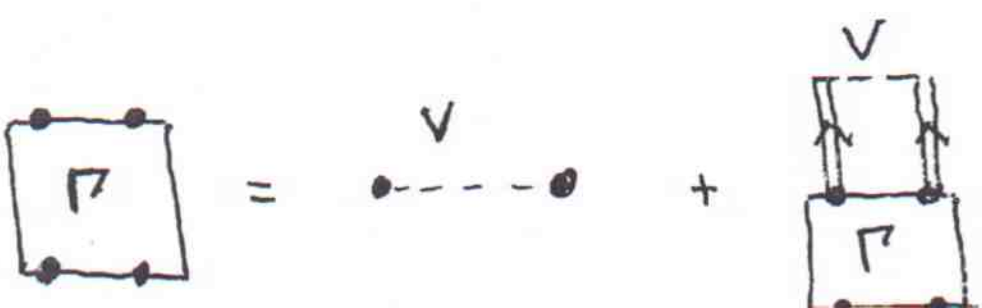
To propagate holes is crucial to get $S_h(k,E)$!

Finally we are facing a self-consistent problem



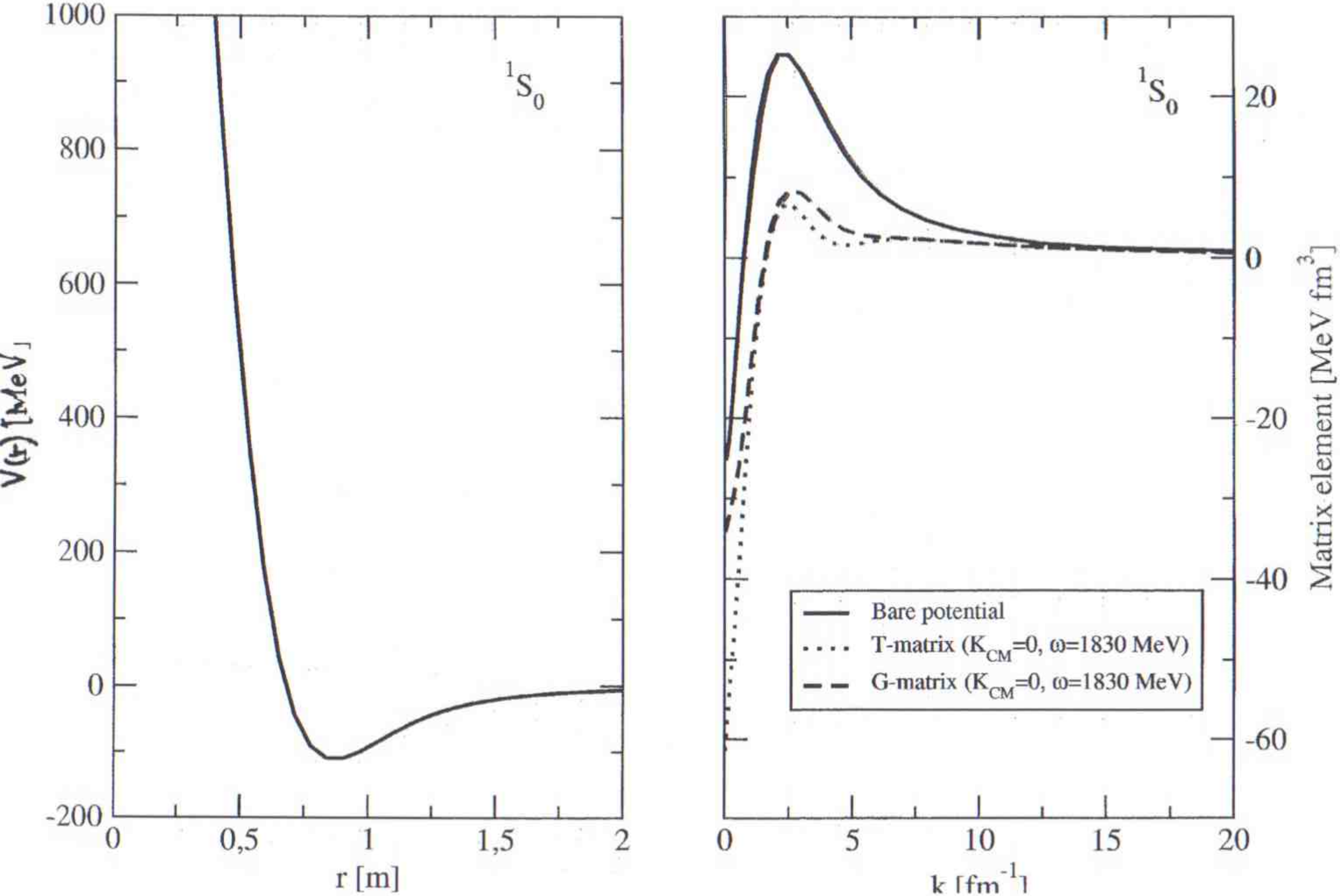
Feynman diagrams propagate particles and holes.

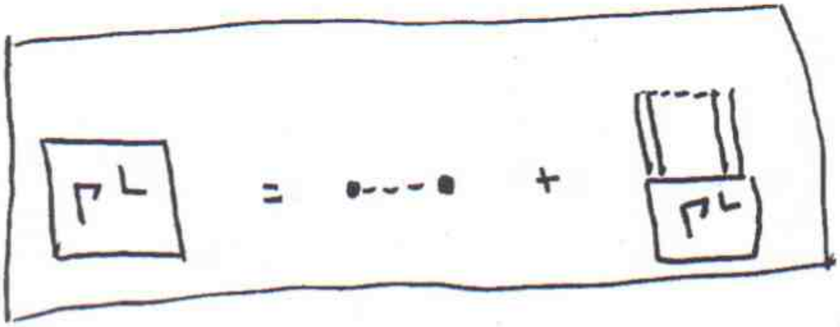
Γ contains propagation of particles and holes to all orders!



Ladder approximation propagating particles and holes

ρ = 0.4 fm⁻³
ω = 2 Mn − 50 MeV





$$\langle \vec{k}_r | \Gamma^L(K_{cm}, \Omega) | \vec{k}'_r \rangle = \langle \bar{k}_r | V | \bar{k}'_r \rangle +$$

$$+ \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3}\, i \int \frac{d\omega}{2\pi} \langle \bar{k}_r | V | \bar{q} \rangle\, g\left(\frac{\bar{K}_{cm}}{2} + \bar{q}, \frac{\Omega}{2} + \omega\right)$$

$$g\left(\frac{\bar{K}_{cm}}{2} - \bar{q}, \frac{\Omega}{2} - \omega\right) \langle \bar{q} | \Gamma^L(K_{cm}, \Omega) | \bar{k}'_r \rangle$$

$g(k, \omega)$ contains the full spectral function.

one gets propagation of two-particles or two-holes

Approximation using Hartree-Fock propagators ...

$$\langle \vec{k}_r | \Gamma^L(K_{cm}, \Omega) | \bar{k}'_r \rangle = \langle \bar{k}_r | V | \bar{k}'_r \rangle +$$

$$+ \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3} \langle \bar{k}_r | V | \bar{q} \rangle\, g^{II}_{HF}(q; K_{cm}, \Omega) \langle \bar{q} | \Gamma^L(\bar{K}_{cm}, \Omega) |$$

two-particles

$$g^{II}_{HF}(\bar{q}, \bar{K}_{cm}, \Omega) = \frac{\theta\left(\left|\frac{\bar{K}_{cm}}{2} + \bar{q}\right| - k_F\right) \theta\left(\left|\frac{\bar{K}_{cm}}{2} - \bar{q}\right| - k_F\right)}{\Omega - \varepsilon\left(\left|\frac{\bar{K}_{cm}}{2} + \bar{q}\right|\right) - \varepsilon\left(\left|\frac{\bar{K}_{cm}}{2} - \bar{q}\right|\right) + i\eta} -$$

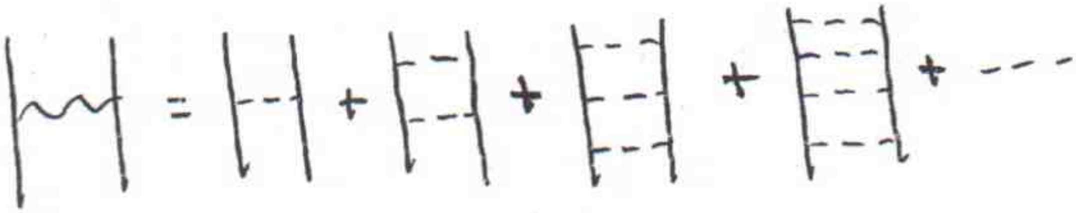
two-holes

$$\frac{\theta\left(k_F - \left|\frac{K_{cm}}{2} + q\right|\right) \theta\left(k_F - \left|\frac{K_{cm}}{2} - q\right|\right)}{\Omega - \varepsilon\left(\left|\frac{\bar{K}_{cm}}{2} + \bar{q}\right|\right) - \varepsilon\left(\left|\frac{\bar{K}_{cm}}{2} - \bar{q}\right|\right) - i\eta}$$

If you consider only propagation of particles ⇒ G matrix

$$\langle \bar{k}_r | G(\bar{K}, \Omega) | \bar{k}_r' \rangle = \langle \bar{k}_r | V | \bar{k}_r' \rangle + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \langle \bar{k}_r | V | \bar{q} \rangle \frac{\Theta\left(\left|\frac{\bar{K}_{cm}}{2} + \bar{q}\right| - k_F\right) \Theta\left(\left|\frac{\bar{K}_{cm}}{2} - \bar{q}\right| - k_F\right)}{\Omega - \varepsilon\left(\left|\frac{\bar{K}_{cm}}{2} + \bar{q}\right|\right) - \varepsilon\left(\left|\frac{\bar{K}_{cm}}{2} - \bar{q}\right|\right)} \langle \bar{q} | G(K, \Omega) | \bar{k}_r' \rangle$$

$$G(E) = V + V \frac{Q_{pp}}{E - H_0 + i\eta} G(E)$$



$$G = V + V \frac{Q_{pp}}{E - H_0 + i\eta} V + V \frac{Q_{pp}}{E - H_0 + i\eta} V \frac{Q_{pp}}{E - H_0 + i\eta} V + \cdots$$

To reduce dimensionality ⇒ Partial wave decomposition. but you need:

- → Average Pauli Operator to eliminate the dependence on the angle between K_{cm} and $\bar{q}$
- → Average the angle in $\varepsilon\left(\left|\frac{K_{cm}}{2} \pm q\right|\right)$

$$H_0 = T + U = \sum_i (t_i + u_i)$$

$$H_1 = V - U = \sum_{i<j} v_{ij} - \sum_i u_i$$

Lowest order of the hole-line expansion.

$$\frac{B}{N} = \langle T \rangle + \langle G \rangle_{FS} = \frac{1}{N}\sum_k \frac{\hbar^2 k^2}{2m} + \frac{1}{2}\frac{1}{N}\sum_{ij<k_F} \langle ij | G(\varepsilon(i)+\varepsilon(j)) | ij \rangle_A$$

BHF sp potential

$$\left.\begin{aligned} U(k) &= \mathrm{Re} \sum_{j<k_F} \langle \bar{k}\bar{j} | G(\varepsilon(k)+\varepsilon(j)) | \bar{k}\bar{j} \rangle \\ G(E) &= V + V \frac{Q_{pp}}{E - H_0 + i\eta} G(E) \end{aligned}\right\} \text{self-consistent problem!}$$

then

$$\frac{B}{N} = \sum_{k<k_F} \left\{ \frac{\hbar^2 k_F^2}{2m} + \frac{1}{2} U(k) \right\}$$

$$S_h(k,E) = \theta(k_F - k)\, \delta(E - \varepsilon(k))$$

$$\varepsilon(k) = U(k) + \frac{\hbar^2 k^2}{2m}$$

standard prescription

$$U(k) = \sum_{j<k_F} \langle kj | G(\varepsilon(k)+\varepsilon(j)) | kj \rangle \qquad k<k_F$$

$$= 0 \qquad k>k_F$$

continuous

$$U(k) = \mathrm{Re} \sum_{j<k_F} \langle kj | G(\varepsilon(k)+\varepsilon(j)) | kj \rangle_A \quad \forall k$$

$$H_0 = T + U = \sum_i (t_i + u_i)$$

$$H_1 = V - U = \sum_{i<j} v_{ij} - \sum_i u_i$$

Lowest order of the hole-line expansion.

$$\frac{B}{N} = \langle T \rangle + \langle G \rangle_{FS} = \frac{1}{N}\sum_k \frac{\hbar^2 k^2}{2m} + \frac{1}{2}\frac{1}{N} \sum_{ij<k_F} \langle ij | G(\varepsilon(i)+\varepsilon(j)) | ij \rangle_A$$

BHF sp potential

$$U(k) = \mathrm{Re} \sum_{j<k_F} \langle \bar{k}\bar{j} | G(\varepsilon(k)+\varepsilon(j)) | \bar{k}\bar{j} \rangle$$

$$G(E) = V + V \frac{Q_{pp}}{E - H_0 + i\eta} G(E)$$

self-consistent problem!

$$S_h(k,E) = \theta(k_F - k)\,\delta(E - \varepsilon(k))$$

$$\varepsilon(k) = U(k) + \frac{\hbar^2 k^2}{2m}$$

then

$$\frac{B}{N} = \sum_{k<k_F} \left\{ \frac{\hbar^2 k_F^2}{2m} + \frac{1}{2} U(k) \right\}$$

standard prescription

$$U(k) = \sum_{j<k_F} \langle kj | G(\varepsilon(k)+\varepsilon(j)) | kj \rangle \qquad k<k_F$$

$$= 0 \qquad k>k_F$$

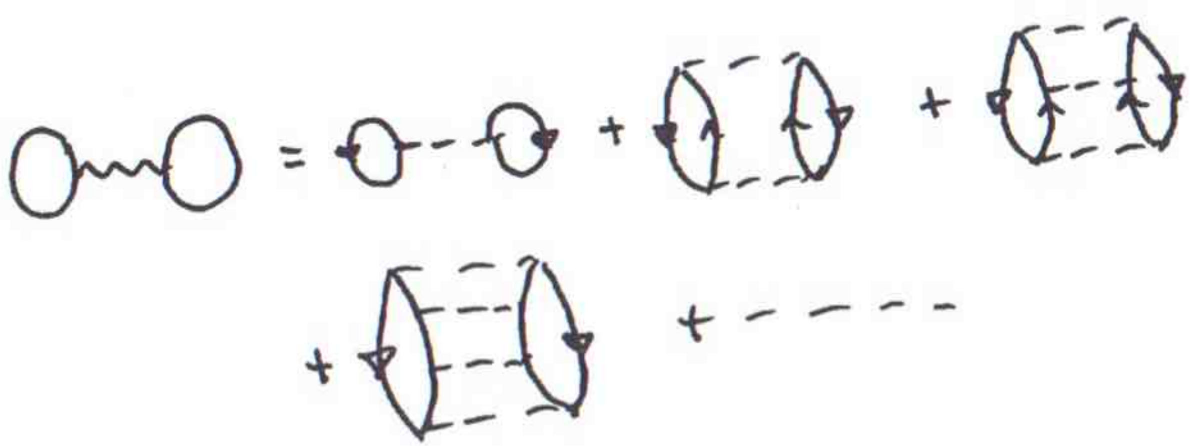
continuous

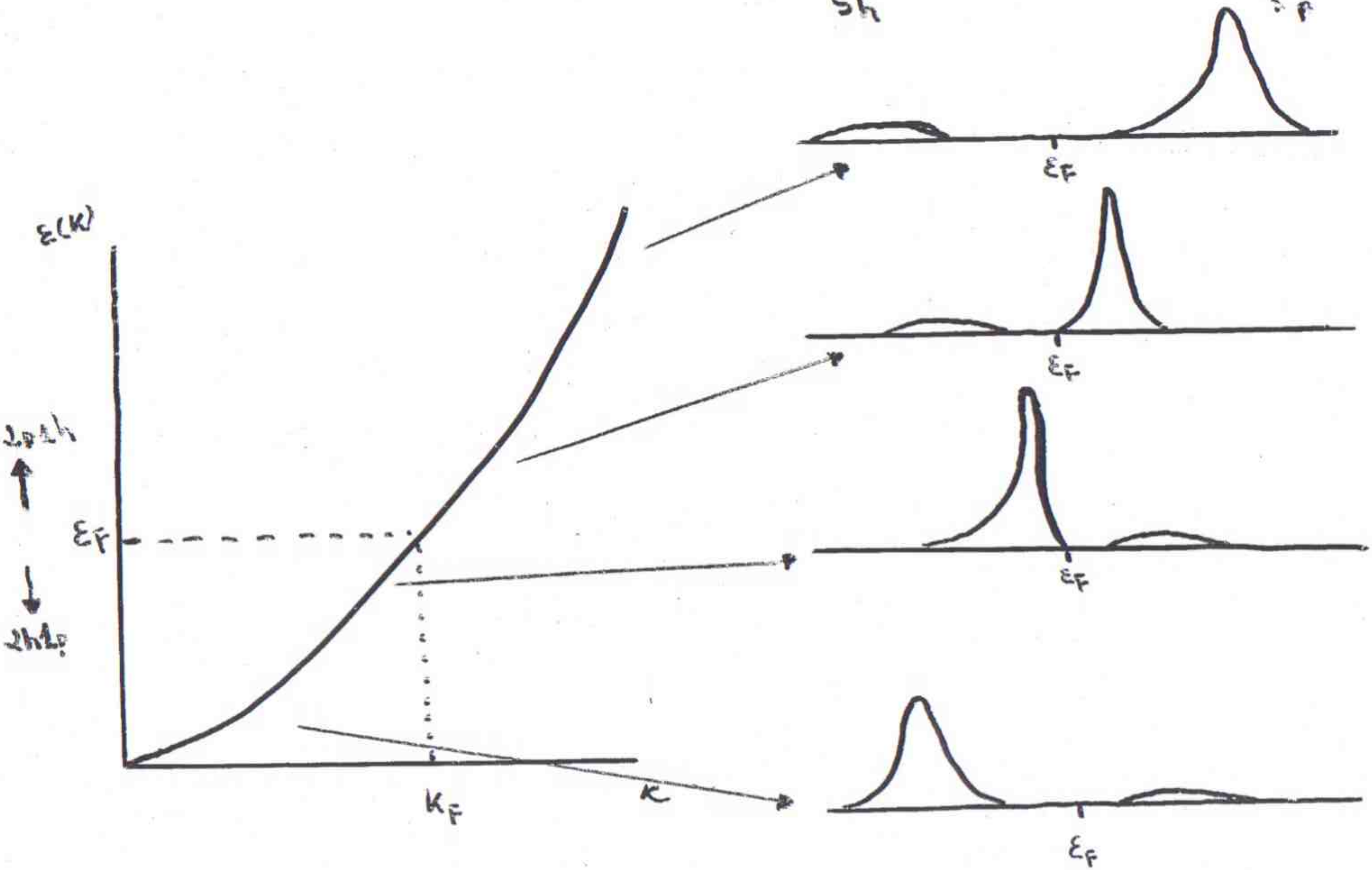
$$U(k) = \mathrm{Re} \sum_{j<k_F} \langle kj | G(\varepsilon(k)+\varepsilon(j)) | kj \rangle_A \quad \forall k$$

In the Brueckner - Goldstone expansion the average binding energy per nucleon is expanded in a series of terms having the same structure as those of the previous series replacing V by G.

The leading term:

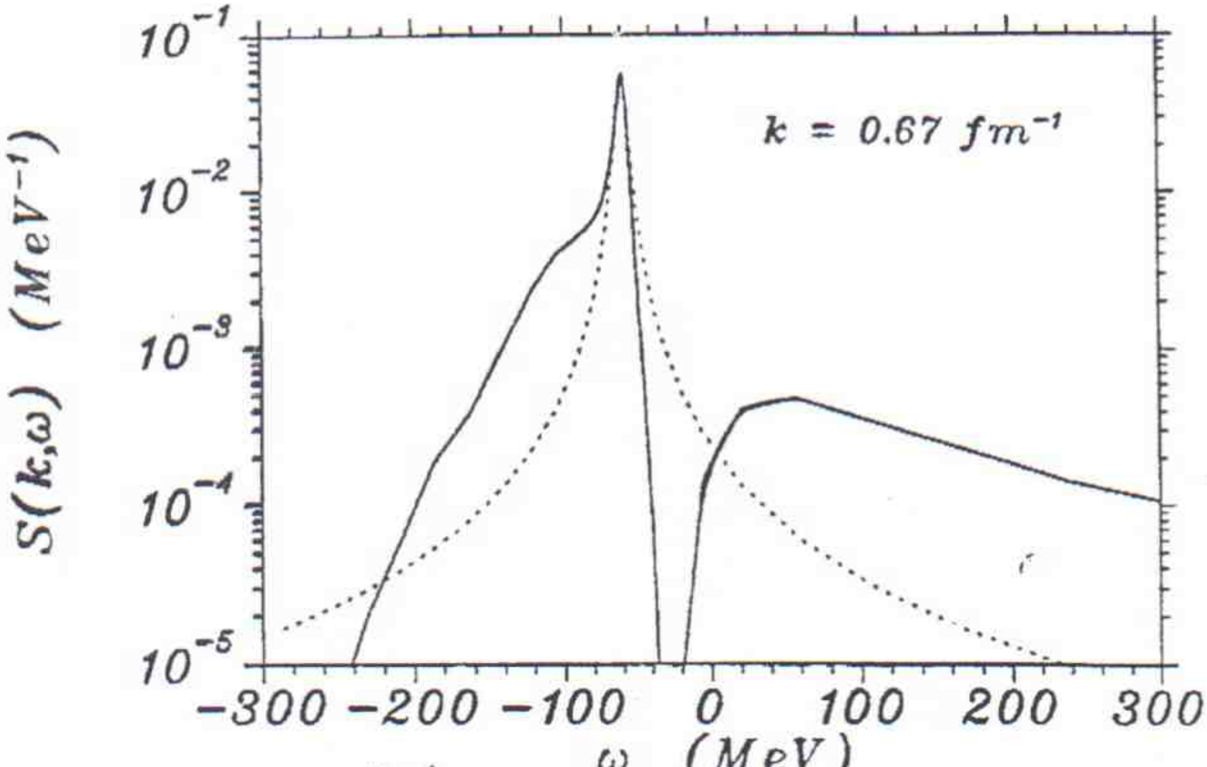
$$\frac{B}{N} = \langle T \rangle + \langle \hat{G} \rangle = \frac{1}{N} \sum_{k}' \frac{\hbar^2 k^2}{2m} + \frac{1}{2} \sum_{k_{ij} < k_F}' \langle kj | G(\epsilon(k) + \epsilon(j)) | kj \rangle_a$$





S_h
S_p
$\varepsilon(k)$
2p1h
ε_F
2h1p
k_F
k
ε_F
ε_F
ε_F
ε_F

Single-particle spectral functions

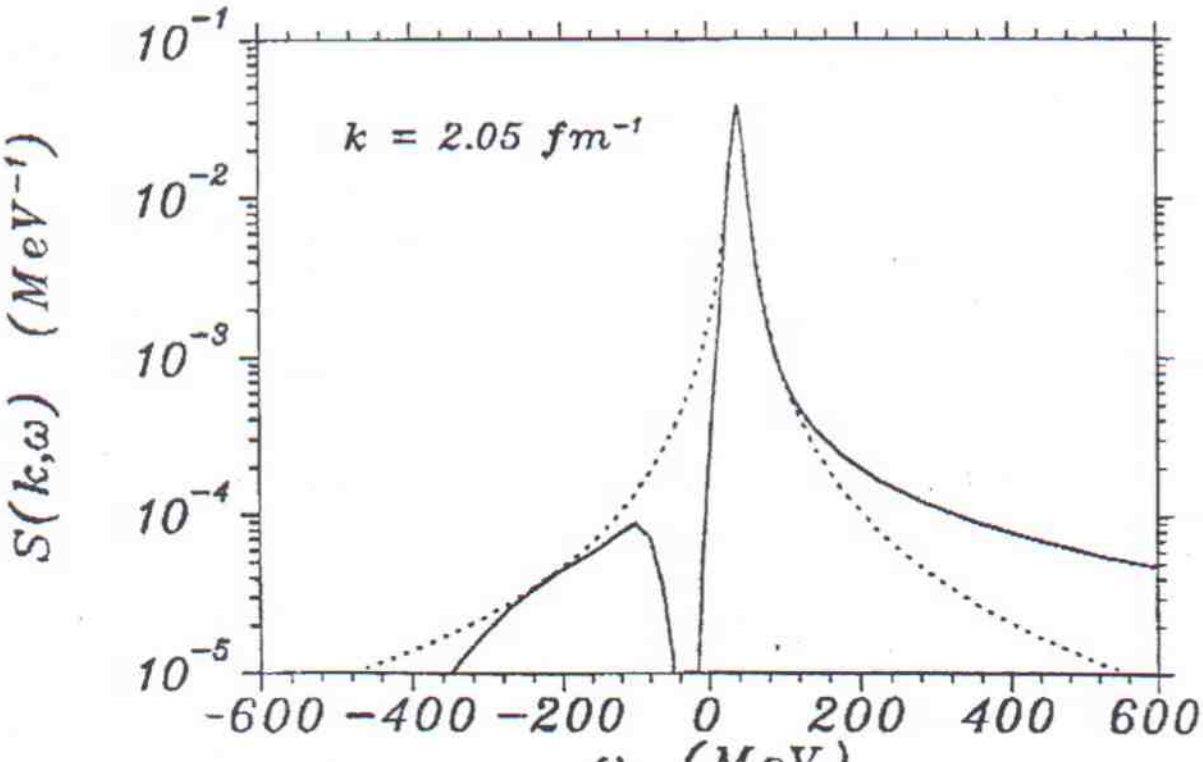


$\varepsilon_{qp} = -59.6$ MeV

$\text{Im}\,\Sigma^1_1(k, \varepsilon_{qp}) = 5.72$ MeV

$n(k) = 0.8$[illegible]

$$S_{qp} = \frac{1}{\pi} \frac{z^2(k)\ |w(k)|}{(\omega - E(k))^2 + (z(k)\,w(k))^2}$$



$\varepsilon_{qp} = 40.38$ MeV

$\text{Im}\,\Sigma^1_1(k, \varepsilon_{qp}) = -8.44$ MeV

$n(k) = 0.0$[illegible]

QUASI-PARTICLE APPROXIMATION

$$\varepsilon(k) = \frac{k^2}{2m} + u(k)$$

$$u(k) \equiv \mathrm{Re}\, \Sigma_1\big(k, \varepsilon(k)\big)$$

$$w(k) \equiv \mathrm{Im}\, \Sigma_1\big(k, \varepsilon(w)\big)$$

$$Z(k) = \left\{ 1 - \frac{\partial \mathrm{Re}\, \Sigma_1(k,w)}{\partial w} \right\}^{-1}$$

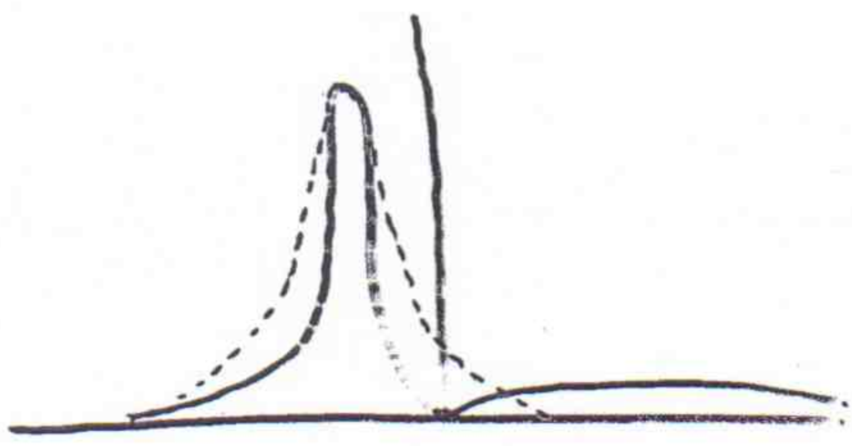
$$g(k,w) = \frac{1}{w - \frac{k^2}{2m} - \Sigma_1(k,w)}$$

$$\downarrow$$

$$g_{QP} = \frac{Z(k)}{w - \varepsilon(k) - i\,Z(k)\cdot w(k)}$$

$$S_{QP} = \frac{1}{\pi} \frac{Z^2(k) \;\; |w(k)|}{(w - \varepsilon(k))^2 + (Z(k)\, w(k))^2}$$

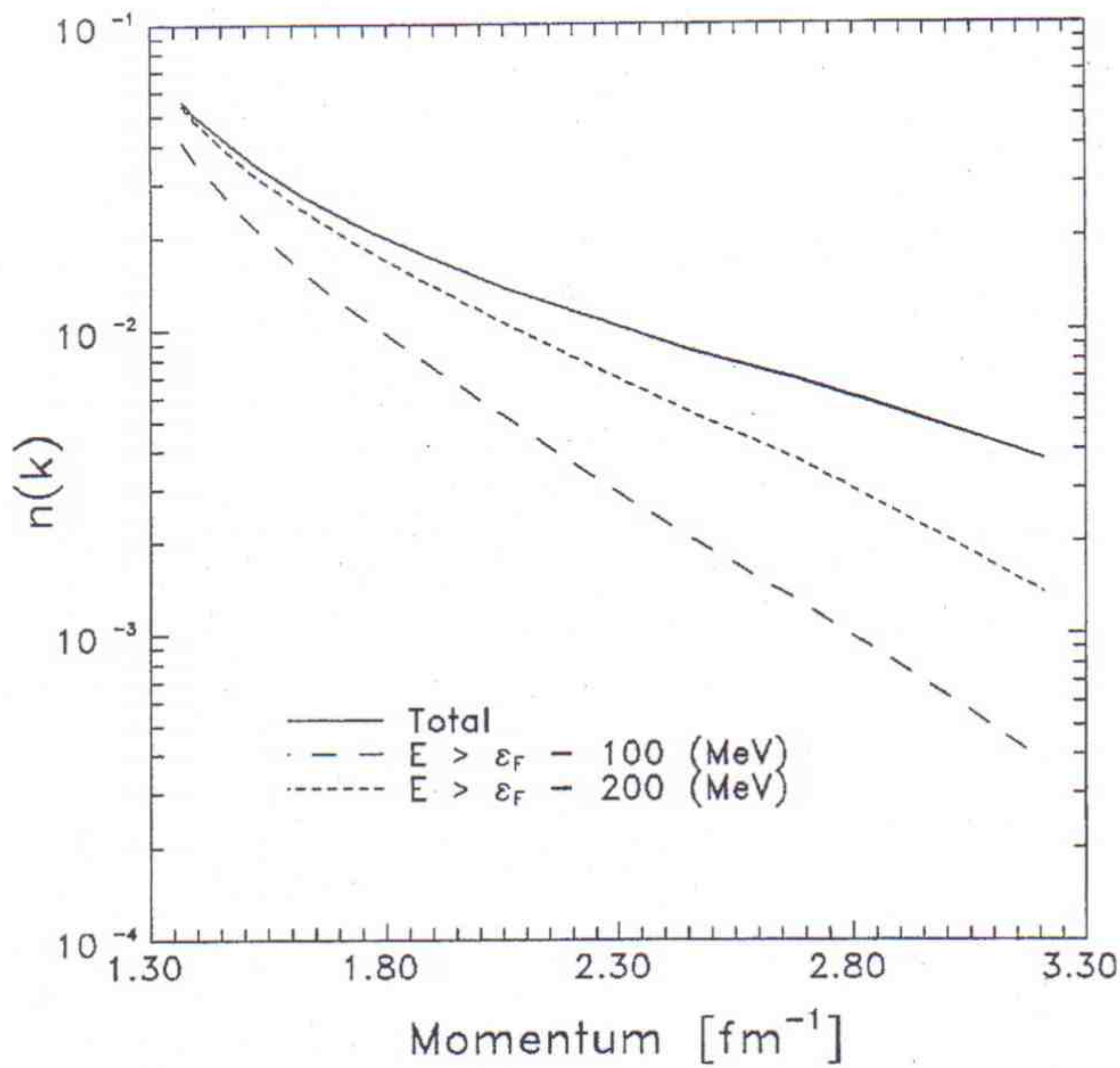
$$S(k,w) = S_{QP}(k,w) + S_{BG}(k,w)$$

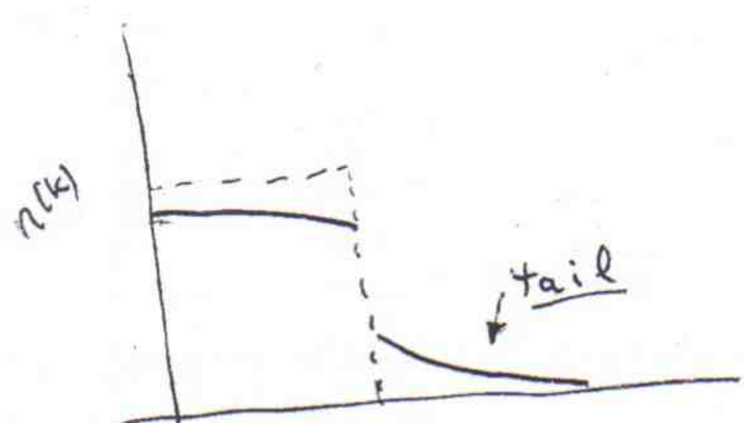


ε_F

* $\int_{-\infty}^{\infty} dw\, S_{QP}(k,w) = Z(k)$
* maximum at $\varepsilon(k)$
* amplitud
* height $\frac{1}{\quad}$

n(k) for nuclear matter   k > k_F





Dynamic Structure function or density response

$$S(q,\omega) = -\frac{1}{\rho\pi}\, \mathrm{Im}\, \Pi(q,\omega)$$ Polarization Propagator

$$= \frac{1}{\rho\pi}\, \mathrm{Im} \langle \Psi_A | \rho^{+}(q) \frac{1}{H - E_A - \omega - i\eta} \rho(q) | \Psi_A \rangle$$

$$\rho(q) = \sum_{k} a^{+}_{\bar{k}}\, a_{\bar{k}-\bar{q}} = \sum_{i=1}^{A} e^{i\bar{q}\bar{r}_i}$$ Density fluctuation operator

The zero order term:

$$S_{DB} = \frac{\nu}{(2\pi)^3\rho} \int_0^{\infty} d^3k \int_{\varepsilon_F}^{\infty} d\omega'' \int_{-\infty}^{\varepsilon_F} d\omega'\; S_h(k,\omega')\; S_p(|\bar{k}+\bar{q}|,\omega'')\; \delta\big(\omega - (\omega''-\omega')\big)$$

$$= \frac{\nu}{(2\pi)^3\rho} \int d^3k \int_{-\infty}^{\varepsilon_F} d\omega'\; S_h(k,\omega')\; S_p(|\bar{k}+\bar{q}|, \omega+\omega')$$

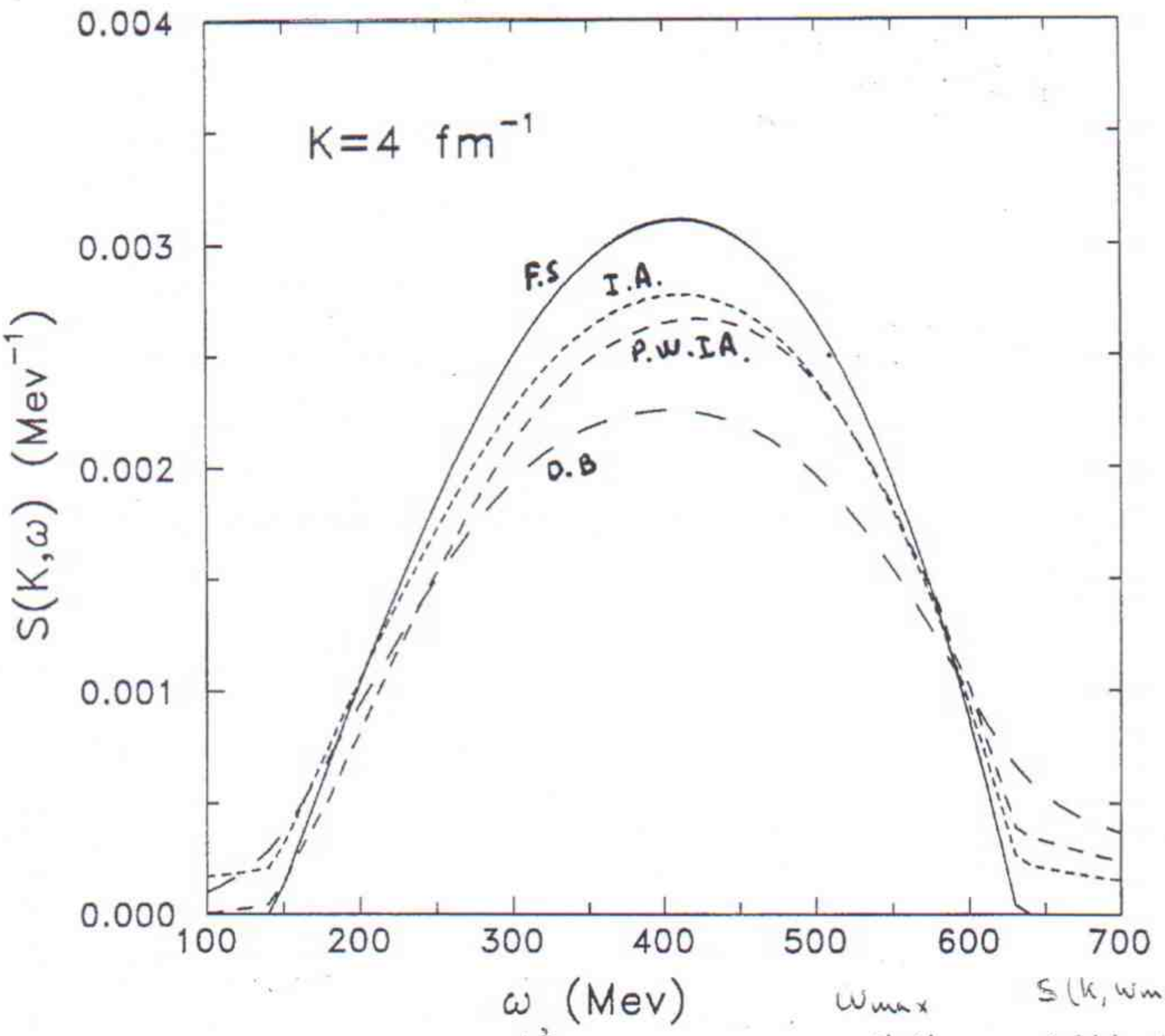
When S_h and S_p are δ's we get the Lindhard function

S_p gives a measure of the Final State Interactions (FSI) of the struck nucleon travelling through the nuclear medium.

$S(k,\omega)$ for Nuclear Matter

Reid Potential $k_F = 1.36\ fm^{-1}$

Depletion, Fragmentation (e,e'p) $\Longrightarrow$ Quenching, Spreading (e,e')



			ω_{max}	$S(k,\omega_{max})$
	———	Free $E(k) = \frac{k^2}{2m} + U(k)$	415 MeV	$3.11 \cdot 10^{-3}$
27%	— — —	D.B.	400 MeV	$2.25 \cdot 10^{-3}$
14%	- - - -	PWIA	420 Mel	$2.66 \cdot 10^{-3}$
11%	--------	I.A.	415 MeV	$2.77 \cdot 10^{-3}$

Strength on the peak

F.S ≃ 100% D.B ≃ 80% PWIA ≃ 90% I.A ≃ 93%

If, $S_p(\vec{k}, E) = \delta(E - \varepsilon(k))\, \Theta(k - k_F)$ i.e. plane wave.

PWIA

$$S_{PWIA}(q, \omega) = \frac{\nu}{(2\pi)^3 \rho} \int d^3k \int_{-\infty}^{\varepsilon_F} d\omega' \; S_h(k, \omega') \cdot \delta\left(\omega - \left(\varepsilon(|\vec{k} + \vec{q}|) - \omega'\right)\right) \Theta(|\vec{k} + \vec{q}| - k_F)$$

* Neglects the spreading of the strength produced by FSI.
* Still includes a real energy shift.

I. A.

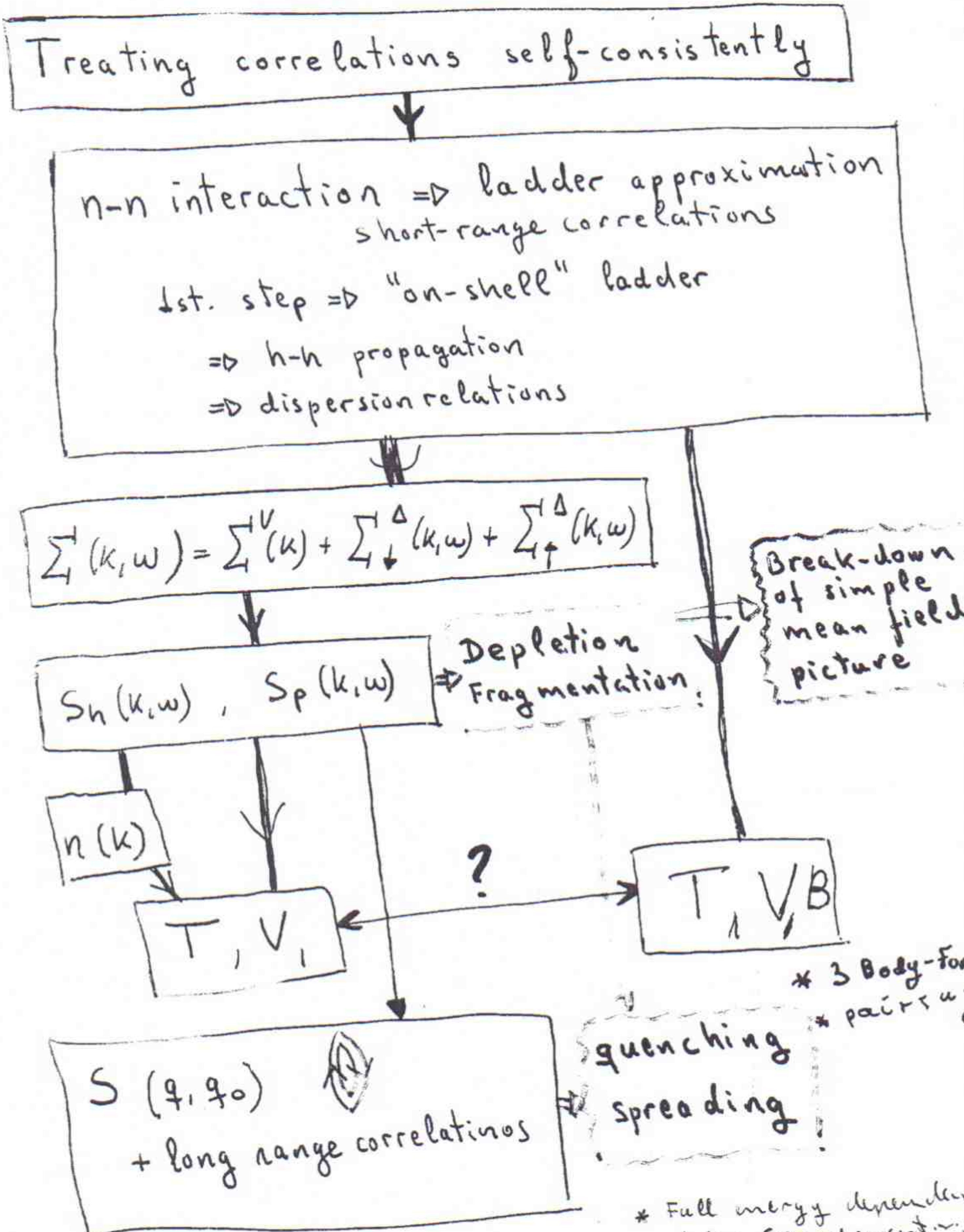
* Neglects the energy dependence of S_h and the correlations of the initial state are averaged by means of the occupation number $n(k)$.

$$S_{IA}(q, \omega) = \frac{\nu}{(2\pi)^3 \rho} \int d^3k \;\; n(k) \;\; \delta\left(\omega - \left(\varepsilon(|\vec{k} + \vec{q}|) - \varepsilon(k)\right)\right)$$

* If free kinematics is used:

$$S_{IA}(q, \omega) = \frac{\nu}{(2\pi)^3 \rho} \int d^3k \;\; n(k) \;\; \delta\left(\omega - \frac{\hbar^2 q^2}{2m} - \frac{\hbar^2}{m} \vec{q} \cdot \vec{k}\right)$$

$$= \frac{\nu}{4\pi^2 \rho} \; \frac{m}{\hbar^2 q} \int_{\bar{q}}^{\infty} dk \;\; n(k)\, k \qquad \bar{q} = \left| \frac{q}{2} - \frac{E m}{\hbar^2 q} \right|$$



* Full energy dependence of the sp propagator in the calculation of the vertex funct

OTher ~~REALISTIC~~ INTERACTIONS

* Calculation of