Electromagnetic structure of A = 2 and 3 nuclei in chiral effective field theory

Maria Piarulli @ R. Anni School 2013, Otranto, Italy

In collaboration with:

Rocco Schiavilla
Luca Girlanda, Laura E. Marcucci, Saori Pastore, Michele Viviani

S. Pastore, R. Schiavilla, and J.L. Goity Phys. Rev. C **78**, 064002 (2008)
S. Pastore *et al.* Phys. Rev. C **80**, 034004 (2009)
L. Girlanda *et al*. Phys. Rev. C **81**, 034005 (2010)
S. Pastore *et al*. Phys. Rev. C **84**, 024001 (2011)
M. Piarulli *et al*. Phys. Rev. C **87**, 014006 (2013)

Outline

- Motivations
- Nuclear χEFT approach
- Formalism:
  - *NN* Potential
  - Current Operators
  - Charge Operators
- Results:
  - Static properties and e.m. f.fs of the deuteron
  - Static properties and e.m. f.fs of the trinucleons
- Outlook

Motivations

❖ Quantitative knowledge of *NN* forces is crucial in order to understand the properties of nuclei and nuclear matter

❖ QCD:
- quarks, gluons
- weak at short distance (asymptotic freedom)
- strong at long distance (1 fm) or low energies (low-energy QCD)

↓

❖ Nuclear physics: difficult to derive it in terms of quarks and gluons

❖ EFT applied to low-energy QCD (χEFT)

Nuclear χEFT Approach

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

❖ χEFT uses the chiral-symmetry to constraint the interactions of π's among themselves, N's and γ-fields

❖ π's couple by powers of its momentum Q, and the Lagrangian ($\mathcal{L}_{eff}$) can be expanded systematically in powers of Q/Λ; ($Q << \Lambda \approx 1$ GeV is the chiral-symmetry breaking scale) allowing for a perturbative treatment in terms of Q expansions

❖ χEFT gives a perturbative expansion of the $\mathcal{L}_{eff}$ in power of Q

$$\mathcal{L}_{eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

❖ The unknown coefficients of the perturbative expansion are called LEC's and are determined fitting the experimental data

❖ The χ-expansion gives rise to chiral potentials (Entem-Machleidt, Epelbaum-Gloeckle-Meissner) and external currents can be naturally incorporated

❖ The χ-expansion can be used to evaluate physical observables to any desired accuracy (example e.m. form factors)

Formalism: Transition Amplitude in TOPT

I. Nucleon-nucleon potential ($NN \rightarrow NN$)

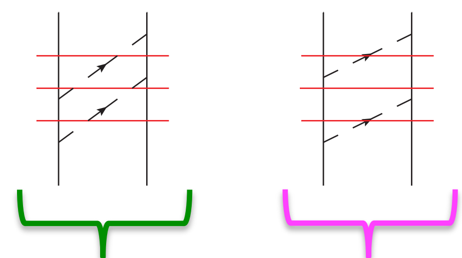
- Time-ordered perturbation theory (TOPT)

$$\langle N'N'|T|NN\rangle = \langle N'N'|H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} |NN\rangle$$

  - H_0 = free N's, π's Hamiltonians
  - H_1 = interacting π, N and γ Hamiltonians implied by $\mathcal{L}_{eff}$
  - E_i = initial energy
  - Completeness: $\sum_{I_i} |I_i\rangle\langle I_i| = 1$ between successive terms of H_1
    - $$\begin{aligned} \langle f|T|i\rangle &= \langle f|H_1|i\rangle + \sum_{I_1} \langle f|H_1|I_1\rangle \frac{1}{E_i - E_1 + i\eta} \langle I_1|H_1|i\rangle \\ &+ \sum_{I_1, I_2} \langle f|H_1|I_2\rangle \frac{1}{E_i - E_2 + i\eta} \langle I_2|H_1|I_1\rangle \frac{1}{E_i - E_1 + i\eta} \langle I_1|H_1|i\rangle + ... \end{aligned}$$

Formalism con't

- Two kinds of diagrams: reducible and irreducible



- n vertices represented by $\langle I_j | H_1 | I_k \rangle$

- n-1 energy denominators $(E_i - E_k + i\eta)^{-1}$, $k = 1, ..., (n-1)$

$n - n_K - 1$ energy denominators:
pion and nucleonic kinetic energies

n_K energy denominators:
only nucleonic kinetic energies (Q^{-2})

small kinetic energies

$$\frac{1}{E_i - E_j + i\eta} \equiv \frac{1}{E_i - E_{I_j} - \omega_k + i\eta} = -\frac{1}{\omega_k}\left[1 + \frac{E_i - E_{I_j}}{\omega_k} + \frac{(E_i - E_{I_j})^2}{\omega_k^2} + ...\right]$$

- The expansion in power of Q is: $-\frac{1}{Q}\Big[\underbrace{1}_{\substack{\text{static limit} \\ (m_N \to \infty)}} + \underbrace{Q + Q^2 + ...}_{\text{non-static corrections}}\Big]$

Power Counting

- Chiral index is determined:

$$m = \prod_{i=1}^{n} Q^{\alpha_i - \beta_i/2} \times Q^{-(n-n_K-1)} Q^{-2n_K} \times Q^{3L}$$

  - α_i = Q-power associated to i vertex
  - β_i = number of pions at each vertex
  - L = number of loops in the diagram (Q^3)

- In chiral-expansion T -matrix can be expanded as:

$$T = T^{(0)} + T^{(1)} + T^{(2)} \ldots, \text{ and } T^{(m)} \equiv T^{(N^m LO)} \sim (Q/\Lambda)^m T^{(0)}$$

From Amplitudes to Potentials

❖ In nuclear physics the two-nucleon potential (v) is introduced, and bound and continuum two-nucleon states are derived from solutions of Schrödinger or Lippmann-Schwinger (LS) equation:

$$v + vG_0v + vG_0vG_0v + ...$$

$$G_0 = \text{two-nucleon propagator} \ \ (Q^{-2})$$

➢ The potential can be written: $v = v^{(0)} + v^{(1)} + v^{(2)} ...$ (with $v^{(m)} \sim Q^m$)

➢ Matching expansion for T with the LS equation order by order:

$$\begin{aligned}
v^{(0)} &= T^{(0)} \\
v^{(1)} &= T^{(1)} - \left[v^{(0)}G_0v^{(0)}\right] \\
v^{(2)}(\nu) &= T^{(2)} - \left[v^{(0)}G_0v^{(0)}G_0v^{(0)}\right] - \left[v^{(1)}G_0v^{(0)} + v^{(0)}G_0v^{(1)}\right] \\
v^{(3)}(\nu) &= T^{(3)} - \left[v^{(0)}G_0v^{(0)}G_0v^{(0)}G_0v^{(0)}\right] - \left[v^{(1)}G_0v^{(0)}G_0v^{(0)} + \text{permutations}\right] \\
&\quad - \left[v^{(1)}G_0v^{(1)}\right] - \left[v^{(2)}G_0v^{(1)} + v^{(0)}G_0v^{(2)}\right]
\end{aligned}$$

➢ A term like $v^{(m)}G_0v^{(n)} \sim Q^{m+n+1}$

➢ Terms accounted in the LS are subtracted from the reducible amplitude

➢ ν-dependence describes the off-the-energy shell prescription adopted for non static OPE and TPE potentials

II. Charge/Current operators ($NN\gamma \rightarrow NN$)

❖ Similar prescription for potential $v_\gamma = A^\mu J_\mu = A^0 \rho - \mathbf{A} \cdot \mathbf{J}$

❖ Time-ordered perturbation theory (TOPT)

➢ In χ-expansion T-matrix: $T_\gamma = T_\gamma^{(-3)} + T_\gamma^{(-2)} + T_\gamma^{(-1)} \ldots$ (charge operators)

$T_\gamma = T_\gamma^{(-2)} + T_\gamma^{(-1)} + T_\gamma^{(0)} \ldots\ldots$ (current operators)

with $T_\gamma^{(m)} \sim e\, Q^m$

❖ In the context of LS: $v_\gamma = v_\gamma^{(-3)} + v_\gamma^{(-2)} + v_\gamma^{(-1)} \ldots$ (charge operators)

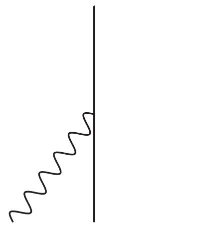
$v_\gamma = v_\gamma^{(-2)} + v_\gamma^{(-1)} + v_\gamma^{(0)} \ldots\ldots$ (current operators)

➢ Determining $v_\gamma^{(m)}$ by matching it with the field theory $T_\gamma^{(m)}$ order by order

➢ From $v_\gamma^{(m)} \rightarrow \rho^{(m)}, \mathbf{J}^{(m)}$ order by order in the χ-expansion

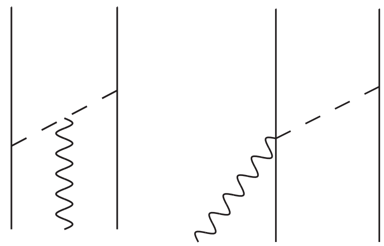
Current Operators up to N2LO (eQ^0)

- LO : $e\,Q^{-2}$



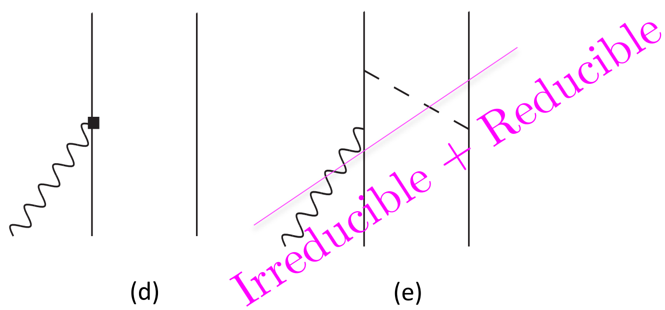
(a)

- NLO : $e\,Q^{-1}$



(b) (c)

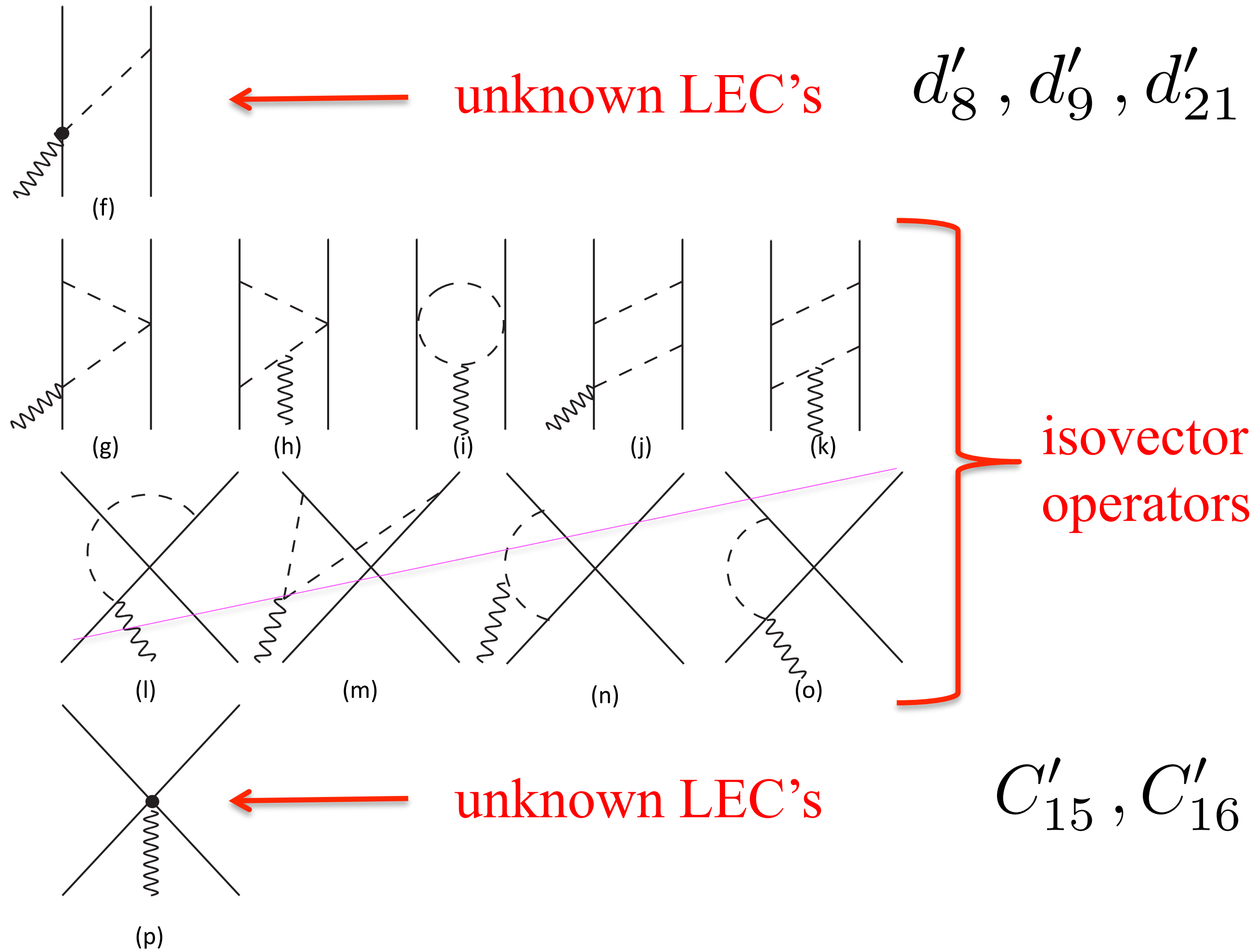
- N2LO : $e\,Q^{0}$



(d) (e)

- Recoil corrections to the reducible diagrams obtained by expanding in power the energy denominators $(E_i - E_{I_j})/\omega_k$
- Diagrams of type (e) vanish since recoil corrections (at N2LO) to the reducible diagrams cancel the static irreducible contributions

Current Operators up to N3LO (eQ)



❖ N3LO : $e\,Q$

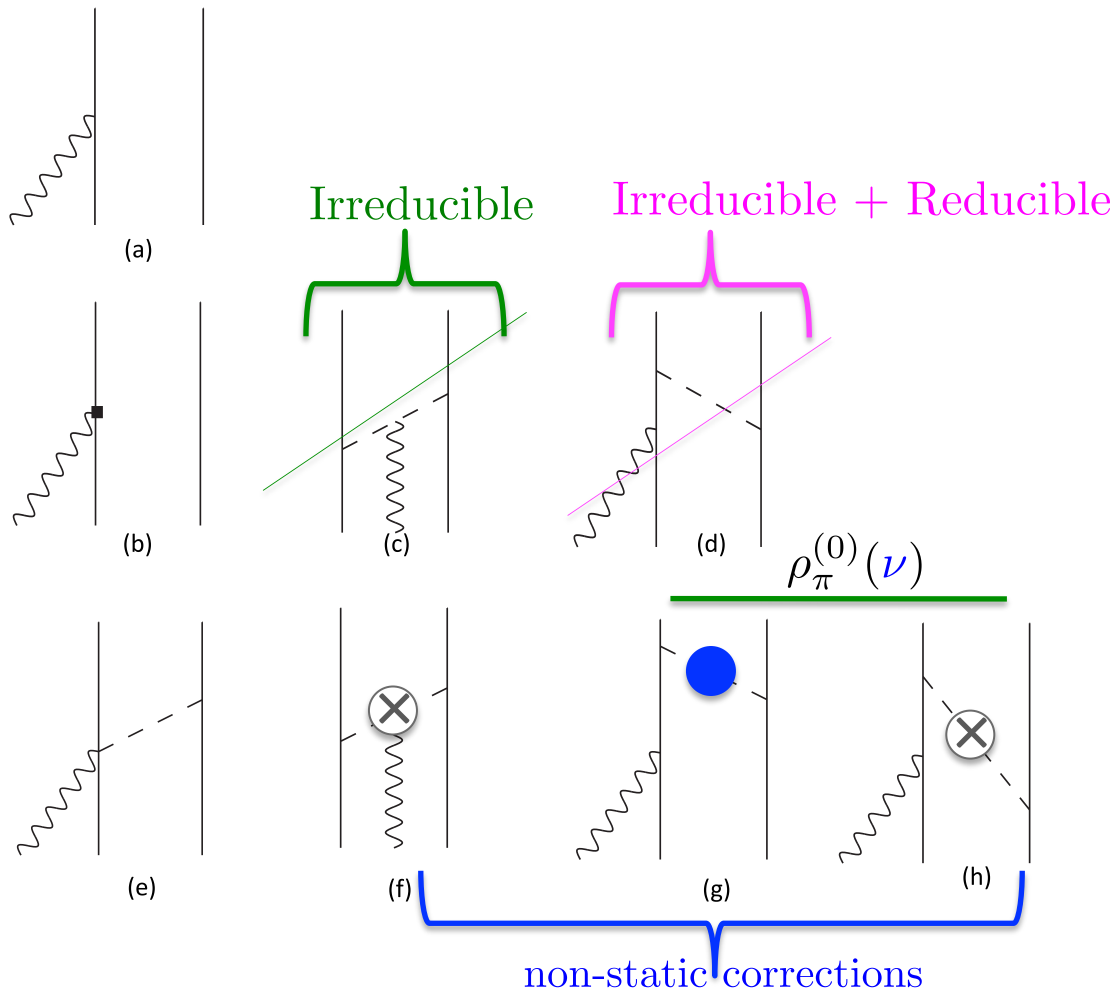
❖ Divergencies associated with loop integrals are reabsorbed by renormalization of contact terms

❖ C'_{15} and d'_9 from expt μ_d and μ_S $(^3\mathrm{He}/^3\mathrm{H})$

❖ C'_{16} from expt μ_V $(^3\mathrm{He}/^3\mathrm{H})$ m.m. and d'_8 (d'_{21}) from Δ-saturation*

Charge Operators up to N3LO (eQ^0)

- LO : $e\,Q^{-3}$
- N2LO : $e\,Q^{-1}$
- N3LO : $e\,Q^{0}$

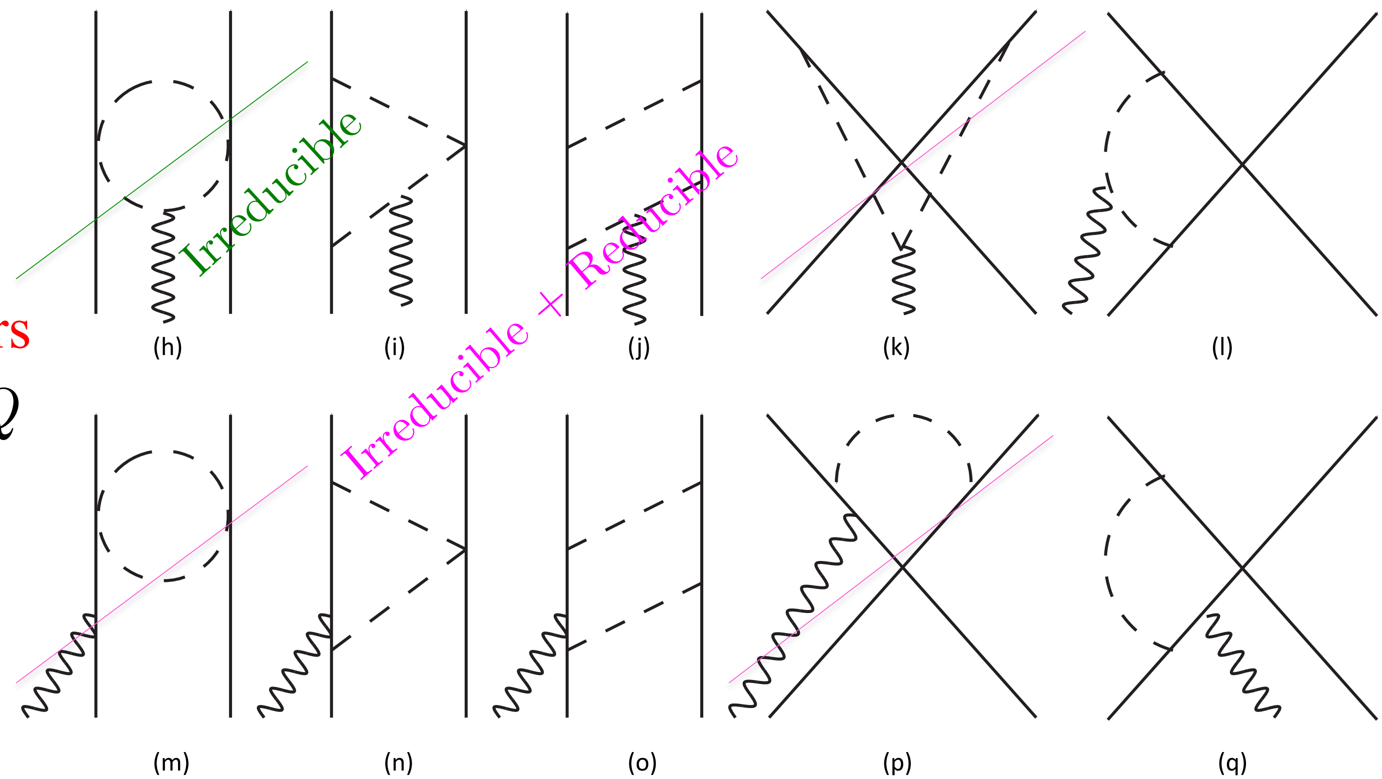


- Diagrams of type (c) vanish when six-time-ordered diagrams are summed up
- Diagrams of type (d) vanish since recoil corrections (at N2LO) to the reducible diagrams cancel the static irreducible contributions
- Non static OPE charge operator $\rho_\pi^{(0)}(\nu)$ depends on $v_\pi^{(2)}(\nu)$

Charge Operators up to N4LO (eQ)

isovector operators

❖ N4LO : $e\,Q$



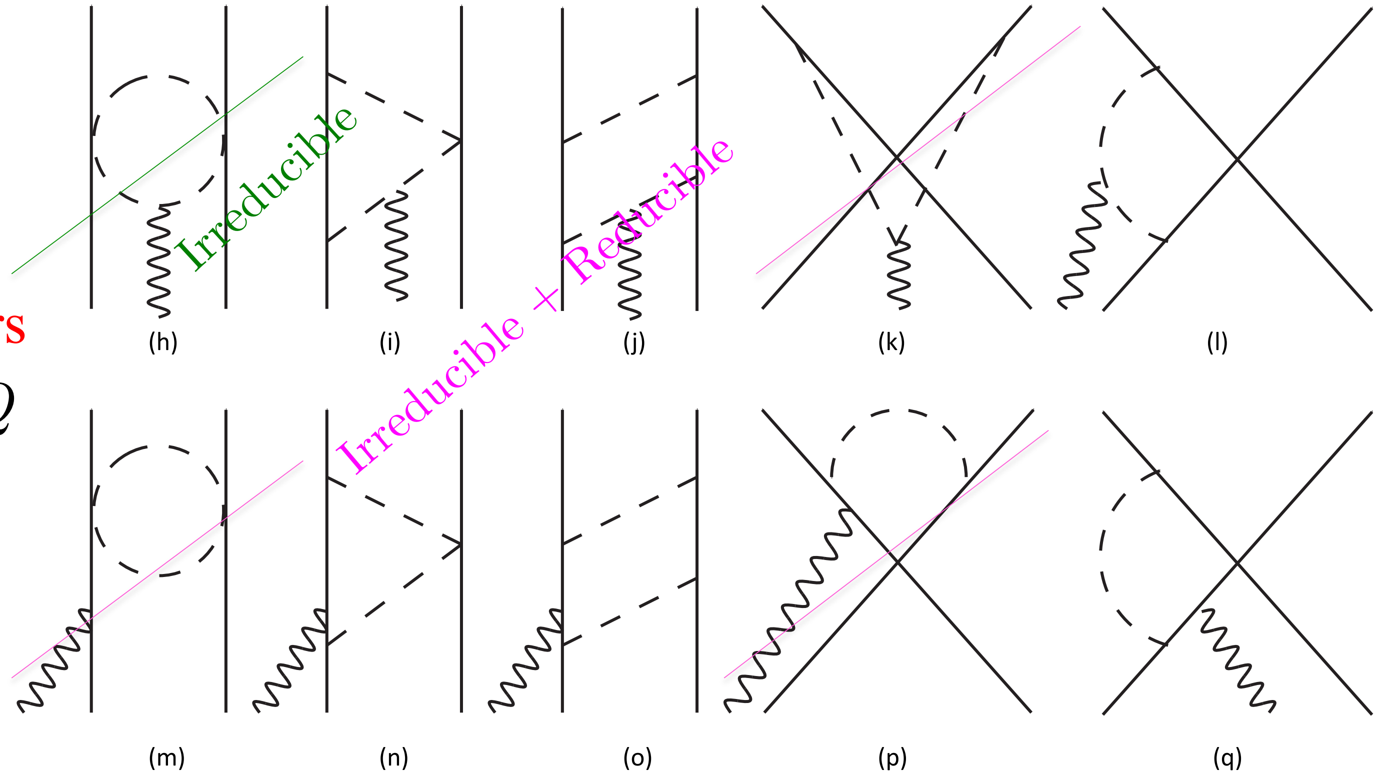
❖ Charge from term involving the second term in the interaction $H_{\gamma\pi N}$ vanishes (therefore no LEC's)

❖ Cancellations between recoil corrections to the reducible diagrams (up to N4LO) and static (N4LO) irreducible diagrams

Charge Operators up to N4LO (eQ)

isovector operators

- N4LO : $e\,Q$



- The loop integrals are ultraviolet divergent: the total charge at N4LO is finite since the divergencies cancel out (in line with absence of LEC's at this order)
- $\rho_o^{(1)}(\nu)$ depends on $v_\pi^{(2)}(\nu)$ and $v_{2\pi}^{(3)}(\nu)$
- Unitary equivalence: $\rho_o^{(1)}(\nu) = \rho_o^{(1)}(\nu=0) + [\rho^{(-3)}, i\,U^{(1)}]$
- $\rho^{(n>-3)}(\mathbf{q}=0) = 0$ as required by charge conservation

Electromagnetic Form Factors : A=2,3

❖ The deuteron charge (G_C), magnetic (G_M), and quadrupole (G_Q) f.f's.:

➤ $$G_C(q) \;=\; \frac{1}{3}\sum_{M=\pm1,0}\langle d;M \mid \rho(q\,\hat{\mathbf{z}}) \mid d;M\rangle$$ → the deuteron state with spin projection $(\pm1, 0)$

$\eta = (q/2\,m_d)^2$

➤ $$G_M(q) \;=\; \frac{1}{\sqrt{2\,\eta}}\,\mathrm{Im}\,[\,\langle d;1 \mid j_y(q\,\hat{\mathbf{z}}) \mid d;0\rangle\,]$$

➤ $$G_Q(q) \;=\; \frac{1}{2\,\eta}\big[\langle d;0 \mid \rho(q\,\hat{\mathbf{z}}) \mid d;0\rangle - \langle d;1 \mid \rho(q\,\hat{\mathbf{z}}) \mid d;1\rangle\big]$$

➤ $$G_C(0) = 1\;,\quad G_M(0) = (m_d/m_N)\,\mu_d\;,\quad G_Q(0) = m_d^2\,Q_d$$

❖ The charge (F_C), and magnetic (F_M) form factors of trinucleons:

➤ $$F_C(q) = \frac{1}{Z}\langle + \mid \rho(q\,\hat{\mathbf{z}}) \mid + \rangle$$ → the ^{3}He state or ^{3}H state in spin projections $\pm1/2$.

➤ $$F_M(q) = -\frac{2\,m_N}{q}\,\mathrm{Im}\,[\,\langle - \mid j_y(q\,\hat{\mathbf{z}}) \mid + \rangle\,]$$

➤ $$F_C(0) = 1\;,\quad F_M(0) = \mu$$ → the ^{3}He or ^{3}H magnetic moment

Technical Issue

❖ Two-body charge and current operators have power law behavior for large momenta

➢ In configuration space this leads to divergencies of type $1/r^n$, $n \geq 2$ (r is the interparticle separation), which need to be regularized to avoid divergencies in the matrix elements of these operators between nuclear wave functions

➢ In momentum space cutoff function:

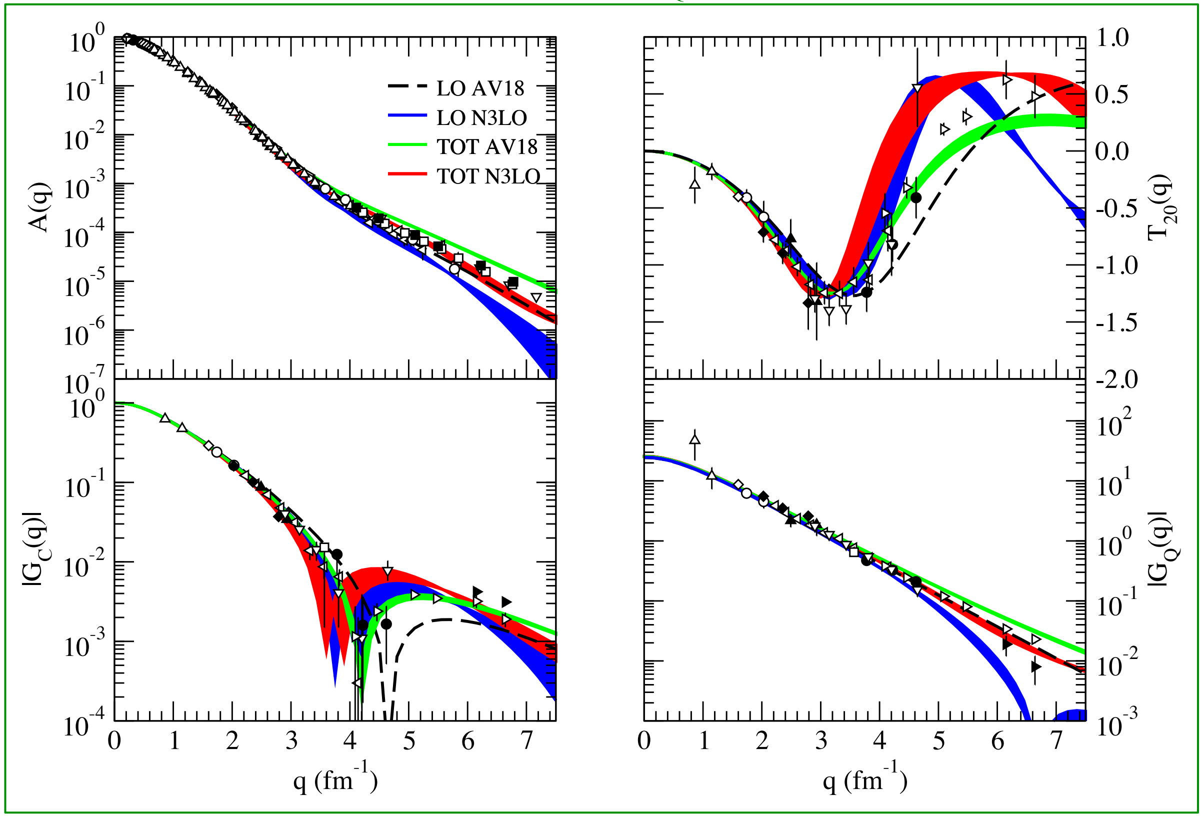
$$C_\Lambda(p) = e^{-(p/\Lambda)^4}$$

❖ The calculations use wave functions derived from either chiral or conventional (AV18/UIX) two- and three-nucleon potential

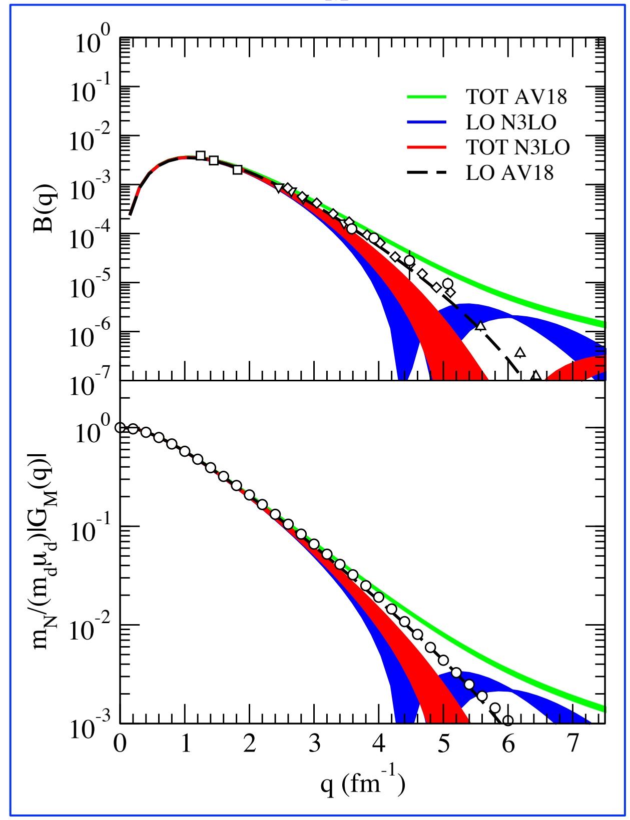
❖ The matrix elements of charge and current operators are calculated in momentum space with Monte Carlo methods.

Static Properties and Form Factors of the Deuteron

- The deuteron A(q) structure function and the tensor polarization T_{20} and charge (G_c) and quadrupole (G_Q) form factors



- The deuteron B(q) structure function and magnetic (G_M) form factor



- Deuteron root-mean-square charge radius and quadrupole moment. Resulting in parenthesis are related to the AV18 Hamiltonian

	r_d (fm)		Q_d (fm^2)	
Λ	500	600	500	600
LO	1.976 (1.969)	1.968 (1.969)	0.2750 (0.2697)	0.2711 (0.2697)
N2LO	1.976 (1.969)	1.968 (1.969)	0.2731 (0.2680)	0.2692 (0.2680)
N3LO(OPE)	1.976 (1.969)	1.968 (1.969)	0.2863 (0.2818)	0.2831 (0.2814)
N3LO($\nu = 1/2$)	1.976 (1.969)	1.968 (1.969)	0.2851 (0.2806)	0.2820 (0.2802)

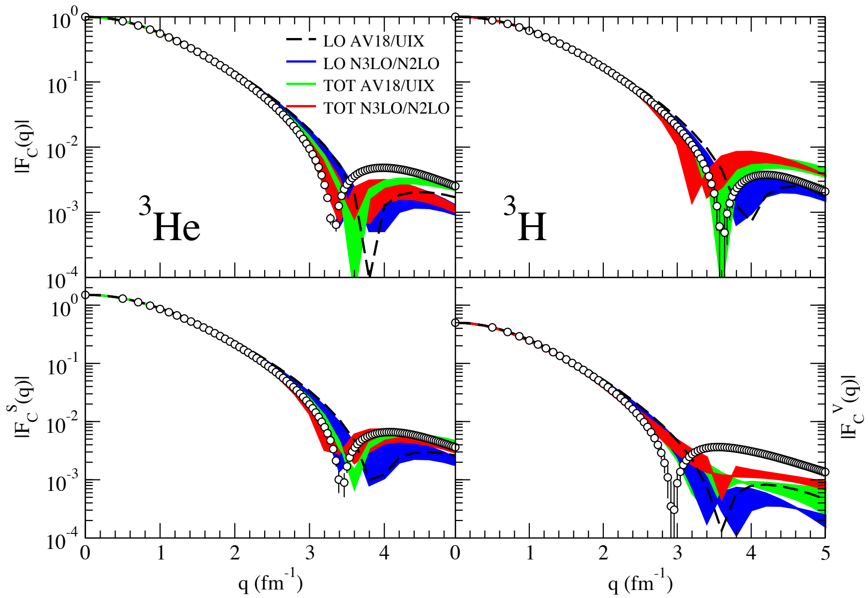
EXP DATA

$r_d = 1.9734(44)$ fm

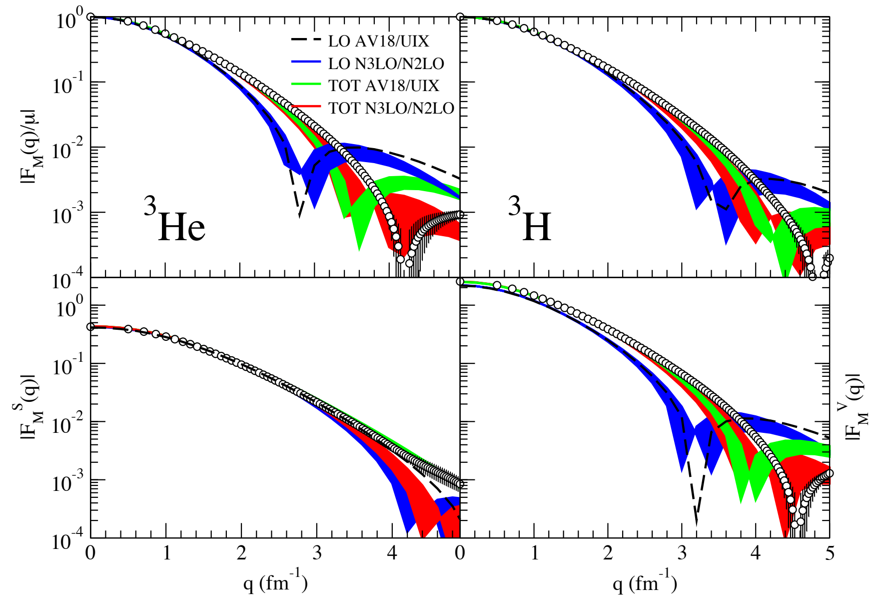
$Q_d = 0.2859(3)$ fm^2

Static Properties and Form Factors of the Trinucleons

❖ The ^{3}He and ^{3}H charge form factors and their isocalar and isovector combinations



❖ The ^{3}He and ^{3}H magnetic form factors and their isocalar and isovector combinations



❖ The ^{3}He and ^{3}H root-mean-square charge radii. Resulting in parenthesis are related to the AV18/UIX Hamiltonian.

	^{3}He		^{3}H	
Λ	500	600	500	600
LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)
N2LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)
N3LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)
N4LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)

EXP DATA:
$$r_c(^3\text{He}) = (1.959 \pm 0.030)\ \text{fm}$$
$$r_c(^3\text{H}) = (1.755 \pm 0.086)\ \text{fm}$$

❖ The ^{3}He and ^{3}H root-mean-square magnetic radii. Resulting in parenthesis are related to the AV18/UIX Hamiltonian.

	^{3}He		^{3}H	
Λ	500	600	500	600
LO	2.098 (2.092)	2.090 (2.092)	1.924 (1.918)	1.914 (1.918)
NLO	1.990 (1.981)	1.983 (1.974)	1.854 (1.847)	1.845 (1.841)
N2LO	1.998 (1.992)	1.989 (1.984)	1.865 (1.859)	1.855 (1.854)
N3LO(I)	1.924 (1.931)	1.910 (1.972)	1.808 (1.800)	1.796 (1.819)
N3LO(II)	1.901 (1.890)	1.883 (1.896)	1.789 (1.774)	1.773 (1.778)
N3LO(III)	1.927 (1.915)	1.913 (1.924)	1.808 (1.792)	1.794 (1.797)

EXP DATA:
$$r_m(^3\text{He}) = (1.965 \pm 0.153)\ \text{fm}$$
$$r_m(^3\text{H}) = (1.840 \pm 0.181)\ \text{fm}$$

Outlook

❖ In the first part we described how to construct nucleon-nucleon potential and charge and current operator in χEFT using TOPT

❖ In the second part of this study, we have provided predictions for the static properties and elastic form factors of the deuteron and trinucleons

❖ The χEFT calculations based on chiral and conventional potentials reproduce very well the observed electromagnetic structure of A=2 and A=3 for momentum transfer up to 2-3 fm^{-1}

Extra Slides

Formalism: TOPT

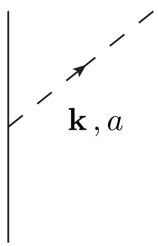
- Degrees of freedom: pions (π) and nucleons (*N*)
- Time-ordered perturbation theory (TOPT)

$$\langle f|T|i\rangle = \langle f|H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} |i\rangle$$

  - H_0 = free *N*'s, π's Hamiltonians
  - H_1 = interacting π, *N* and γ Hamiltonians implied by $\mathcal{L}_{eff}$
  - E_i = initial energy

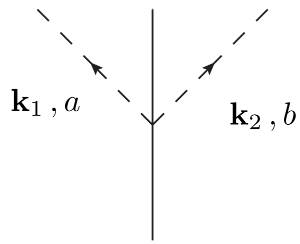
Strong Interaction Vertices up to Q^2

$H_{\pi NN}$



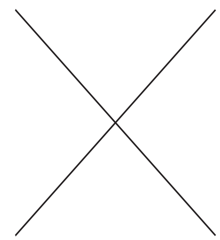
$\sim Q$

$H_{\pi\pi NN}$



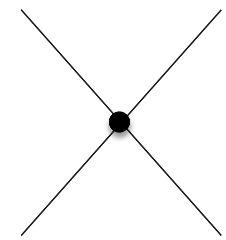
$\sim Q$

H_{CT0}



$\sim Q^0$

H_{CT2}



$\sim Q^2$

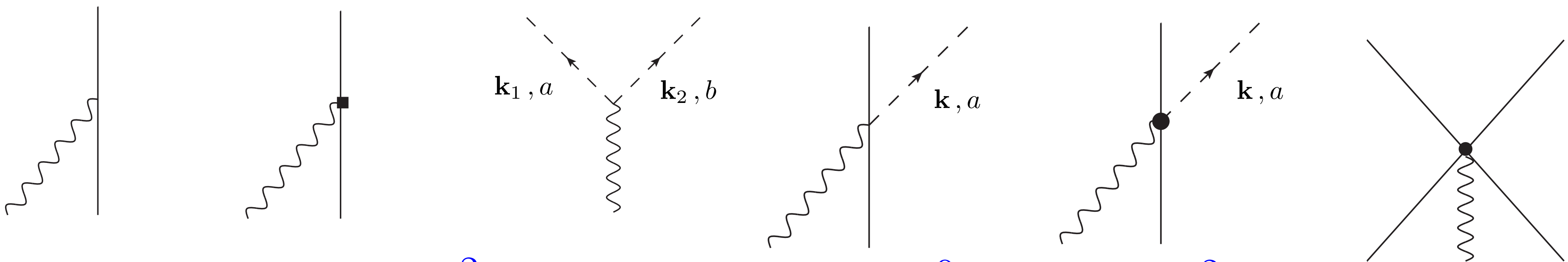
- $H_{\pi NN}$ and $H_{\pi\pi NN}$ are known: $g_A \approx 1.25$ (axial coupling constant), $F_\pi \approx 186$ MeV (pion decay amplitude)
- H_{CT0}: 4N contact terms, 2 LEC's (C_T, C_S : fitting *np* *S*-wave phase shift[1])
- H_{CT2} : 4N contact terms with two gradients, 7 LEC's (C_i : fitting *np* and *pp* elastic scattering data and the deuteron binding energy[1]; relativistic corrections have been ignored)

---

[1] R. Machelidt and D. R. Etem, Phys. Rep. **503**, 1 (2011)

Current Interaction Vertices up to eQ^2

$H_{\gamma NN}$ $\quad H^{(2)}_{\gamma NN}$ $\quad H_{\gamma\pi\pi}$ $\quad H_{\gamma\pi NN}$ $\quad H^{(2)}_{\gamma\pi NN}$ $\quad H_{CT\gamma}$



$\sim e\,Q$ $\quad \sim e\,(Q/m_N)^2$ $\quad \sim e\,Q$ $\quad \sim e\,Q^0$ $\quad \sim e\,Q^2$ $\quad \sim e\,Q$

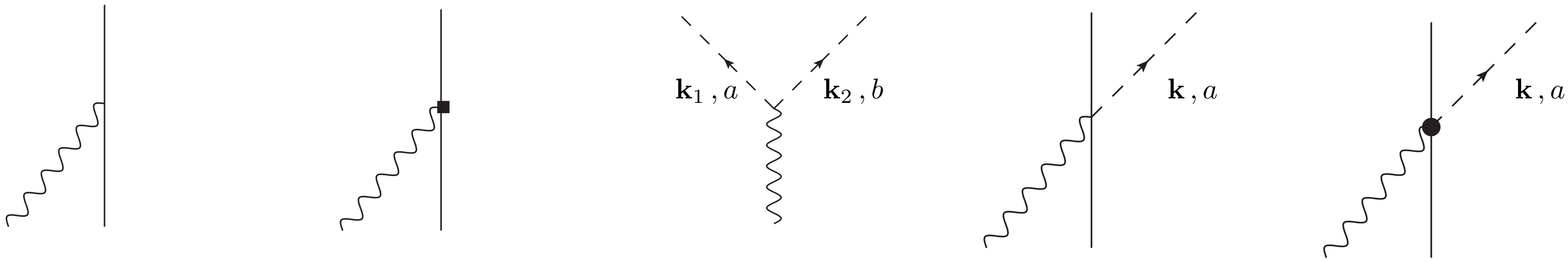
- $H_{\gamma NN}$, $H^{(2)}_{\gamma NN}$ and $H_{\gamma\pi NN}$ depend on g_A, F_π and proton and neutron magnetic moments ($\mu_p = 2.793\,\mu_N$ and $\mu_n = -1.913\,\mu_N$)

- $H_{CT\gamma}$: terms from minimal substitution in H_{CT2} known, two additional LEC's due to non minimal substitution (C'_{15} and C'_{16})

- $H^{(2)}_{\gamma\pi NN}$ from $\mathcal{L}_{\gamma\pi N}$ of Fettes *et al*. (1998) depends on F_π and 3 LEC's d'_8, d'_9, d'_{21} ; two multiplying isovector structure and one isoscalar structure

Charge Interaction Vertices up to eQ^2

$H_{\gamma NN}$ | $H^{(2)}_{\gamma NN}$ | $H_{\gamma\pi\pi}$ | $H_{\gamma\pi NN}$ | $H^{(2)}_{\gamma\pi NN}$



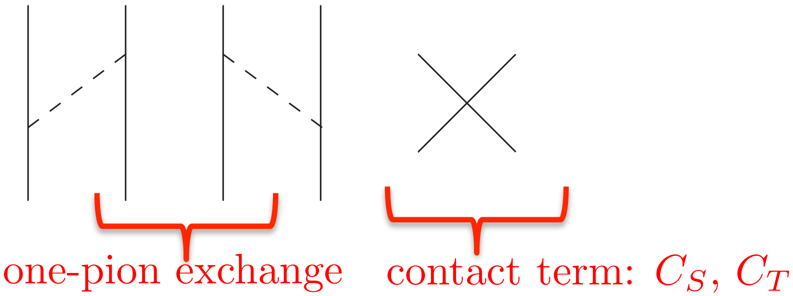
$\sim e\,Q^0$ | $\sim e\,(Q/m_N)^2$ | $\sim e\,Q$ | $\sim e\,Q$ | $\sim e\,Q^2$

- $H_{\gamma NN}$, $H^{(2)}_{\gamma NN}$ and $H_{\gamma\pi NN}$ depend on g_A, F_π and proton and neutron magnetic moments ($\mu_p = 2.793\,\mu_N$ and $\mu_n = -1.913\,\mu_N$)

- $H^{(2)}_{\gamma\pi NN}$ depends on: F_π and 3 LEC's (d_{20}, d_{21}, d_{21})

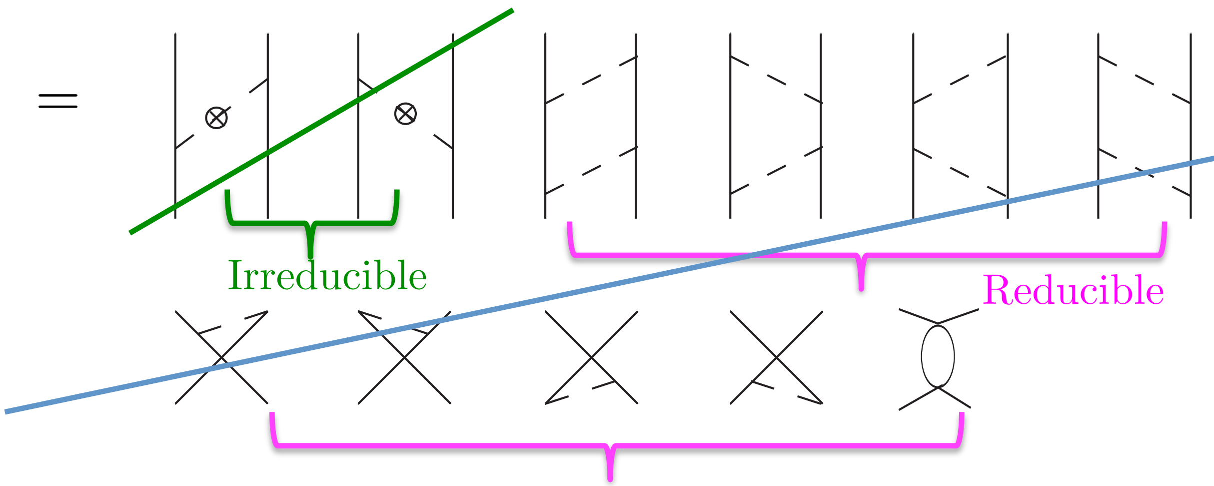
From Amplitudes to Potentials: Example up to NLO

$$v^{(0)} = \qquad T^{(0)} =$$



$$v^{(1)} = \qquad T^{(1)} - \left[v^{(0)} G_0 v^{(0)} \right]$$

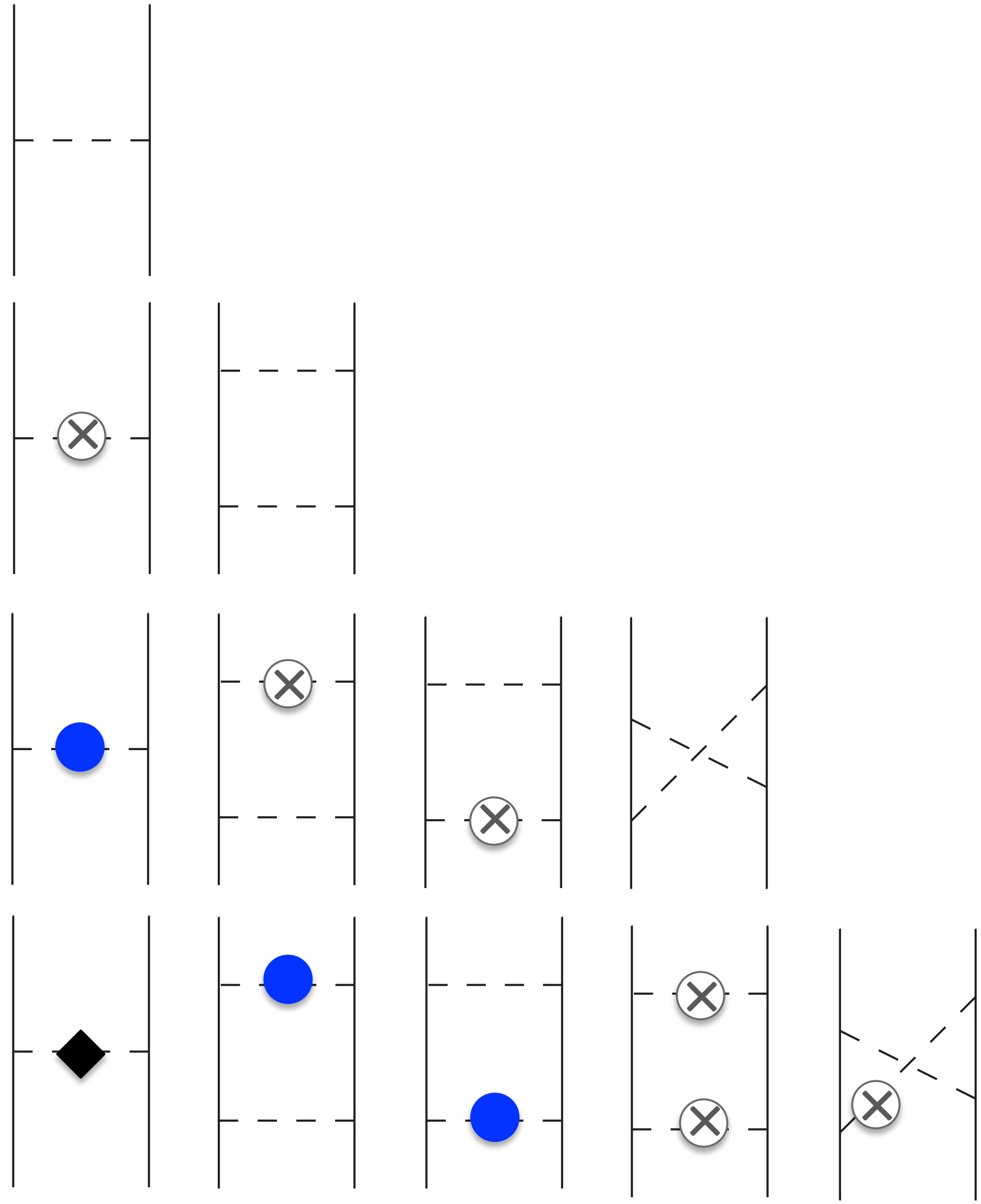
$=$



$$\cancel{-\left[v^{(0)} G_0 v^{(0)} \right]}$$

- Irreducible: first non-static corrections cancel each other
- Reducible: iterations of $v^{(0)}$, they are completely canceled by $\left[v^{(0)} G_0 v^{(0)} \right]$
- Complete and partial cancellations persist at higher orders

OPE and TPE(box) only beyond the Static Limit



$T^{(0)}$ ➢ $v_\pi^{(0)}$

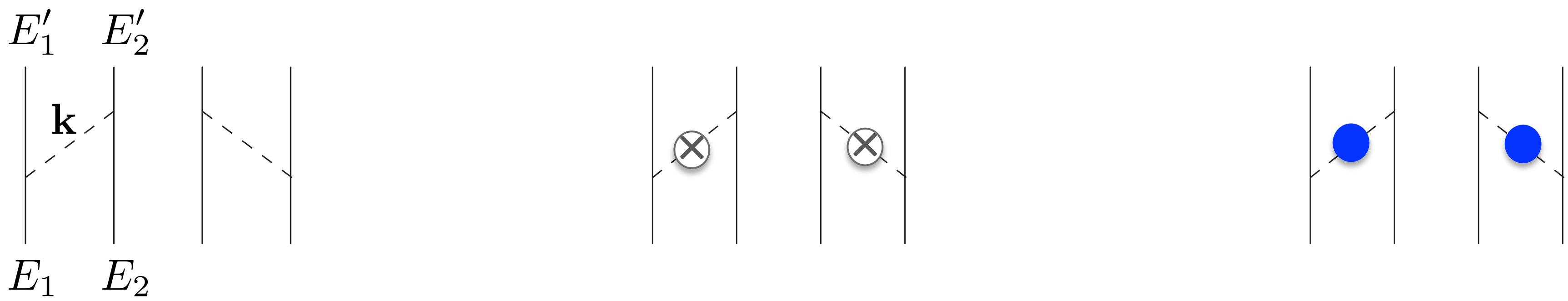
$T^{(1)}$ ➢ $v_\pi^{(1)} + \text{LS terms}$

$T^{(2)}$ ➢ $v_\pi^{(2)}(\nu) + v_{2\pi}^{(2)} + \text{LS terms}$

$T^{(3)}$ ➢ $v_\pi^{(3)} + v_{2\pi}^{(3)}(\nu) + \text{LS terms}$

❖ ν-dependence in $v_\pi^{(2)}(\nu)$ and therefore in $v_{2\pi}^{(3)}(\nu)$

OPE beyond the Static Limit



$v_\pi^{(0)} \sim Q^0$ $\qquad$ $v_\pi^{(1)} \sim Q^1$ $\qquad$ $v_\pi^{(2)}(\nu) \sim Q^2$

- On-the-shell-energy, non-static OPE at N2LO can equivalently written as

$$v_\pi^{(2)}(\nu=0) = v_\pi^{(0)}(\mathbf{k}) \frac{(E_1' - E_1)^2 + (E_2' - E_2)^2}{2\,\omega_k^2}$$

$$v_\pi^{(2)}(\nu=1) = -v_\pi^{(0)}(\mathbf{k}) \frac{(E_1' - E_1)(E_2' - E_2)}{\omega_k^2}$$

These corrections are different off-the energy-shell $E_1 + E_2 \neq E_1' + E_2'$

$$v_\pi^{(0)}(\mathbf{k}) = -\frac{g_A^2}{F_\pi^2}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\, \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k}\, \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2}$$

$$\omega_k^2 = k^2 + m_\pi^2$$

Unitary Equivalence of $v_\pi^{(2)}(\nu)$ and $v_{2\pi}^{(3)}(\nu)$

❖ Different off-shell parameterizations lead to unitarily equivalent two-nucleon Hamiltonians (limiting our consideration to OPE and box TPE potentials only)

$$H(\nu) = \underbrace{K^{(-1)}}_{\text{Kinetic energy}} + \underbrace{v_\pi^{(0)} + v_{2\pi}^{(2)}}_{\text{Static OPE and TPE(box)}} + \underbrace{v_\pi^{(2)}(\nu) + v_{2\pi}^{(3)}(\nu)}_{\text{Recoil OPE and TPE(box)}}$$

❖ The Hamiltonians are related each other by unitary transformation

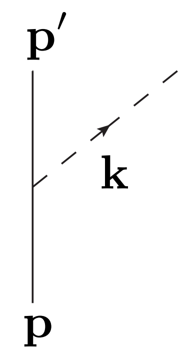
$$H(\nu) = e^{-i\,U(\nu)} H(\nu=0) e^{i\,U(\nu)} \qquad i\,U(\nu) \simeq i\,U^{(0)}(\nu) + i\,U^{(1)}(\nu)$$

❖ There exist an infinite class of 2nd order recoil correction to OPE which are equivalent on-shell, parameterized by a parameter ν :

$$v_\pi^{(2)}(\mathbf{k},\mathbf{K};\nu) = (1-2\,\nu)\;\frac{v_\pi^{(0)}(\mathbf{k})}{\omega_k^2}\,\frac{(\mathbf{k}\cdot\mathbf{K})^2}{4\,m_N^2}$$

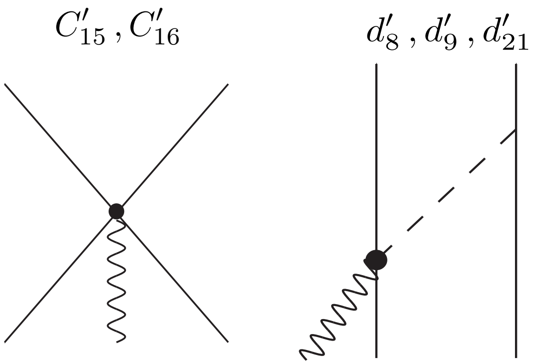
$$\mathbf{k} = \mathbf{p}' - \mathbf{p}$$

$$\mathbf{K} = (\mathbf{p}' + \mathbf{p})/2$$



❖ The off-shell ambiguities will affect successive terms: for each $v_\pi^{(2)}(\nu)$ there is a corresponding $v_{2\pi}^{(3)}(\nu)$

EM Observables at N3LO: fixing LECs



❖ Five LECs: it is convenient to introduce the adimensional set $d_i^{S,V}$

$$C'_{15} = d_1^S/\Lambda^4\ , \qquad d'_9 = d_2^S/\Lambda^2,$$
$$C'_{16} = d_1^V/\Lambda^4\ , \qquad d'_8 = d_2^V/\Lambda^2\ , \qquad d'_{21} = d_3^V/\Lambda^2$$

❖ Five LECs: fixed in A=2-3 nucleons' sector

❖ Isoscalar sector:

- ➢ d_1^S and d_2^S from expt μ_d and $\mu_S\,(^3\mathrm{He}/^3\mathrm{H})$

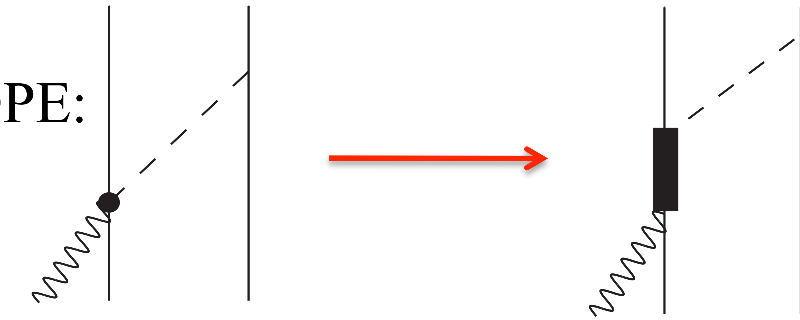
Λ	d_1^S	$d_2^S \times 10$
500	4.072 (2.522)	2.190 (–1.731)
600	11.38 (5.238)	3.231 (–2.033)

EM Observables at N3LO: fixing LECs

❖ Isovector OPE:

$d_2^V \,, d_3^V$

$\gamma N\Delta$ – excitation current



❖ d_2^V and d_3^V are knowing from Δ-resonance saturation*
$(d_2^V / d_3^V = 1/4)$

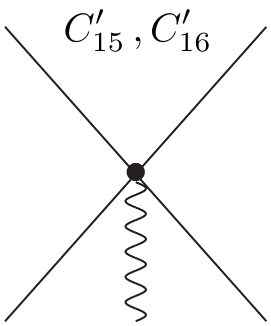
❖ Two LECs left: fixed in A=2-3 nucleons sector

❖ Isovector sector:

- I $= d_1^V$ and d_2^V from expt μ_V $(^3\mathrm{He}/^3\mathrm{H})$ m.m. and $npd\gamma$ xsec.
- II $= d_1^V$ from expt $npd\gamma$ xsec. and d_2^V from Δ-saturation*
- III $= d_1^V$ from expt μ_V $(^3\mathrm{He}/^3\mathrm{H})$ m.m. and d_2^V from Δ-saturation*

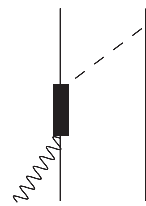
Λ	d_1^V (I)	d_2^V (I)	d_1^V (II)	d_2^V (II)	d_1^V (III)	d_2^V (III)
500	10.36 (45.10)	17.42 (35.57)	–13.30 (–9.339)	3.458	–7.981 (–5.187)	3.458
600	41.84 (257.5)	33.14 (75.00)	–22.31 (–11.57)	4.980	–11.69 (–1.025)	4.980

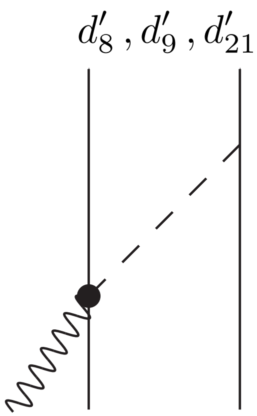
$$* \; d_2^V = \frac{4\,\mu_{\gamma N\Delta}\, h_A\, \Lambda^2}{9\, m_N\, (m_\Delta - m_N)}$$



- Terms from gauging the subleading two nucleon contact Lagrangian (minimal substitution). These can expressed in terms of the same LECs entering the *NN* potential (constrained by fitting *np*, *pp* elastic scattering data and the deuteron binding energy)
- Terms involving the electromagnetic field tensor $F_{\mu\nu}$ (1 isoscalar and 1 isovector terms)

$$\mathbf{j}^{(1)}_{\rm a,nm} = -i\,e\Big[G_E^S(q^2)\,C'_{15}\,\boldsymbol{\sigma}_1 + G_E^V(q^2)\,C'_{16}\times(\tau_{1,z}-\tau_{2,z})\,\boldsymbol{\sigma}_1\Big]\times\mathbf{q} + 1 \rightleftharpoons 2$$

- Isovector: →  $\gamma N\Delta$ – excitation current

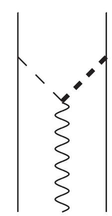


$$\mathbf{j}^{(1)}_{\rm b,IV} = i\,e\,\frac{g_A}{F_\pi^2}\,\frac{G_{\gamma N\Delta}(q^2)}{\mu_{\gamma N\Delta}}\,\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{\omega^2_{k_2}}\Big[d'_8\tau_{2,z}\,\mathbf{k}_2 - d'_{21}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\,\boldsymbol{\sigma}_1\times\mathbf{k}_2\Big]\times\mathbf{q} + 1 \rightleftharpoons 2$$

$$G_{\gamma N\Delta}(q^2) = \frac{\mu_{\gamma N\Delta}}{(1+q^2/\Lambda^2_{\Delta,1})^2\sqrt{1+q^2/\Lambda^2_{\Delta,2}}}$$

analysis of γN data in the Δ-resonance region

- $\mu_{\gamma N\Delta} \sim 3\,\mu_N$
- $\Lambda_{\Delta,1} = 0.84\,\text{GeV}$
- $\Lambda_{\Delta,2} = 1.20\,\text{GeV}$

- Isoscalar: →  $\gamma\pi\rho$ – transition current

$$\mathbf{j}^{(1)}_{\rm b,IS} = i\,e\,\frac{g_A}{F_\pi^2}\,d'_9\,G_{\gamma\pi\rho}(q^2)\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{\omega^2_{k_2}}\,\mathbf{k}_2\times\mathbf{q} + 1 \rightleftharpoons 2$$

$$G_{\gamma\pi\rho}(q^2) = \frac{1}{1+q^2/m^2_\omega}$$

Deuteron E.M. Form Factors

- The deuteron charge (G_C), magnetic (G_M), and quadrupole (G_Q) f.f's.:
  - $$G_C(q) = \frac{1}{3} \sum_{M=\pm 1,0} \langle d; M \mid \rho(q\,\hat{\mathbf{z}}) \mid d; M \rangle$$ → the deuteron state with spin projection $(\pm 1, 0)$
  - $$G_M(q) = \frac{1}{\sqrt{2\,\eta^*}} \mathrm{Im}\left[\langle d; 1 \mid j_y(q\,\hat{\mathbf{z}}) \mid d; 0 \rangle \right]$$
  - $$G_Q(q) = \frac{1}{2\,\eta^*}\left[\langle d; 0 \mid \rho(q\,\hat{\mathbf{z}}) \mid d; 0 \rangle - \langle d; 1 \mid \rho(q\,\hat{\mathbf{z}}) \mid d; 1 \rangle \right]$$
- Normalization:
  - $$G_C(0) = 1\ ,\quad G_M(0) = (m_d/m_N)\,\mu_d\ ,\quad G_Q(0) = m_d^2\, Q_d$$

$m_d =$ the deuteron mass
$m_N =$ the nucleon mass

$\mu_d, Q_d =$ the deuteron magnetic moment (in units of μ_N) and quadrupole moment

- Structure functions A(q) and B(q) and polarization tensor $T_{20}(q)$

$* \; \eta = (q/2\,m_d)^2$

Trinucleons E.M. Form Factors

- The charge (F_C), and magnetic (F_M) form factors of trinucleons:

  - $F_C(q) = \frac{1}{Z} \langle + | \, \rho(q\,\hat{\mathbf{z}}) \, | + \rangle$ → the ^{3}He state or ^{3}H state in spin projections $\pm 1/2$.
  - $F_M(q) = -\frac{2\, m_N}{q} \, \mathrm{Im} \left[\langle - | \, j_y(q\,\hat{\mathbf{z}}) \, | + \rangle \right]$

- Normalization: $F_C(0) = 1 \; , \quad F_M(0) = \mu$ → the ^{3}He or ^{3}H magnetic moment

- Isoscalar (F_M^S) and isovector (F_M^V) combinations

- Wave function in r-space

$$\Psi_d(M) \equiv \Psi(\mathbf{r}, M) = \left[\frac{u(r)}{r}\mathcal{Y}_{011M}(\hat{\mathbf{r}}) + \frac{w(r)}{r}\mathcal{Y}_{211M}(\hat{\mathbf{r}})\right]\eta_{00}$$

  - ⋆ Vector spherical harmonics: $\mathcal{Y}_{LSJM}(\hat{\mathbf{r}}) \equiv \left[Y_{LM_L}(\hat{\mathbf{r}}) \otimes \chi_{SM_S}\right]_{JM}$
  - ⋆ Radial functions: $u(r)$ and $w(r)$
  - ⋆ Isospin state: η_{00}

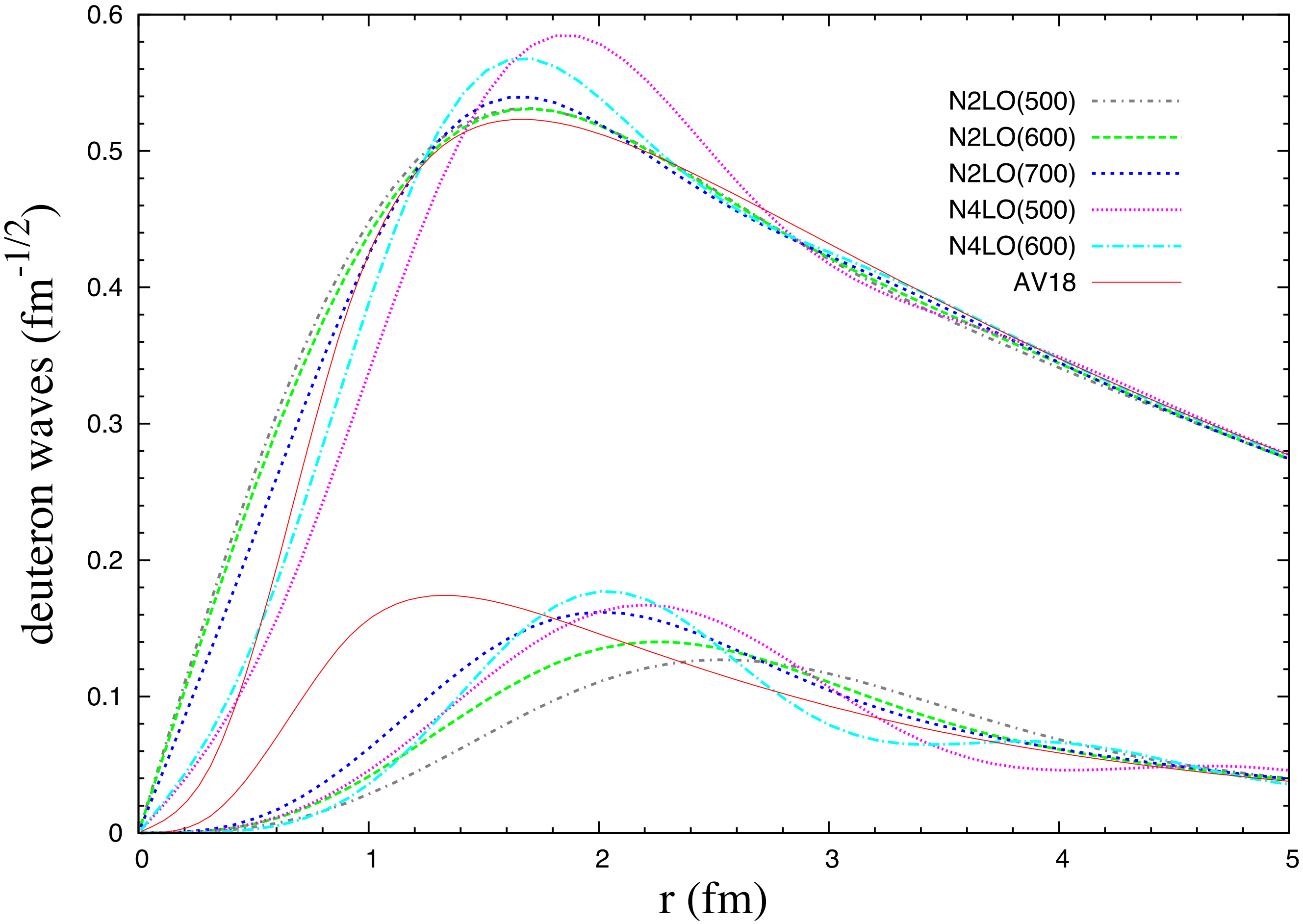
- Numerically, we solve

$$\bar{u}_L(p) = \frac{1}{E_d - p^2/(2\,\mu)}\frac{2}{\pi}\int_0^\infty dk\, k^2 \sum_{L'} v_{LL'}^{SJ}(p,k)\bar{u}_{L'}(k)$$

  - ⋆ E_d = binding energy, μ = reduced mass
  - ⋆ $J = S = 1,\ L = L' = 0, 2$
  - ⋆ $v_{LL'}^{SJ}(p,k) = \int d\mathbf{r}\, j_{L'}(pr)\mathcal{Y}^{\dagger}_{L'SJM} v(\mathbf{r})\mathcal{Y}_{LSJM} j_L(kr)$

(two-nucleon potential)

Results 1: Deuteron properties



	N2LO(500)	N2LO(600)	N2LO(700)	N4LO(500)	N4LO(600)	AV18	Empirical
B_d (MeV)	-2.224	-2.225	-2.224	-2.225	-2.225	-2.225	-2.224575(9)
r_d (fm)	1.944	1.948	1.951	1.975	1.967	1.968	1.97535(85)
P_D (%)	3.44	3.87	4.77	4.51	4.43	5.76	-