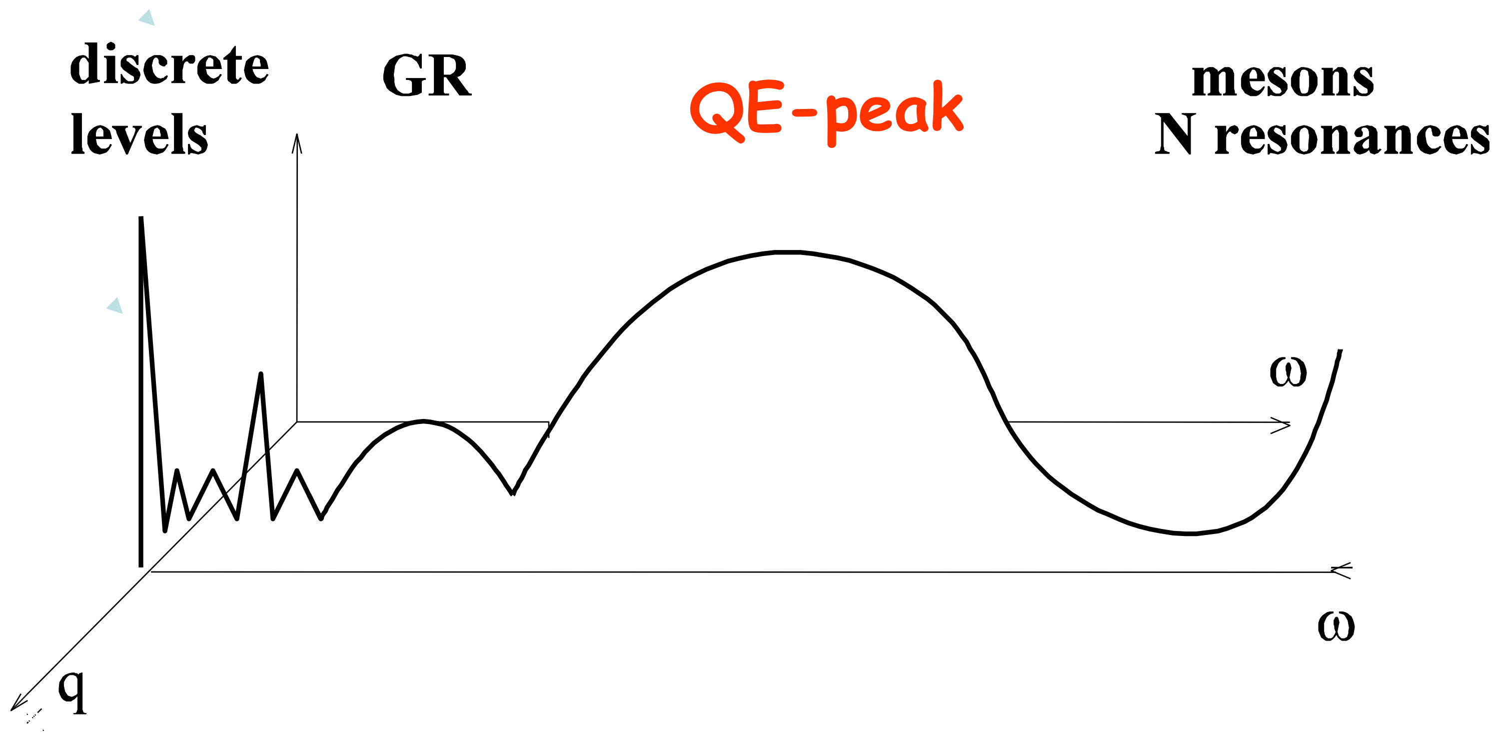
ELECTRON AND NEUTRINO SCATTERING IN THE QUASIELASTIC REGIME

Carlotta Giusti
Università and INFN, Pavia

Scuola di Fisica Nucleare Raimondo Anni Otranto May 27 - 31 2013

nuclear response to the electroweak probe



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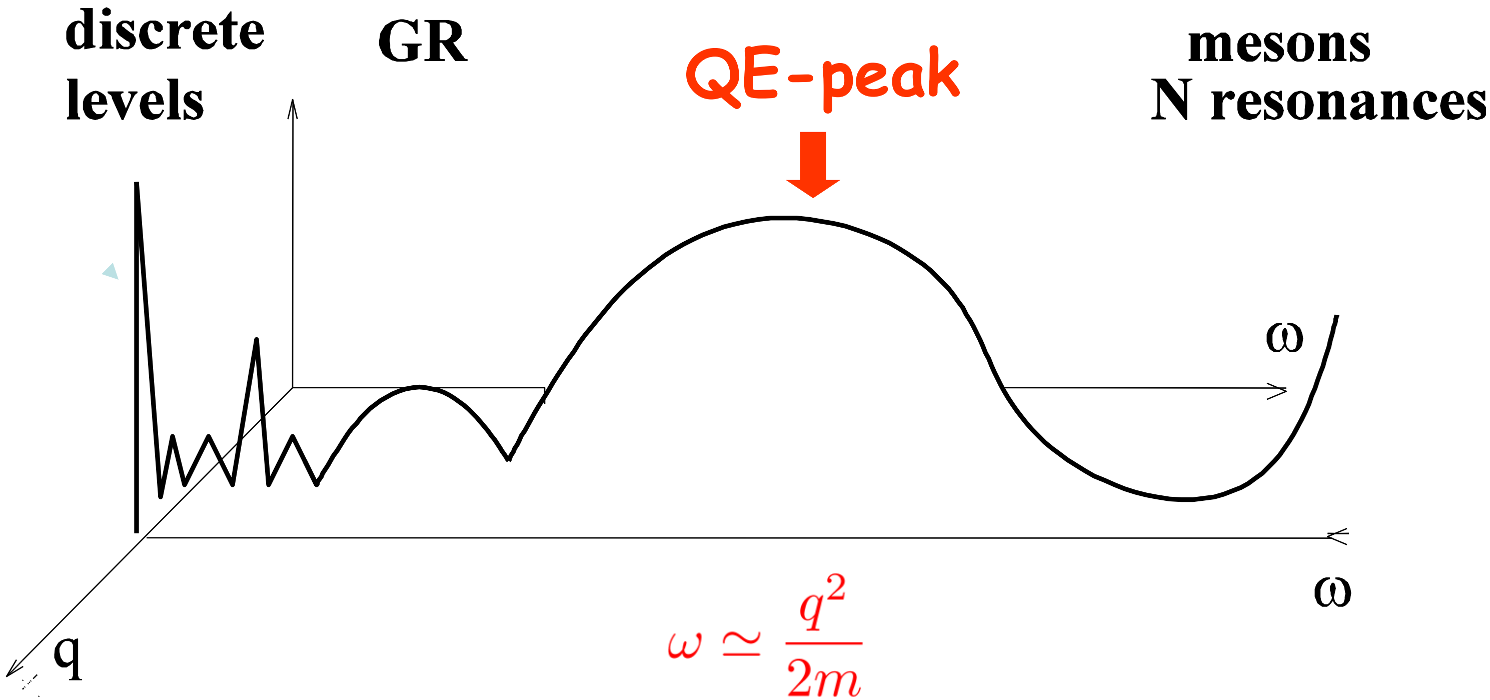
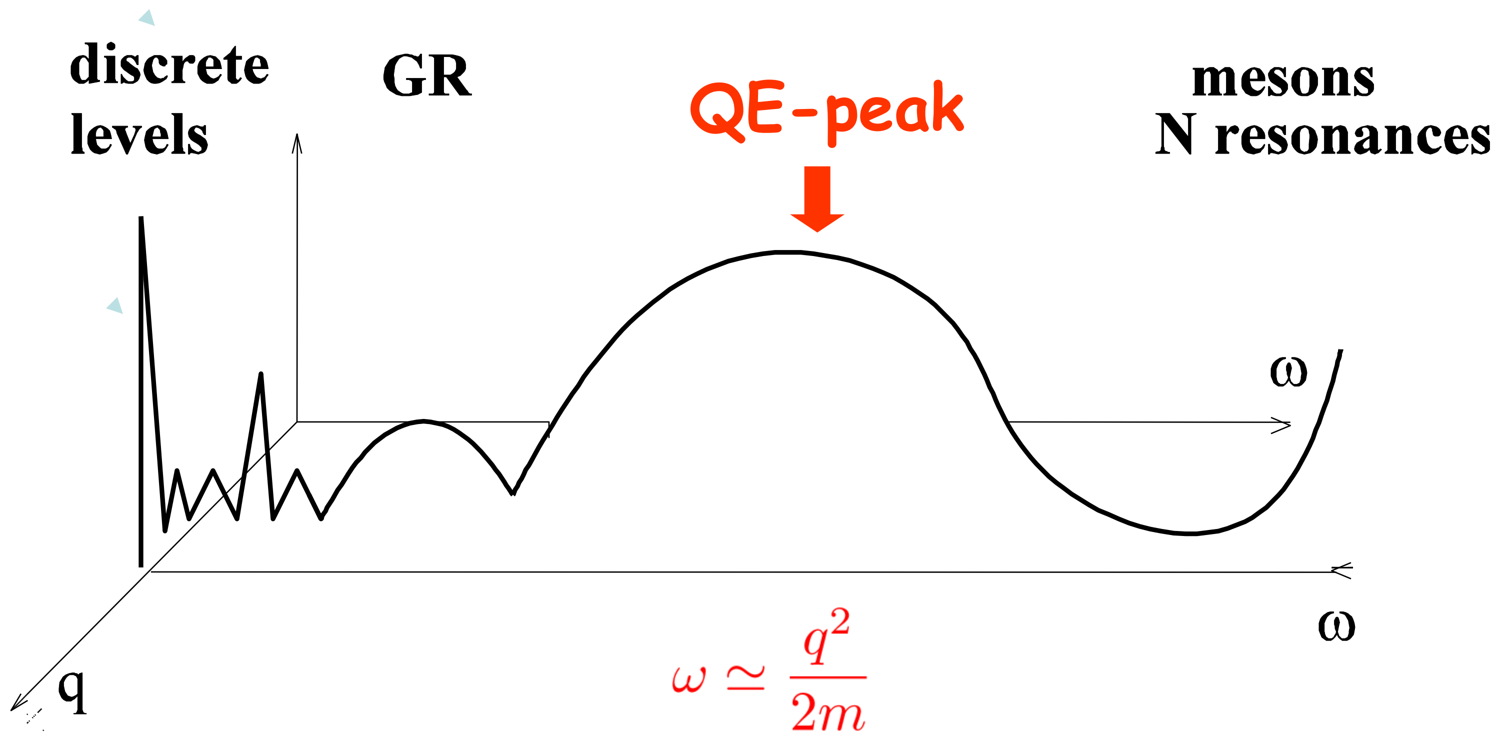


$$\omega \simeq \frac{q^2}{2m}$$

nuclear response to the electroweak probe



QE-peak dominated by one-nucleon knockout

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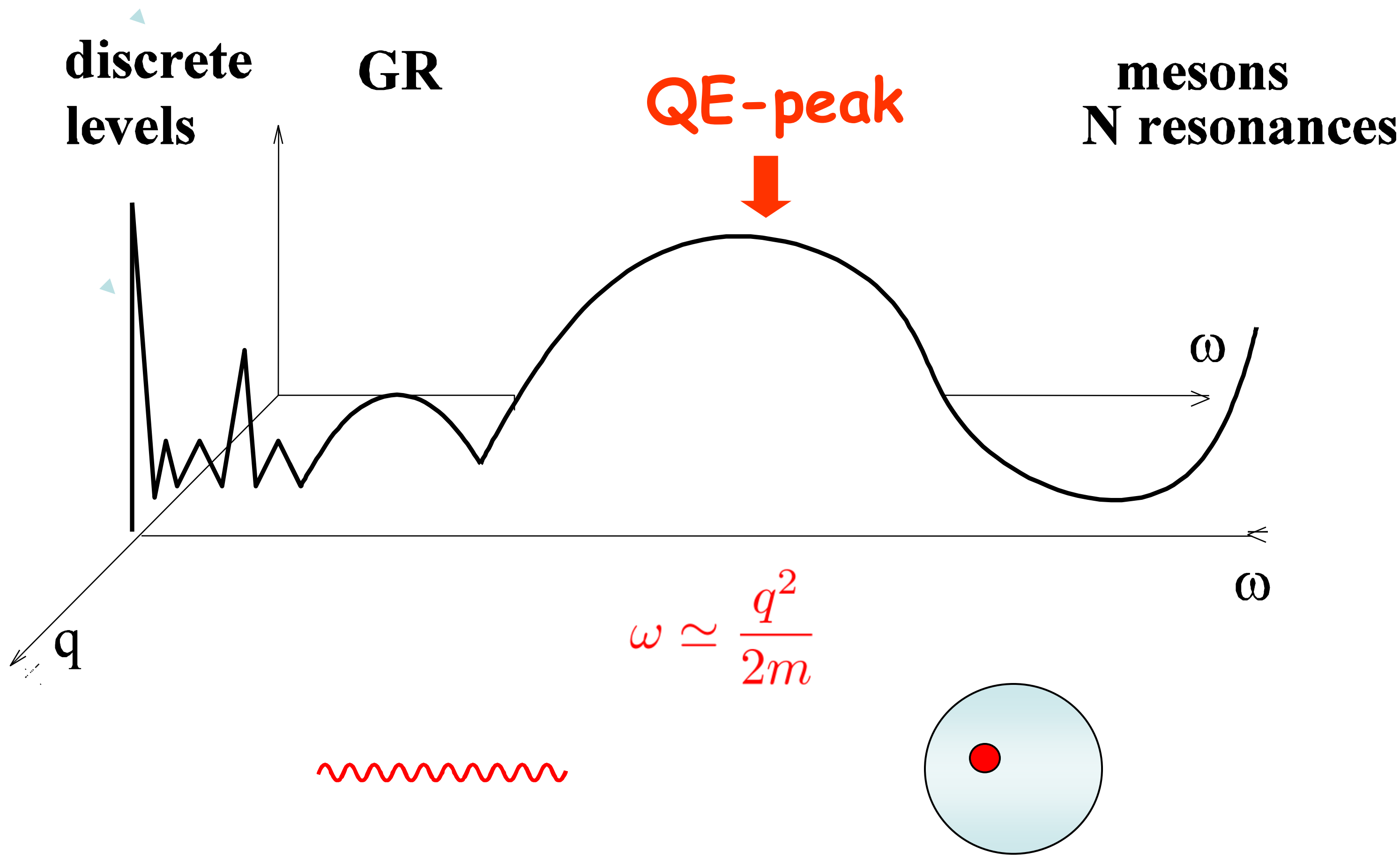

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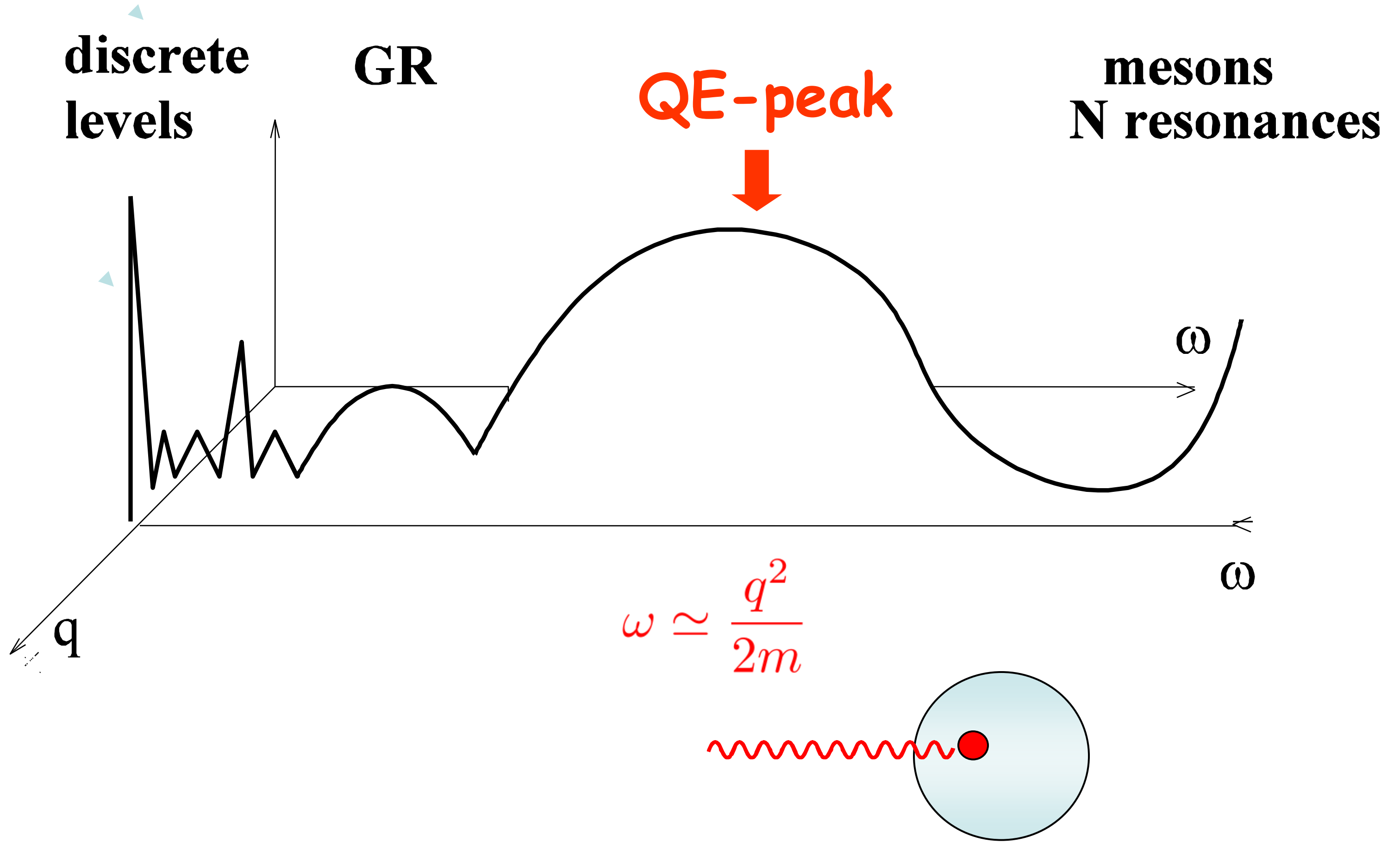

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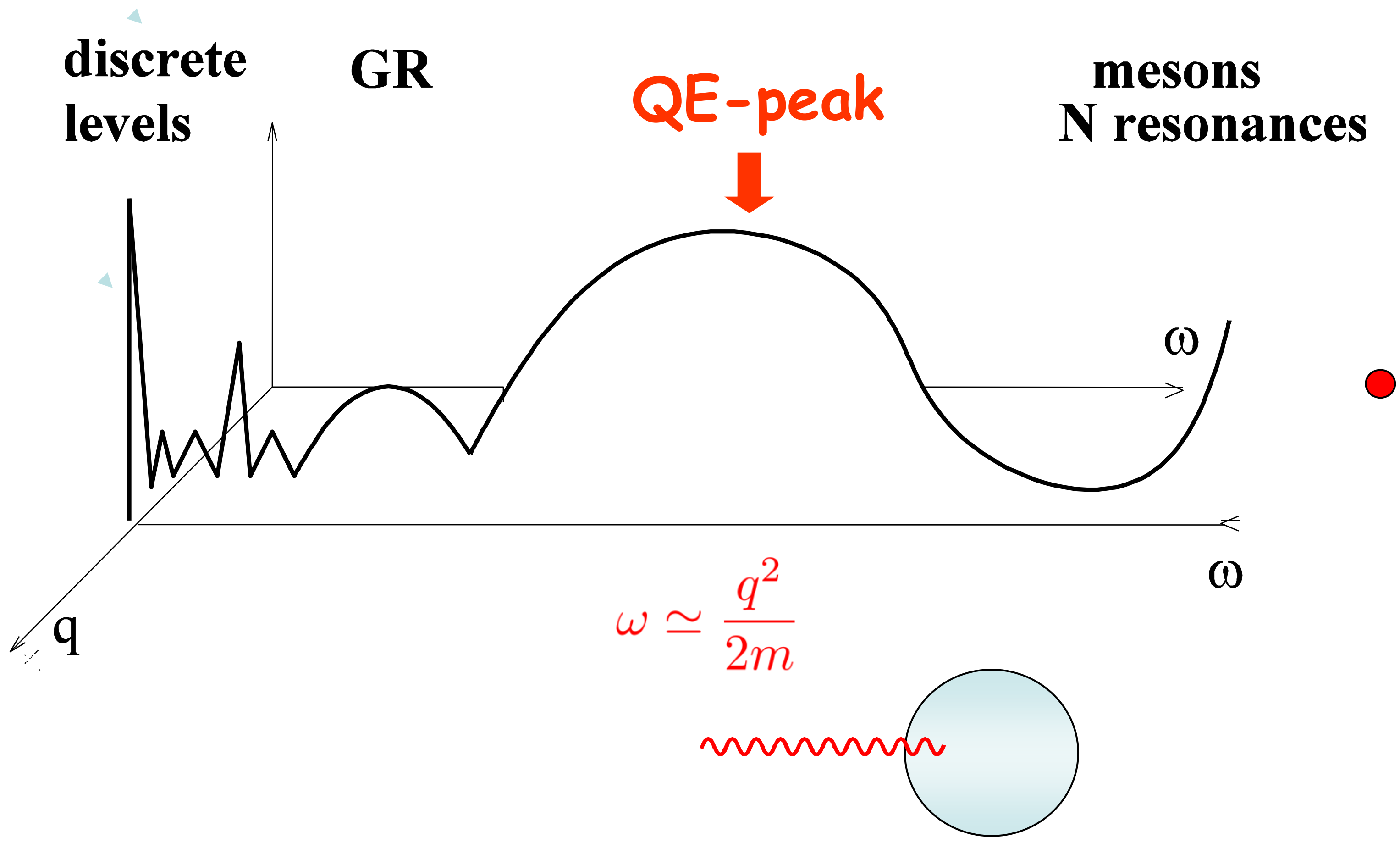

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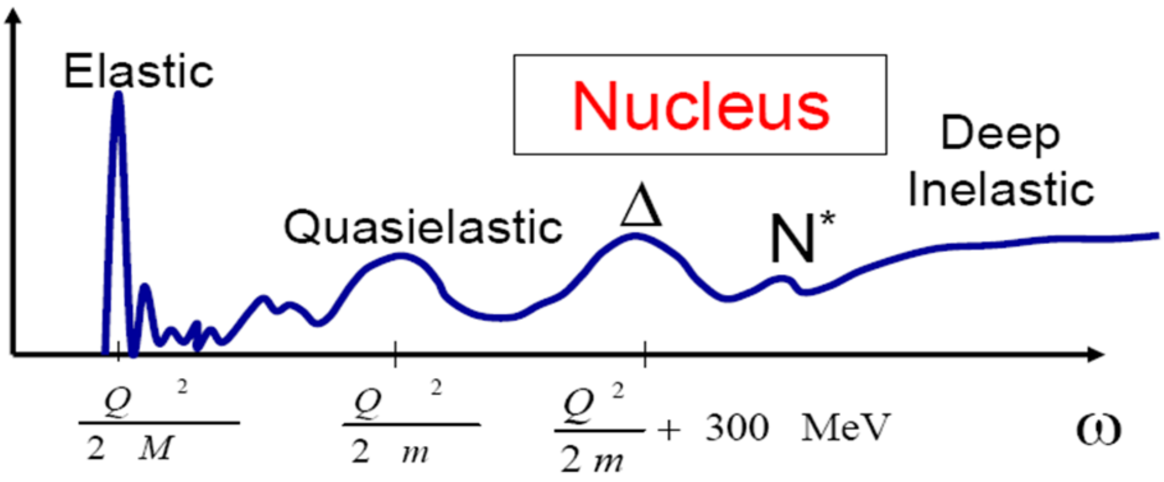

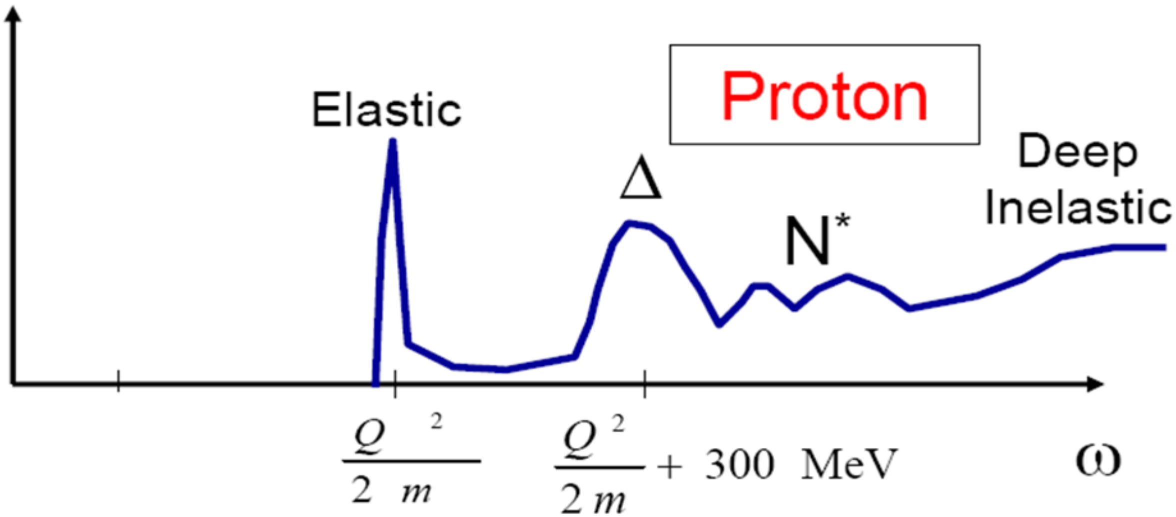

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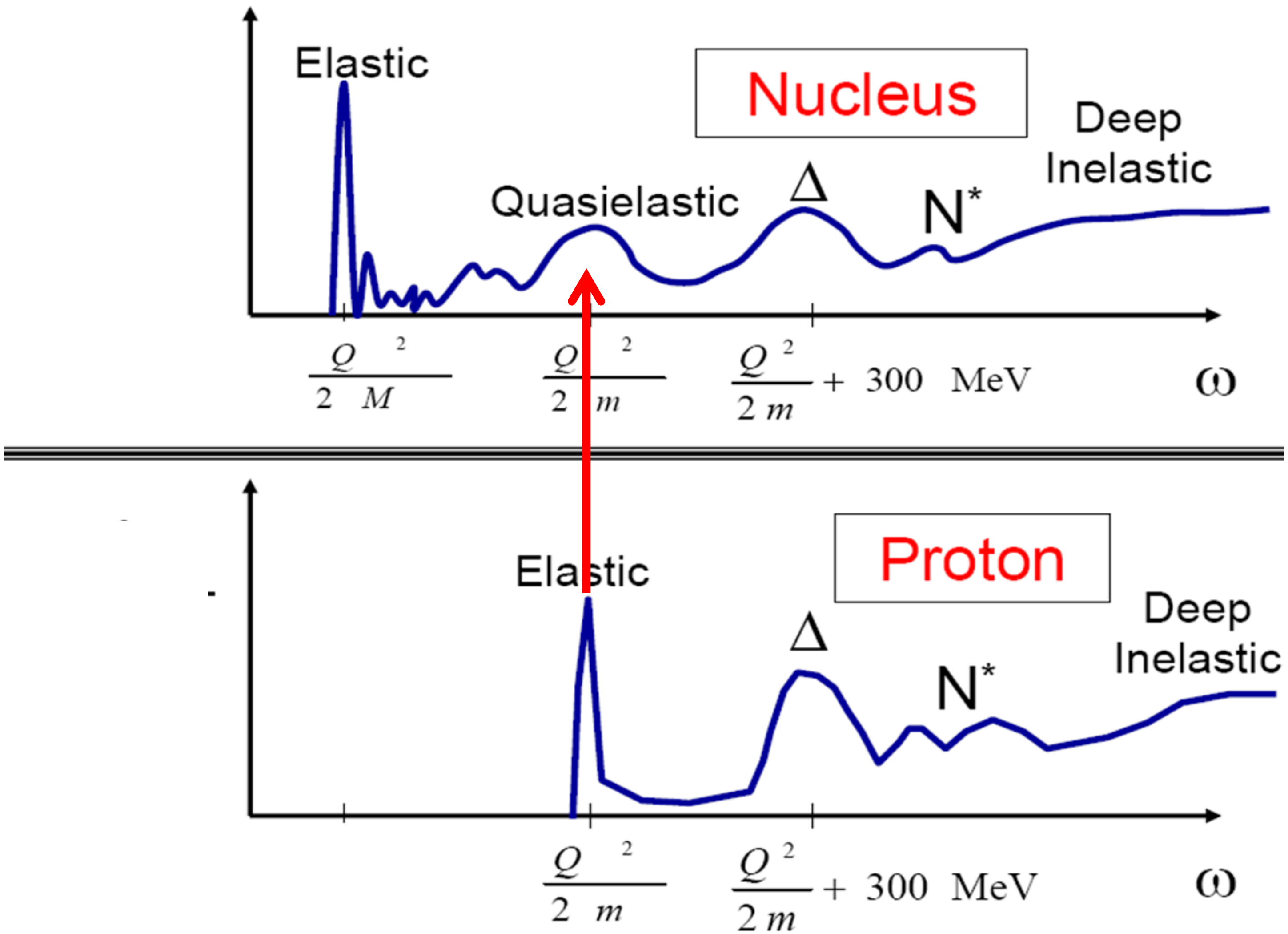
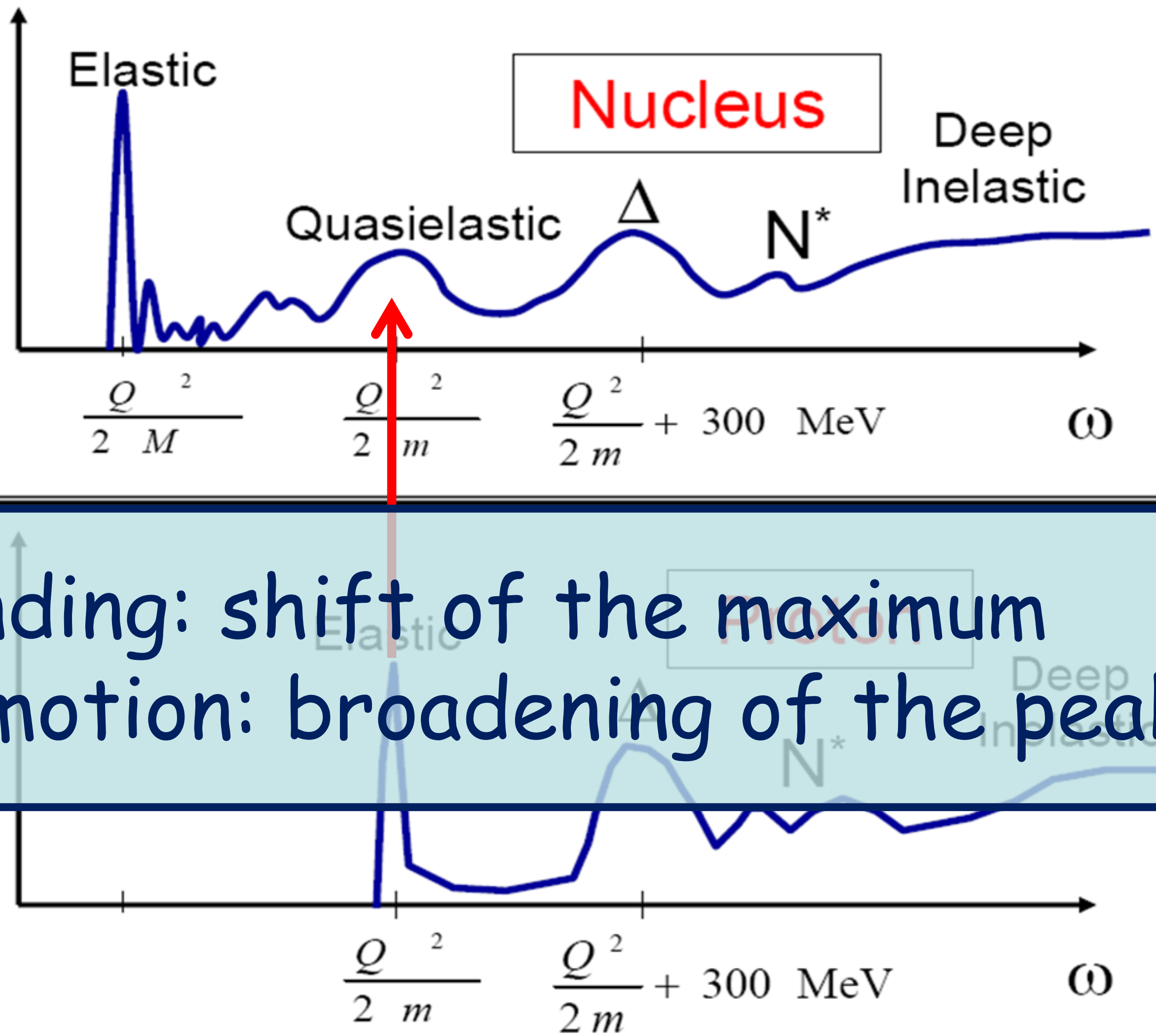
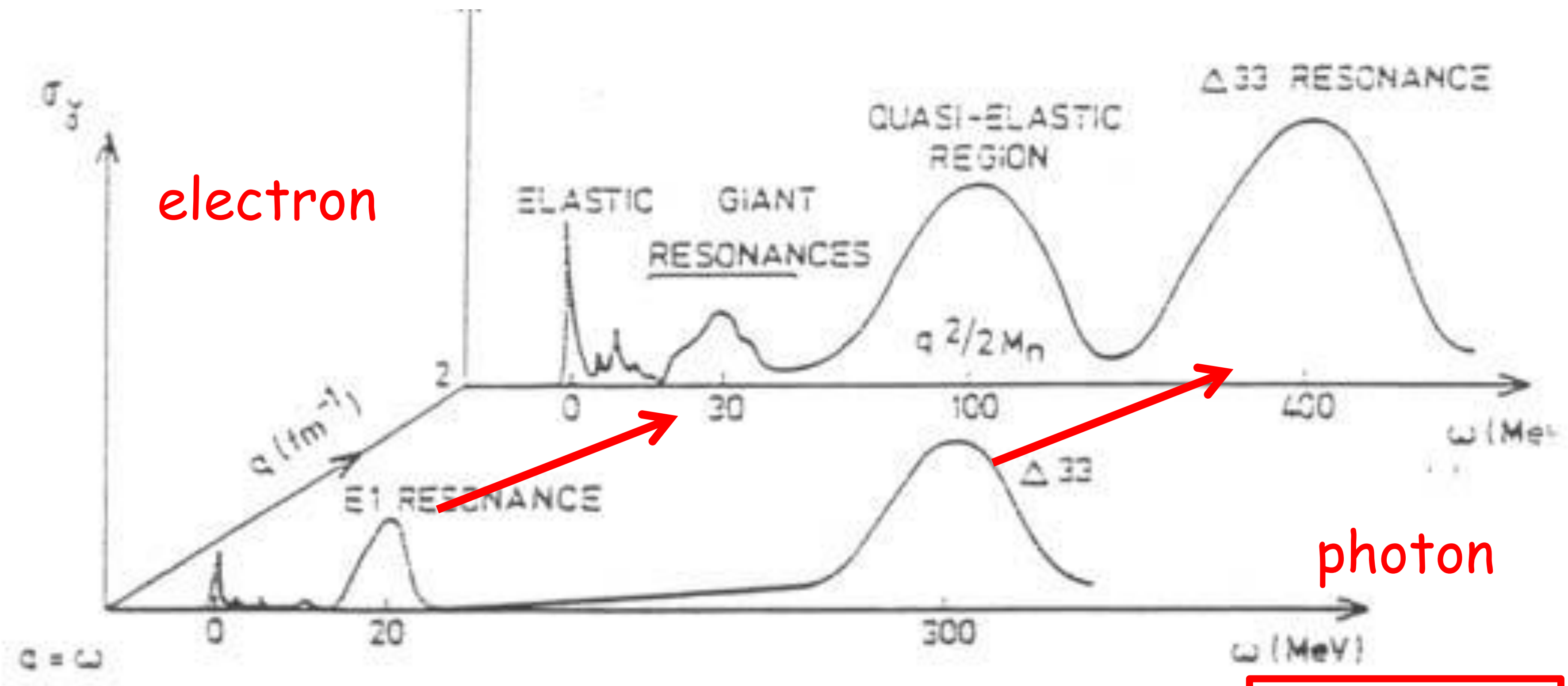


response to the electroweak probe



binding: shift of the maximum
Fermi motion: broadening of the peak

nuclear response to electrons (virtual) and real photons



$$q = \omega = E_\gamma$$

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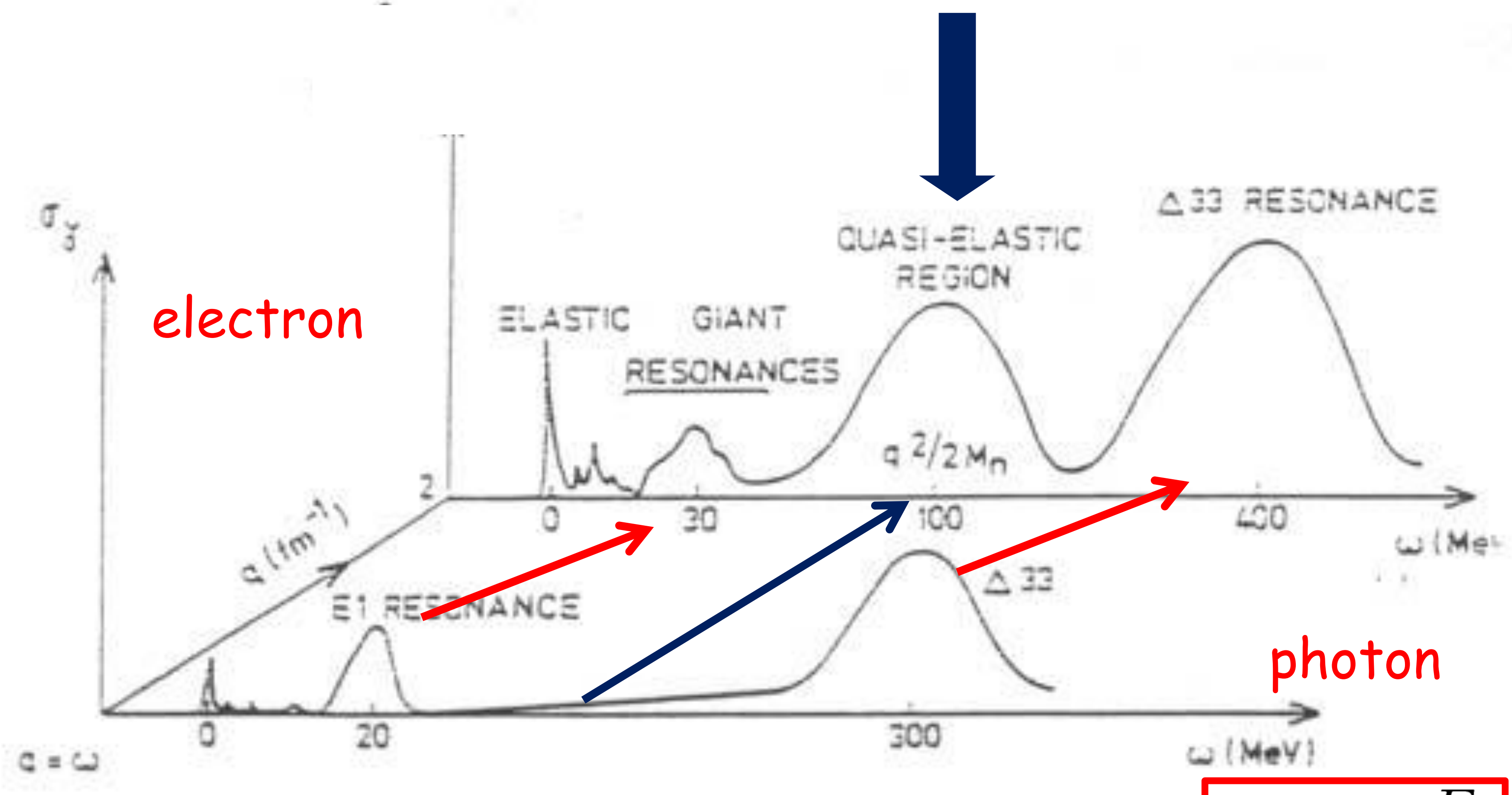

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QE e-nucleus scattering

$$e + A \Longrightarrow e' + N + (A-1)$$

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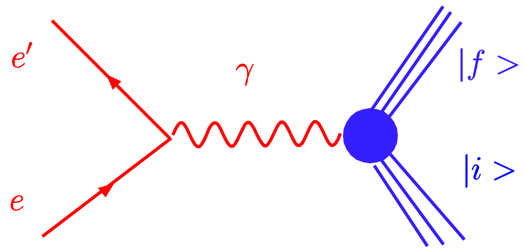
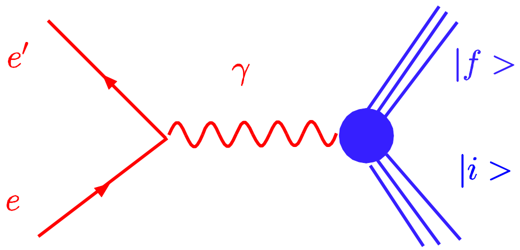
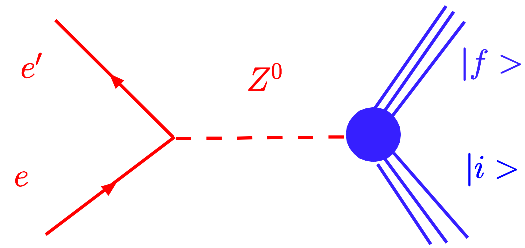


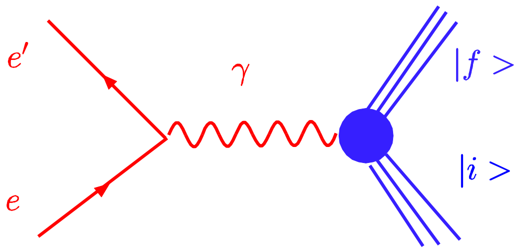
electron scattering

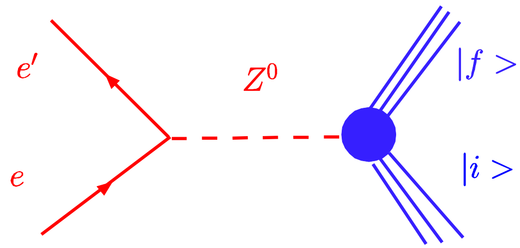




PVES

electron scattering

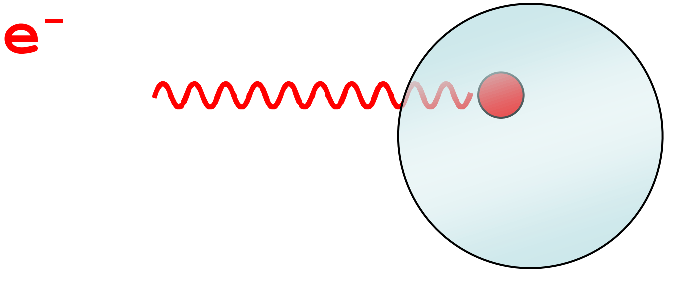




PVES

electron scattering

(e,e'p) one-nucleon knockout



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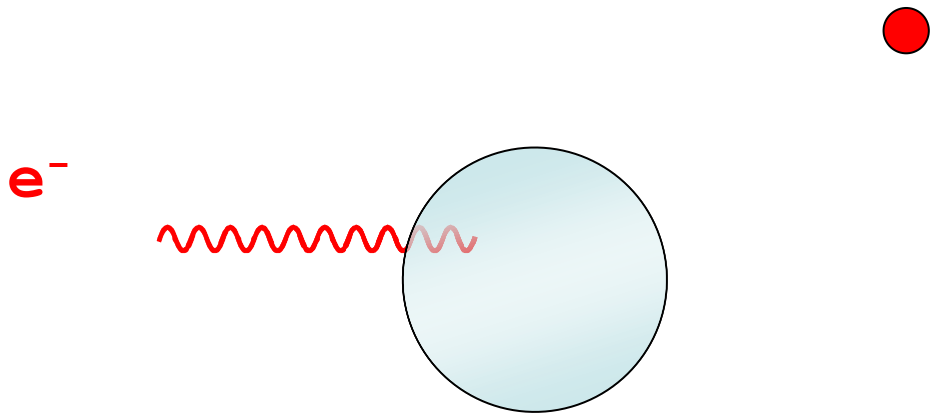
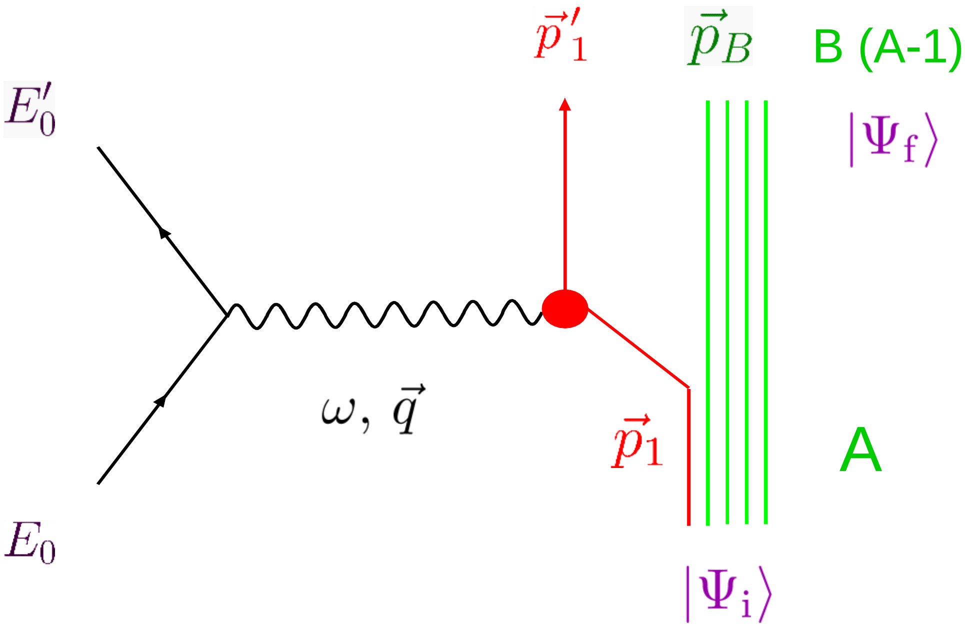


properties of bound protons

independent particle shell model
validity and limits

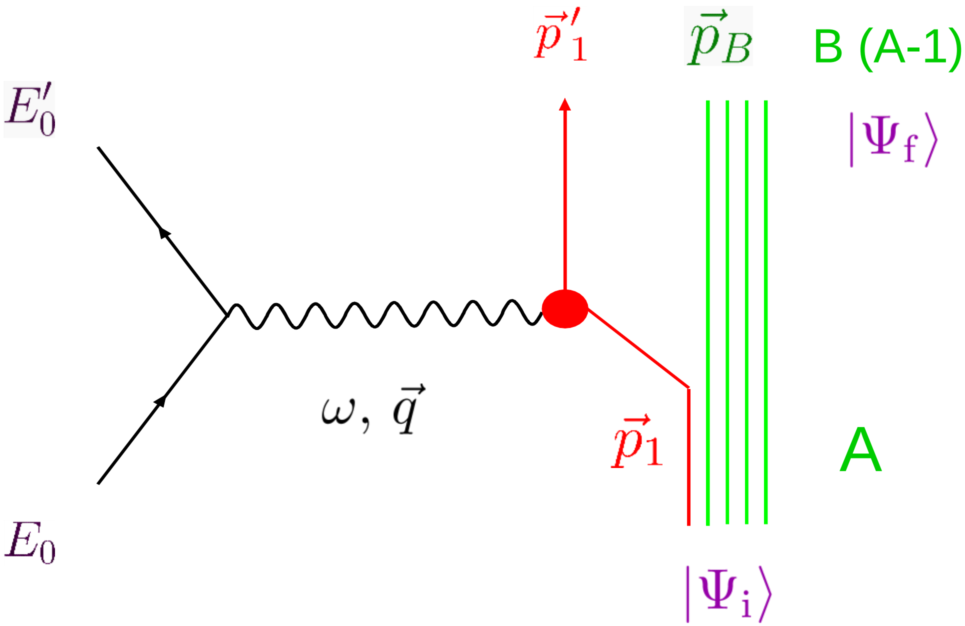
nuclear correlations

(e,e'p)



$$E_m = \omega - \frac{{p'_1}^2}{2m} - \frac{{p_B}^2}{2m(A-1)} = W_B^* - W_A$$ missing energy

$$\vec{p}_m = \vec{q} - \vec{p'}_1 = \vec{p}_B = -\vec{p}_1$$ missing momentum



(e,e'p)

$$E_m = \omega - \frac{p_1'^2}{2m} - \frac{p_B{}^2}{2m(A-1)} = W_B^* - W_A$$ missing energy

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one-hole spectral function

$$S(\vec{p_1}, \vec{p_1}; E_m) = \langle \Psi_i | a^+_{\vec{p_1}} \delta(E_m - H) a_{\vec{p_1}} | \Psi_i \rangle$$

E_m

exclusive reaction

ONE-HOLE SPECTRAL FUNCTION

$$S(\vec{p_1},\vec{\bar{p}_1};E_m) = \langle \Psi_{\rm i}| a^{+}_{\vec{\bar{p}}_1} \delta(E_m - H) a_{\vec{p_1}} |\Psi_{\rm i}\rangle$$

$\vec{p}_1 = \vec{\bar{p}}_1$

joint probability of removing from the target a nucleon p_1 leaving the residual nucleus in a state with energy E_m

E_m

exclusive reaction

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$$S(\vec{p_1},\vec{\bar{p}_1};E_m) = \langle \Psi_{\rm i}|a^{+}_{\vec{\bar{p}}_1}\delta(E_m - H)a_{\vec{p_1}}|\Psi_{\rm i}\rangle$$

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joint probability of removing from the target a nucleon p_1
leaving the residual nucleus in a state with energy E_m

$$\int S(\vec{p_1},\vec{\bar{p}_1};E_m){\rm d}E_m = \rho(\vec{p_1},\vec{\bar{p}_1})$$

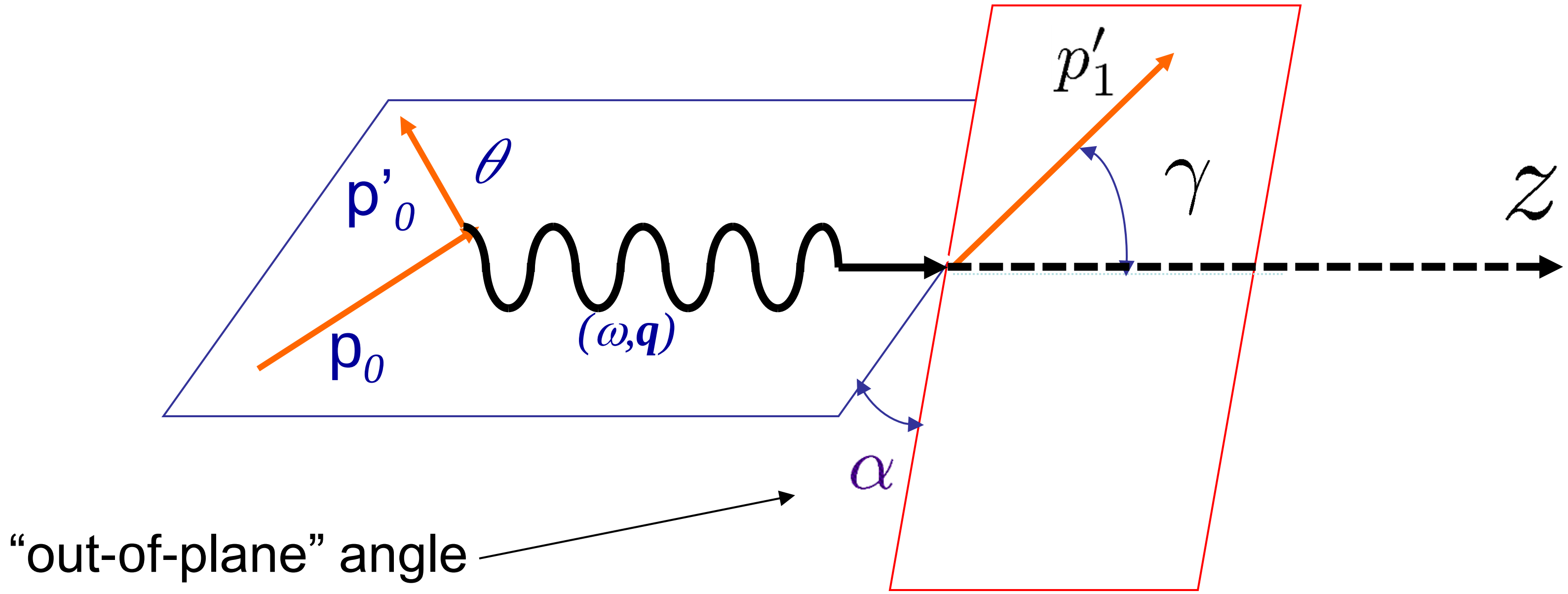
inclusive reaction : one-body density

$\vec{p}_1 = \vec{\bar{p}}_1$ ⇒ $\rho(\vec{p_1},\vec{\bar{p}_1}) = F(\vec{p_1})$

MOMENTUM DISTRIBUTION

$$F(\vec{p_1}) = \int |\Psi_{\rm i}(\vec{p_1},\vec{p_2},....,\vec{p_A}|^2{\rm d}\vec{p_2}....{\rm d}\vec{p_A}$$

ONE-NUCLEON KNOCKOUT

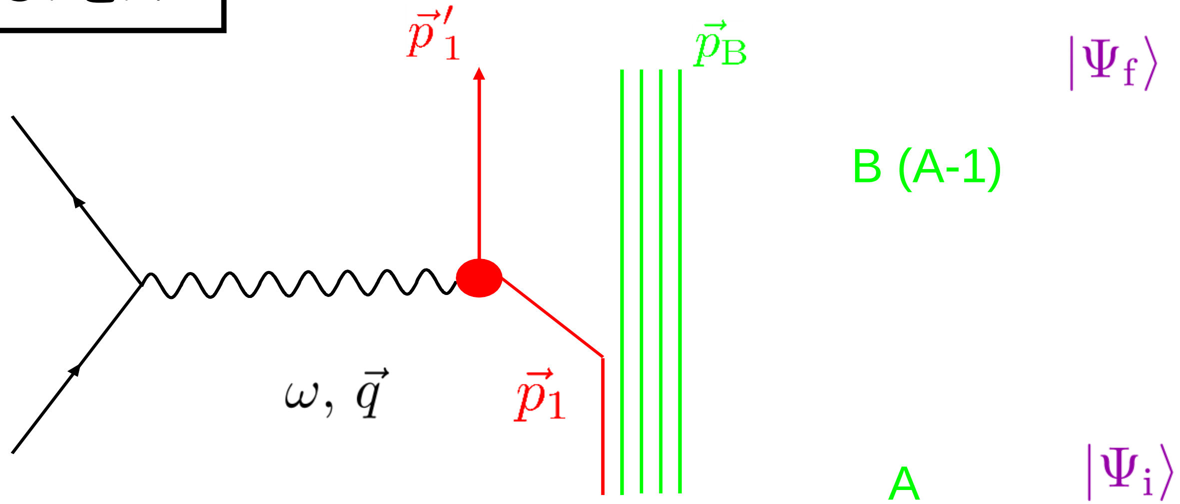


q// z xz electron plane

α out-of plane angle

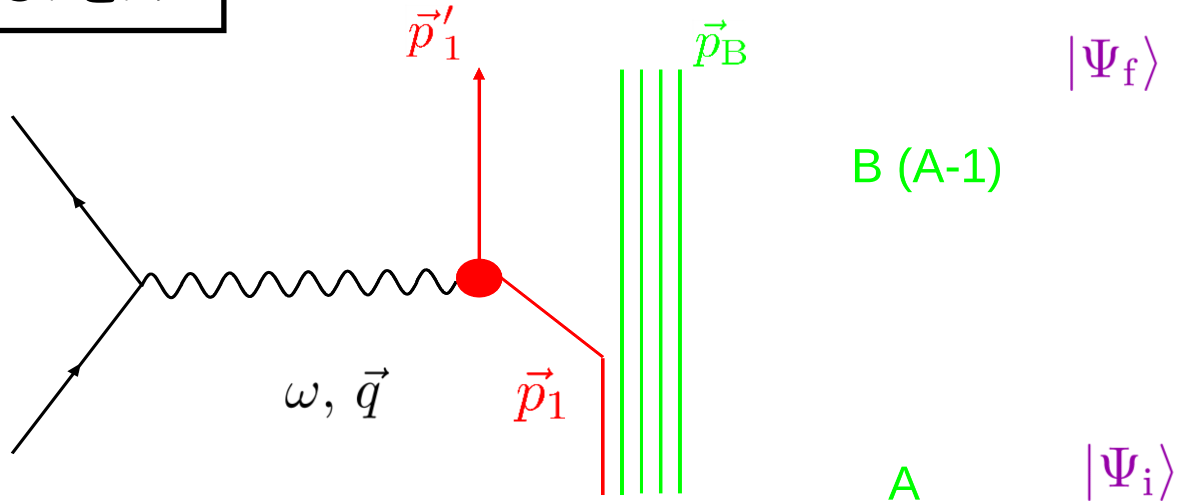
γ angle between q and p'$_1$

OPEA



$$\sigma = K L_{\mu\nu} \, W^{\mu\nu}$$

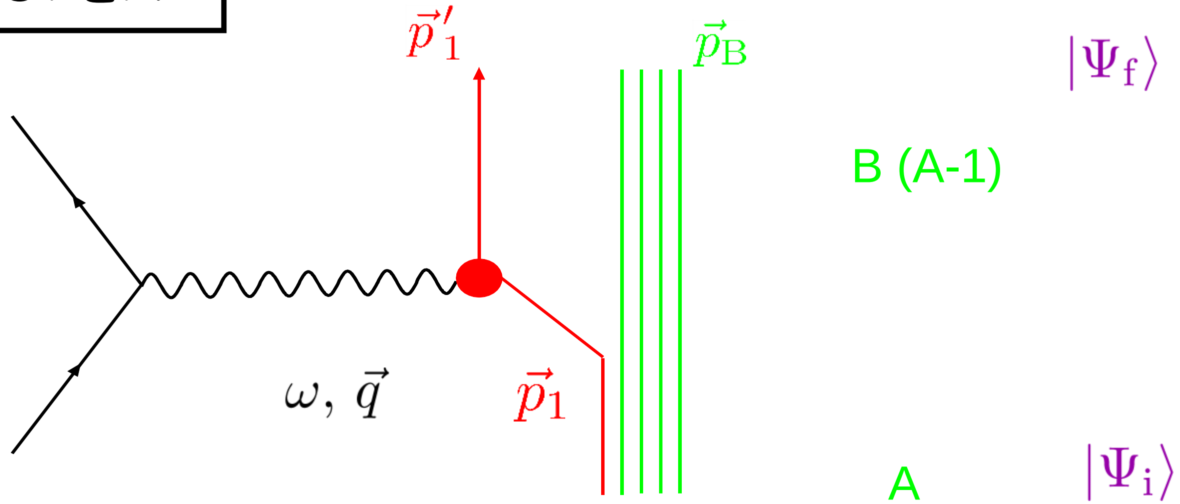
OPEA



$$\sigma = K\, L_{\mu\nu}\, W^{\mu\nu}$$

kinematical factor

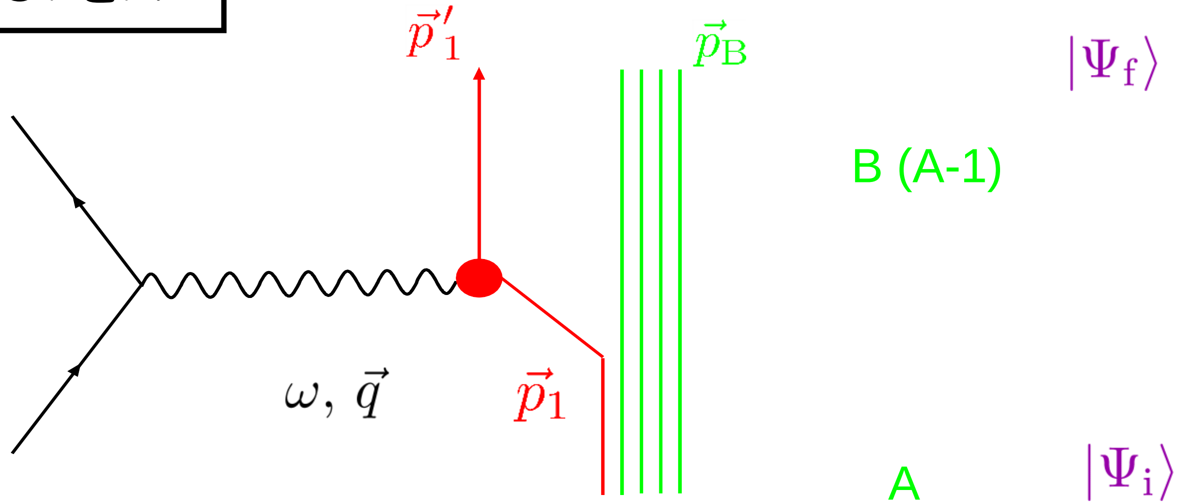
OPEA



$$\sigma = K L_{\mu\nu} W^{\mu\nu}$$

lepton tensor

OPEA

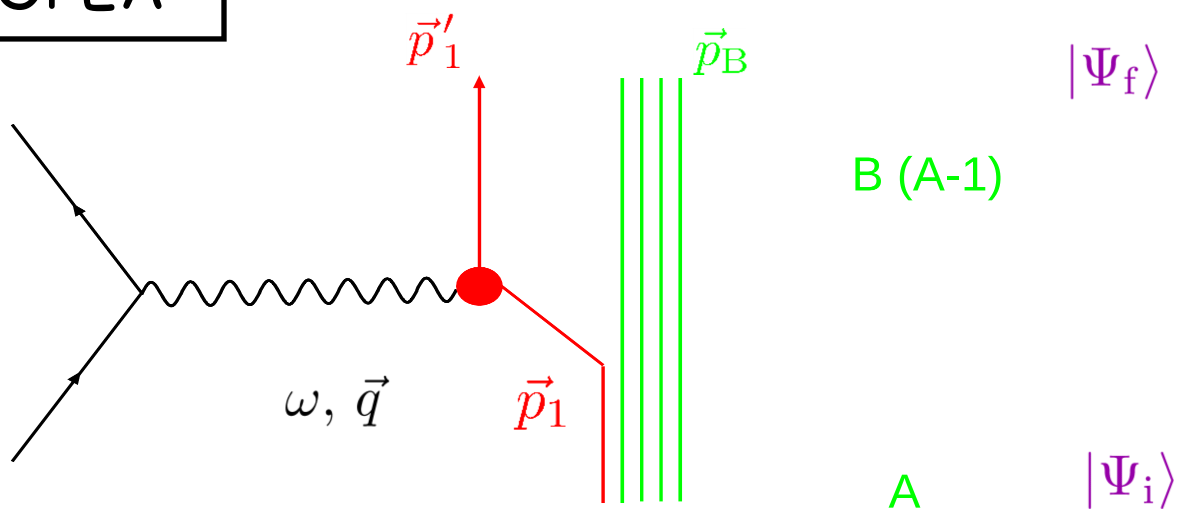


$$\sigma = K L_{\mu\nu} \ W^{\mu\nu}$$

↓

hadron tensor

OPEA



$$\sigma = K L_{\mu\nu} W^{\mu\nu}$$

plane-wave approximation for electrons:

$L_{\mu\nu}$ kinematic factors

hadron tensor

$$W^{\mu\nu} = \overline{\sum_{\mathrm{i,f}}} J^{\mu}(\vec{q}) J^{\nu *}(\vec{q}) \delta(E_{\mathrm{i}} - E_{\mathrm{f}})$$

$$J^{\mu}(\vec{q}) = \int \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \langle \Psi_{\mathrm{f}} \mid \hat{J}^{\mu}(\vec{r}) \mid \Psi_{\mathrm{i}} \rangle \mathrm{d}\vec{r}$$

$$W^{\mu\nu} = W^{\mu\nu S} + W^{\mu\nu A}$$

$$W^{\mu\nu S} = W^{\nu\mu S} \quad W^{\mu\nu A} = -W^{\nu\mu A}$$

$$L_{\mu\nu} = L^S_{\mu\nu} + hL^A_{\mu\nu}$$

h helicity
for unpolarised electrons $L_{\mu\nu} = L^S_{\mu\nu}$

$\Rightarrow L_{\mu\nu}W^{\mu\nu A} = 0 \Rightarrow W^{\mu\nu} = W^{\mu\nu S}$

J^μ is a 4-vector and therefore $W^{\mu\nu}$ is a rank 2 tensor
Its most general form can be built from invariance arguments
$W^{\mu\nu}$ depends on the only independent 4-vectors $q^\mu, p_1^{;\mu}, P_A^\mu$

$$W^{\mu\nu} = W^{\nu\mu} = Ag^{\mu\nu} + Bq^\mu q^\nu + CP_A^\mu P_A^\nu + D(P_A^\mu q^\nu + P_A^\nu q^\mu) + E(P_A^\mu p_1'^\nu + P_A^\nu p_1'^\mu) + F(p_1'^\mu q^\nu + p_1'^\nu q^\mu) + Gp_1'^\mu p_1'^\nu$$

A, B,..... G 7 coefficients dependent on the only independent scalar invariants that can be built with q^μ, $p_1^{\mu,}$, P_A^μ: $q_\mu^2, q^\mu \cdot P_A^\mu, q^\mu \cdot p_1'^\mu, p'^\mu \cdot P_A^\mu$

$$(p'_{1\mu} p_1'^\mu = m^2 \quad P_{A\mu} P_A^\mu = M_A^2)$$

Conservation of nuclear current $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$ 3 relations

A........ G → $W_1\ W_2\ W_3\ W_4$

7 4

$W_1 = A$ $W_2 = C$ $W_3 = E$ $W_4 = G$ ←

$$W^{\mu\nu} = -W_1\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q_\mu^2}\right) + \frac{W_2}{M_A^2}\left(P_A^\mu - \frac{q\cdot P_A}{q_\mu^2}q^\mu\right)\left(P_A^\nu - \frac{q\cdot P_A}{q_\mu^2}q^\nu\right) + \frac{W_4}{m^2}\left(p_1^{'\mu} - \frac{q\cdot p_1'}{q_\mu^2}q^\mu\right)\left(p_1^{'\nu} - \frac{q\cdot p_1'}{q_\mu^2}q^\nu\right) + \frac{W_3}{2p_1'\cdot P_A}\left[\left(P_A^\mu - \frac{q\cdot P_A}{q_\mu^2}q^\mu\right)\left(p_1^{'\nu} - \frac{q\cdot p_1'}{q_\mu^2}q^\nu\right) + \left(P_A^\nu - \frac{q\cdot P_A}{q_\mu^2}q^\nu\right)\left(p_1^{'\mu} - \frac{q\cdot p_1'}{q_\mu^2}q^\mu\right)\right]$$

Conservation of electron current $L_{\mu\nu} q^\mu = L_{\mu\nu} q^\nu = 0$ ←
suppresses terms linear in q^μ which do not contribute when contracting with the hadron tensor

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M_A^2}P_A^\mu P_A^\nu + \frac{W_4}{m^2}p_1^{'\mu}p_1^{'\nu} + \frac{W_3}{2p_1'\cdot P_A}\left(P_A^\mu p_1^{'\nu} + P_A^\nu p_1^{'\mu}\right)$$

Introducing spherical components

$$\vec{e}_{\pm 1} = \mp \frac{1}{\sqrt{2}}(\hat{x} \pm \hat{y}) \qquad \vec{e}_0 = \hat{z} = \frac{\vec{q}}{|\vec{q}|}$$

$$\vec{e}_\lambda{}^\dagger \cdot \vec{e}_{\lambda'} = \delta_{\lambda\lambda'}$$

$$\vec{e}_\lambda{}^* = (-1)^\lambda \vec{e}_{-\lambda}$$

$$\vec{J} = \vec{J}_L + \vec{J}_T = J_L \vec{e}_0 + (J_1 \vec{e}_1 + J_{-1}\vec{e}_{-1})$$

- current continuity

$$q_\mu J^\mu = 0 \qquad J_L = \frac{\omega}{q} J^0$$

- Lorentz condition

$$q_\mu A^\mu = 0 \qquad A_L = \frac{\omega}{q} A_0$$

A Moller potential

$$A_\mu J^\mu = -\frac{q_\mu^2}{\vec{q}^2} A_0 J_0 - \vec{A}_T \cdot \vec{J}_T$$

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$$\frac{1}{2} L_{\mu\nu} W^{\mu\nu} = \rho_{00} F_{00} + \rho_{11} F_{11} + \rho_{01} F_{01} + \rho_{1-1} F_{1-1}$$

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$$\rho_{00} = \frac{1}{2}\frac{q_\mu^4}{\vec{q}^4} L_{00} = \frac{q_\mu^4}{\vec{q}^4} 2E_0 E_0' \cos^2\frac{\theta}{2}$$

$$\rho_{11} = -\frac{1}{2} L_{11} = -\frac{1}{2}\frac{q_\mu^2}{\vec{q}^2}(\vec{q}^2 + 2E_0 E_0' \cos^2\frac{\theta}{2})$$

$$\rho_{01} = \frac{1}{2}\frac{q_\mu^2}{\vec{q}^2} L_{01} = -\frac{q_\mu^2}{\vec{q}^2}\frac{1}{\sqrt{2}}(E_0 + E_0')\frac{|\vec{p}_0 \times \vec{p}_0'|}{|\vec{q}|}$$

$$\rho_{1-1} = -\frac{1}{2} L_{1-1} = -\frac{|\vec{p}_0 \times \vec{p}_0'|^2}{\vec{q}^2}$$

$$F_{00} = W^{00}$$
$$F_{11} = -W^{-1-1} - W^{11}$$
$$F_{01} = W^{01} + W^{10} - W^{0-1} - W^{-10}$$
$$F_{1-1} = W^{1-1} - W^{-11}$$

$$F_{00} = W_1 \frac{q^2}{q_\mu^2} + \frac{q^4}{q_\mu^4}(W_2 + cW_3 + \frac{E_1' p_1^{12}}{m^2} c^2 W_4)$$

$$F_{11} = 2W_1 + W_4 \frac{p_1^{'2}}{m^2} \sin^2\gamma$$

$$F_{01} = \frac{q^2}{q_\mu^2} \frac{\sqrt{2} p_1'}{E_1'} (W_3 + 2\frac{E_1^{'2}}{m^2} cW_4) \sin\gamma \cos\alpha$$

$$F_{1-1} = -\frac{W_4}{m^2} \vec{p}_1^{\,\prime 2} \sin\gamma \cos 2\alpha \qquad c = 1 - \frac{\omega p_1'}{qE_1'} \cos\gamma$$

functions of $W_{1,2,3,4}$ depend on ω, q, $p'_1 \gamma$

$$
\begin{aligned}
F_{00} &= W^{00} \\
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\end{aligned}
$$

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F_{1-1} &= -\frac{W_4}{m^2} \vec{p}_1'^2 \sin\gamma \cos 2\alpha \qquad c = 1 - \frac{\omega p_1'}{q E_1'} \cos\gamma
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$$F_{11} = 2W_1 + W_4 \frac{p_1'^2}{m^2} \sin^2\gamma$$

$$F_{01} = \frac{q^2}{q_\mu^2} \frac{\sqrt{2} p_1'}{E_1'} (W_3 + 2\frac{E_1'^2}{m^2} cW_4) \sin\gamma \cos\alpha$$

$$F_{1-1} = -\frac{W_4}{m^2} \vec{p}_1'^2 \sin\gamma \cos 2\alpha \qquad c = 1 - \frac{\omega p_1'}{q E_1'} \cos\gamma$$

functions of $W_{1,2,3,4}$ depend on ω, q, $p'_1 \gamma$

$$\sigma \;\; \alpha \;\; (\rho_{00} f_{00} + \rho_{11} f_{11} + \rho_{1-1} f_{1-1} \cos 2\alpha + \rho_{01} f_{01} \cos\alpha)$$

$$F_{00} = W^{00}$$
$$F_{11} = -W^{-1-1} - W^{11}$$
$$F_{01} = W^{01} + W^{10} - W^{0-1} - W^{-10}$$
$$F_{1-1} = W^{1-1} - W^{-11}$$

$$F_{00} = W_1 \frac{q^2}{q_\mu^2} + \frac{q^4}{q_\mu^4}(W_2 + cW_3 + \frac{E_1' p_1^{12}}{m^2} c^2 W_4)$$

$$F_{11} = 2W_1 + W_4 \frac{p_1'^2}{m^2} \sin^2\gamma$$

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functions of $W_{1,2,3,4}$ depend on ω, q, $p'_1 \gamma$

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$$\sigma \;\; \alpha \;\; (\rho_{00} f_{00} + \rho_{11} f_{11} + \rho_{1-1} f_{1-1} \cos 2\alpha + \rho_{01} f_{01} \cos \alpha)$$

4 nuclear structure functions

their separation requires non coplanar kinematics $\alpha \neq 0, 180$

σ (α=0) - σ (α = 180) $\rightarrow$ f_{01}

parallel kinematics q // p'_1 $\sin\gamma = 0$

f_{01} = $f_{1\text{-}1}$ = 0 , only f_{00} and f_{11} survive

Rosenbluth separation

$$
\begin{aligned}
F_{00} &= W^{00} \\
F_{11} &= -W^{-1-1} - W^{11} \\
F_{01} &= W^{01} + W^{10} - W^{0-1} - W^{-10} \\
F_{1-1} &= W^{1-1} - W^{-11}
\end{aligned}
$$

$$
\begin{aligned}
F_{00} &= W_1 \frac{q^2}{q_\mu^2} + \frac{q^4}{q_\mu^4}\left(W_2 + cW_3 + \frac{E_1' p_1^{12}}{m^2} c^2 W_4\right) \\
F_{11} &= 2W_1 + W_4 \frac{p_1'^2}{m^2} \sin^2\gamma \\
F_{01} &= \frac{q^2}{q_\mu^2} \frac{\sqrt{2} p_1'}{E_1'} \left(W_3 + 2\frac{E_1'^2}{m^2} c W_4\right) \sin\gamma \cos\alpha \\
F_{1-1} &= -\frac{W_4}{m^2} \vec{p}_1'^2 \sin\gamma \cos 2\alpha
\end{aligned}
$$

$$
c = 1 - \frac{\omega p_1'}{q E_1'} \cos\gamma
$$

functions of $W_{1,2,3,4}$ depend on ω, q, $p'_1 \gamma$

$$
\sigma \;\; \alpha \;\; (\rho_{00} f_{00} + \rho_{11} f_{11} + \rho_{1-1} f_{1-1} \cos 2\alpha + \rho_{01} f_{01} \cos\alpha)
$$

$$\sigma \;\alpha\; (\rho_{00} f_{00} + \rho_{11} f_{11} + \rho_{1-1} f_{1-1} \cos 2\alpha + \rho_{01} f_{01} \cos \alpha)$$

4 nuclear structure functions

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parallel kinematics q // p'_1 $\sin\gamma = 0$
$f_{01} = f_{1-1} = 0$, only f_{00} and f_{11} survive
Rosenbluth separation

EXCLUSIVE ⟹ INCLUSIVE

$$\int \mathrm{d}\vec{p'_1} \quad \Longrightarrow \quad \begin{aligned} \int f_{00}\, \mathrm{d}\vec{p'_1} &= R_L(\omega, q) \\ \int f_{11}\, \mathrm{d}\vec{p'_1} &= R_T(\omega, q) \\ \int f_{1-1}\, \mathrm{d}\vec{p'_1} &= 0 \\ \int f_{01}\, \mathrm{d}\vec{p'_1} &= 0 \end{aligned}$$

$$\Longrightarrow \quad \int \mathrm{d}\vec{p'_1}(e, e'p) = (e, e')$$

EXCLUSIVE ⟹ INCLUSIVE

$$\int \mathrm{d}\vec{p_1'} \quad \Longrightarrow \quad \begin{aligned} \int f_{00}\,\mathrm{d}\vec{p_1'} &= R_L(\omega,q) \\ \int f_{11}\,\mathrm{d}\vec{p_1'} &= R_T(\omega,q) \\ \int f_{1-1}\,\mathrm{d}\vec{p_1'} &= 0 \\ \int f_{01}\,\mathrm{d}\vec{p_1'} &= 0 \end{aligned}$$

$$\Longrightarrow \quad \int \mathrm{d}\vec{p_1'}(e,e'p) = (e,e')$$

$$\Longrightarrow \quad \sigma = K\left(R_T(\omega,q) + 2\varepsilon_L R_L(\omega,q)\right)$$

$$\varepsilon_L = \frac{Q^2}{q^2}\left[1 + 2\frac{q^2}{Q^2}\tan^2\frac{\theta}{2}\right]$$

(e,e'p)

DWIA model

$$J^{\mu}(\vec{q}) = \int \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \langle \Psi_{\mathrm{f}} \mid \hat{J}^{\mu}(\vec{r}) \mid \Psi_{\mathrm{i}} \rangle \mathrm{d}\vec{r}$$

Ψ_i g.s of the target nucleus A

Ψ_f A-body nuclear state asymptotically corresponding to the KO nucleon 1 (E'_1, p'_1) and a residual A-1 nucleus B

(e,e'p)

DWIA model

$$J^{\mu}(\vec{q}) = \int e^{i\vec{q}\cdot\vec{r}} \langle \Psi_f \mid \hat{J}^{\mu}(\vec{r}) \mid \Psi_i \rangle d\vec{r}$$

Ψ_i g.s of the target nucleus *A*

Ψ_f *A*-body nuclear state asymptotically corresponding to the KO nucleon 1 (E'_1, p'_1) and a residual *A*-1 nucleus B

- for $E_m = \omega - T'_1 - T_B = W_B^* - W_A$ corresponding to a peak in the E_m distr. we assume that the residual nucleus is in a discrete state n with exc energy $W_B^* = E_n$ EXCLUSIVE REACTION ★

(e,e'p)

DWIA model

$$J^{\mu}(\vec{q}) = \int \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \langle \Psi_{\mathrm{f}} \mid \hat{J}^{\mu}(\vec{r}) \mid \Psi_{\mathrm{i}} \rangle \mathrm{d}\vec{r}$$

Ψ_i g.s of the target nucleus A

Ψ_f A-body nuclear state asymptotically corresponding to the KO nucleon 1 (E'_1, p'_1) and a residual A-1 nucleus B

- for $E_m = \omega - T'_1 - T_B = W_B^* - W_A$ corresponding to a peak in the E_m distr. we assume that the residual nucleus is in a discrete state n with exc energy $W_B^* = E_n$ EXCLUSIVE REACTION ★
- the final state is projected $|\Psi_{\mathrm{f}}\rangle = |E'_1, n\rangle \simeq P_n |E'_1 n\rangle$ ★

$$P_n = \int \mathrm{d}\vec{p}' a^{+}_{\vec{p}'} |n><n| a_{\vec{p}'} \qquad P_n^2 = P_n \qquad P_n + Q_n = 1$$

(e,e'p)

DWIA model

$$J^\mu(\vec{q}) = \int \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \langle \Psi_\mathrm{f} \mid \hat{J}^\mu(\vec{r}) \mid \Psi_\mathrm{i} \rangle \mathrm{d}\vec{r}$$

Ψ_i g.s of the target nucleus A

Ψ_f A-body nuclear state asymptotically corresponding to the KO nucleon 1 (E'_1, p'_1) and a residual A-1 nucleus B

- for $E_m = \omega - T'_1 - T_B = W_B^* - W_A$ corresponding to a peak in the E_m distr. we assume that the residual nucleus is in a discrete state n with exc energy $W_B^* = E_n$ EXCLUSIVE REACTION ★
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$$P_n = \int \mathrm{d}\vec{p}' a^+_{\vec{p}'} |n><n| a_{\vec{p}'} \qquad P_n^2 = P_n \qquad P_n + Q_n = 1$$

$$\langle \Psi_\mathrm{f} \mid \hat{J}^\mu \mid \Psi_\mathrm{i} \rangle \simeq \int \langle n E'_1 \mid a^+_{\vec{p}'} \mid n \rangle \langle n \mid a_{\vec{p}'} \hat{J}^\mu \mid \Psi_\mathrm{i} \rangle \mathrm{d}\vec{p}'$$

(e,e'p)

DWIA model

$$J^{\mu}(\vec{q}) = \int \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \langle \Psi_{\mathrm{f}} \mid \hat{J}^{\mu}(\vec{r}) \mid \Psi_{\mathrm{i}} \rangle \mathrm{d}\vec{r}$$

Ψ_i g.s of the target nucleus A

Ψ_f A-body nuclear state asymptotically corresponding to the KO nucleon 1 (E'_1, p'_1) and a residual A-1 nucleus B

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$$P_n = \int \mathrm{d}\vec{p}' a^{+}_{\vec{p}'} |n><n| a_{\vec{p}'} \qquad P_n^2 = P_n \qquad P_n + Q_n = 1$$

$$\langle \Psi_{\mathrm{f}} \mid \hat{J}^{\mu} \mid \Psi_{\mathrm{i}} \rangle \simeq \int \underbrace{\langle n E'_1 \mid a^{+}_{\vec{p}'} \mid n \rangle}_{\chi^{(-)*}_{n\vec{p}'_1}(\vec{p}'_1)} \langle n \mid a_{\vec{p}'} \hat{J}^{\mu} \mid \Psi_{\mathrm{i}} \rangle \mathrm{d}\vec{p}'$$

FINAL STATE: product of a s.p. DW $\chi^{(-)}$ the outgoing proton and residual nucleus n

★

(e,e'p)

DWIA model

$$J^{\mu}(\vec{q}) = \int e^{i\vec{q}\cdot\vec{r}} \langle \Psi_f \mid \hat{J}^{\mu}(\vec{r}) \mid \Psi_i \rangle d\vec{r}$$

Ψ_i g.s of the target nucleus A

Ψ_f A-body nuclear state asymptotically corresponding to the KO nucleon 1 (E'_1, p'_1) and a residual A-1 nucleus B

- for $E_m = \omega - T'_1 - T_B = W_B^* - W_A$ corresponding to a peak in the E_m distr. we assume that the residual nucleus is in a discrete state n with exc energy $W_B^* = E_n$ EXCLUSIVE REACTION ★
- the final state is projected $|\Psi_f\rangle = |E'_1, n\rangle \simeq P_n |E'_1 n\rangle$ ★

$$P_n = \int d\vec{p}' a^{+}_{\vec{p}'} |n><n| a_{\vec{p}'} \qquad P_n^2 = P_n \qquad P_n + Q_n = 1$$

$$\langle \Psi_f \mid \hat{J}^{\mu} \mid \Psi_i \rangle \simeq \int \langle n E'_1 \mid a^{+}_{\vec{p}'} \mid n \rangle \langle n \mid a_{\vec{p}'} \hat{J}^{\mu} \mid \Psi_i \rangle d\vec{p}'$$

$\chi^{(-)*}_{n\vec{p}'_1}(\vec{p}'_1)$

eigenfunction of Feshbach optical potential $\mathcal{H}^{+}(T'_1)$ with eigenvalue T'_1

$$\mathcal{H}_n(E) = P_n H P_n + P_n H Q_n \frac{1}{E - Q_n H Q_n + i\eta} Q_n H P_n$$

$$\hat{J}^{\mu} = \sum_i \hat{j}_i^{\mu} \simeq \hat{j}_1^{\mu}$$

one-body nuclear current

interaction only on the quasi-free proton 1 ★

$\hat{J}^{\mu} = \sum_i \hat{j}_i^{\mu} \simeq \hat{j}_1^{\mu}$ one-body nuclear current

interaction only on the quasi-free proton 1 ★

$P_n \hat{j}_1^{\mu} P_n$ does not connect different channel subspaces

→ **IA DKO** ★

$\hat{J}^{\mu} = \sum_i \hat{j}_i^{\mu} \simeq \hat{j}_1^{\mu}$ one-body nuclear current

interaction only on the quasi-free proton 1 ★

$P_n \hat{j}_1^{\mu} P_n$ does not connect different channel subspaces

IA DKO ★

$$\langle n \mid a_{\vec{p}'} \hat{J}^{\mu} \mid \Psi_{\rm i} \rangle = \int n^*(\vec{p}_2, \cdots, \vec{p}_A) j^{\mu}(\vec{p}_1, \vec{q})\, \delta(\vec{p}_1' - \vec{p}_1 - \vec{q}) \Psi_{\rm i}(\vec{p}_1, \cdots \vec{p}_A)\, {\rm d}\vec{p}_1 \cdots {\rm d}\vec{p}_A$$

$P_n \hat{j}^{\mu}{}_1 P_n$

$$= j^{\mu}(\vec{p}_1' - \vec{q}, \vec{q}) \int n^*(\vec{p}_2, \cdots, \vec{p}_A) \Psi_{\rm i}(\vec{p}_1, \cdots \vec{p}_A)\, {\rm d}\vec{p}_1 \cdots {\rm d}\vec{p}_A$$

$$= \quad j^{\mu}(\vec{p}_1, \vec{q}) \qquad \lambda_n^{1/2} \phi_n(p_1)$$

$$\hat{J}^\mu = \sum_i \hat{j}_i^\mu \simeq \hat{j}_1^\mu$$ one-body nuclear current

interaction only on the quasi-free proton 1 ★

$P_n \hat{j}_1^\mu P_n$ does not connect different channel subspaces

IA DKO ★

$$\langle n \mid a_{\vec{p}\,'} \hat{J}^\mu \mid \Psi_\mathrm{i} \rangle = \int n^*(\vec{p}_2, \cdots, \vec{p}_A) j^\mu(\vec{p}_1, \vec{q})\, \delta(\vec{p}\,'_1 - \vec{p}_1 - \vec{q}) \Psi_\mathrm{i}(\vec{p}_1, \cdots \vec{p}_A)\, \mathrm{d}\vec{p}_1 \cdots \mathrm{d}\vec{p}_A$$

$P_n \hat{j}^\mu{}_1 P_n$

$$= j^\mu(\vec{p}\,'_1 - \vec{q}, \vec{q}) \int n^*(\vec{p}_2, \cdots, \vec{p}_A) \Psi_\mathrm{i}(\vec{p}_1, \cdots \vec{p}_A)\, \mathrm{d}\vec{p}_1 \cdots \mathrm{d}\vec{p}_A$$

$$= j^\mu(\vec{p}_1, \vec{q}) \qquad \lambda_n^{1/2} \phi_n(p_1)$$

overlap spectroscopic amplitude

ϕ_n eigenstate of $\mathcal{H}_n(\omega$ -T'$_1$ = - E$_m$) with eigenvalue - E$_m$

λ_n spectroscopic factor

$\hat{J}^\mu = \sum_i \hat{j}_i^\mu \simeq \hat{j}_1^\mu$ one-body nuclear current

interaction only on the quasi-free proton 1 ★

$P_n \hat{j}_1^\mu P_n$ does not connect different channel subspaces

IA DKO ★

$$\langle n \mid a_{\vec{p}'} \hat{J}^\mu \mid \Psi_{\rm i}\rangle = \int n^*(\vec{p}_2, \cdots, \vec{p}_A) j^\mu(\vec{p}_1, \vec{q})\, \delta(\vec{p}_1' - \vec{p}_1 - \vec{q}) \Psi_{\rm i}(\vec{p}_1, \cdots \vec{p}_A)\, {\rm d}\vec{p}_1 \cdots {\rm d}\vec{p}_A$$

$P_n \hat{j}^\mu{}_1 P_n$

$$= j^\mu(\vec{p}_1' - \vec{q}, \vec{q}) \int n^*(\vec{p}_2, \cdots, \vec{p}_A) \Psi_{\rm i}(\vec{p}_1, \cdots \vec{p}_A)\, {\rm d}\vec{p}_1 \cdots {\rm d}\vec{p}_A$$

$$= j^\mu(\vec{p}_1, \vec{q}) \qquad \lambda_n^{1/2} \phi_n(p_1)$$

overlap spectroscopic amplitude

ϕ_n eigenstate of $\mathcal{H}_n(\omega - T'_1 = -E_m)$ with eigenvalue $-E_m$

λ_n spectroscopic factor

$$J^\mu(\vec{q}) = \lambda_n^{1/2} \int \chi_{n\vec{p}_1'}^{(-)*}(\vec{q} + \vec{p})\, j^\mu(\vec{p}, \vec{q})\, \phi_n(\vec{p}) {\rm d}\vec{p}$$

$$\hat{J}^\mu = \sum_i \hat{j}_i^\mu \simeq \hat{j}_1^\mu$$

one-body nuclear current

interaction only on the quasi-free proton 1 ★

$$P_n \hat{j}_1^\mu P_n$$

does not connect different channel subspaces

IA DKO ★

$$\langle n \mid a_{\vec{p}'} \hat{J}^\mu \mid \Psi_\mathrm{i} \rangle = \int n^*(\vec{p}_2, \cdots, \vec{p}_A) j^\mu(\vec{p}_1, \vec{q})\, \delta(\vec{p}_1' - \vec{p}_1 - \vec{q}) \Psi_\mathrm{i}(\vec{p}_1, \cdots \vec{p}_A)\, \mathrm{d}\vec{p}_1 \cdots \mathrm{d}\vec{p}_A$$

$P_n \hat{j}^\mu{}_1 P_n$

$$= j^\mu(\vec{p}_1' - \vec{q}, \vec{q}) \int n^*(\vec{p}_2, \cdots, \vec{p}_A) \Psi_\mathrm{i}(\vec{p}_1, \cdots \vec{p}_A)\, \mathrm{d}\vec{p}_1 \cdots \mathrm{d}\vec{p}_A$$

$$= j^\mu(\vec{p}_1, \vec{q}) \qquad \lambda_n^{1/2} \phi_n(p_1)$$

overlap spectroscopic amplitude

ϕ_n eigenstate of $\mathcal{H}_n(\omega - T'_1 = -E_m)$ with eigenvalue $-E_m$

λ_n spectroscopic factor

$$J^\mu(\vec{q}) = \lambda_n^{1/2} \int \chi_{n\vec{p}_1'}^{(-)*}(\vec{q} + \vec{p})\, j^\mu(\vec{p}, \vec{q})\, \phi_n(\vec{p}) \mathrm{d}\vec{p}$$

DWIA

$$J^{\mu}(\vec{q}) = \lambda_n^{1/2} \int \chi_{n\vec{p}_1'}^{(-)*}(\vec{q}+\vec{p})\, j^{\mu}(\vec{p},\vec{q})\, \phi_n(\vec{p}) \mathrm{d}\vec{p}$$

DWIA

EXCLUSIVE REACTION
DKO MECHANISM

DWIA model

exclusive reaction: for a missing energy value corresponding to a peak in the energy distr. we assume that the residual nucleus is in a discrete state **n**

the final state is projected $|\Psi_{\rm f}\rangle = P|\Psi_{\rm f}\rangle \quad P = \int {\rm d}\vec{r}_1 |\vec{r}_1 n\rangle\langle n\vec{r}_1|$

DKO mechanism: one-body nuclear current does not connect different channel subspaces

$$Pj^\mu Q = 0 \quad Q = 1 - P$$

DKO mechanism, IA: the probe interacts through a one-body current with one nucleon which is then emitted the remaining nucleons are spectators

$$\int {\rm e}^{{\rm i}\vec{q}\cdot\vec{r}} \langle \Psi_{\rm f} \mid P j^\mu(\vec{r}) P \mid \Psi_{\rm i}\rangle {\rm d}\vec{r} = \int {\rm e}^{{\rm i}\vec{q}\cdot\vec{r}} \underbrace{\chi^{(-)*}(\vec{r}_1)}_{\langle \Psi_f|\vec{r}_1 n\rangle} \, j^\mu(\vec{r}_1,\vec{r}) \, \underbrace{\lambda_n{}^{1/2}\phi_n(\vec{r}_1)}_{\langle \vec{r}_1 n|\Psi_i\rangle} {\rm d}\vec{r}_1 {\rm d}\vec{r}$$

Direct knockout DWIA (e,e′p)

$$\lambda_n^{1/2} \; \langle \chi^{(-)} \mid j^{\mu} \mid \phi_n \rangle$$

- j^{μ} one-body nuclear current
- $\chi^{(-)}$ s.p. scattering w.f. $H^{+}(\omega+E_m)$
- ϕ_n s.p. bound state overlap function $H(-E_m)$
- λ_n spectroscopic factor
- $\chi^{(-)}$ and ϕ consistently derived as eigenfunctions of a Feshbach optical model Hamiltonian

$$\mathcal{H}(E) = PHP + PHQ \frac{1}{E - QHQ + \mathrm{i}\eta} QHP$$

Hadron tensor ⇒ Spectral function

$$
\begin{aligned}
W^{\mu\nu} &= \overline{\sum_{\mathrm{i,f}}} J^{\mu}(\vec{q}) J^{\nu *}(\vec{q}) \delta(E_{\mathrm{i}} - E_{\mathrm{f}}) \\
&= \lambda_n(E_m) \int \chi^{(-)*}_{n\vec{p}'_1}(\vec{q}+\vec{p})\, j^{\mu}(\vec{p},\vec{q})\, \phi_n(\vec{p}) \\
&\times \chi^{(-)}_{n\vec{p}'_1}(\vec{q}+\vec{\overline{p}})\, j^{\nu *}(\vec{\overline{p}},\vec{q})\, \phi^{*}_n(\vec{\overline{p}})\, \mathrm{d}\vec{p}\, \mathrm{d}\vec{\overline{p}}\, \delta(\omega - T'_1 - T_B - E_m) \\
&= \int \chi^{(-)*}_{n\vec{p}'_1}(\vec{q}+\vec{p})\, j^{\mu}(\vec{p},\vec{q})\; S(E_m;\vec{p}\vec{\overline{p}}) j^{\nu *}(\vec{\overline{p}},\vec{q})\, \chi^{(-)}_{n\vec{p}'_1}(\vec{q}+\vec{\overline{p}}) \mathrm{d}\vec{p}\, \mathrm{d}\vec{\overline{p}}
\end{aligned}
$$

Hadron tensor ⟹ Spectral function

$$
\begin{aligned}
W^{\mu\nu} &= \overline{\sum_{\mathrm{i,f}}} J^{\mu}(\vec{q}) J^{\nu *}(\vec{q}) \delta(E_{\mathrm{i}} - E_{\mathrm{f}}) \\
&= \lambda_n(E_m) \int \chi^{(-)*}_{n\vec{p}_1'}(\vec{q}+\vec{p})\, j^{\mu}(\vec{p},\vec{q})\, \phi_n(\vec{p}) \\
&\times \chi^{(-)}_{n\vec{p}_1'}(\vec{q}+\vec{\overline{p}})\, j^{\nu *}(\vec{\overline{p}},\vec{q})\, \phi^{*}_n(\vec{\overline{p}})\, \mathrm{d}\vec{p}\, \mathrm{d}\vec{\overline{p}}\, \delta(\omega - T_1' - T_B - E_m) \\
&= \int \chi^{(-)*}_{n\vec{p}_1'}(\vec{q}+\vec{p})\, j^{\mu}(\vec{p},\vec{q})\; S(E_m;\vec{p}\vec{\overline{p}}) j^{\nu *}(\vec{\overline{p}},\vec{q})\, \chi^{(-)}_{n\vec{p}_1'}(\vec{q}+\vec{\overline{p}}) \mathrm{d}\vec{p}\, \mathrm{d}\vec{\overline{p}}
\end{aligned}
$$

Hadron tensor ⟹ Spectral function

$$
\begin{aligned}
W^{\mu\nu} &= \overline{\sum_{\mathrm{i,f}}} J^{\mu}(\vec{q}) J^{\nu *}(\vec{q}) \delta(E_{\mathrm{i}} - E_{\mathrm{f}}) \\
&= \lambda_n(E_m) \int \chi^{(-)*}_{n\vec{p}'_1}(\vec{q}+\vec{p})\, j^{\mu}(\vec{p},\vec{q})\, \phi_n(\vec{p}) \\
&\times \chi^{(-)}_{n\vec{p}'_1}(\vec{q}+\vec{\overline{p}})\, j^{\nu *}(\vec{\overline{p}},\vec{q})\, \phi^{*}_n(\vec{\overline{p}})\, \mathrm{d}\vec{p}\, \mathrm{d}\vec{\overline{p}}\, \delta(\omega - T'_1 - T_B - E_m) \\
&= \int \chi^{(-)*}_{n\vec{p}'_1}(\vec{q}+\vec{p})\, j^{\mu}(\vec{p},\vec{q})\; S(E_m;\vec{p}\vec{\overline{p}}) j^{\nu *}(\vec{\overline{p}},\vec{q})\, \chi^{(-)}_{n\vec{p}'_1}(\vec{q}+\vec{\overline{p}}) \mathrm{d}\vec{p}\, \mathrm{d}\vec{\overline{p}}
\end{aligned}
$$

$$
\langle \Psi_{\mathrm{i}} \mid a_{\overline{p}}\, \delta(H_B - E_m - W_A)\, a_p \mid \Psi_{\mathrm{i}} \rangle
$$

ONE-HOLE SPECTRAL FUNCTION

ONE-HOLE SPECTRAL FUNCTION

$$S(E;\vec{p},\vec{p}') = \langle \Psi_{\rm i} | a^+_{\vec{p}'} \;\; \delta(H_{A-1} - E - W_A) \;\; a_{\vec{p}} | \Psi_{\rm i} \rangle$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$\sum_\alpha \int {\rm d}\epsilon \mid \epsilon\alpha\rangle\langle\epsilon\alpha \mid \qquad\qquad \sum_{\alpha'} \int {\rm d}\epsilon' \mid \epsilon'\alpha'\rangle\langle\epsilon'\alpha' \mid$$

$$= \sum_\alpha \int {\rm d}\epsilon \langle \Psi_{\rm i} \mid a^+_{\vec{p}'} \mid \epsilon\alpha\rangle \delta(\epsilon - E - W_A) \langle \epsilon\alpha \mid a_{\vec{p}} \mid \Psi_{\rm i}\rangle$$

$$= \sum_\alpha \phi^*_{E\alpha}(\vec{p}') \;\; \lambda_\alpha(E) \;\; \phi_{E\alpha}(\vec{p})$$

It is defined only for discrete energy values corresponding to bound states of the residual nucleus and for a continuum spectrum starting from the particle emission threshold of the residual nucleus

ONE-HOLE SPECTRAL FUNCTION

$$S(E;\vec{p},\vec{\bar{p}}) = \langle \Psi_\mathrm{i} | a^+_{\vec{\bar{p}}} \;\; \delta(H_{A-1} - E - W_A) \;\; a_{\vec{p}} | \Psi_\mathrm{i} \rangle$$

$$= \sum_\alpha \int \mathrm{d}\epsilon \langle \Psi_\mathrm{i} \mid a^+_{\vec{\bar{p}}} \mid \epsilon\alpha \rangle \delta(\epsilon - E - W_A) \langle \epsilon\alpha \mid a_{\vec{p}} \mid \Psi_\mathrm{i} \rangle$$

$$= \sum_\alpha \phi^*_{E\alpha}(\vec{\bar{p}}) \; \lambda_\alpha(E) \; \phi_{E\alpha}(\vec{p})$$

It is related to the hole s.p. Green's function

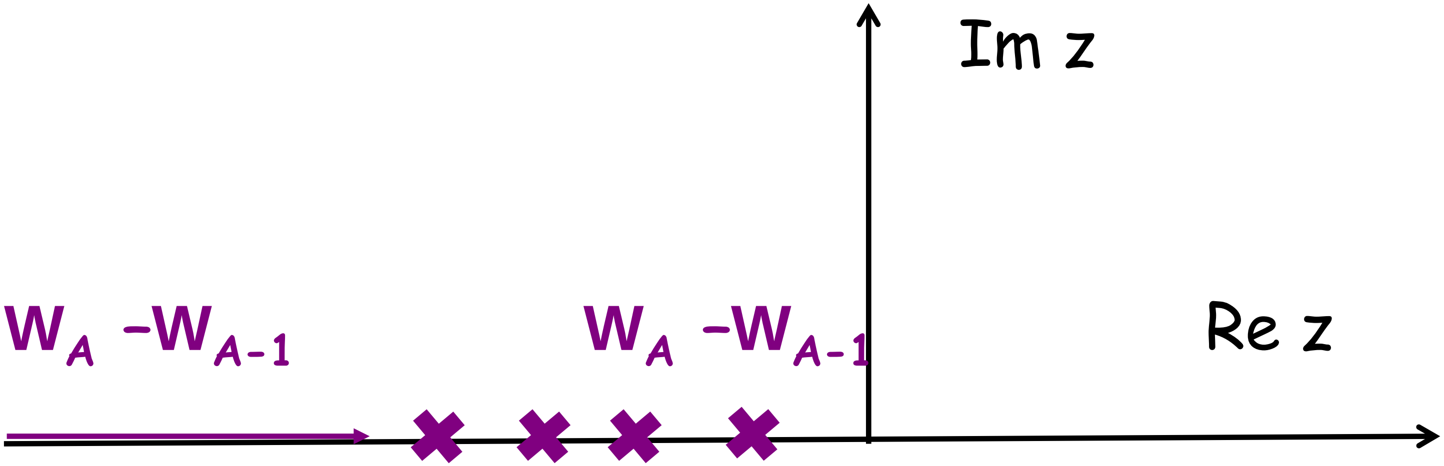
$$G^h(E - \mathrm{i}\eta; \vec{p}, \vec{\bar{p}}) \quad = \quad \langle \Psi_\mathrm{i} \mid a^+_{\vec{\bar{p}}} \frac{1}{E - \mathrm{i}\eta - W_A + H_{A-1}} a_{\vec{p}} \mid \Psi_\mathrm{i} \rangle$$

$$= \quad \lim_{\eta \to +0} \frac{1}{2\pi\mathrm{i}} \{ G^h(E - \mathrm{i}\eta; \vec{p}, \vec{\bar{p}} - G^h(-E + \mathrm{i}\eta; \vec{p}, \vec{\bar{p}} \}$$

and provides direct information on the propagation of proton holes in the target

It can be calculated with the help of the hole Green's function and considering its analytic structure

Analytic structure of the hole Green's function $G^h(z)$



poles bound states |n> of the (A-1) system

λ_n (E) = residue of the corresponding pole

left-hand cut continuum states of the (A-1) system

$$\lambda_c(E) = \frac{1}{2\pi} \frac{\Gamma_c(E)}{[E - W_A + F_c(E)]^2 + [\frac{\Gamma_c(E)}{2}]^2}$$

F_c(E) = <T+V(E)> average values of the hermitean

Γ_c(E) = 2 <W(E)> and antihermitean part of the hole selfenergy operator (V + iW)

ϕ_α **spectroscopic amplitudes** eigenfunctions of a nonlocal energy dependent Hamiltonian involving the mass operator or of the Feshbach optical model Hamiltonian

PURE SHELL MODEL

only **real poles** at energies corresponding to the various bound states occupied in the target

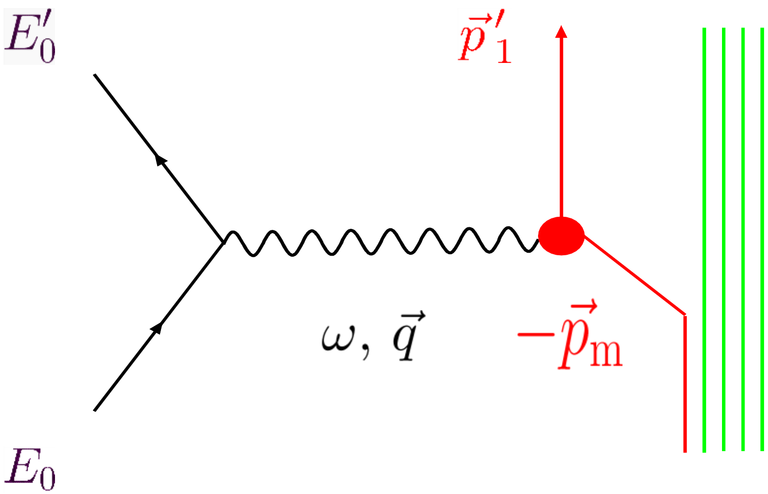
ϕ_n s.p. bound state wave function

λ_n occupation probability of the s.p. bound state

In general the calculation of the spectral function is a complicated many-body problem

FSI=0

PW



PWIA

PW

$$\chi^{(-)}_{\vec{p}'_1}(\vec{p}+\vec{q}) \to \delta(\vec{q}+\vec{p}-\vec{p}'_1)$$

$$\vec{p} = \vec{\overline{p}} = \vec{p}'_1 - \vec{q} = -\vec{p}_m$$

PW PW

$$\int \chi^{(-)*}_{n\vec{p}'_1}(\vec{q}+\vec{p})\, j^{\mu}(\vec{p},\vec{q})\ S(E_m;\vec{p}\vec{\overline{p}}) j^{\nu *}(\vec{\overline{p}},\vec{q})\ \chi^{(-)}_{n\vec{p}'_1}(\vec{q}+\vec{\overline{p}}) \mathrm{d}\vec{p}\,\mathrm{d}\vec{\overline{p}}$$

$$\sigma = K\ \ \sigma_{\rm ep}\ \ S(E_{\rm m}, -\vec{p}_{\rm m})$$

factorized c.s

$$S(E_{\rm m}, -\vec{p}_{\rm m}) = \sum_n\ \lambda_n(E_{\rm m})\ \ |\phi_n(-\vec{p}_{\rm m})|^2$$

$$S(E_{\mathrm{m}}, -\vec{p}_{\mathrm{m}}) = \sum_n \lambda_n(E_{\mathrm{m}}) \; |\phi_n(-\vec{p}_{\mathrm{m}})|^2$$

spectroscopic factor ($\lambda_n(E_{\mathrm{m}})$) — overlap function ($\phi_n(-\vec{p}_{\mathrm{m}})$)

For each E_m the mom. dependence of the SF is given by the mom. distr. of the quasi-hole states n produced in the target nucleus at that energy and described by the normalized OVF

The spectroscopic factor is the norm of the OVF and gives the probability that n is a pure hole state in the target.

IPSM

ϕ_n s.p. SM state

λ_n 1 occupied SM states

0 empty SM states

There are correlations and the strength of the quasi-hole state is fragmented over a set of s.p. states $0 \leq \lambda_n \leq 1$

DWIA calculations

- selected transitions to discrete states with quantum numbers l, j
- non relativistic DWIA
- non relativistic 1-b nuclear current with relativistic corrections
- phenomenological ingredients for s.p. bound and scattering w.f.
- $\chi^{(-)}$ phenomenological optical potential
- ϕ_n phenomenological s.p. wave functions WS, HF (some calculations including correlations are available)
- λ_n extracted in comparison with data: reduction factor applied to the calculated c.s. to reproduce the magnitude of the experimental data

Experimental data: E_m and p_m distributions

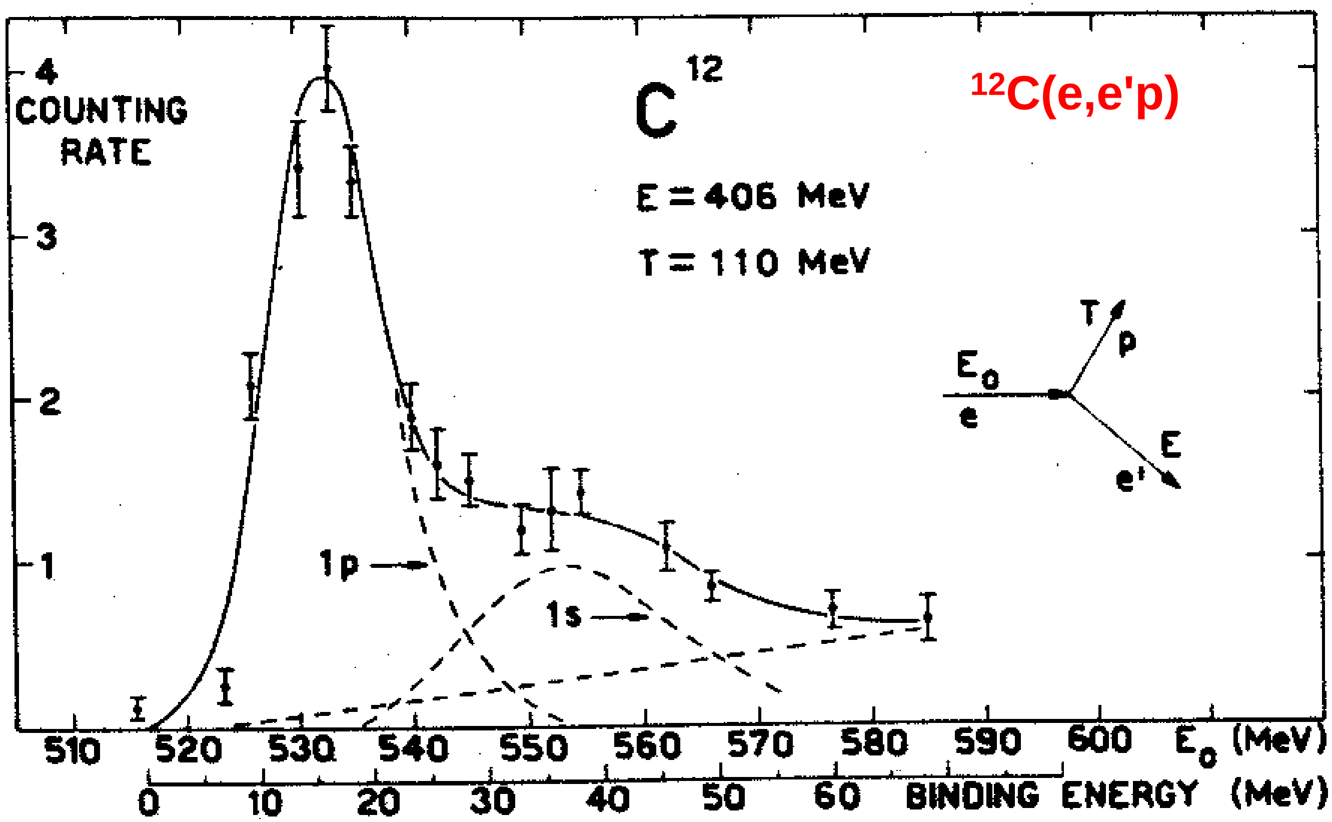
DWIA comparison with data

- σ^{exp} (E_0, E'_0, θ, E'_1, γ, α) ⇒ σ^{exp} (E_m, p_m)
- for a peak in the E_m distribution

$$\int_{\Delta E_M} \frac{d\sigma^{exp}}{d\vec{p}_0' d\vec{p}_1'} / K\sigma_{ep} \Leftrightarrow \int_{\Delta E_M} \frac{d\sigma^{th}}{d\vec{p}_0' d\vec{p}_1'} / K\sigma_{ep}$$

- **REDUCED CROSS SECTION**
- the shapes of the reduced c.s. as a function of p_m are compared, the exp. spectroscopic factor is extracted in comparison with data
- the reduction factor applied to the calculated reduced c.s. in order to reproduce the exp reduced c.s. is identified with the spectroscopic factor ⇒ $\lambda^{exp} < 1$
- λ gives a measurement of correlation effects but in these analyses it is extracted through a fit to the data and may include also the uncertainties and the approximations of the theor. model

1964: Frascati

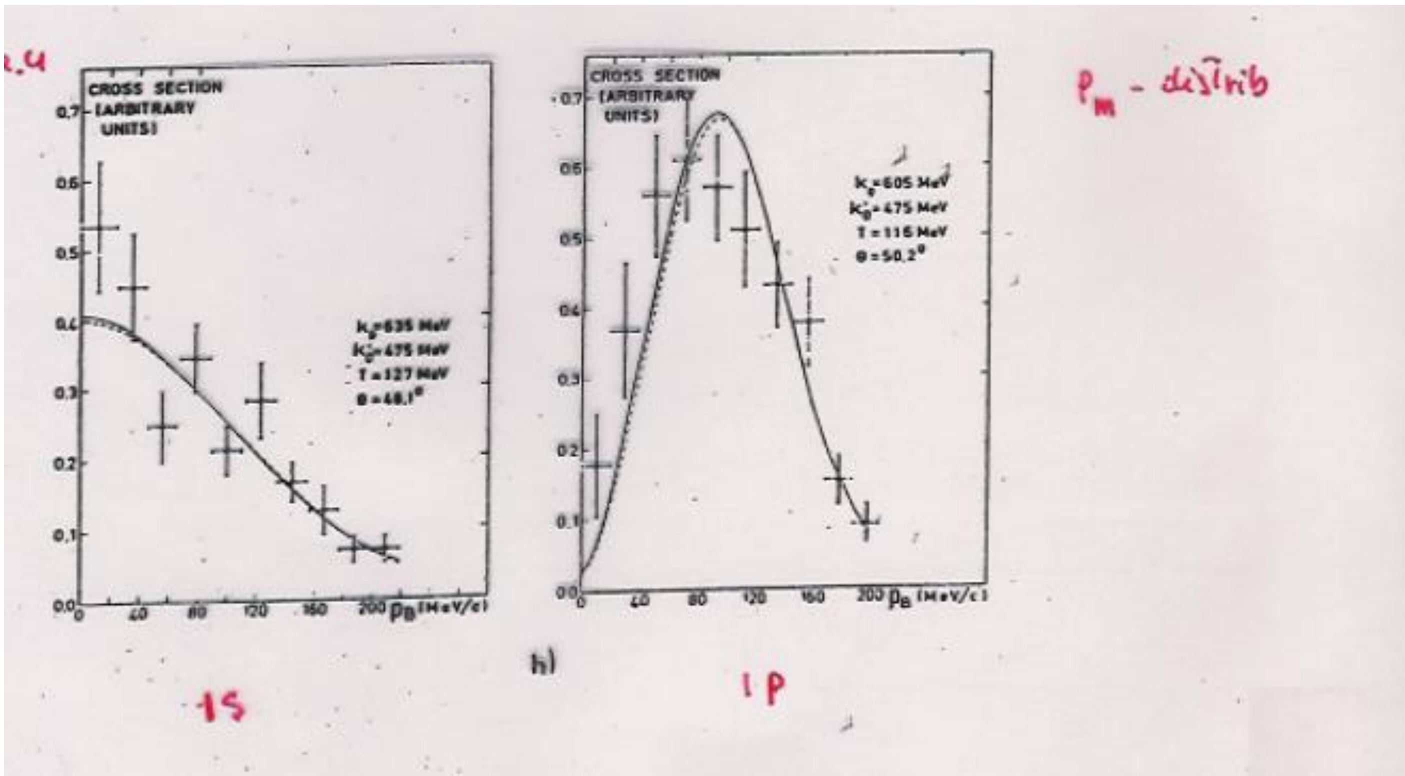


U. Amaldi, Jr. *et al.*, Phys. Rev. Lett. **13**, 341 (1964).

1964: Frascati

$^{12}C(e,e'p)$

1s | 1p | p_m distribution



U. Amaldi, Jr. *et al.*, Phys. Rev. Lett. **13**, 341 (1964).

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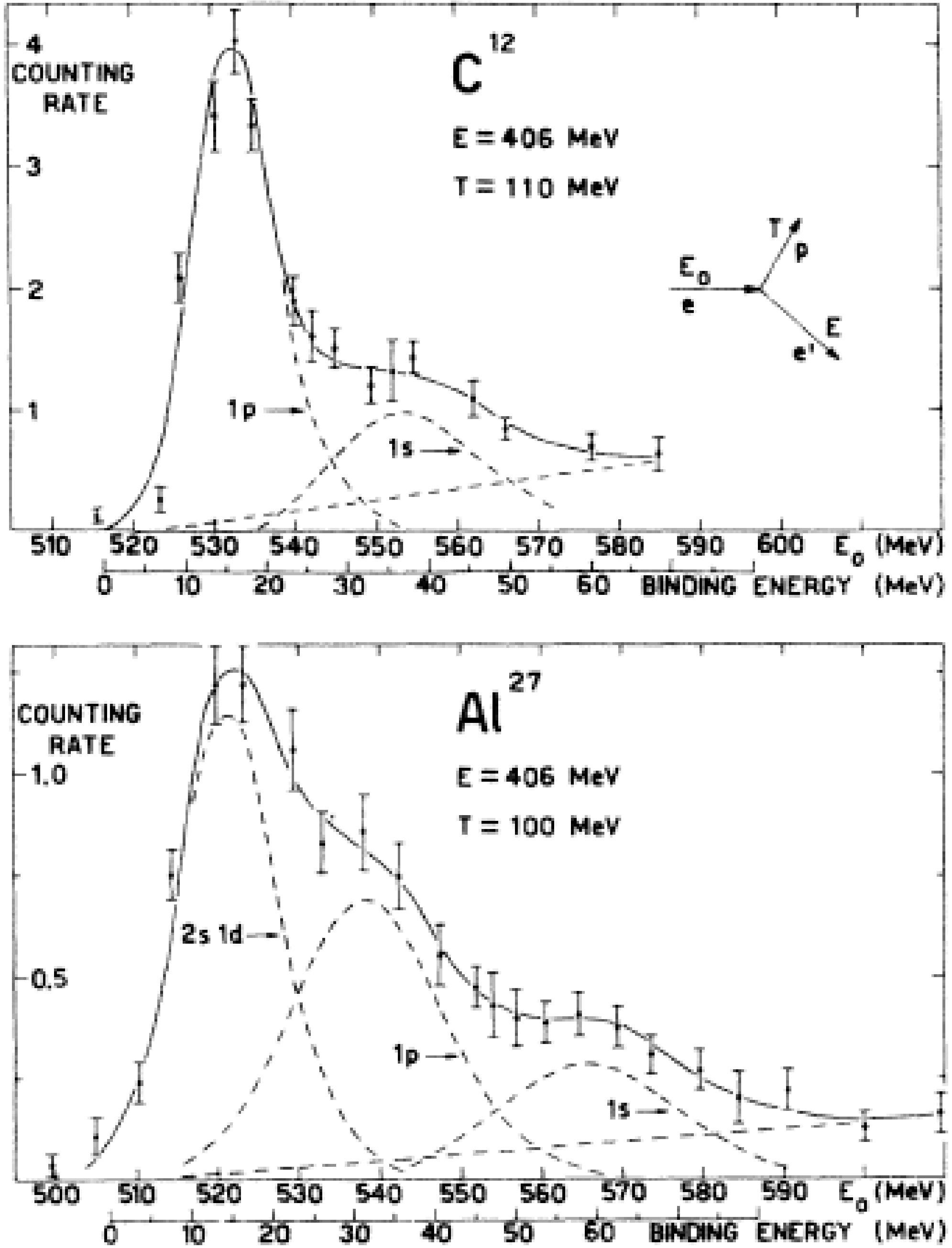
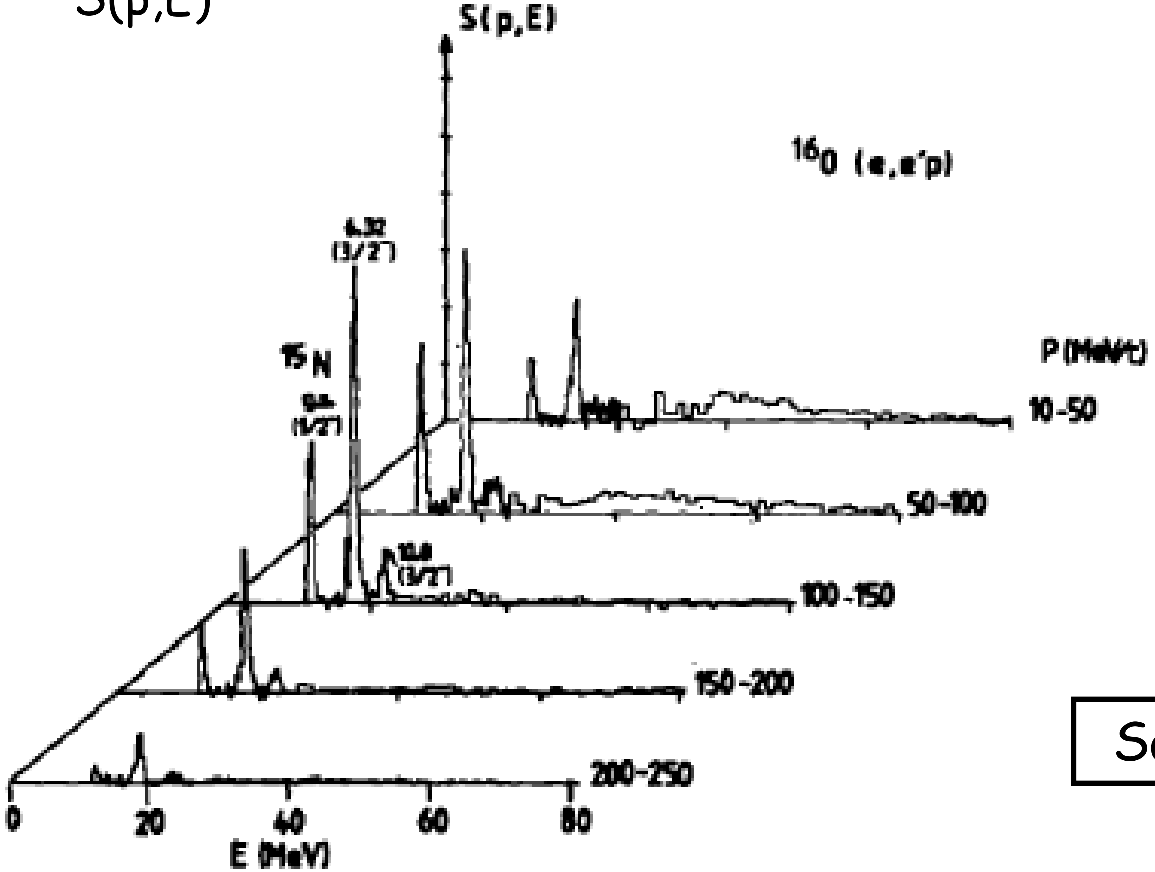


U. Amaldi, Jr. *et al.,* Phys. Rev. Lett. **13**, 341 (1964).

Experimental data: E_m and p_m distributions

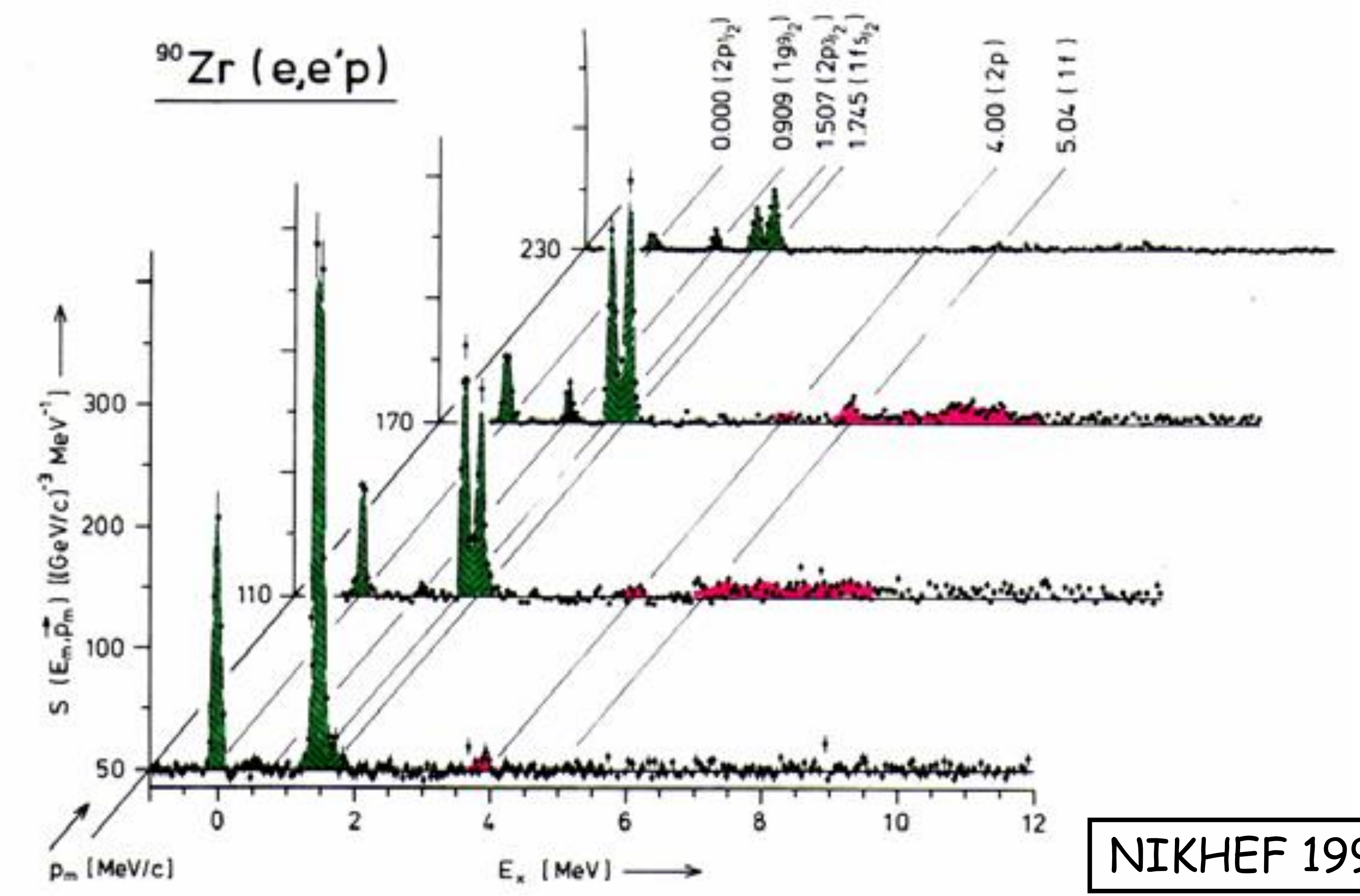
$$^{16}\mathrm{O}(e,e'p)$$

S(p,E)



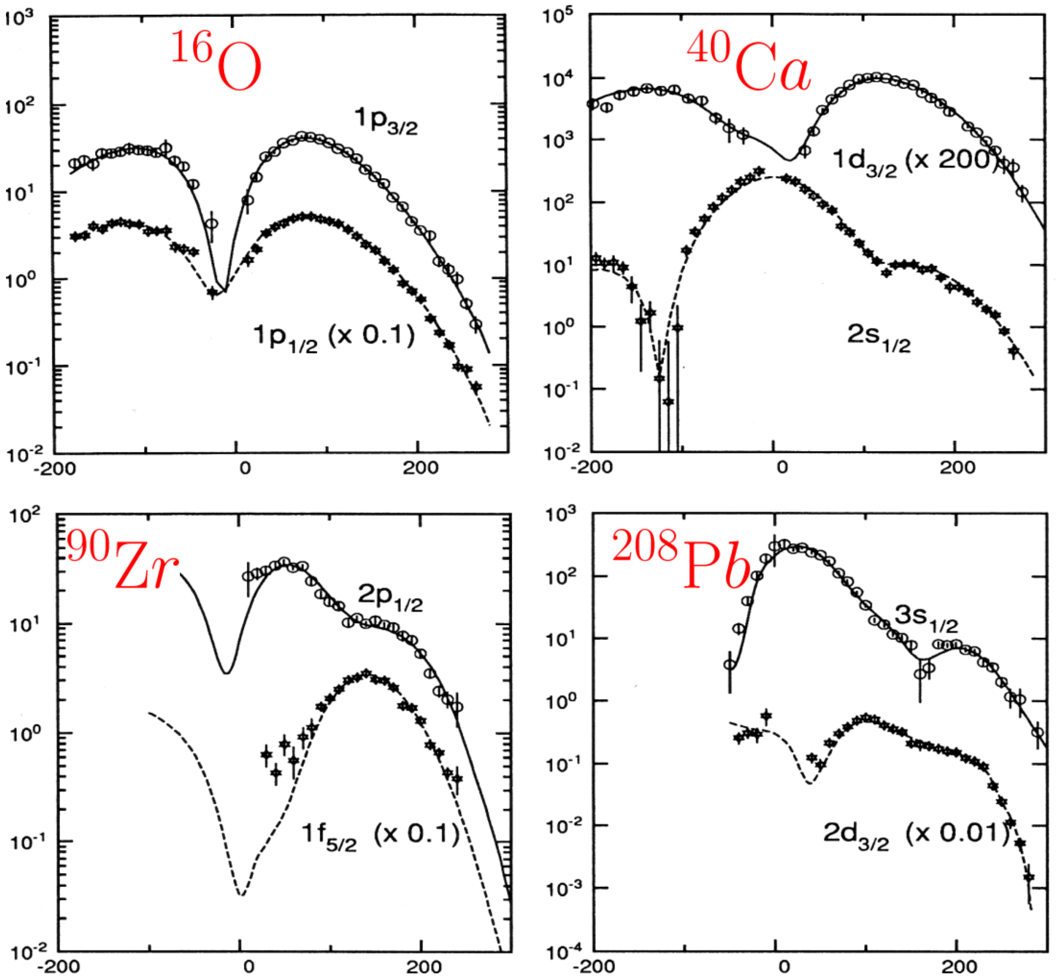
Saclay 1980

Experimental data: E_m and p_m distributions



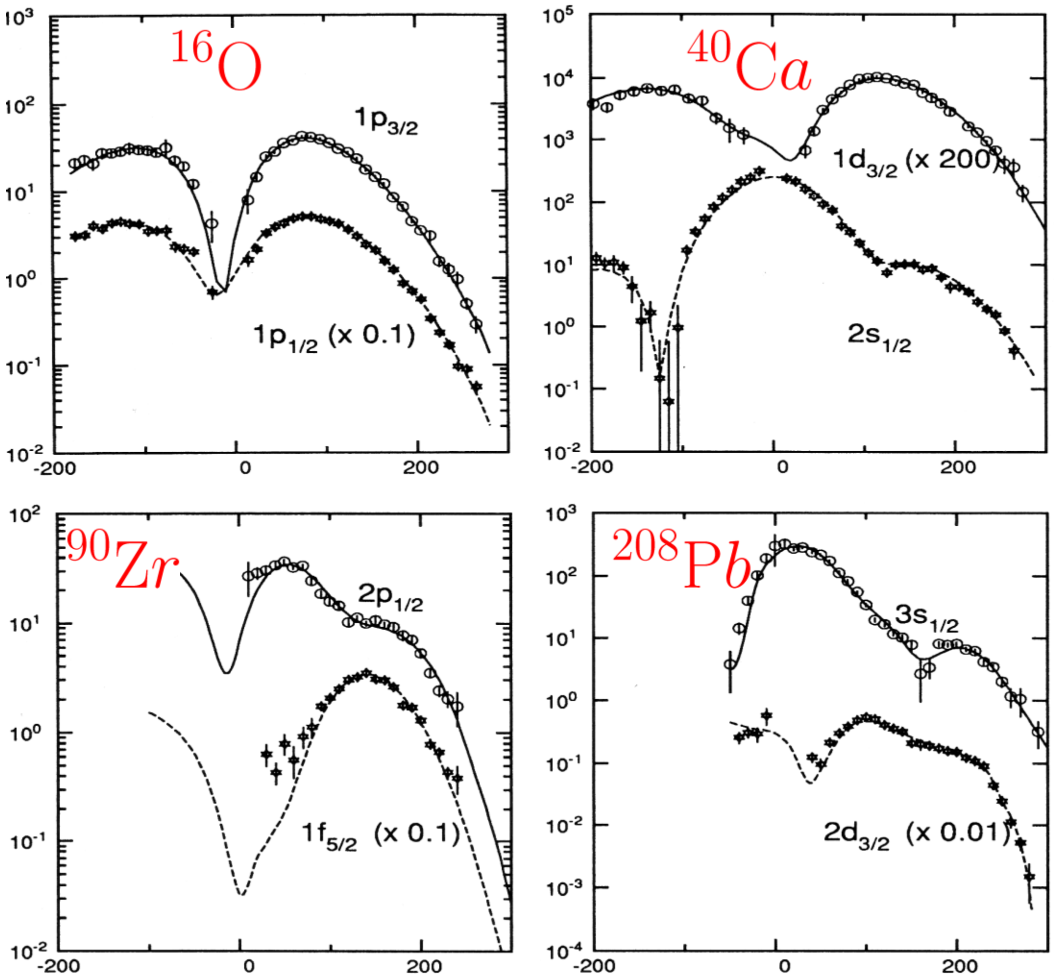
NIKHEF 1990

Experimental data: p_m distributions



NIKHEF data & CDWIA calculations
1993

Experimental data: p_m distributions

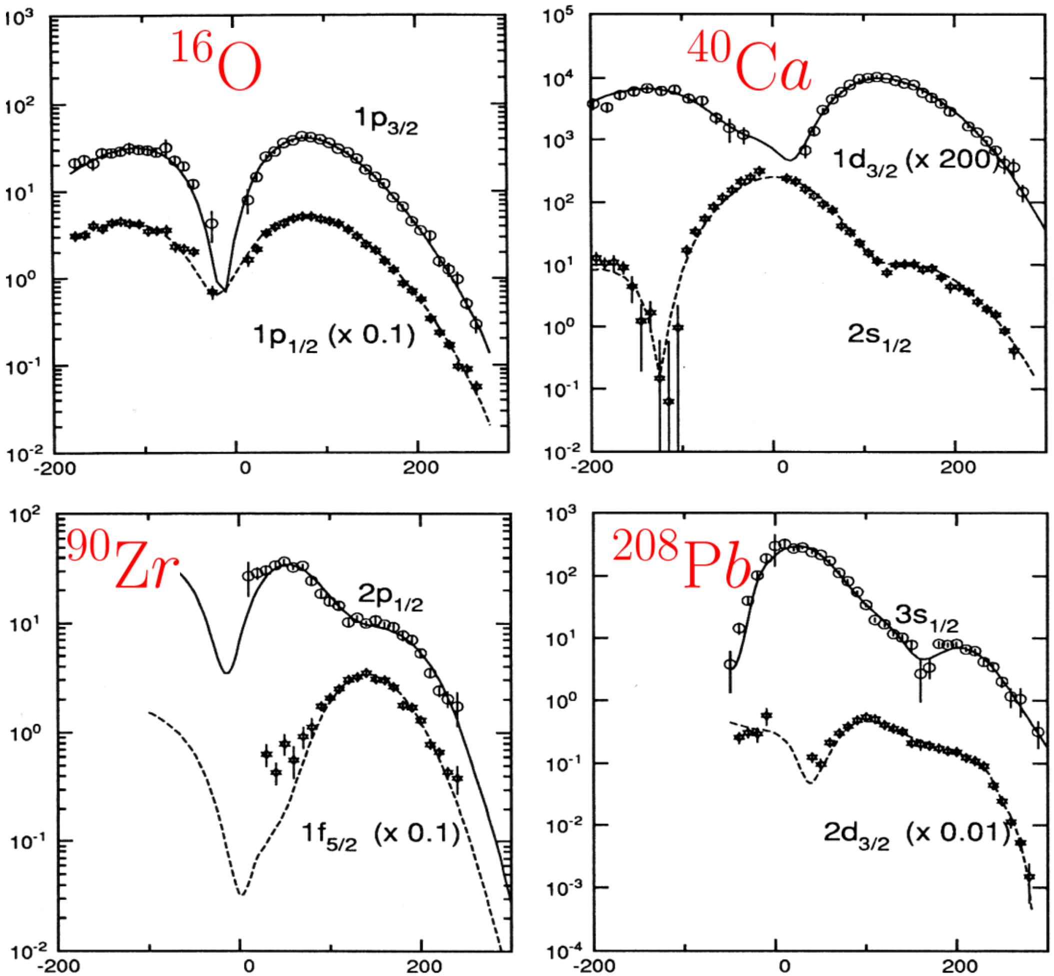


reduction factors applied:
spectroscopic factors

0.6 - 0.7

NIKHEF data & CDWIA calculations
1993

Experimental data: p_m distributions



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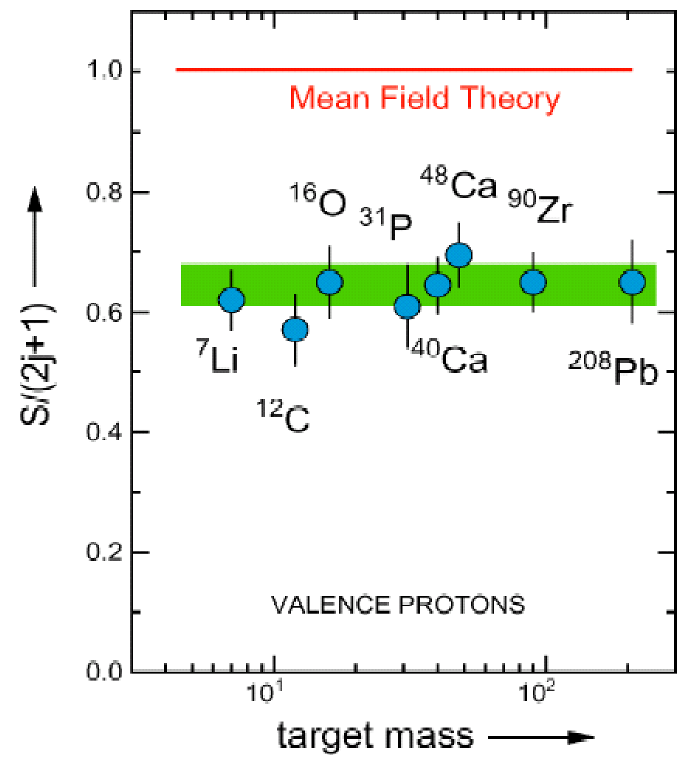

NIKHEF data & CDWIA calculations
1993

(e,e'p) data

- information on the hole structure of target nuclei
- validity and limit of IPSM MFA
- SM orbitals
- DWIA calculations good agreement with the shape of p_m distributions
- spectroscopic factors about 65% of the value predicted by the MFA CORRELATIONS
- calculations able to reproduce the magnitude of the experimental c.s. without the need to apply a reduction factor not available in general for complex nuclei
- calculations including correlations .....

- Short-Range Correlations (short-range repulsion of NN interaction) give a depletion up to 10%, 15% with tensor correlations

- The rest of the depletion is due to Long-Range Correlations: (long-range part of NN interaction collective excitations of nucleons at the nuclear surface)

The reduction (experimental spectroscopic) factors extracted from the comparison of DWIA calculations with (e,e′p) data can be affected by uncertainties in the theoretical ingredients of the calculation or by effects neglected or not adequately described by the model

- choice of the phen. OP: differences within e few %
- two-body MEC: very small effects in the usual kinematics of (e,e′p) experiments
- other effects have been evaluated CM-motion….. relativistic effects

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- other effects have been evaluated CM-motion….. relativistic effects

Relativistic RDWIA models

Relativistic RDWIA models

RDWIA models have been developed based on the same assumptions and approximations, but with

relativistic one-body current

relativistic (Dirac spinors) for s.p. bound (RMF) and scattering wave functions (ROP)

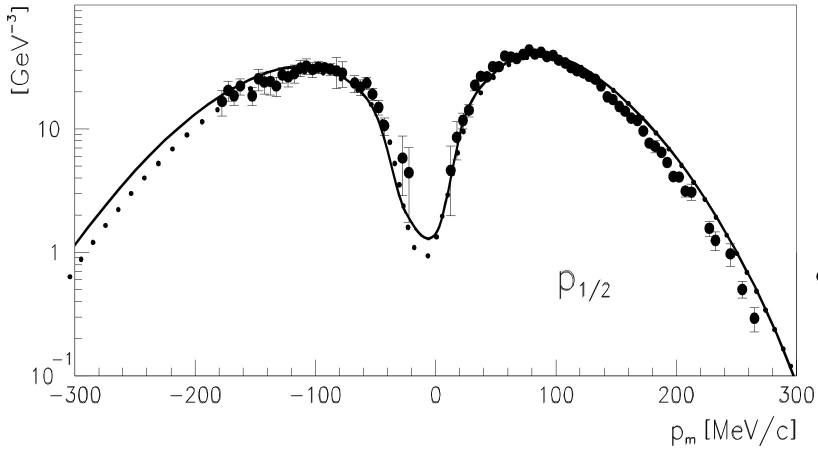
RDWIA necessary for the analysis of (e,e′p) at higher energies (JLab)

The relevance of relativistic effects can be investigated also in the kinematics of NKHEF exp.

DWIA ⟷ RDWIA

Relativistic RDWIA

$^{16}O(e,e'p)$



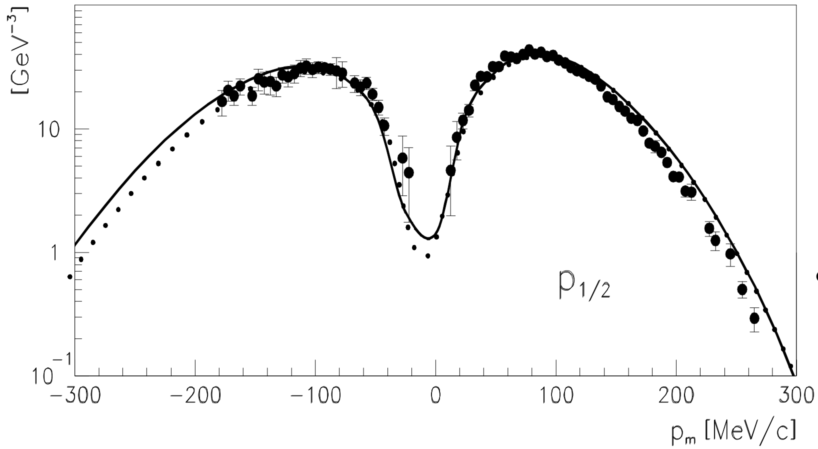
NIKHEF parallel kin E_0 = 520 MeV T_p = 90 MeV

rel RDWIA

nonrel DWIA

Relativistic RDWIA

$^{16}O(e,e'p)$



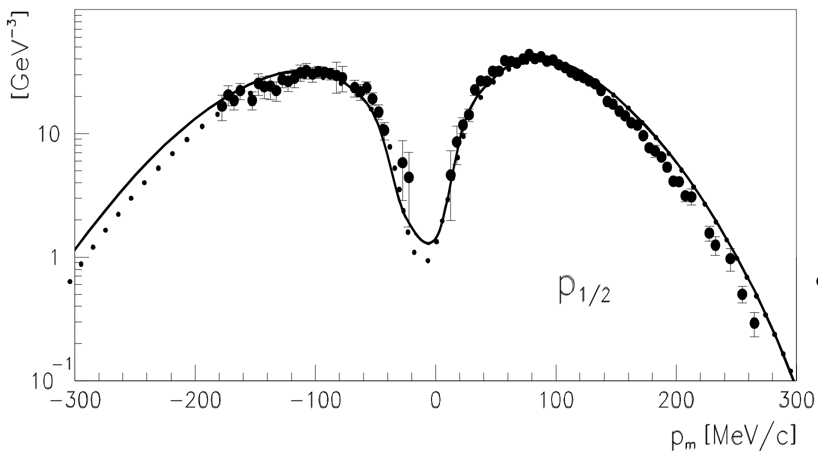
NIKHEF parallel kin $E_0 = 520$ MeV $T_p = 90$ MeV

rel RDWIA $\lambda_n = 0.7$

nonrel DWIA $\lambda_n = 0.65$

Relativistic RDWIA

$^{16}O(e,e'p)$

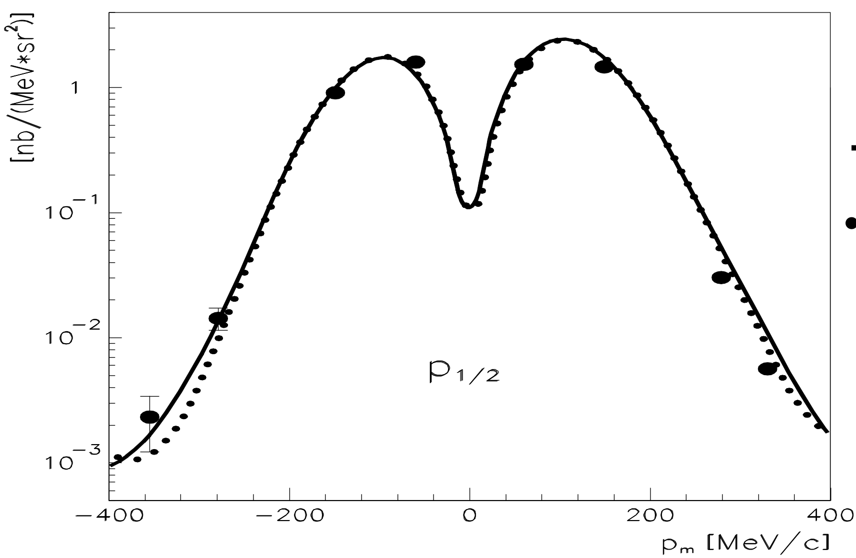


NIKHEF parallel kin E_0 = 520 MeV T_p = 90 MeV

rel RDWIA λ_n = 0.7

nonrel DWIA λ_n =0.65

JLab (ω,q) const kin E_0 = 2445 MeV ω =439 MeV T_p= 435 MeV

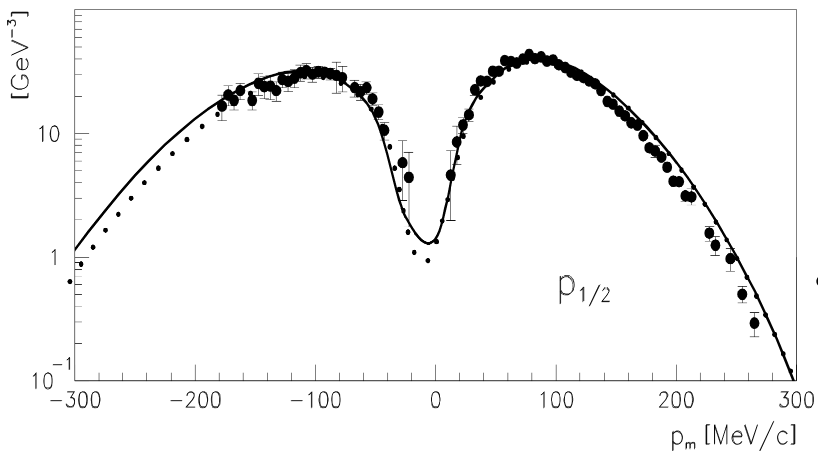


RDWIA diff opt.pot.

Jlab 2000

Relativistic RDWIA

$^{16}O(e,e'p)$



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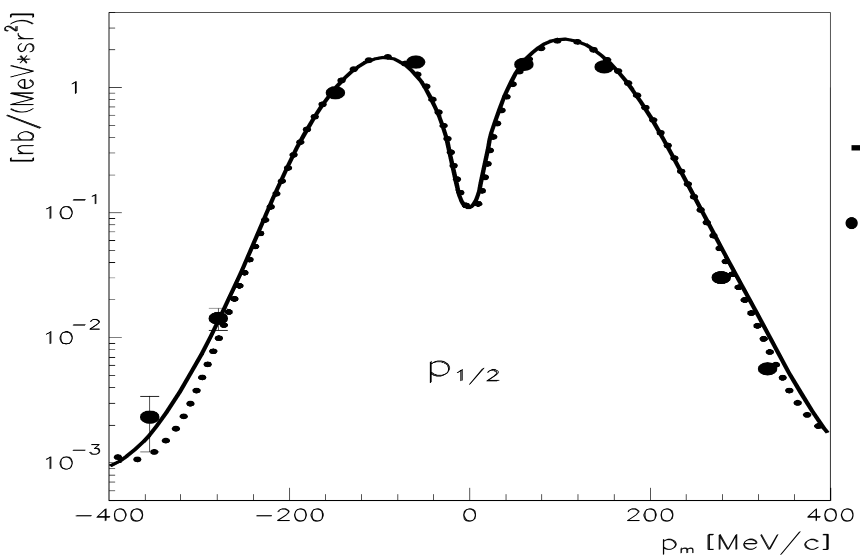

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Relativistic RDWIA

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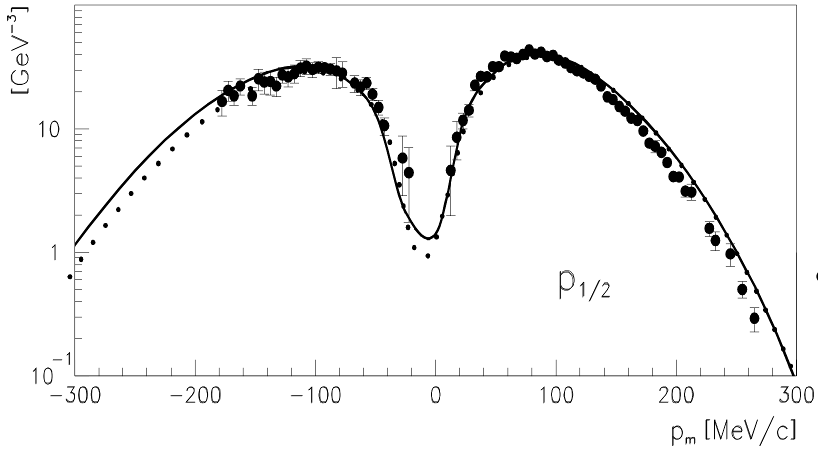

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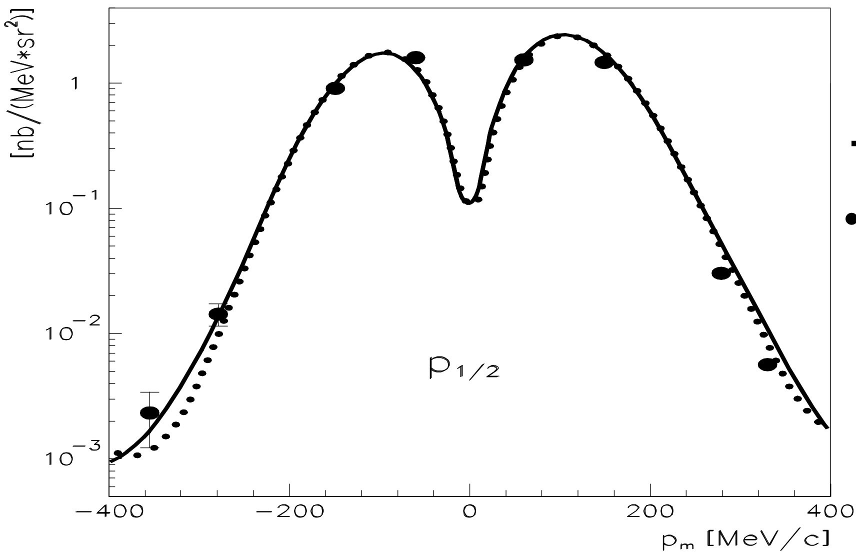
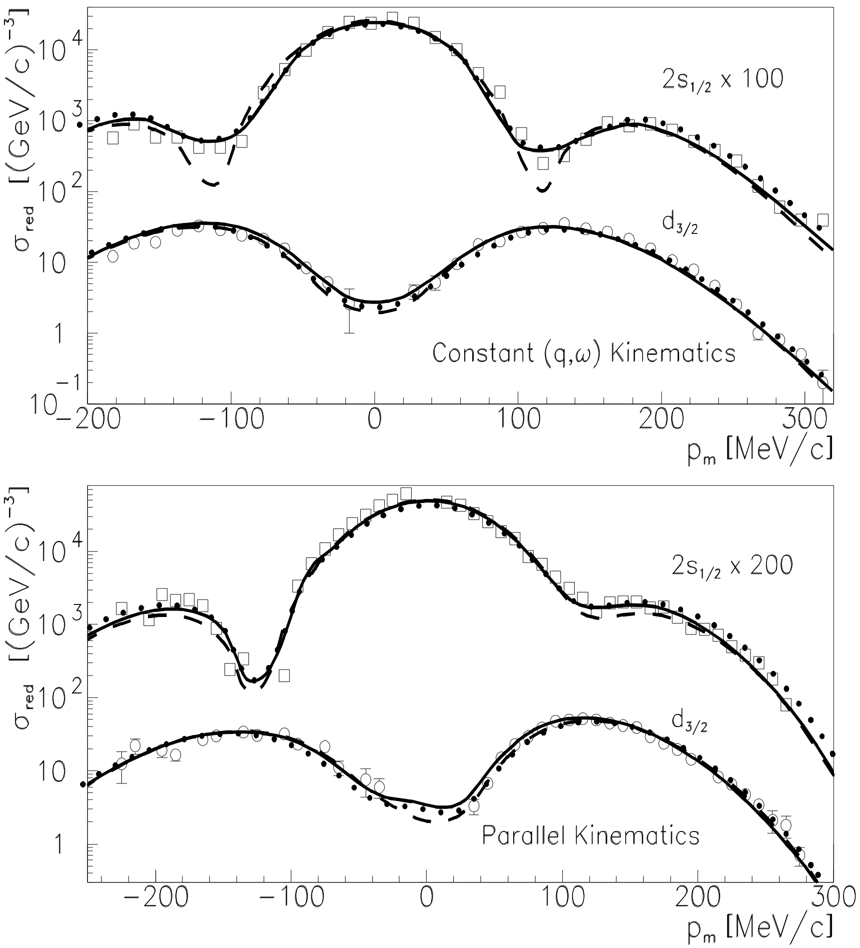


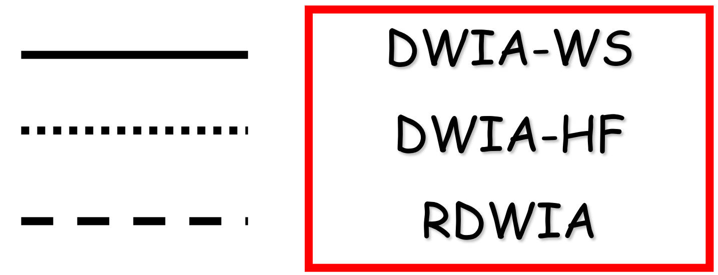
RDWIA diff opt.pot. λ_n = 0.7

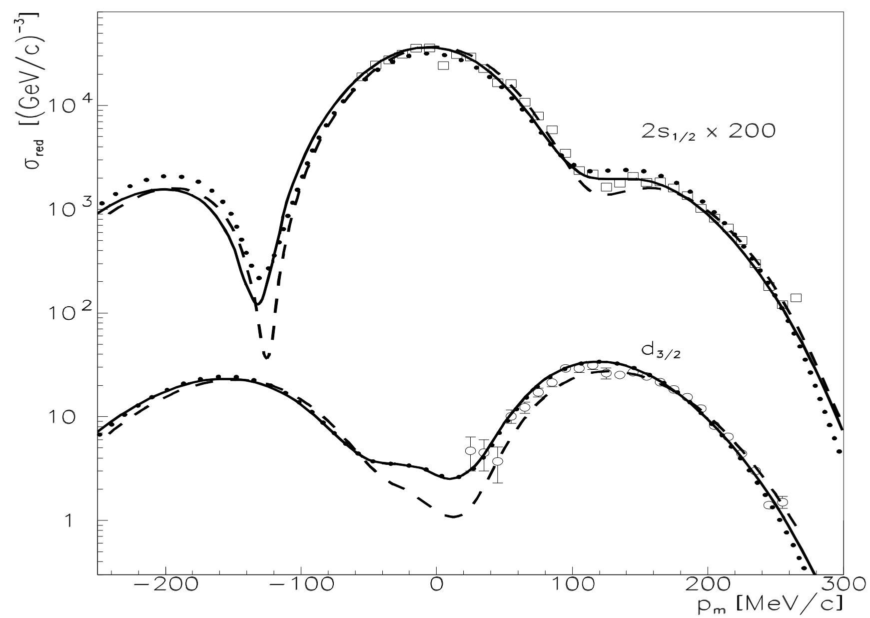
Jlab 2000

$^{40}Ca(e,e'p)$

$^{48}Ca(e,e'p)$







C. Giusti et al. PRC 84 024615 (2011)

$^{40}Ca(e,e'p)$

$1d_{3/2}$

$^{48}Ca(e,e'p)$

λ_n = 0.49 DWIA-WS

0.51 DWIA-HF

0.49 RDWIA

(ω,q) const kin

λ_n = 0.65 DWIA-WS

0.64 DWIA-HF

0.69 RDWIA

parallel kin

λ_n = 0.56 DWIA-WS

0.55 DWIA-HF

0.52 RDWIA

(e,e’p) preferential tool to study proton-hole states, bound protons, validity and limits of MFA

- large amount of exp end theor work on (e,e’p) has provided accurate information on stable nuclei

- advantages of the elm probe and (e,e’p) studies can be extended to exotic nuclei

- understanding the evolution of nuclear properties as a function of N/Z is one of the major topics of interest in modern nuclear physics

- in the next years the advent of RIB facilities will provide data on unstable nuclei

- a new generation of electron RIB colliders that use storage rings under construction (GSI, RIKEN) will offer unprecedented opportunities to study exotic nuclei with electron scattering (ELISe at FAIR, SCRIT at RIKEN)

- elastic: global properties, nuclear density distribution

- quasi-elastic: dynamical properties, proton-hole states, 1hSF (exclusive) integral of 1hSF over all the final states (inclusive)

INCLUSIVE QUASIELASTIC SCATTERING (e,e')

INCLUSIVE QUASIELASTIC SCATTERING (e,e')

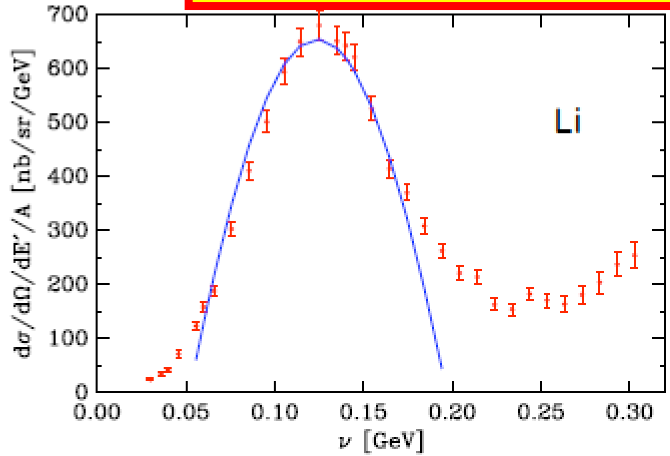
- only scattered electron detected
- all final nuclear states are included
- in the QE region the main contribution is given by the interaction on single nucleons and direct one-nucleon emission

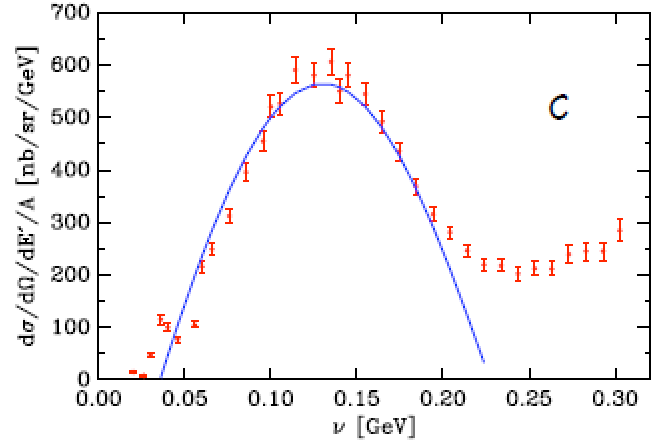
SIMPLE MODEL: FERMI GAS MODEL

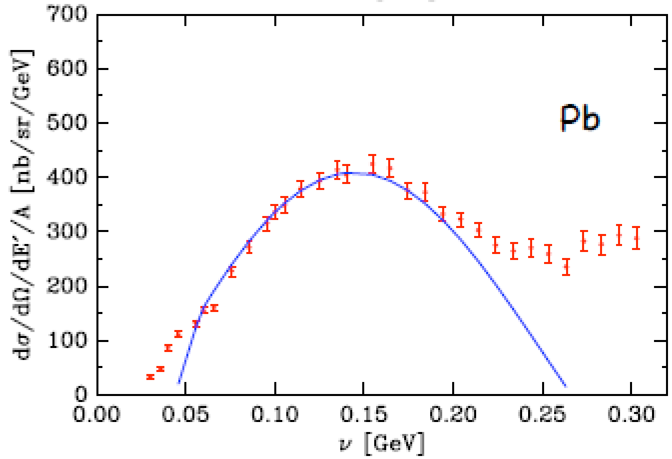
- c.s given by the sum over all the nucleons of incoherent processes involving only one-nucleon scattering

- Fermi Gas Model (nucleus viewed as a collection of non-interacting fermions) with two parameters: energy shift $\bar{\epsilon}$ (accounts for nuclear binding) and Fermi momentum k_F (width of the peak is proportional to k_F)

QUASIELASTIC (e,e') CROSS SECTION
FERMI GAS MODEL







Nucleus	k_F	$\bar{\epsilon}$
^{6}Li	169	17
^{12}C	221	25
^{24}Mg	235	32
^{40}Ca	251	28
^{nat}Ni	260	36
^{89}Y	254	39
^{nat}Sn	260	42
^{181}Ta	265	42
^{208}Pb	265	44

compared to Fermi model:fit parameter k_F and ϵ

R.R. Whitney. *et al.*, Phys. Rev. *C* **9**, 2230 (1974)

$$\sigma = K\left(R_T(\omega,q) + 2\varepsilon_L R_L(\omega,q)\right)$$

$$\varepsilon_L = \frac{Q^2}{q^2}\left[1 + 2\frac{q^2}{Q^2}\tan^2\frac{\theta}{2}\right] \qquad Q^2 = q^2 - \omega^2$$

$$R_L = W^{00} \qquad R_T = W^{xx} + W^{yy}$$

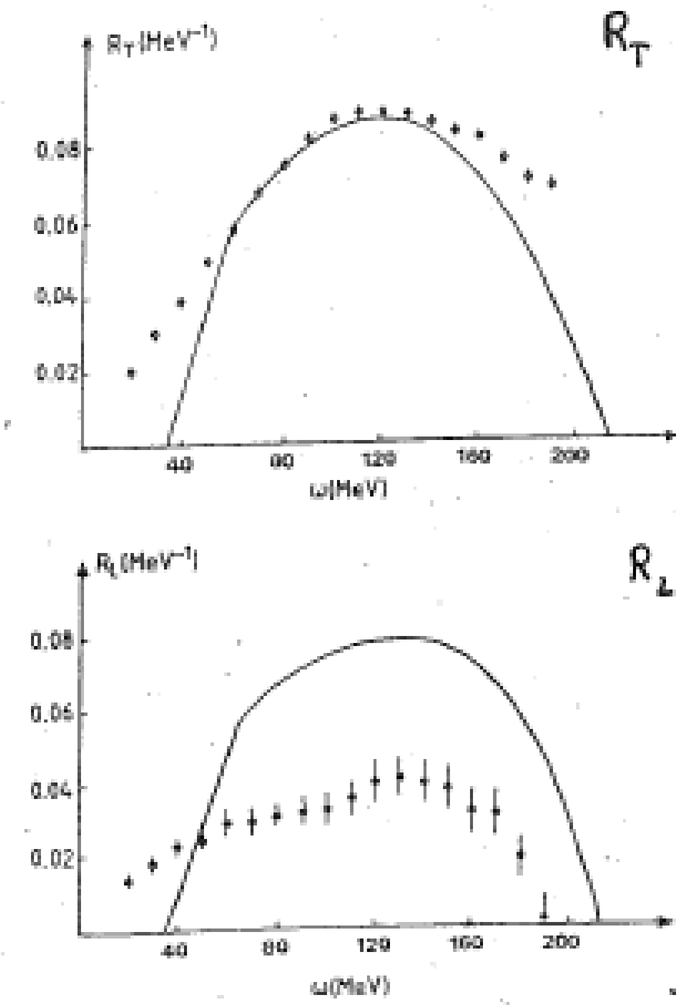
Rosenbluth separation:

Plot of the c.s./K at fixed ω,q as a function of ϵ_L

⇨ slope ⇨ R_L

intercept at ϵ_L =0 ⇨ R_T

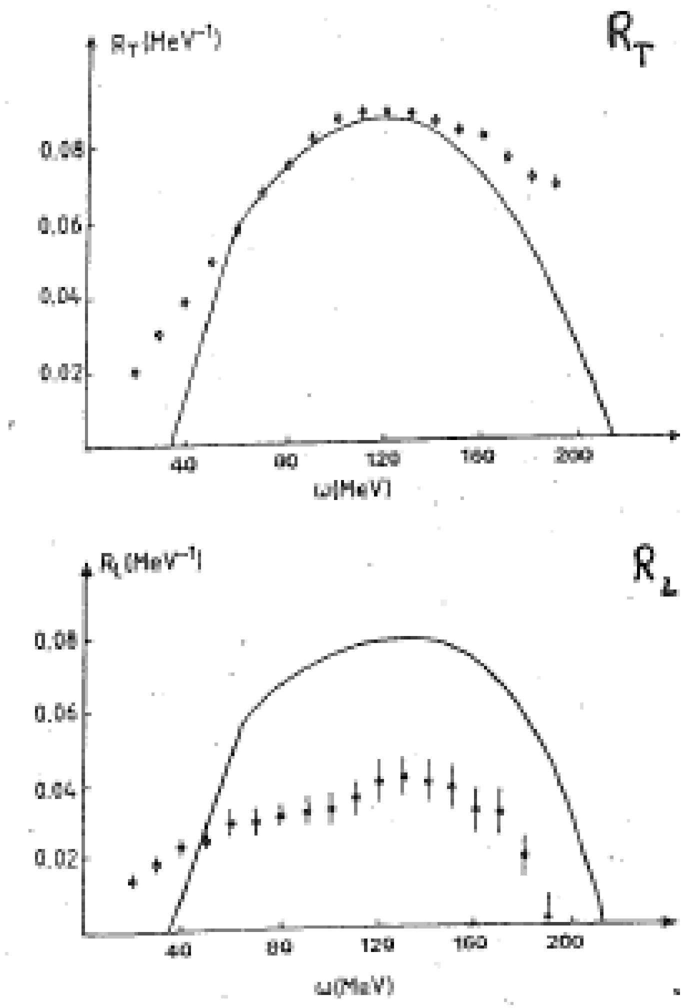
ROSENBLUTH SEPARATION R_L R_T
FERMI GAS MODEL



$^{56}Fe(e,e')$

Z.E. Meziani (1984)

ROSENBLUTH SEPARATION R_L R_T
FERMI GAS MODEL



$^{56}Fe(e,e')$

a more sophisticated model is needed!

Z.E. Meziani (1984)

Large amount of theoretical work, different models developed with the aim to explain experimental data for R_L and R_T

Unified and consistent description of R_L and R_T data has not been achieved

Experiments of separation difficult, discrepancies in the exp. results from different laboratories, new and more data with improved accuracy would be helpful to make the situation clear

Different models based on the IA seem generally able to reproduce R_L but underestimate R_T

Indications that effects beyond IA can be important for R_T

Role of 2-body MEC should be carefully evaluated before drawing conclusions

In the following models based on the IA are considered....

INCLUSIVE SCATTERING : IMPULSE APPROXIMATION

- IA : c.s given by the sum of integrated direct one-nucleon emission over all the nucleons
- IPSM : $\sum_n$ over all occupied states in the SM, all the nucleons are included but correlations are neglected
- partial occupancy can be included, spectral function

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INCLUSIVE SCATTERING: FSI

DWIA RDWIA

sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

INCLUSIVE SCATTERING: FSI

DWIA RDWIA

sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

PWIA RPWIA

FSI neglected

INCLUSIVE SCATTERING: FSI

DWIA RDWIA

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PWIA RPWIA

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REAL POTENTIAL

INCLUSIVE SCATTERING: FSI

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sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

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REAL POTENTIAL

rOP rROP

only the real part of the OP: conserves the flux but it is conceptually wrong

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RMF

RELATIVISTIC MEAN FIELD: same real energy-independent potential of bound states

Orthogonalization, fulfills dispersion relations and maintains the continuity equation

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DWIA RDWIA

sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

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RELATIVISTIC MEAN FIELD: same real energy-independent potential of bound states

Orthogonalization, fulfills dispersion relations and maintains the continuity equation

GF RGF

GREEN'S FUNCTION complex OP conserves the flux consistent description of FSI in exclusive and inclusive QE electron scattering

FSI for the inclusive scattering : Green's Function Model

Y. Horikawa, F. Lenz, N.C. Mukhopadhyay PRC 22 (1980) 1680

multiple scattering theory

F. Capuzzi, C. Giusti, F.D. Pacati, Nucl. Phys. A 524 (1991) 281

Feshbach projection operator formalism ⟸

FSI for the inclusive scattering : Green's Function Model

(e,e') nonrelativistic

F. Capuzzi, C. Giusti, F.D. Pacati, Nucl. Phys. A 524 (1991) 281

F. Capuzzi, C. Giusti, F.D. Pacati, D.N. Kadrev Ann. Phys. 317 (2005) 492 (AS CORR)

(e,e') relativistic

A. Meucci, F. Capuzzi, C. Giusti, F.D. Pacati, PRC (2003) 67 054601

A. Meucci, C. Giusti, F.D. Pacati Nucl. Phys. A 756 (2005) 359 (PVES)

A. Meucci, J.A. Caballero, C. Giusti, F.D. Pacati, J.M. Udias PRC (2009) 80 024605 (RGF-RMF)

CC relativistic

A. Meucci, C. Giusti, F.D. Pacati Nucl. Phys. A739 (2004) 277

A. Meucci, J.A Caballero, C. Giusti, J.M. Udias PRC (2011) 83 064614 (RGF-RMF)

comparison with MiniBooNE data

A. Meucci, M.B. Barbaro, J.A. Caballero, C. Giusti, J.M. Udias PRL (2011) 107 172501

A. Meucci, C. Giusti, F.D. Pacati PRD (2011) 84 113003

A. Meucci, C. Giusti, PRD (2012) 85 093002

FSI for the inclusive scattering : Green's Function Model

- the components of the inclusive response are expressed in terms of the Green's operator
- under suitable approximations can be written in terms of the s.p. optical model Green's function
- the explicit calculation of the s.p. Green's function can be avoided by its spectral representation that is based on a biorthogonal expansion in terms of the eigenfunctions of the non Herm optical potential V and V^{+}
- matrix elements similar to DWIA
- scattering states eigenfunctions of V and V^{+} (absorption and gain of flux): the imaginary part redistributes the flux and the total flux is conserved
- consistent treatment of FSI in the exclusive and in the inclusive scattering

NUCLEAR RESPONSE

$$
\begin{aligned}
W^{\mu\mu} &= \sum_{\rm f} \langle \Psi_{\rm i} \mid J^{\mu +} \mid \Psi_{\rm f} \rangle \langle \Psi_{\rm f} \mid J^{\mu} \mid \Psi_{\rm i} \rangle \delta(\omega + E_{\rm i} - E_{\rm f}) \\
&= \sum_{\rm f} \langle \Psi_{\rm i} \mid J^{\mu +} \delta(\omega + E_{\rm i} - H) \mid \Psi_{\rm f} \rangle \langle \Psi_{\rm f} \mid J^{\mu} \mid \Psi_{\rm i} \rangle \\
&= \langle \Psi_{\rm i} \mid J^{\mu +} \delta(\omega + E_{\rm i} - H) J^{\mu} \mid \Psi_{\rm i} \rangle \\
&= \frac{1}{\pi} {\rm Im} \langle \Psi_{\rm i} \mid J^{\mu +} G^{+}(\omega + E_{\rm f}) J^{\mu} \mid \Psi_{\rm i} \rangle
\end{aligned}
$$

$$
\frac{1}{x \pm {\rm i}\varepsilon} = \mathcal{P}\left(\frac{1}{x}\right) \mp {\rm i}\pi\delta(x)
$$

$$
G^{+}(\omega + E_{\rm f}) = \frac{1}{\omega + E_{\rm i} - H - {\rm i}\varepsilon}
$$

GREEN'S FUNCTION

H nuclear Hamiltonian

The diagonal components of the hadron tensor are expressed in terms of the Green function G^+ the full A-body propagator. Only an approximate treatment reduces the problem to a tractable form with suitable approximations the components of the nuclear response are written in terms of **the s.p. optical model Green's function**

$$W^{\mu\mu} = \frac{1}{\pi} \mathrm{Im} \langle \Psi_{\mathrm{i}} \mid J^{\mu +} G^{+}(\omega + E_{\mathrm{f}}) J^{\mu} \mid \Psi_{\mathrm{i}} \rangle$$

- one-body current $J^{\mu} \simeq \sum_{k=1}^{A} j_k^{\mu}$
- non diagonal terms neglected (high enough q) $j_k^{\mu +} G^{+} j_l^{\mu} \quad k \neq l$

$$W^{\mu\mu} \simeq \frac{1}{\pi} \sum_k \mathrm{Im} \langle \Psi_{\mathrm{i}} \mid j_k^{\mu +} G^{+}(E_{\mathrm{f}}) j_k^{\mu} \mid \Psi_{\mathrm{i}} \rangle$$

$$= \frac{Z}{\pi} \mathrm{Im} \langle \Psi_{\mathrm{i}} \mid j_{1p}^{\mu +} G^{+}(E_{\mathrm{f}}) j_{1p}^{\mu} \mid \Psi_{\mathrm{i}} \rangle + \frac{N}{\pi} \mathrm{Im} \langle \Psi_{\mathrm{i}} \mid j_{1n}^{\mu +} G^{+}(E_{\mathrm{f}}) j_{1n}^{\mu} \mid \Psi_{\mathrm{i}} \rangle$$

- $j_1^{\mu} \mid \Psi_{\mathrm{i}} \rangle \simeq \sum_n P_n j_1^{\mu} \mid \Psi_{\mathrm{i}} \rangle \qquad P_n = \int \mathrm{d}\vec{r}_1 \mid \vec{r}_1 n \rangle \langle \vec{r}_1 n \mid$

n discrete eigenstate of H_{A-1} or isolated resonance in the continuum

$$P_n + Q_n = 1 \quad Q_n^2 = Q_n \quad \sum_n P_n + P' = 1 \quad Q_n = \sum_{m \neq n} Q_m + P'$$

$$W^{\mu\mu} = \frac{A}{\pi} \mathrm{Im} \sum_{n,m} \langle \Psi_\mathrm{i} \mid j_1^{\mu +} P_n G^+(E_\mathrm{f}) P_m j_1^{\mu} \mid \Psi_\mathrm{i} \rangle$$

$$P_n G^+(E_\mathrm{f}) P_m j_1^{\mu} \mid \Psi_\mathrm{i} \rangle \simeq 0 \quad \text{if} \quad n \neq m$$

$$W^{\mu\mu} \simeq \frac{A}{\pi} \mathrm{Im} \sum_{n} \langle \Psi_\mathrm{i} \mid j_1^{\mu +} \underbrace{P_n G^+(E_\mathrm{f}) P_n}_{G_n^+(E_\mathrm{f})} j_1^{\mu} \mid \Psi_\mathrm{i} \rangle$$

$$= A \sum_n W_n^{\mu\mu}$$

G^+ replaced by $\sum_n G_n^+$

$G_n^+(E_\mathrm{f})$ is the s.p. Green's function related to the Feshbach optical model Hamiltonian $\mathcal{H}_n^+$

$$(E-H)\ G(E) = 1$$

$$P_n + Q_n$$

$$P_n \Rightarrow (E - P_nHP_n)P_nG(E)P_n - P_nHQ_nG(E)P_n = P_n$$

$$Q_n \Rightarrow (E - Q_nHQ_n)Q_nG(E)P_n - Q_nHQ_nG(E)P_n = P_n$$

$$\Rightarrow \quad Q_nG(E)P_n = \frac{1}{E - Q_nHQ_n + \mathrm{i}\eta} Q_nHP_nG(E)P_n$$

$$P_nG(E)P_n = G_n(E) = \frac{P_n}{E - \mathcal{H}_n^A(E) + \mathrm{i}\eta} \qquad G_n^+(E) = \frac{P_n}{E - \mathcal{H}_n^{A+}(E) - \mathrm{i}\eta}$$

$$\mathcal{H}_n^A(E) = P_nHP_n + P_nHQ_n \frac{1}{E - Q_nHQ_n + \mathrm{i}\eta} Q_nHP_n$$

is the OP A-body Hamiltonian which describes the elastic scattering of a nucleon by an (A-1)-system in the state n

the matrix elements of G_n give the s.p. optical model Green's function

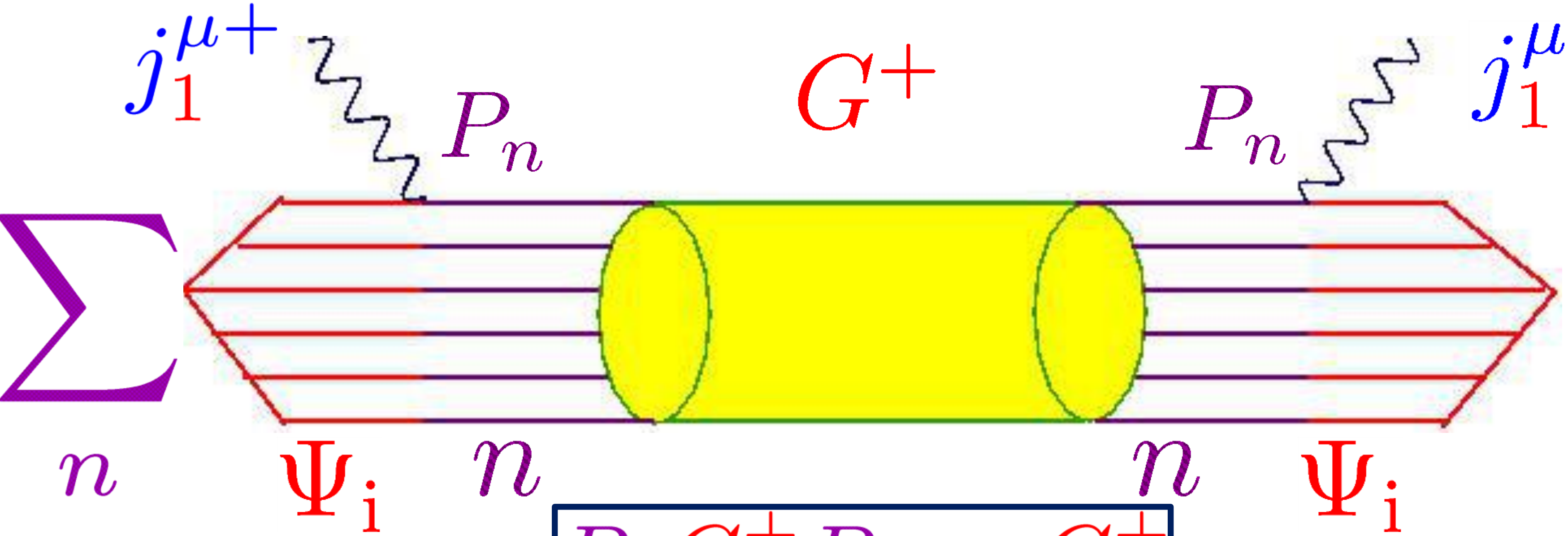
$$W^{\mu\mu} = \sum_n W_n^{\mu\mu} = \overline{\sum_{\mathrm{i}}} \sum_n \frac{1}{\pi} \mathrm{Im} \langle \Psi_{\mathrm{i}} \mid j_1^{\mu+} \underbrace{P_n G^+(E_{\mathrm{f}}) P_n}_{G_n^+(E_{\mathrm{f}})} j_1^{\mu} \mid \Psi_{\mathrm{i}} \rangle$$

1. J^{μ} 1-body $\quad j_1^{\mu}$

2. $j_1^{\mu} \mid \Psi_{\mathrm{i}} \rangle \simeq \sum_n P_n j_1^{\mu} \mid \Psi_{\mathrm{i}} \rangle$ ⟹ j_1^{μ} produces only states $\mid \vec{r_1}\, n \rangle$ and their combination

3. $P_n G^+(E_{\mathrm{f}})\, P_m j_1^{\mu} \mid \Psi_{\mathrm{i}} \rangle \simeq 0 \quad \text{if} \quad n \neq m$

$P_n G^+(E_{\mathrm{f}}) Q_n$



$$P_n G^+ P_n = G_n^+$$

ALL final states are included in G^+ which contains the total nuclear Hamiltonian H

Spectral decomposition of the nuclear response

The eigenfunctions of $\mathcal{H}_n$ and $\mathcal{H}_n^+$

$$\begin{aligned} \mathcal{H}_n^+(E_\mathrm{f}) \mid \Phi_E^{(\mp)}\rangle &= E \mid \Phi_E^{(\mp)}\rangle \\ \mathcal{H}_n(E_\mathrm{f}) \mid \tilde{\Phi}_E^{(\mp)}\rangle &= E \mid \tilde{\Phi}_E^{(\mp)}\rangle \end{aligned} \qquad \forall n \;\; E_f$$

form a biorthogonal system

$$\int \mathrm{d}E \mid \Phi_E^{(\mp)}\rangle\langle\tilde{\Phi}_E^{(\mp)}| = \int \mathrm{d}E \mid \tilde{\Phi}_E^{(\mp)}\rangle\langle\Phi_E^{(\mp)}| = 1$$ completeness

$$\langle\tilde{\Phi}_E^{(\mp)} \mid \Phi_E^{(\mp)}\rangle = \delta(E - E')$$ orthogonality

$$
\begin{aligned}
W_n^{\mu\mu} &= \frac{1}{\pi}\,\mathrm{Im}\langle\Psi_\mathrm{i} \mid j_1^{\mu+} G_n^+(E_\mathrm{f}) j_1^{\mu} \mid \Psi_\mathrm{i}\rangle \\
&= \frac{1}{2\pi\mathrm{i}}\langle\Psi_\mathrm{i} \mid j_1^{\mu+}\left[G_n^+(E_\mathrm{f} - G_n(E_\mathrm{f})\right] j_1^{\mu} \mid \Psi_\mathrm{i}\rangle \\
&= \frac{1}{2\pi\mathrm{i}}\int \mathrm{d}E\left[\langle\Psi_\mathrm{i} \mid j_1^{\mu+} \mid \Phi_E^{(-)}\rangle \frac{1}{E_\mathrm{f} - E - \mathrm{i}\varepsilon}\langle\tilde{\Phi}_E^{(-)} \mid j_1^{\mu} \mid \Psi_\mathrm{i}\rangle\right. \\
&- \left.\langle\Psi_\mathrm{i} \mid j_1^{\mu+} \mid \tilde{\Phi}_E^{(-)}\rangle \frac{1}{E_\mathrm{f} - E + \mathrm{i}\varepsilon}\langle\Phi_E^{(-)} \mid j_1^{\mu} \mid \Psi_\mathrm{i}\rangle\right]
\end{aligned}
$$

Spectral representation of G_n and $G+_n$

$$
\frac{1}{x \pm \mathrm{i}\varepsilon} = \mathcal{P}\left(\frac{1}{x}\right) \mp \mathrm{i}\pi\delta(x)
$$

$$
\begin{aligned}
W_n^{\mu\mu} &= \frac{1}{\pi}\,\mathrm{Im}\langle\Psi_\mathrm{i} \mid j_1^{\mu+} G_n^{+}(E_\mathrm{f}) j_1^{\mu} \mid \Psi_\mathrm{i}\rangle \\
&= \frac{1}{2\pi\mathrm{i}}\langle\Psi_\mathrm{i} \mid j_1^{\mu+}\left[G_n^{+}(E_\mathrm{f} - G_n(E_\mathrm{f})\right] j_1^{\mu} \mid \Psi_\mathrm{i}\rangle \\
&= \frac{1}{2\pi\mathrm{i}}\int \mathrm{d}E \Big[\langle\Psi_\mathrm{i} \mid j_1^{\mu+} \mid \Phi_E^{(-)}\rangle \frac{1}{E_\mathrm{f} - E - \mathrm{i}\varepsilon}\langle\tilde{\Phi}_E^{(-)} \mid j_1^{\mu} \mid \Psi_\mathrm{i}\rangle \\
&\quad - \langle\Psi_\mathrm{i} \mid j_1^{\mu+} \mid \tilde{\Phi}_E^{(-)}\rangle \frac{1}{E_\mathrm{f} - E + \mathrm{i}\varepsilon}\langle\Phi_E^{(-)} \mid j_1^{\mu} \mid \Psi_\mathrm{i}\rangle\Big]
\end{aligned}
$$

Spectral representation of G_n and $G+_n$

$$
\frac{1}{x \pm \mathrm{i}\varepsilon} = \mathcal{P}\left(\frac{1}{x}\right) \mp \mathrm{i}\pi\delta(x)
$$

$$
\begin{aligned}
W_n^{\mu\mu} &= \mathrm{Re}\Big[\underbrace{\langle\Phi_{E_\mathrm{f}}^{(-)} \mid j_1^\mu \mid \Psi_\mathrm{i}\rangle(\langle\tilde{\Phi}_{E_\mathrm{f}}^{(-)} \mid j_1^\mu \mid \Psi_\mathrm{i}\rangle)^*}\Big] \\
&- \frac{1}{\pi}\mathcal{P}\int \frac{\mathrm{d}E}{E_\mathrm{f}-E}\mathrm{Im}\Big[\underbrace{\langle\Phi_{E_\mathrm{f}}^{(-)} \mid j_1^\mu \mid \Psi_\mathrm{i}\rangle(\langle\tilde{\Phi}_{E_\mathrm{f}}^{(-)} \mid j_1^\mu \mid \Psi_\mathrm{i}\rangle)^*}_{T_n^{\mu\mu}(E)}\Big]
\end{aligned}
$$

$$
W^{\mu\mu} = \sum_n \left[\mathrm{Re}T_n^{\mu\mu}(E_\mathrm{f}) - \frac{1}{\pi}\mathcal{P}\int \frac{\mathrm{d}E}{E_\mathrm{f}-E}\,\mathrm{Im}T_n^{\mu\mu}(E_\mathrm{f})\right]
$$

$$
\langle\Phi_{E_\mathrm{f}}^{(-)} \mid P_n j_1^\mu P_n \mid \Psi_\mathrm{i}\rangle = \int \mathrm{d}\vec{r}\mathrm{d}\vec{r}_1\, e^{\mathrm{i}\vec{q}\cdot\vec{r}}\,\chi_{E,n}^{(-)*}(\vec{r}_1) j^\mu(\vec{r}_1,\vec{r})\lambda_n^{1/2}\phi_n(\vec{r}_1)
$$

$$
\langle\tilde{\Phi}_{E_\mathrm{f}}^{(-)} \mid P_n j_1^\mu P_n \mid \Psi_\mathrm{i}\rangle = \int \mathrm{d}\vec{r}\mathrm{d}\vec{r}_1\, e^{\mathrm{i}\vec{q}\cdot\vec{r}}\,\tilde{\chi}_{E,n}^{(-)*}(\vec{r}_1) j^\mu(\vec{r}_1,\vec{r})\lambda_n^{1/2}\phi_n(\vec{r}_1)
$$

The components of the inclusive response are written in terms of the same ingredients appearing in the DWIA approach of the exclusive 1NKO

The components of the inclusive response are written in terms of the same ingredients appearing in the DWIA approach of the exclusive 1NKO

$\chi^{(-)*}$ ⇒ DW

j^{μ} ⇒ 1-body nuclear current

$\lambda^{1/2}\phi$ ⇒ overlap

INCLUSIVE SCATTERING

$$T_n^{\mu\mu}(E) = \lambda_n \int \mathrm{d}\vec{r}\mathrm{d}\vec{r}_1 \, e^{\mathrm{i}\vec{q}\cdot\vec{r}} \, \chi_{E,n}^{(-)*}(\vec{r}_1) j^{\mu}(\vec{r},\vec{r}_1)\phi_n(\vec{r}_1)$$
$$\times \left(\int \mathrm{d}\vec{r}^{\,\prime}\mathrm{d}\vec{r}_1^{\,\prime} \, e^{\mathrm{i}\vec{q}\cdot\vec{r}^{\,\prime}} \, \tilde{\chi}_{E,n}^{(-)*}(\vec{r}_1^{\,\prime}) j^{\mu}(\vec{r}^{\,\prime},\vec{r}_1^{\,\prime})\phi_n(\vec{r}_1^{\,\prime}) \right)^*$$

$\chi^{(-)}$ ⇨ eigenstate of $\mathcal{H}_n^{+}$
absorption of flux

$\tilde{\chi}^{(-)}$ ⇨ eigenstate of $\mathcal{H}_n$
gain of flux

The imaginary part of the optical potential is responsible for the redistribution of the strength in the different channels

Interference between different channels

In the model

$$\langle \Psi_\mathrm{i} \mid j^{\mu +} G^+(E_\mathrm{f}) j^\mu \mid \Psi_\mathrm{i} \rangle \simeq \sum_n \langle \Psi_\mathrm{i} \mid j^{\mu +} G_n^+(E_\mathrm{f}) j^\mu \mid \Psi_\mathrm{i} \rangle$$

$$G^+(E) = \sum_n P_n G^+(E)(P_n + Q_n) = \sum_n (P_n + Q_n) G^+(E) P_n$$

$$Q_n = \sum_{m \neq n} P_m + P'$$

If we set $G^{+2}(E) \simeq \sum_n G_n^{+2}(E)$

The exact relation $G^{+2}(E) = -\dfrac{\mathrm{d}G^+(E)}{\mathrm{d}E}$ is not satisfied

$$-\frac{\mathrm{d}G^+(E)}{\mathrm{d}E} \simeq -\sum_n \frac{\mathrm{d}G_n^+(E)}{\mathrm{d}E} = \sum_n G_n^+(E)\left(1 - \mathcal{H}_n^{+'}(E)\right) G_n^+(E)$$

$$= \sum_n G_n^{+2}(E) - \sum_n G_n^+(E) \mathcal{V}_n^{+'}(E) G_n^+(E)$$

When terms $Q_n G P_n$ are neglected a discrepancy with the exact relation is obtained due to the energy depen. of the Feshbach OP that describes processes $P_n H Q_n$

Interference between different channels

In the model

$$\langle \Psi_\mathrm{i} \mid j^{\mu +} G^{+}(E_\mathrm{f}) j^{\mu} \mid \Psi_\mathrm{i} \rangle \simeq \sum_n \langle \Psi_\mathrm{i} \mid j^{\mu +} G_n^{+}(E_\mathrm{f}) j^{\mu} \mid \Psi_\mathrm{i} \rangle$$

$$G^{+}(E) = \sum_n P_n G^{+}(E)(P_n + \cancel{Q_n}) = \sum_n (P_n + \cancel{Q_n}) G^{+}(E) P_n$$

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$$= \sum_n G_n^{+2}(E) - \sum_n G_n^{+}(E) \mathcal{V}_n^{+'}(E) G_n^{+}(E)$$

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When terms $Q_n\, G\, P_n$ are neglected a discrepancy with the exact relation is obtained due to the energy dependence of the Feshbach OP that describes processes of the type $P_n\, H\, Q_n$

The discrepancy can be eliminated and the approach improved

$$G(E) \simeq \sum_n \hat{G}_n(E)$$

$$\hat{G}_n(E) = \sqrt{1 - v'_n(E)}\, G_n(E) \sqrt{1 - v'_n(E)} \qquad v''_n(E) \simeq 0$$

$$G^2(E) \simeq \sum_n \hat{G}^2_n(E) = \sum_n \sqrt{1 - v'_n(E)}\, G_n(E)\, (1 - v'_n(E))\, G_n(E)\, \sqrt{1 - v'_n(E)}$$

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$$G^2(E) \simeq \sum_n \hat{G}_n^2(E) = \sum_n \sqrt{1 - v_n'(E)}\, G_n(E)\, (1 - v_n'(E))\, G_n(E)\, \sqrt{1 - v_n'(E)}$$

operates in the same channel subspace and under the assumption of an almost linear energy dependence of the OP restores consistency with the exact relationship and includes most of the contributions of interference between different channels

When terms $Q_n\, G\, P_n$ are neglected a discrepancy with the exact relation is obtained due to the energy dependence of the Feshbach OP that describes processes of the type $P_n\, H\, Q_n$

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$$-\frac{\mathrm{d}G(E)}{\mathrm{d}E} \simeq -\frac{\mathrm{d}}{\mathrm{d}E} \sum_n \hat{G}_n(E)$$

$$= \sum_n \sqrt{1 - v_n'(E)}\, G_n(E)\, (1 - v_n'(E))\, G_n(E)\, \sqrt{1 - v_n'(E)}$$

$$\hat{G}_n(E) = \frac{P_n}{E - \hat{\mathcal{H}}_n + \mathrm{i}\varepsilon}$$

$$\hat{\mathcal{H}}_n = (1 - v_n'(E))^{-1/2}(\mathcal{H}_n(E) - E v_n'(E))(1 - v_n'(E))^{-1/2}$$

is energy independent if $v''_n(E) \simeq 0$

$$G_n(E) \Rightarrow \hat{G}_n(E) = \sqrt{1 - v_n'(E)}\, G_n(E) \sqrt{1 - v_n'(E)}$$

$$\mathcal{H}_n(E) \Rightarrow \hat{\mathcal{H}}_n = \sqrt{1 - v_n'(E)}(\mathcal{H}_n(E) - E v_n'(E)) \sqrt{1 - v_n'(E)}$$

$$\chi_E^{(-)} \Rightarrow \hat{\chi}_E^{(-)} = \sqrt{1 - v_n^{\dagger\prime}(E)}\, \chi_E^{(-)}$$

$$\tilde{\chi}_E^{(-)} \Rightarrow \hat{\tilde{\chi}}_E^{(-)} = \sqrt{1 - v_n'(E)}\, \tilde{\chi}_E^{(-)}$$

The eigenfunctions of a non local energy independent potential can be written

$$\hat{\chi}_{E}^{(-)} = \sqrt{1 - v_{\mathrm{L}}^{*\prime}(E)}\, \chi_{\mathrm{L},E}^{(-)}$$

$$\hat{\tilde{\chi}}_{E}^{(-)} = \sqrt{1 - v_{\mathrm{L}}^{\prime}(E)}\, \tilde{\chi}_{\mathrm{L},E}^{(-)}$$

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$$\hat{\chi}_E^{(-)} = \sqrt{1 - v_{\mathrm{L}}^{*\prime}(E)}\,\chi_{\mathrm{L},E}^{(-)}$$

$$\hat{\tilde{\chi}}_E^{(-)} = \sqrt{1 - v_{\mathrm{L}}^{\prime}(E)}\,\tilde{\chi}_{\mathrm{L},E}^{(-)}$$

eigenfunction of the local equivalent energy dependent potential

$$v_{\mathrm{L}}^{+}(E) \quad v_{\mathrm{L}}(E)$$

The eigenfunctions of a non local energy independent potential can be written

$$\hat{\chi}_E^{(-)} = \sqrt{1 - v_L^{*\prime}(E)}\,\chi_{L,E}^{(-)}$$

$$\hat{\tilde{\chi}}_E^{(-)} = \sqrt{1 - v_L^{\prime}(E)}\,\tilde{\chi}_{L,E}^{(-)}$$

eigenfunction of the local equivalent energy dependent potential

$$v_L^{+}(E)\ v_L(E)$$

takes into account terms of interference between different channels and removes the whole energy dependence of $v_L^{+}(E)\ v_L(E)$