Nuclear Physics School 2013

Scuola di Fisica Nucleare Raimondo Anni

Neutrino Physics

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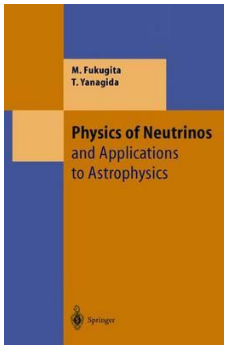
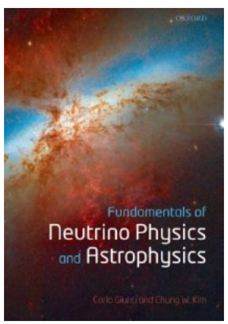
Part One: Theory

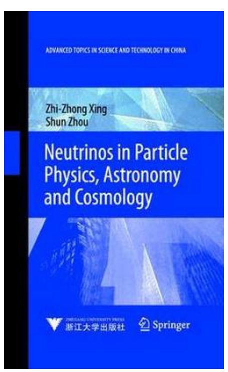
Suggested books



M. Fukugita, T. Yanagida:
Physics of Neutrinos (Springer)



C. Giunti, C.W. Kim:
Fundamentals of Neutrino Physics and Astrophysics (Oxford)



Z. Xing, S. Zhou:
Neutrinos in Particle Physics, Astronomy and (Springer)

A lot of reviews…

Among them:

- A. Strumia, F. Vissani, “Neutrino masses and mixings and...”, (hep-ph/0606054)
- M. C. Gonzalez-Garcia, Y. Nir, “Neutrino masses and mixing: evidence and implications”, Rev. Mod. Phys.75 (2003) p. 345

Dedicated sites

- The Neutrino Unbound (by C. Giunti) http://www.nu.to.infn.it
- The Neutrino Oscillation Industry http://www.hep.anl.gov/ndk/hypertext/

Some reminders: Dirac equation

Fermions are described by a four component spinor field Ψ. The free field is described by the Dirac equation

$$(\gamma^k P_k - M)\Psi = 0$$

were P_k is the quadri-momentum operator and the gamma's fulfill the anti-commutation properties

$$\{\gamma_k, \gamma_l\} = 2g_{kl}$$

<u>Weil representation for the gamma matrices:</u>

$$\gamma_0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_i = \begin{pmatrix} 0 & -\sigma_i \\ \sigma_i & 0 \end{pmatrix}, \ (i = 1, 2, 3)$$

with I 2×2 identity matrix and σ_i Pauli matrices. We define also

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

which anticommutes with all the gamma's

Let us define the chiral projector operators

$$P_L = \frac{1-\gamma_5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}, \quad P_R = \frac{1+\gamma_5}{2} = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

(notice that $P_L P_R = P_R P_L = 0$ and $P_R + P_L = 1$) and the chiral components

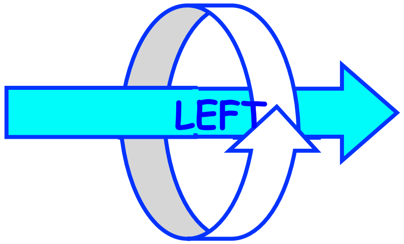
$$\Psi_{L,R} = P_{L,R}\Psi \qquad \Psi = \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix} = \Psi_R + \Psi_L$$

with ψ_{LR} 2-components spinor fields. For massles fermions from Dirac equation we see that the two components evolve independently

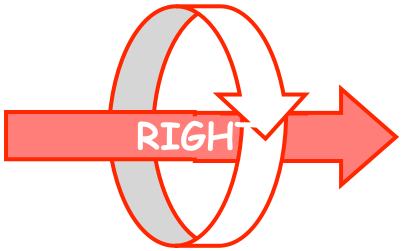
$$\sigma \cdot \mathbf{P}\psi_{R,L} = \pm|\mathbf{P}|\psi_{R,L}$$

i.e. the two components are also eigenstates of helicity with eigenvalues ±1/2.

Ψ_L:



Ψ_R:



For massive fermions the two chiral components are combinations of the helicity eigenstates $\phi_\pm$

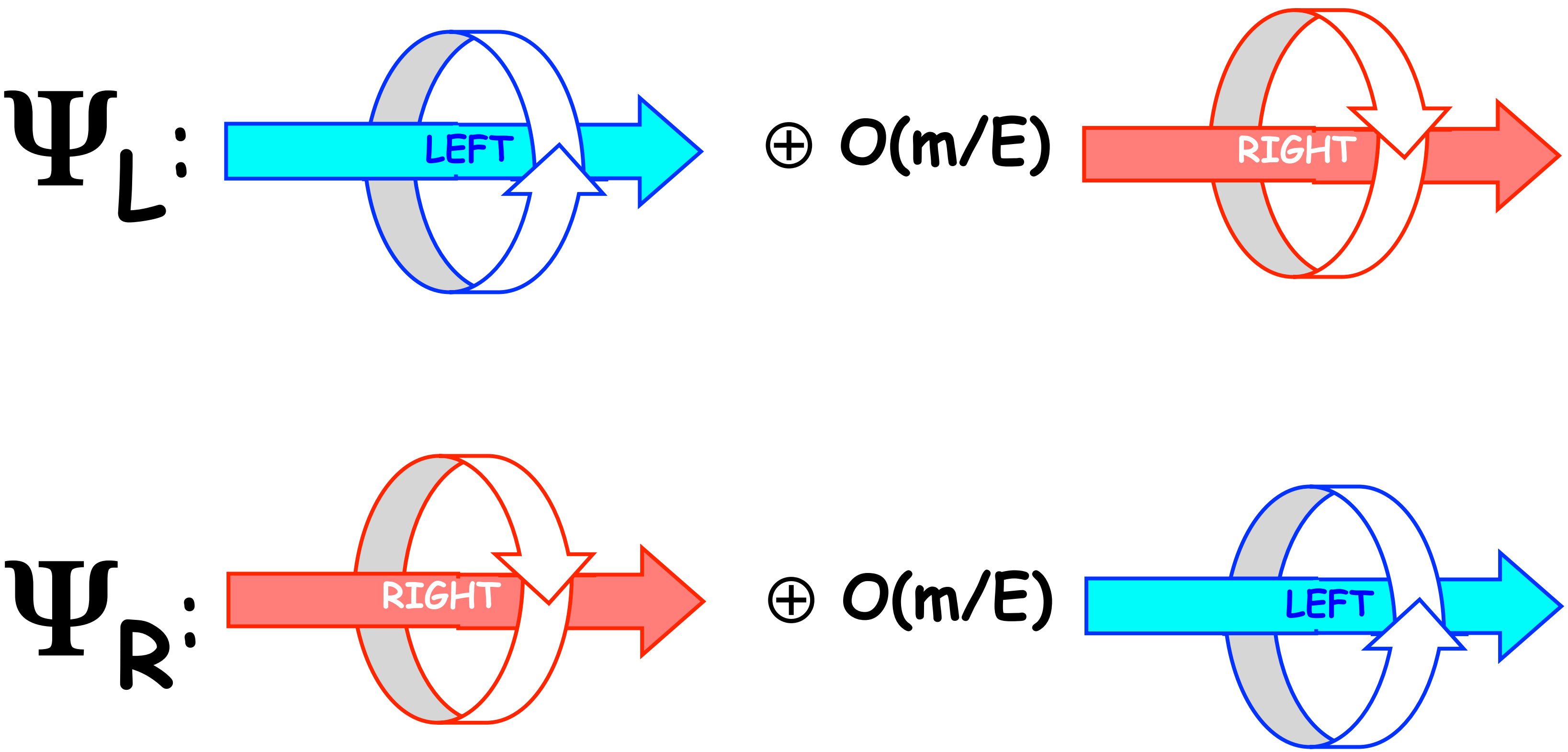
$$\psi_R = \sqrt{\frac{E+|\mathbf{P}|}{2E}}\phi_+ + \sqrt{\frac{E-|\mathbf{P}|}{2E}}\phi_-$$
$$\psi_L = -\sqrt{\frac{E-|\mathbf{P}|}{2E}}\phi_+ + \sqrt{\frac{E+|\mathbf{P}|}{2E}}\phi_-$$

Notice that chirality is Lorentz invariant while helicity is not (unless m=0). For ultrarelativistic fermions, i.e. $E \gg M$ we have

$$\psi_R \simeq \phi_+ + \frac{m}{2E}\phi_-$$
$$\psi_L \simeq \phi_- - \frac{m}{2E}\phi_+$$

Chiral components carry out a small component of “wrong” helicity.

<u>Dirac</u> spinor: Left and Right components are independent



“Left –Right” oscillations (with very small amplitude m/E) are possible

Discrete symmetries

1) Parity:

$$P\psi(\mathbf{x},t) = \psi(-\mathbf{x},t)$$

from Dirac equation is easy to show that $P \equiv \gamma^0$. Notice that $PP_L = P_R P$: P changes Left handed into right handed states and vice versa.

2) Time reversal

$$T\psi(\mathbf{x},t) = \psi^*(\mathbf{x},-t)$$

$T = i\gamma^1\gamma^3$

3) Charge conjugation: changes the charge(s) of the particle

$$(\gamma^k(P_k - eA_k) - M)\psi = 0 \Rightarrow (\gamma^k(P_k + eA_k) - M)\psi^c = 0$$

With a little of algebra it can be shown that

$$\psi^c = C\bar{\psi}^T = i\gamma^2\gamma^0\bar{\psi}^T \qquad \bar{\psi} = \psi^\dagger\gamma^0$$

negative energy states ("holes") for the field ψ are positive energy states for ψ^c. Notice that charge conjugation reverses chirality: ψ^c_L is indeed a "Right" fermion.

Majorana spinors

Majorana spinors are spinors which are invariant under the charge conjugation symmetry: $\chi=\chi^c$. It can be shown that there are two kinds of Majorana spinors

$$\chi_R = \begin{bmatrix} \psi_R \\ i\sigma_2\psi_R^* \end{bmatrix} = \psi_R + \psi_R^c$$

$$\chi_L = \begin{bmatrix} -i\sigma_2\psi_L^* \\ \psi_L \end{bmatrix} = \psi_L + \psi_L^c$$

Notice that despite the "L" and "R" superscript, Majorana states have not definite chirality since the two Weyl components are not independent (L and R refers to what component of the spinor we choose as the independent one).

Mass term lagrangian

Dirac mass Lagrangian:

$$L_M^D = M^D \bar{\psi}\psi = M^D(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

where $\psi=\psi_L+\psi_R$ is a usual Weil field with independent chiral components. The Dirac mass term mixes Left and Right states.

Majorana mass Lagrangian:

$$\begin{aligned} L_M^M &= M_L^M \bar{\chi}_L \chi_L + M_R^M \bar{\chi}_R \chi_R \\ &= M_L^M \bar{\psi}_L^c \psi_L + M_R^M \bar{\psi}_R^c \psi_R + h.c. \end{aligned}$$

Majorana mass terms mix particle and antiparticle states. This mass term is forbidden for charged fermions since violates charge by two units.

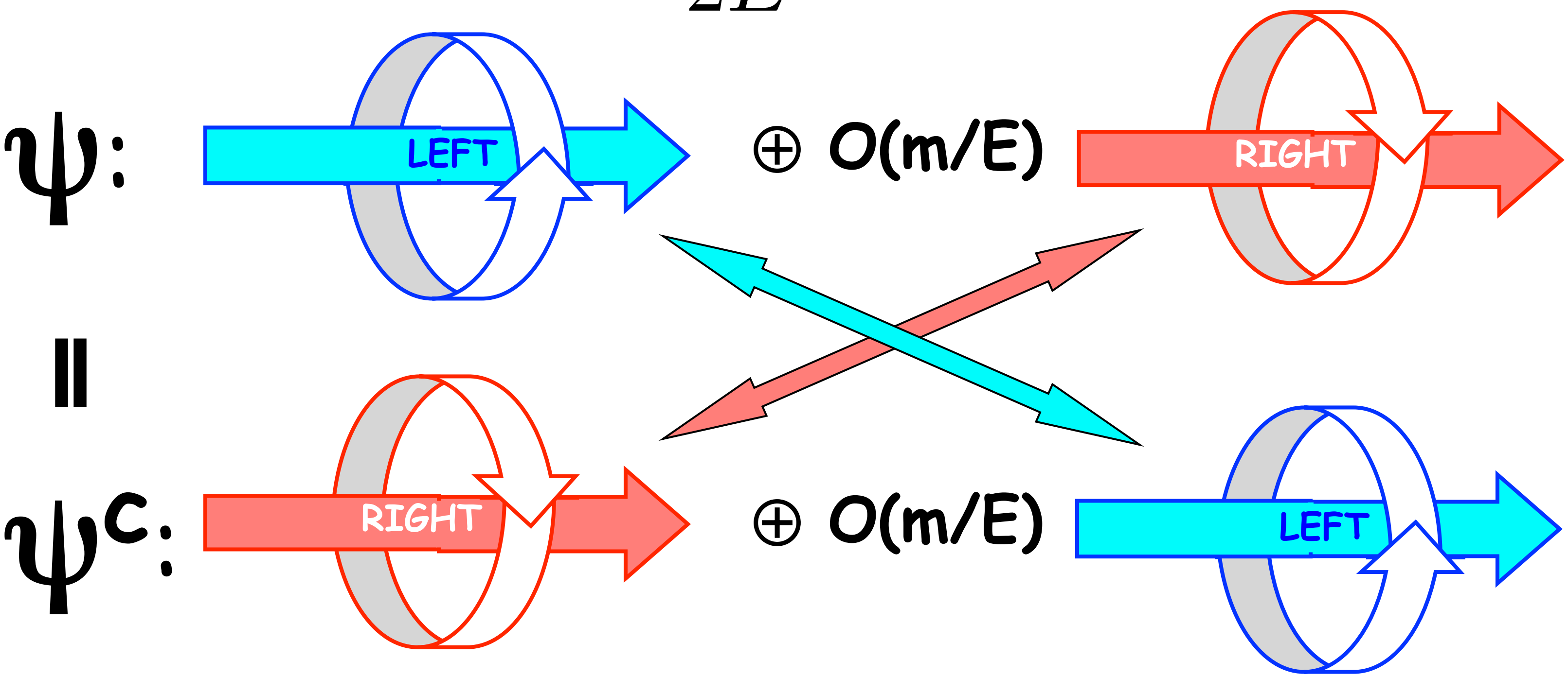
We will see that neutrinos can be Majorana fermions. Also other non standard (hypothetical) particles can be Majorana particles, e.g. SUSY gauginos.

There is not a clear distinction between Majorana particles and antiparticles. However, for $E \gg M$ we can distinguish the "particle" from the "antiparticle" by its helicity

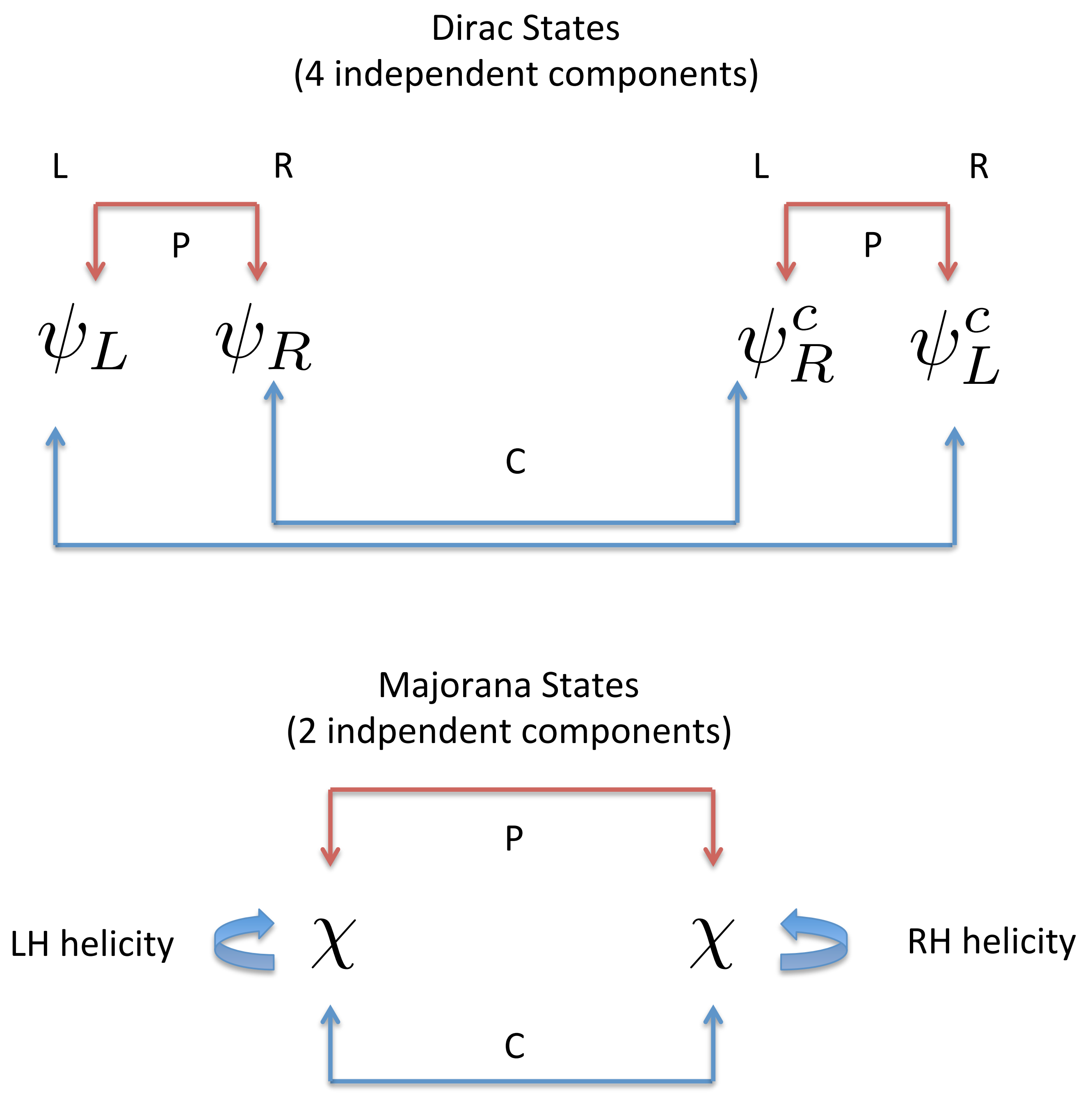
$$\psi_L \equiv \phi_- - \frac{m}{2E}\phi_+$$

$$\psi_L^c \equiv \phi_+ + \frac{m}{2E}\phi_-$$

$$i\sigma_2\phi_\pm^* = \pm\phi_\mp$$



Notice that for Dirac fermions the $\phi_\pm$ states are different from particles to antiparticles while for Majorana fermions are the same. "Particle-antiparticle" oscillations (with small amplitude m/E) are possible.


Dirac States
(4 independent components)
L
R
L
R
P
P
ψ_L
ψ_R
ψ_R^c
ψ_L^c
C
Majorana States
(2 indpendent components)
P
LH helicity
χ
χ
RH helicity
C

Standard Model neutrino interactions

Fermion-Gauge fields interaction Lagrangian in the Standard Model

$$L_{int} = L_{CC} + L_{NC} + L_{EM}$$

$$L_{CC} = \frac{g}{\sqrt{2}}\left[\mathbf{J}_{CC}^{(+)} \cdot \mathbf{W}^{(-)} + \mathbf{J}_{CC}^{(-)} \cdot \mathbf{W}^{(+)}\right]$$

Charge current

$$L_{NC} = \frac{g}{\cos\theta_W}\mathbf{J}_{NC} \cdot \mathbf{Z}$$

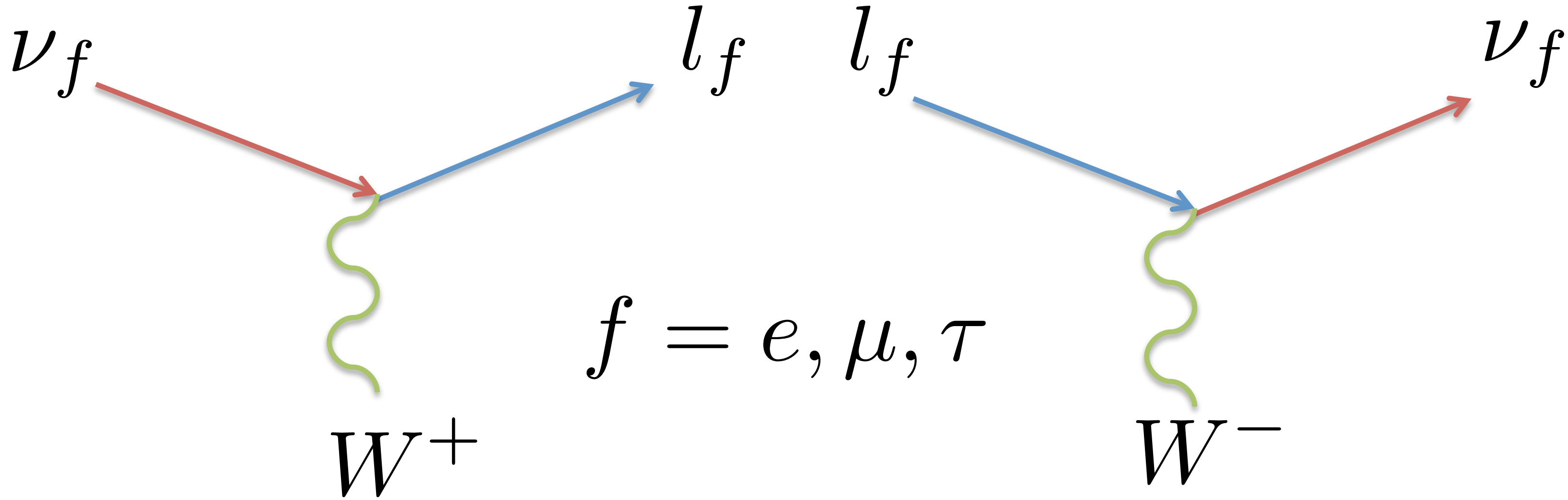
Neutral current

$$L_{EM} = e\mathbf{J}_{EM} \cdot \mathbf{A}$$

Electromagnetic

Charge current interactions (lepton sector)

$$J^{(\pm)}_{CC,i} = \sum_f \left(\bar{\nu}_{L,f}\gamma_i l_{L,f} + \bar{l}_{L,f}\gamma_i \nu_f\right)$$

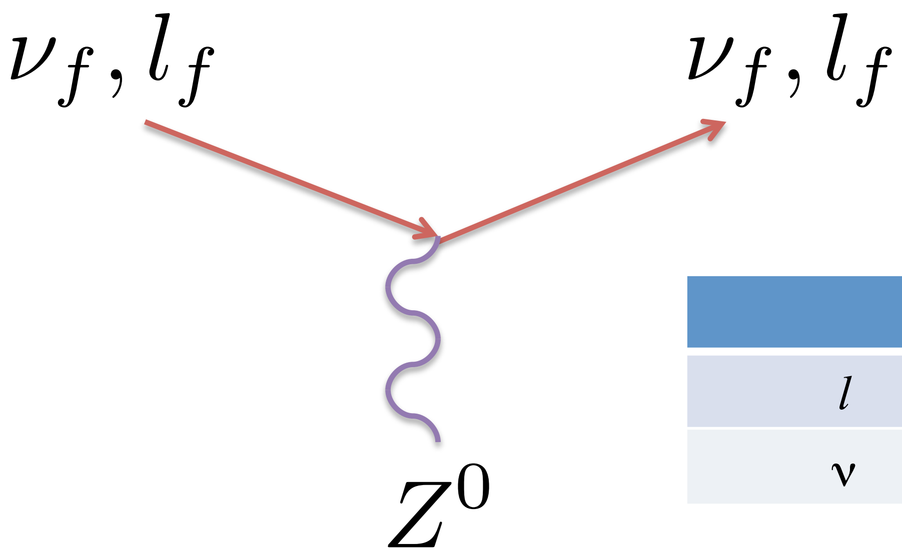


$$f = e, \mu, \tau$$

Only left-handed fields are involved in the interactions. No flavor changing interactions allowed

Neutral current interactions (lepton sector)

$$J_{NC,i}^{(\pm)} = \sum_f \left[\bar{\nu}_f \gamma_i (g_V^\nu - g_A^\nu \gamma_5) \nu_f + \bar{l}_f \gamma_i (g_V^l - g_A^l \gamma_5) l_f \right]$$



	g_V	g_A
l	$-1/2+2\sin^2\theta_W$	-1/2
ν	1/2	-1/2

Fermion masses

In the SM fermions get mass from Yukawa and SSB

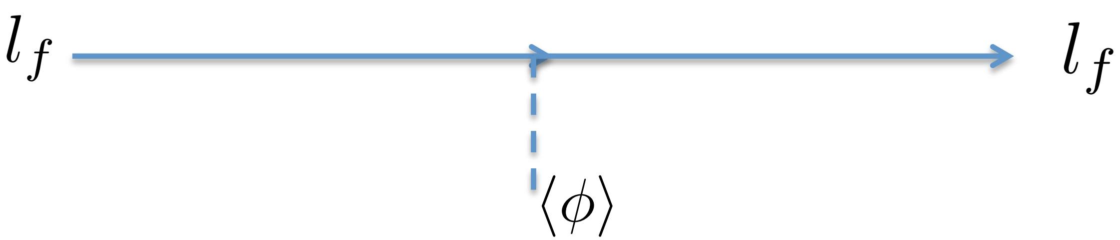
$$L_Y = \sum_{ff'} y_{ff'} \bar{E}_f \phi l_{R,f'} + h.c. \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

(for simplicity we consider here only leptons). Here l_R are the right handed SU(2) singlets and f is the Higgs doublet field. After SSB the Higgs field evolves a VeV

$$\phi = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

The Yukawa coupling thus becomes a (Dirac) mass term for charged leptons. Without loss of generality we can choose the matrix **y** as diagonal. With this choice the charged leptons become massive

$$L_{M,l} = \sum_f M_f^l \bar{l}_{L,f} l_{R,f'} + h.c. \qquad \text{with} \qquad M_f^l = y_{ff} v/\sqrt{2}$$



Neutrino masses

From oscillations (see later) we have now evidence that neutrinos are massive. How neutrinos can get mass in the SM? In principle we can add right handed singlets ν_R in and write down a Yukawa Lagrangian similar to those for charged fermions

$$L'_Y = \sum_{ff'} y'_{ff'} \bar{E}_f \hat{\phi} \nu_{R,f'} + h.c. \qquad \hat{\phi} = i\sigma_2 \phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix}$$

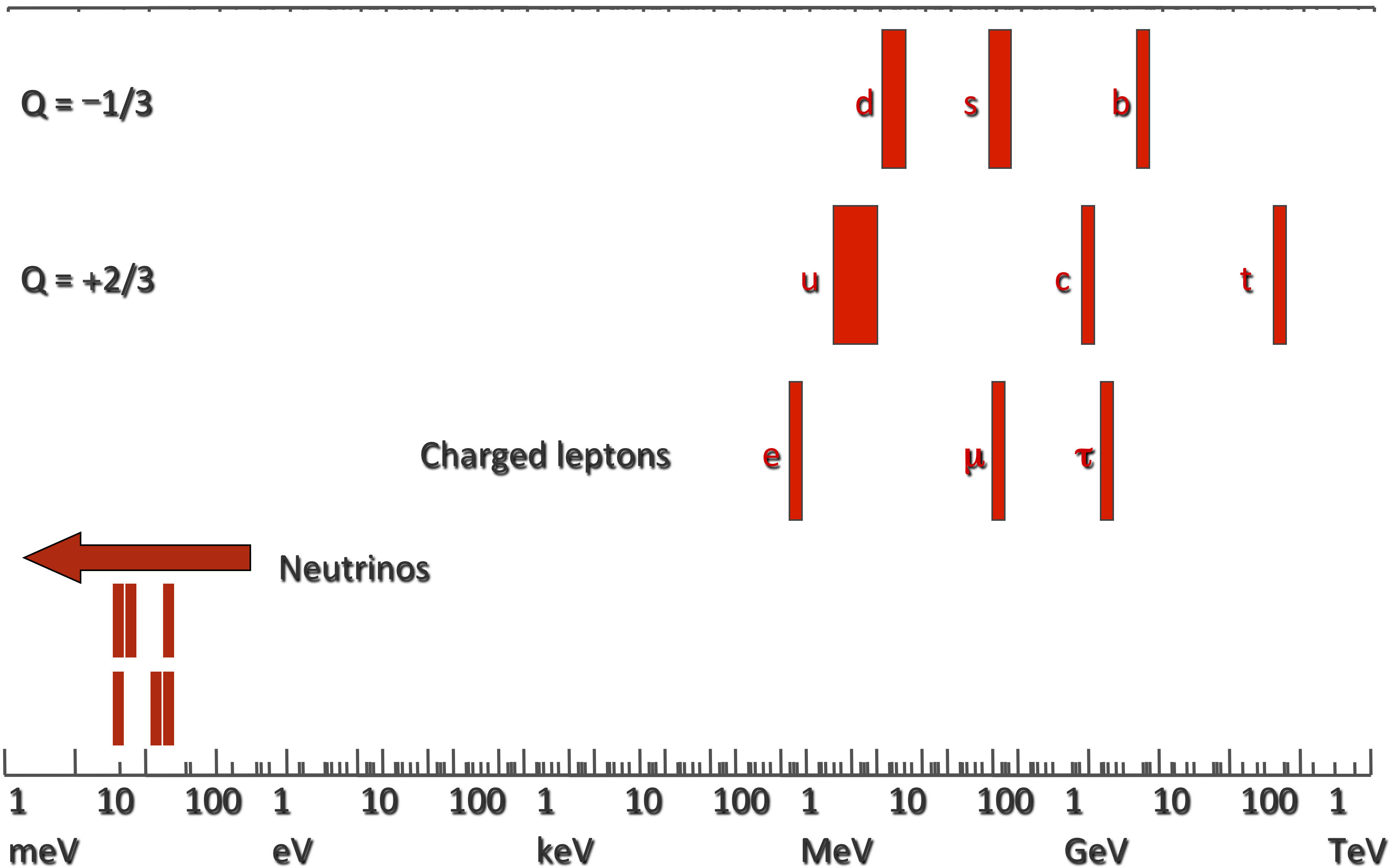
(the last transformation is necessary to make L'_Y singlet of hypercharge). After SSB we can get a (Dirac) mass Lagrangian for neutrinos

$$L_{M,\nu} = \sum_{ff'} M^{\nu}_{ff'} \bar{\nu}_{L,f} \nu_{R,f'} + h.c.$$

$$M^{\nu}_{ff'} = y'_{ff'} v/\sqrt{2}$$

However there is a problem…

Neutrinos masses are unnaturally small!



this means that the «**y**'»'s must be order of magnitude smaller than the «**y**»'s. This poses a problem of naturalness. However, forget this for the moment. We come back on it later.

Consider these unitary transformations on the flavor basis (i.e. on the “f” indices)

$$\nu_{L,R} \to O^{\nu} \cdot \nu_{L,R} \qquad\qquad l_{L,R} \to O^{l} \cdot l_{L,R}$$

It is straightforward to show that NC and EM currents are invariant under independent transformations of the neutrino and lepton fields (exercise).
Conversely, the CC current is in general no longer invariant under independent transformations of the fields. For example:

$$J^{(+)}_{CC,i} = \sum_{f} \bar{\nu}_{L,f}\gamma_i l_{L,f} + h.c. = \sum_{ff'} \left[O^{\nu\dagger} O^{l}\right]_{ff'} \bar{\nu}_{L,f}\gamma_i l_{L,f'} + h.c.$$

With these transformations CC interactions are no longer diagonal <u>unless $O^l=O^\nu$</u>.

We can choose O^l as the matrix that diagonalizes the charged lepton mass matrix

$$O^{l\dagger} M^{l} O^{l} = M^{l}_{D} = \mathrm{diag}\{m_e, m_\mu, m_\tau\}$$

With this choice the charged leptons are also mass eigenstates (i.e. they have definite masses). However, under this transformation in general the neutrinos mass matrix is not diagonal

$$M'^{\nu} = O^{l\dagger} M^{\nu} O^{l} \neq \mathrm{diag}$$

Let O^ν the matrix thad diagonalizes M^ν, we can write

$$M'^\nu = (O^{l\dagger} O^\nu) M^\nu_D (O^{\nu\dagger} O^l) \equiv U \cdot \mathrm{diag}\{m_1, m_2, m_3\} \cdot U^\dagger$$

The matrix U is defined as “mixing matrix” and connect the neutrino mass eigenstates to the flavor (interaction) eigenstates. In fact if we define

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1^m \\ \nu_2^m \\ \nu_3^m \end{pmatrix}$$

we have that the neutrino mass Lagrangian is diagonal in the mass basis

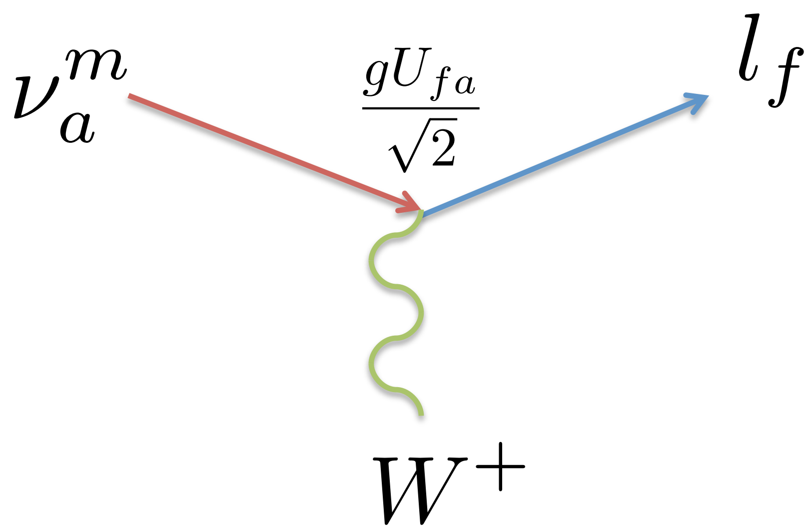
$$L_{M,\nu} = \sum_a m_a \bar{\nu}_a^m \nu_a^m + h.c.$$

while the CC interactions are still diagonal in the flavor basis

$$L_{CC} = \frac{g}{\sqrt{2}} (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau) \gamma^i \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} W_i^{(+)} + h.c.$$

Technical note: from the point of view of QFT the mass fields ν^m are the real physical fields (they describe asymptotic states) while flavor eigenstates are not. In this case the CC interactions are no longer diagonal

$$L_{CC} = \frac{g}{\sqrt{2}}(\bar{\nu}_1^m, \bar{\nu}_2^m, \bar{\nu}_3^m)U^\dagger\gamma^i \left(\begin{array}{c} e \\ \mu \\ \tau \end{array}\right) W_i^{(+)} + h.c.$$



However, the flavor eigenstates are a convenient convention.

Flavor violating processes?

With neutrino mixing family lepton number is no longer conserved
Flavor violating processes are in principle allowed at 1-loop, for example

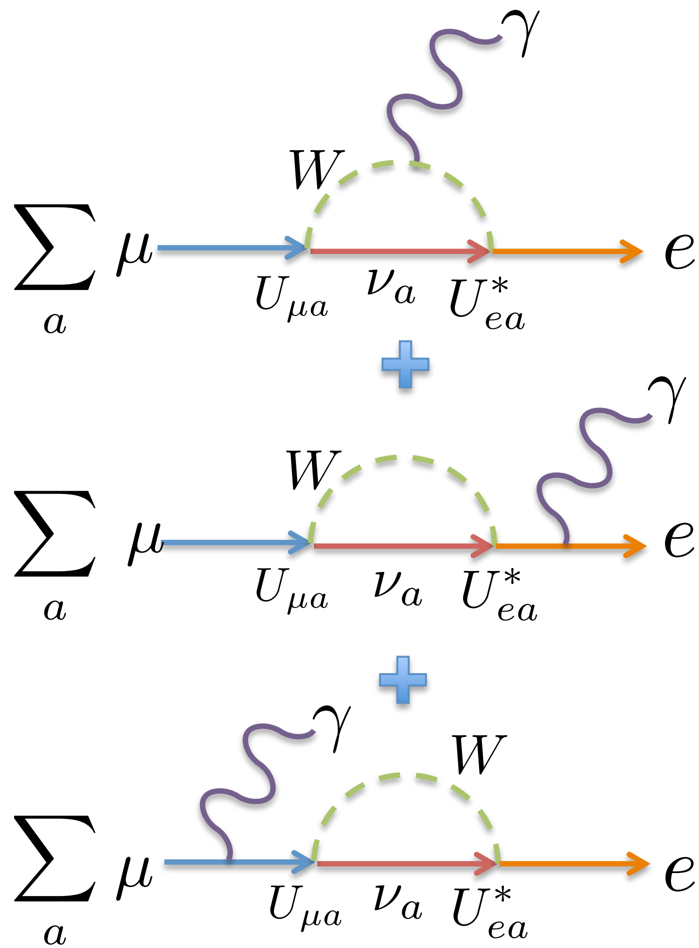
$$\mu \to e\gamma$$

however, the calculation gives

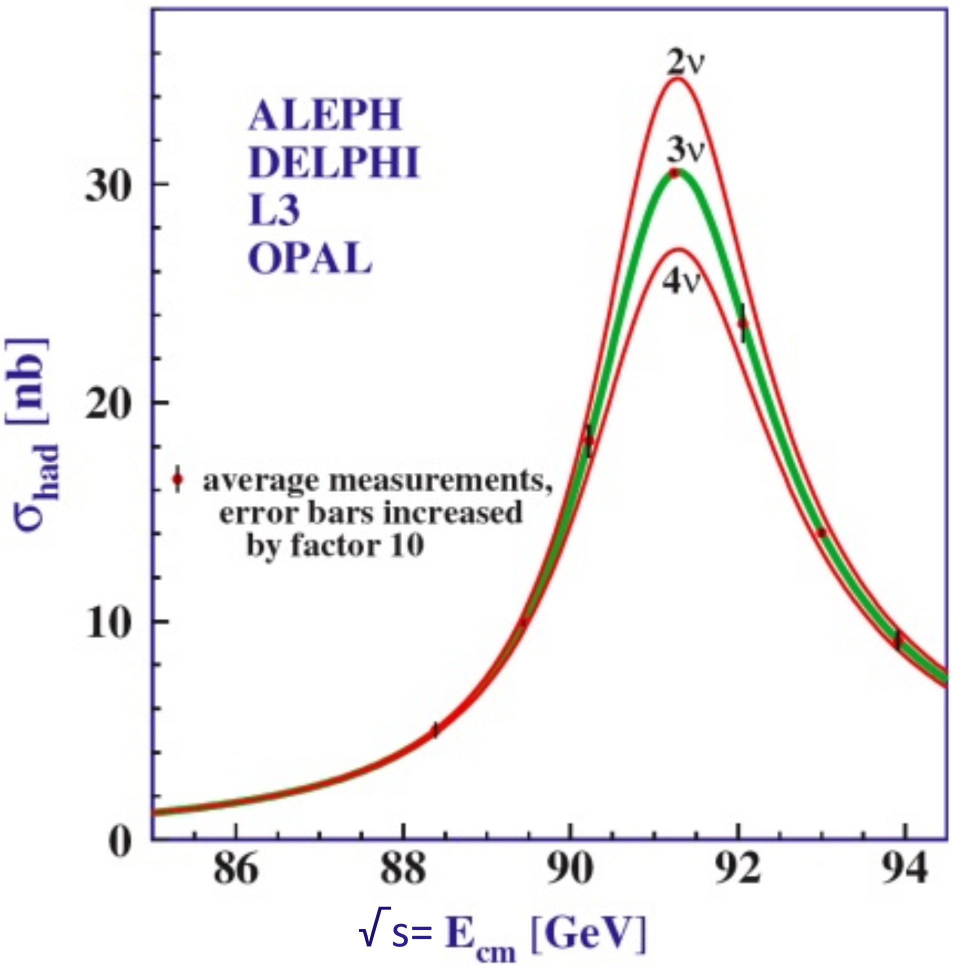
$$R = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\bar{\nu}_e \nu_\mu)} = \frac{3\alpha}{32\pi} \left| \sum_a U^*_{\mu a} U_{ea} \frac{m_a^2}{M_W^2} \right|^2$$

$$= 5.2 \times 10^{-48} \left| \sum_a U^*_{\mu a} U_{ea} \left(\frac{m_a}{1\mathrm{eV}} \right)^2 \right|^2$$

36 order of magnitude lower than the present limits! Nevertheless, the previous proces could be enhanced by other non-standard processes.



How many neutrinos?



From the measure of the $e^+e^- \rightarrow Z^0$ resonance at LEP is possible to fit the the "hidden width" (that is, all the decays in hidden particles, such as neutrinos)

$$\sigma_{e^+e^-\rightarrow Z^0} \propto \frac{s}{(s-M_Z)^2 + s^2\Gamma_Z^2/M_Z}$$

$$\Gamma_Z \ni N_\nu \Gamma_{\nu\bar{\nu}}$$

$$\Gamma_{\nu\bar{\nu}} = 167.1\,\mathrm{MeV}$$

$$N_\nu = 2.9841 \pm 0.0083$$

No more than 3 "active" neutrinos with mass lower than $M_Z/2$.
Limits on other weak particles coupled with the Z boson.

…however, the LEP measure in principle does not forbid the existence of one or more singlet fermions (that we can conveniently call them “sterile” neutrinos). They would not couple to any SM gauge and do not contribute to triangular anomalies. They could manifest their existence through the mixing with active neutrinos.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_{s,1} \\ \nu_{s,2} \\ \ldots \end{pmatrix} = U \begin{pmatrix} \nu_1^m \\ \nu_2^m \\ \nu_3^m \\ \nu_4^m \\ \nu_5^m \\ \ldots \end{pmatrix}$$

The interest for sterile neutrinos have raised recently for some anomalies in the data. However, the existence of these extra states is still a matter of debate.

For the sake of simplicity in the following we consider only three generations of active neutrinos.

The mixing matrix

Any unitary N×N matrix can be parameterized by the product of a diagonal phase matrix and N(N-1)/2 block matrices containing rotations and phases (Murnagham, 1962)

$$U = \left(\prod_{b>a} \mathcal{U}_{ab} \right) \cdot \Gamma$$

with

$$\Gamma = \mathrm{diag}\{e^{i\alpha_1}, \cdots, e^{i\alpha_N}\}$$

$$\mathcal{U}_{ab} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cos\theta_{ab} & \cdots & \sin\theta_{ab} e^{-i\delta_{ab}} & \cdots \\ \cdots & \cdots & \ddots & \cdots & \cdots \\ \cdots & -\sin\theta_{ab} e^{i\delta_{ab}} & \cdots & \cos\theta_{ab} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

$$\theta_{ab} \in [0, \pi/2) \qquad \alpha_a, \delta_{ab} \in [0, 2\pi)$$

Apparently, there are N(N+1)/ independent phases. If we look at the structure of interactions, we see that under the gauge transformations

$$\nu_a^m \to \exp(i\alpha_a)\nu_a^m,\ l_f \to \exp(i\beta_f)l_f$$

the NC and the EM terms are invariant, while the CC term changes as

$$L_{CC} \to \frac{g}{\sqrt{2}} U_{af} e^{i(\alpha_a-\beta_f)} \bar{\nu}_{L,a}^m \gamma^i l_{L,f} W_i^{(+)} + h.c.$$

Notice the Lagrangian is no longer invariant under global gauge transformations of each family: the family lepton number is violated! However, the Lagrangian is invariant under the global transformation

$$\nu_a^m \to \exp(i\Lambda)\nu_a^m,\ l_f \to \exp(i\Lambda)l_f$$

This means that the total lepton number is conserved (this will be no longer true when we will consider Majorana mass terms).

We can use the previous relation to remove 2N-1 unphysical phases (because the previous relation is invariant under the change $\alpha_a \to \alpha_a + \Lambda$ and $\beta_f \to \beta_f + \Lambda$). We remain thus with only N(N+1)/2-(2N-1)=(N-1)(N-2)/2 physical phases.

For N=2 the matrix U is completely real. For N=3 there is only 1 physical phase.

Without loss of generality we can choose

$$\mathcal{U}_{23}(\theta_{23}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$$\mathcal{U}_{13}(\theta_{13}, \delta) = \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix}$$

$$\mathcal{U}_{12}(\theta_{12}) = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

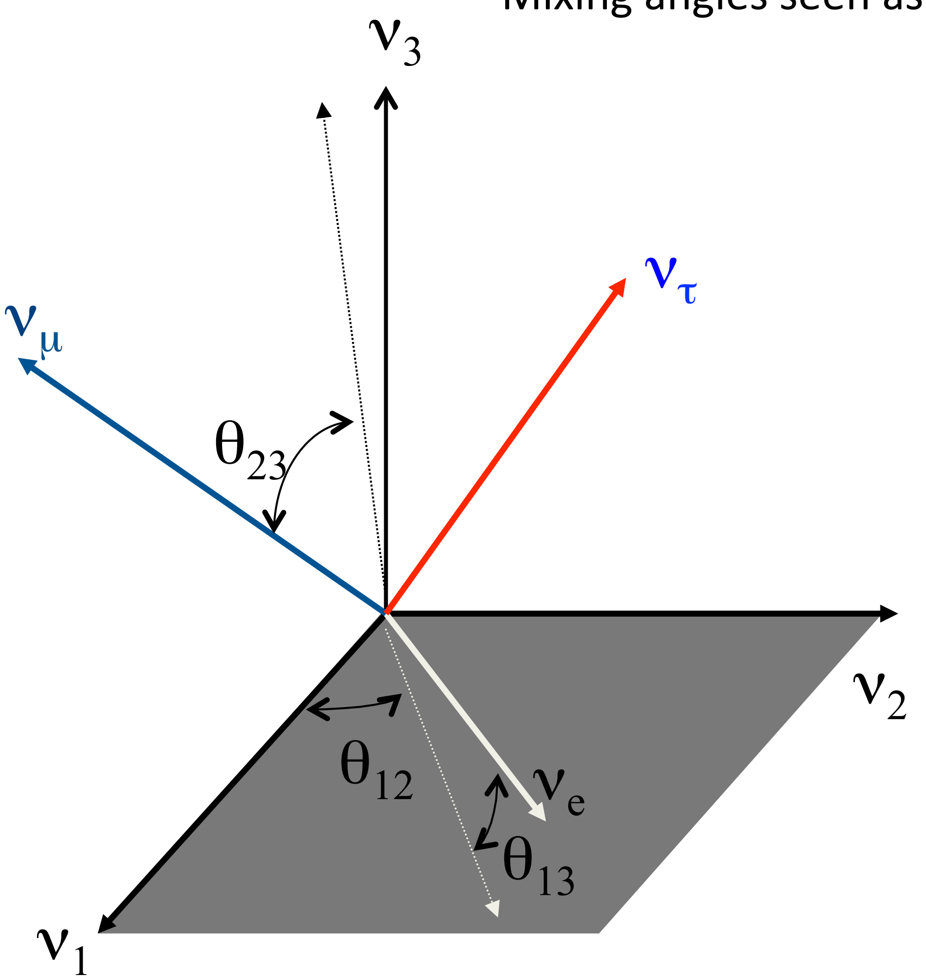
with the shorthand $c_{ij} \equiv \cos\theta_{ij},\ s_{ij} \equiv \sin\theta_{ij}$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}c_{23} \end{pmatrix}$$

Pontecorvo–Maki–Nakagawa–Sakata matrix (**PMNS matrix**)

“Geometrical” meaning of U

Mixing angles seen as “Euler” rotations



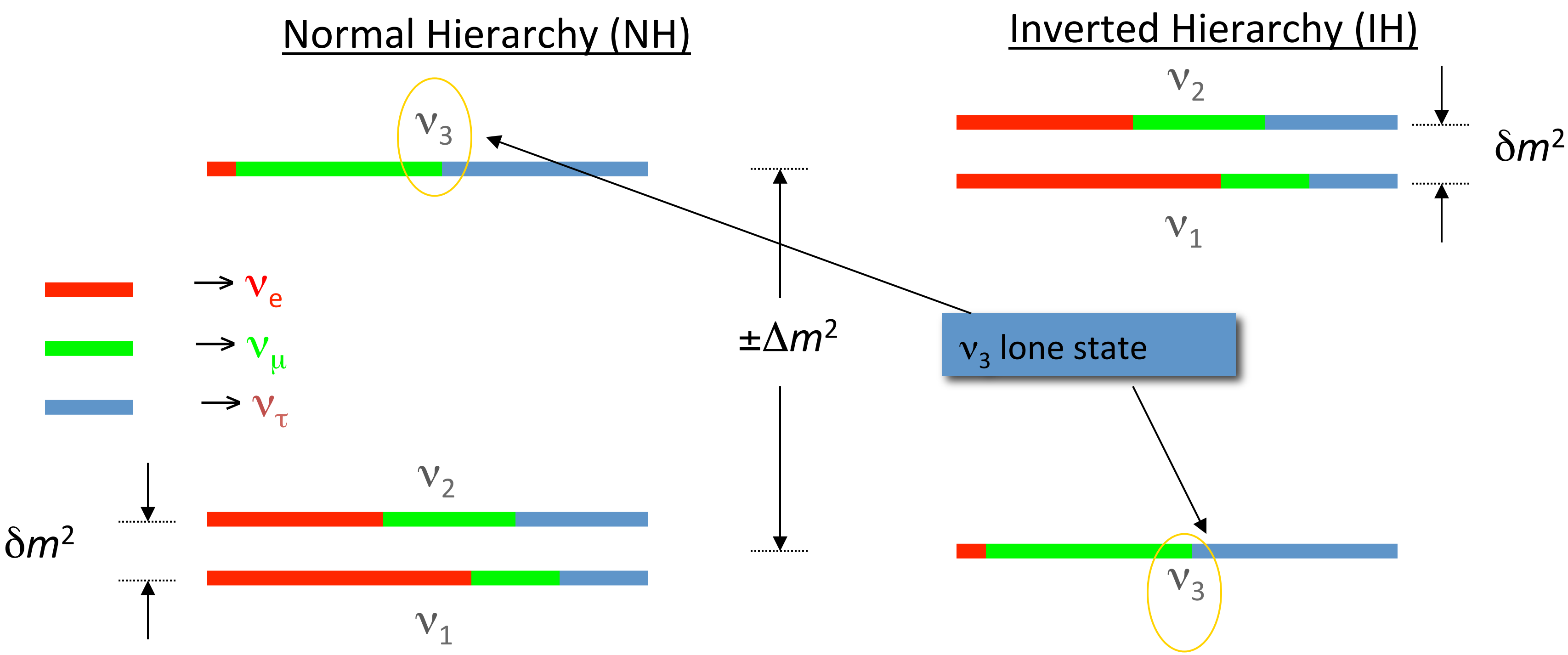
Notice that if θ_{13}=0

- ν_e linear combination of $\nu_{1,2}$
- ν_3 linear combination of $\nu_{\mu,\tau}$

We will see that θ_{13} is small but non-vanishing.

Neutrino masses

From neutrino oscillations we will see that neutrino masses are organized in a "doublet" (conventionally ν_1 and ν_1) and a "lone state" (ν_3). The absolute scale of mass as well the hierarchy of masses are unknown.



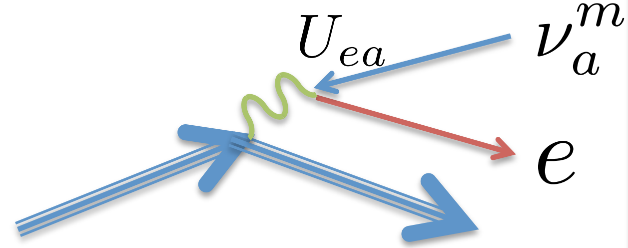
$$\mathrm{diag}\{m_a^2\} = M_\nu^2 + \mathrm{diag}\left\{-\frac{1}{2}\delta m^2, +\frac{1}{2}\delta m^2, \pm\Delta m^2\right\}$$

Neutrino oscillations are sensitive only to the differences of the mass^2.

Nuclear β decay and neutrino mass

The most sensitive way to measure the neutrino masses is to measure the recoil spectrum of the electron of the nuclear beta decay.

$$(A,Z) \rightarrow (A,Z+1) + e^- + \bar{\nu}_e \qquad \sum_a \Big| \quad \Big|^2$$



The recoil spectrum can be calculated through the Fermi theory and gives

$$\frac{d\Gamma}{dT_e} = \frac{G_F^2 m_e^5}{2\pi^3} \cos^2\theta_c |\mathcal{M}|^2 F(Z,T_e) \sum_a |U_{ea}|^2 E_e p_e E_a p_a \Theta(Q - T_e - m_a)$$

where Q is the energy available in the process, G_F is the Fermi constant, θ_c the Cabibbo angle, $\mathcal{M}$ the matrix element, F the Fermi function (which accounts for the EM interaction between the electron and the nucleus), p_e, E_e (T_e) the momentum and total (kinetic) energy of the electron, and p_a, E_a the same for the a-th mass eigenstate. We have summed incoherently on all final states. The step Θ function ensures that the neutrino state ν^m_a is produced if its total energy is larger than its mass.

For practical reasons it is convenient not to measure directly the recoil spectrum but the "normalized" quantity (Kurie function)

$$K(T_e) = \left[\frac{d\Gamma/dT_e}{\frac{G_F^2 m_e^5}{2\pi^3} \cos^2\theta_c |\mathcal{M}|^2 F(Z,T_e) E_e p_e} \right]^{1/2}$$

For massless neutrinos the Kurie function is linear: $K(T_e) = Q - T_e$

For massive neutrinos we have

$$K(T_e) = \left[(Q - T_e) \sum_a |U_{ea}|^2 \sqrt{(Q - T_e)^2 - m_a^2} \right]^{1/2}$$

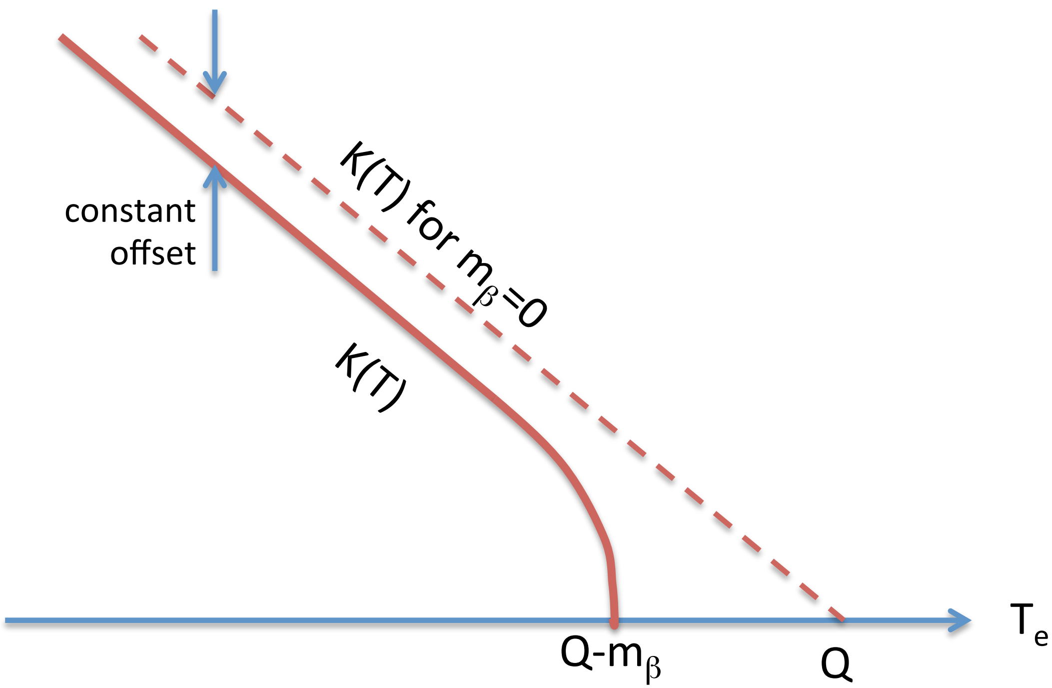
In general the measure is made fitting the spectrum for value of T_e not too close to the Q value (were the decay rate tends to zero). For this reason we can expand the square root

$$\begin{aligned} K^2 &\simeq (Q - T_e)^2 \sum_a |U_{ea}|^2 \left[1 - \frac{1}{2} \frac{m_a^2}{(Q - T_e)^2} \right] \\ &= (Q - T_e)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q - T_e)^2} \right] \\ &\simeq (Q - T_e) \sqrt{(Q - T_e)^2 - m_\beta^2} \end{aligned}$$

with

$$m_\beta^2 = \sum_a |U_{ea}|^2 m_a^2 = c_{13}^2(c_{12}^2 m_1^2 + s_{12}^2 m_2^2) + s_{13}^2 m_3^2$$

This is the same expression that we can obtain if only one generation of neutrinos is considered.

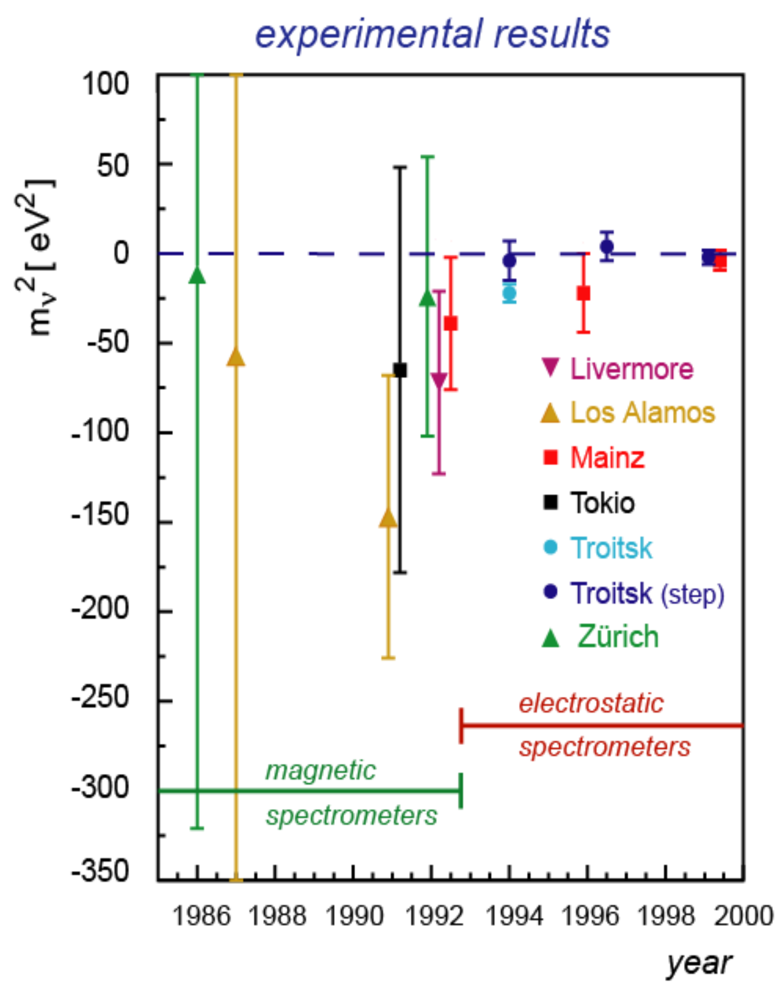


Unfortunately the measure is complicated by: 1) excitations of the parent nuclei in the lattice 2) detector resolution and bias uncertainty .

Tritium β decay: the lowest Q-value known in nature

$$^3H \rightarrow ^3He + e^- + \bar{\nu}_e \ \ (Q = 18.57\text{keV}, \ \ T_{1/2} = 12.32\text{y})$$

	m_ν
ITEP *T_2 in complex molecule* *magn. spectrometer (Tret'yakov)*	17-40 eV
Los Alamos *gaseous T_2 - source* *magn. spectrometer (Tret'yakov)*	< 9.3 eV
Tokio *T - source* *magn. spectrometer (Tret'yakov)*	< 13.1 eV
Livermore *gaseous T_2 - source* *magn. spectrometer (Tret'yakov)*	< 7.0 eV
Zürich *T_2 - source impl. on carrier* *magn. spectrometer (Tret'yakov)*	< 11.7 eV
Troitsk (1994-today) *gaseous T_2 - source* *electrostat. spectrometer*	< 2.2 eV
Mainz (1994-today) *frozen T_2 - source* *electrostat. spectrometer*	< 2.3 eV



Neutrino oscillations

Heuristic derivation: Let us consider a neutrino moving as a plane wave in the "x" direction. The equation of motion can be expressed in form of Klein Gordon equation

$$\left(\partial_x^2 - \partial_t^2 - (M^\nu)^2\right) \nu(x,t) = 0$$

Let us suppose that the neutrino has a definite energy and is ultra-relativistic (i.e. its energy is much greater than its mass). Ansatz:

$$\nu(x) = \Psi_\nu(x) \exp(iE \cdot (x-t))$$

that is, the neutrino wavefunction is written as the product of a plane "carrier" wave moving (almost) at the speed of light and a "modulating" wave, function of the distance. Inserting in the equation of motion we have

$$i\Psi'_\nu(x) \simeq \frac{1}{2E}(M^\nu)^2\Psi_\nu$$

where we have supposed that the modulating wave varies very slowly respect to the carrier wave, i.e. $\Psi'_{\nu k} \ll E\psi_{\nu k}$ (we will verify this a posteriori).

The previous equation is a Shrödinger-like equation describing the evolution of the flavor content of the neutrino. In the flavor basis the equation become

$$i\Psi'(x) = \frac{1}{2E} U \mathrm{diag}\{m_a^2\} U^\dagger \Psi \qquad \Psi_\nu = \begin{bmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{bmatrix}$$

which has solution

$$\Psi(x) = \exp\left(-i\frac{U\mathrm{diag}\{m_a^2\}U^\dagger}{2E}x\right) \cdot \Psi(0) = U\mathrm{diag}\left\{e^{-i\frac{m_a^2 x}{2E}}\right\}U^\dagger \cdot \Psi(0)$$

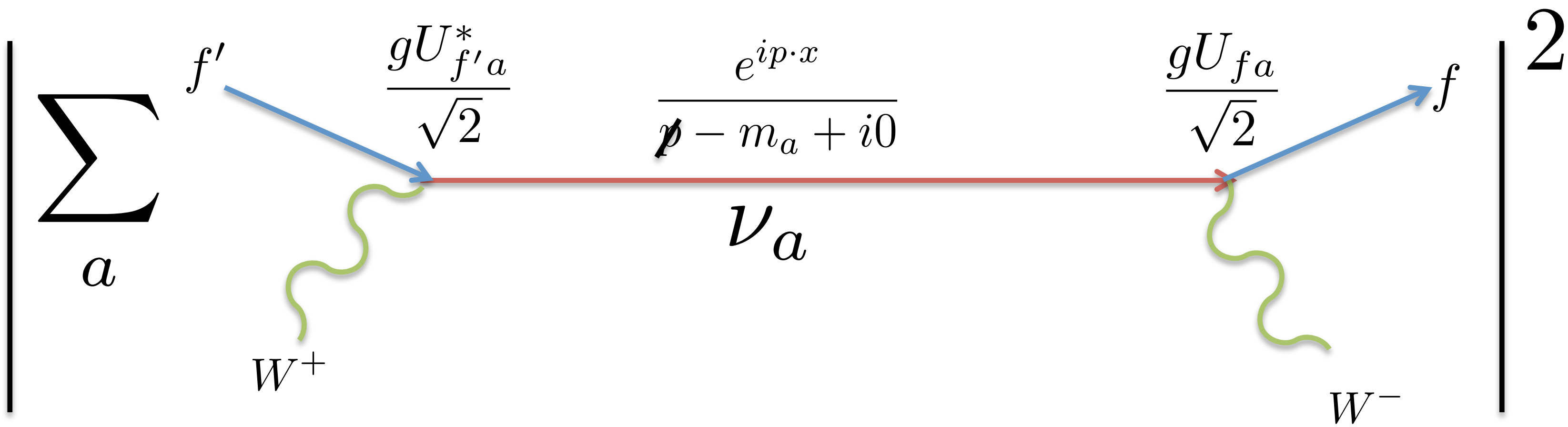
that is

$$\Psi_f(x) = \sum_a U_{af} U^*_{af'} e^{-i\frac{m_a^2}{2E}x} \Psi_{f'}(0)$$

Taking the square modulus we have the probability that a neutrino with an initial flavor f' is observed with a flavor f at a given distance x from the source

$$P(\nu_{f'} \to \nu_f; x) = \left| \sum_a U_{af} U^*_{af'} e^{-i\frac{m_a^2}{2E}x} \right|^2$$

Remark: the previous derivation is only heuristic because flavor eigenstates are not physical fields. Nevertheless the result is (almost) correct. A correct derivation must take in account QFT. The process can be regarded as a sigle process in which we sum coherently on all the amplitudes of the processes in which the mass eigenstates are seen as propagators



the calculation is much more complicated but the result is the same to those obtained with the simple derivation, apart small corrections negligible for all practical purposes.

With a little algebra, using the unitarity of U we obtain (exercise)

$$P(\nu_{f'} \to \nu_f; x) = \delta_{ff'} - 4\sum_{a<b} \Re[U_{fa}U^*_{fb}U^*_{f'a}U_{f'b}] \sin^2\left(\frac{\Delta m^2_{ab} x}{4E}\right)$$
$$+2\sum_{a<b} \Im[U_{fa}U^*_{fb}U^*_{f'a}U_{f'b}] \sin\left(\frac{\Delta m^2_{ab} x}{2E}\right)$$

where $\Delta m^2_{ab} = m^2_b - m^2_a$

Notice that oscillations are sensitive only to the difference of the square masses of the mass eigenstates. Oscillations are thus insenstitive to absolute neutrino masses.

For the sake of simplicity let us consider only two generations. In this case (exercise)

$$P(\nu_{f'} \to \nu_f; x) = \begin{cases} 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} x\right) \text{ if } f' = f \\ \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} x\right) \text{ if } f' \neq f \end{cases}$$

The oscillation wavelength is (exercise)

$$\lambda = \frac{4\pi E}{\Delta m^2} \simeq 2.47\mathrm{m} \times \frac{E}{\mathrm{MeV}} \times \frac{\mathrm{eV}^2}{\Delta m^2}$$

sugg.: use $\hbar c \simeq 197\,\mathrm{MeV} \cdot \mathrm{fm}$

Notice that the de Broglie wavelength (i.e., the "carrier" wavelength) of the neutrino is given by

$$\tilde{\lambda} = \frac{2\pi}{E}$$

the ratio between the two wavelengths is

$$\frac{\tilde{\lambda}}{\lambda} = \frac{2E^2}{\Delta m^2}$$

We will see from phenomenology that the value of the Δm^2's are $\lesssim 10^{-3}\,\mathrm{eV}^2$ while in almost the process of interest the neutrino energy is always $E \gtrsim \mathrm{MeV}$. This means that the previos ratio is always greater than 10^{15}! This fully justifies the previous assumption that the amplitude of the wave varies very slowly respect to the wavelength of the plane wave.

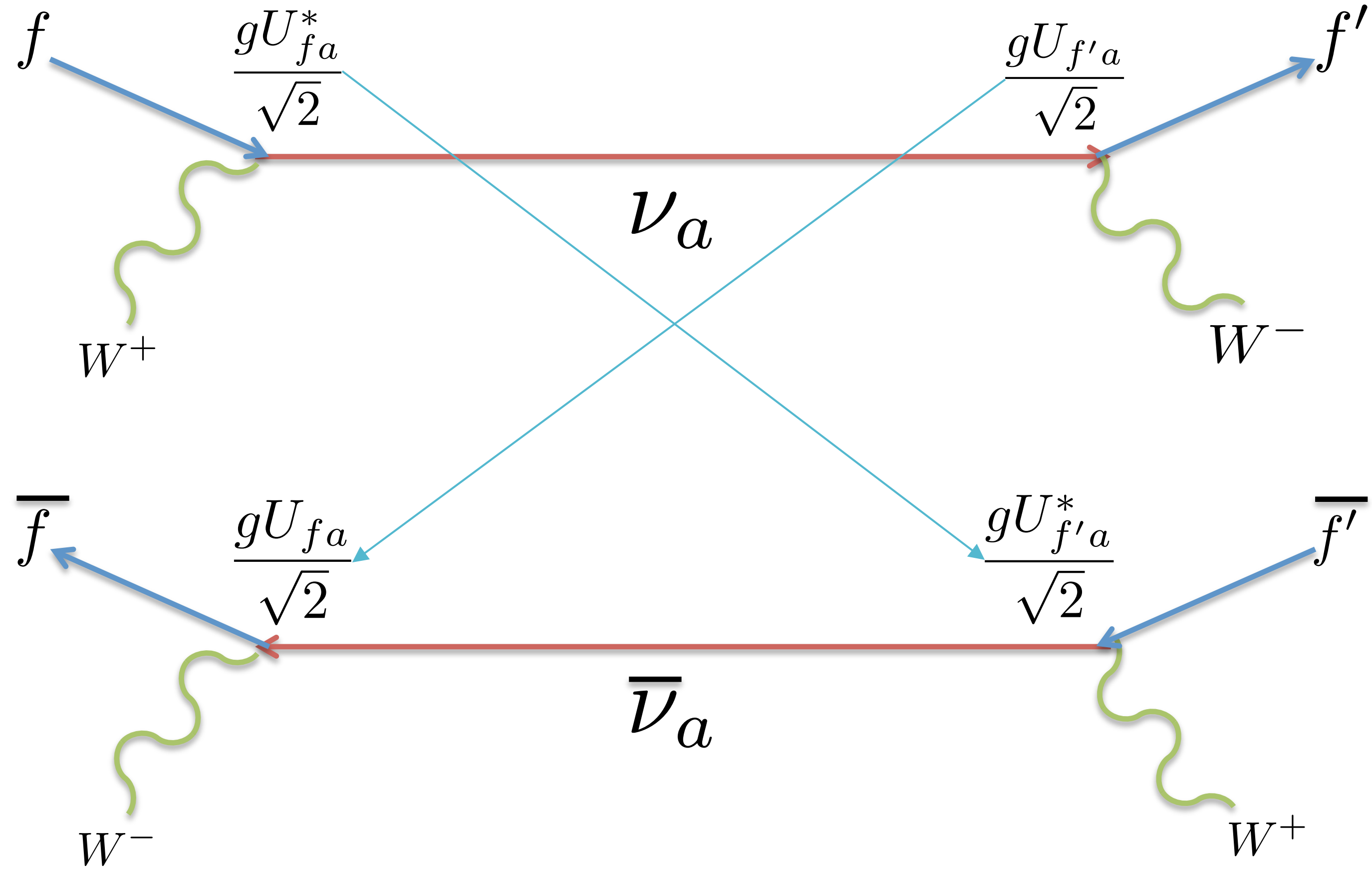
CP violations in oscillations

Under CP transformations we "reverse" the diagram of the oscillation process

$$T(\nu_f \to \nu_{f'}) = \sum_a$$

CP

$$T(\bar{\nu}_f \to \bar{\nu}_{f'}) = \sum_a$$



This is equivalent to interchange U with its conjugate in the oscillation formula

$$P(\nu_{f'} \to \nu_f; x) = \delta_{ff'} - 4\sum_{a<b} \Re[U_{fa}U^*_{fb}U^*_{f'a}U_{f'b}]\sin^2\left(\frac{\Delta m^2_{ab}x}{4E}\right)$$

$$+2\sum_{a<b} \Im[U_{fa}U^*_{fb}U^*_{f'a}U_{f'b}]\sin\left(\frac{\Delta m^2_{ab}x}{2E}\right)$$

$$P(\bar{\nu}_{f'} \to \bar{\nu}_f; x) = \delta_{ff'} - 4\sum_{a<b} \Re[U^*_{fa}U_{fb}U_{f'a}U^*_{f'b}]\sin^2\left(\frac{\Delta m^2_{ab}x}{4E}\right)$$

$$+2\sum_{a<b} \Im[U^*_{fa}U_{fb}U_{f'a}U^*_{f'b}]\sin\left(\frac{\Delta m^2_{ab}x}{2E}\right)$$

invariant

changes sign

Since in general the two probabilities are different (unless U is real): CP violations in neutrino oscillations.

The same result can be obtained exchanging the initial and the final flavors in the oscillation formula (T violations)

$$P(\bar{\nu}_f \to \bar{\nu}_{f'}; x) = P(\nu_{f'} \to \nu_f; x)$$

while interchanging neutrinos with antineutrinos and initial and final flavors the oscillation probability is invariant (CPT invariance)

$$P(\bar{\nu}_{f'} \to \bar{\nu}_f; x) = P(\nu_f \to \nu_{f'}; x)$$

the differences between the two probabilities is

$$\Delta P = P(\bar{\nu}_f \to \bar{\nu}_{f'}; x) - P(\nu_f \to \nu_{f'}; x)$$
$$= 16J \sum_{f"} \epsilon_{f f' f"} \sin \frac{\Delta m^2_{21} x}{4E} \sin \frac{\Delta m^2_{31} x}{4E} \sin \frac{\Delta m^2_{32} x}{4E}$$

where the "Jarlskog invariant" J is given by

$$J = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos \theta_{13} \sin \delta_{CP}$$

Observe CP violation is hard! To observe CP violations we need

1) $\delta_{CP} \neq 0, \pi$
2) $f \neq f'$ (appearance experiment)
3) all mixing angles non zero
4) all Δm^2 non zero
5) (almost) single energy experiment and accurate energy resolution of the detector in order to avoid smearing in the (linear) “sin” terms over a broad energy spectrum

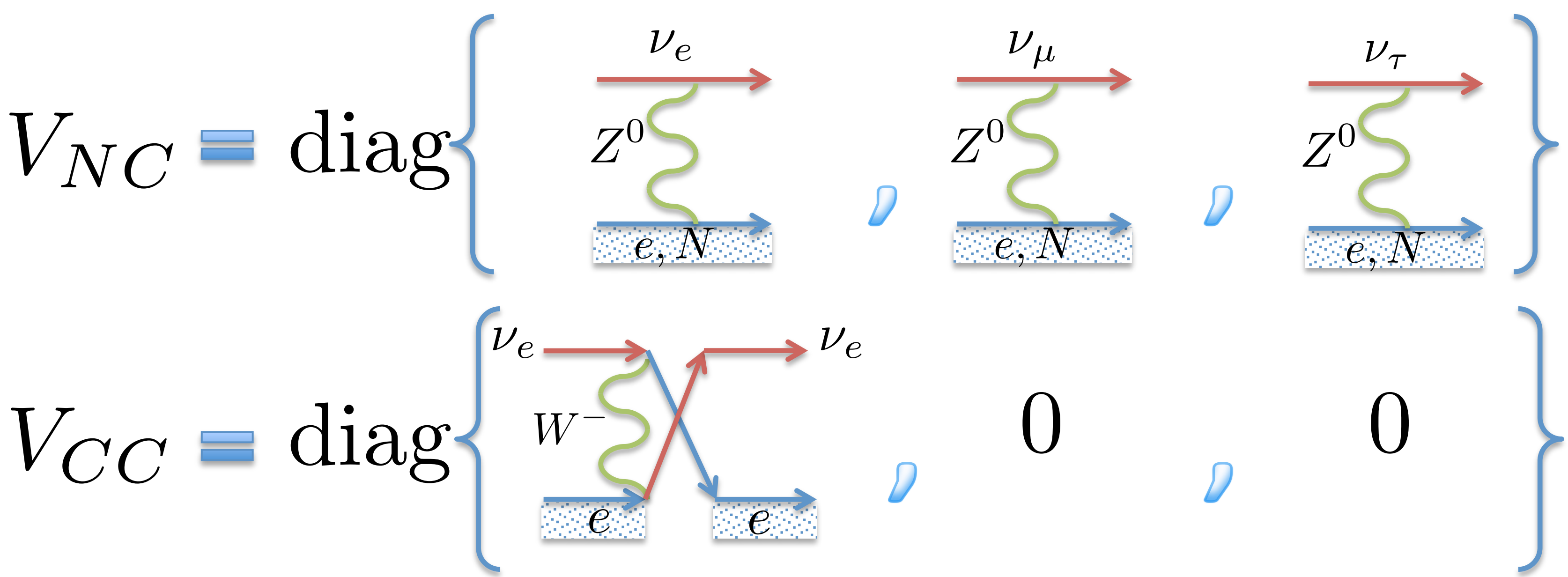
The smallness of θ_{13} and the fact that there is one mass scale dominance ($\Delta m^2_{21} \ll \Delta m^2_{23}$) make the measure of CP violations a big challenge for the future.

Neutrino oscillations in matter

Neutrinos travelling in matter receive contributions from coherent forward scattering on electrons and quarks. The propagation equation (in flavor basis) is modified as follows

$$i\Psi'(x) = \left[\frac{1}{2E}U\mathrm{diag}\{m_a^2\}U^\dagger + V\right]\Psi$$

where the potential matrix V contains the interactions with the matter background. For standard interactions (no FCNC currents) the matrix V is the sum of two diagonal matrices V=VNC+VCC containing Neutral Current and Charge Current interactions with matter



Matter effects can eventually be used also to probe non standard FCNC and FDNC interactions. Here we consider only standard interactions.

V_{NC} matrix is diagonal (up to very small 1-loop corrections) and can be removed by a overall phase redefinition

$$\Psi \to \exp(-iV_{NC})\Psi$$

The only nonzero CC component can be calculated by averaging the interaction Lagrangian on the electron background. For low energy neutrinos ($E \ll M_W^2/m_e \sim 10^{16}$eV) we can neglect the W propagator, so the effective Lagrangian is

$$\begin{aligned} L_{CC} &= -\frac{G_F}{\sqrt{2}} \langle \bar{e}\gamma^i(1-\gamma^5)\nu_e \cdot \bar{\nu}_e\gamma_i(1-\gamma^5)e\rangle_{bkr} \\ &= \frac{G_F}{\sqrt{2}} \langle \bar{e}\gamma^i(1-\gamma^5)e\rangle_{bkr} \cdot \bar{\nu}_e\gamma_i(1-\gamma^5)\nu_e \end{aligned}$$

where we have performed a Fierz transformation. The average over the non relativistic unpolarized electron background gives

$$L_{CC}(x) = -\sqrt{2}G_F N_e(x)[\bar{\nu}_{e,L}\gamma_0\nu_{e,L}]$$

where N_e is the number density of electrons.

From this Lagrangian we obtain the equation of motion of neutrinos

$$\left[i\gamma^k\partial_k - M - \sqrt{2}G_F N_e(x)\gamma_0\frac{1-\gamma^5}{2}\right]\nu = 0$$

Reasoning as in the case of free neutrinos we obtain the evolution equation

$$i\frac{d}{dx}\nu(x) = \left[\frac{1}{2E}U\mathrm{diag}\{m_a^2\}U^\dagger + \mathrm{diag}\{V_e(x),0,0\}\right]\nu(x)$$

with $V_e(x) = \sqrt{2}G_F Ne(x)$

This is called Mikheyev-Smirnov-Wolfenstein (MSW) equation

For antineutrinos V_e reverses the sign.

Exercise: prove that $V_e = 7.63\times 10^{-14}\mathrm{eV}\cdot\left(\frac{N_e}{\mathrm{mol}\cdot\mathrm{cm}^{-3}}\right)$

Despite the "smallness" of the potential, the matter effect is a "cumulative" effect that can modify deeply the dynamics of flavor oscillations for neutrinos moving into matter.

Neutrino potential in the more general case (from T. Kuo and J. Pantaleone, Rev. Mod. Phys. 61 (1989) p. 937)

TABLE II. Potentials induced for a neutrino traveling through background matter. The upper sign refers to neutrinos, the lower sign to antineutrinos. N_f is the number density of fermion f in the background matter. For nonrelativistic background electrons, $\langle E_e \rangle \to \frac{3}{4} m_e$.

Neutrino flavor	Background flavor	Potential V
ν_e	e	$\pm G_F(4\sin^2\theta_W+1)(N_e-N_{\bar{e}})/\sqrt{2}-\frac{8\sqrt{2}G_F E_\nu}{3M_W^2}(\langle E_e \rangle N_e+\langle E_{\bar{e}} \rangle N_{\bar{e}})$
ν_μ, ν_τ	e	$\pm G_F(4\sin^2\theta_W-1)(N_e-N_{\bar{e}})/\sqrt{2}$
ν_e, ν_μ, ν_τ	n	$\pm G_F(N_{\bar{n}}-N_n)/\sqrt{2}$
ν_e, ν_μ, ν_τ	p	$\pm G_F(1-4\sin^2\theta_W)(N_p-N_{\bar{p}})/\sqrt{2}$
ν_e	ν_e	$\pm 2\sqrt{2}G_F(N_\nu^L-N_{\bar{\nu}}^L)-\frac{8\sqrt{2}G_F E_\nu}{3M_Z^2}(\langle E_\nu \rangle N_\nu^L+\langle E_{\bar{\nu}} \rangle N_{\bar{\nu}}^L)$
ν_μ, ν_τ	ν_e	$\pm\sqrt{2}G_F(N_\nu^L-N_{\bar{\nu}}^L)$

very interesting for neutrino dense medium, such as supernova core

For constant matter density we can define the mass eigenstates in matter

$$\nu^m = (U^m)^\dagger \nu$$

where U^m is the unitary matrix that diagonalizes the hamiltonian in the propagation equation

$$(U^m)^\dagger \mathcal{H} U^m = (U^m)^\dagger \left[U \frac{M^2}{2E} U^\dagger + V \right] U^m = \mathrm{diag} \left\{ \frac{M_a^2}{2E} \right\}$$

the M_a are the "effective" masses in matter. After this we can easily calculate the oscillation probability in matter in the usual way

$$P(\nu_f \to \nu_{f'}; x) = \left| \sum_a U^m_{af} U^{m*}_{af'} e^{-i \frac{M_a^2}{2E} x} \right|^2$$

For two generations the diagonalization is straightforward

$$U^m = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix}$$

with $\sin 2\theta_m = \dfrac{s_{2\theta}}{\sqrt{(2EV/\delta m^2 - c_{2\theta})^2 + s_{2\theta}^2}}$

Notice the existence of the critical condition $2EV = \delta m^2 c_{2\theta}$

When this condition is fulfilled the matter mixing angle becomes maximal ($\theta_m=\pi/4$): this condition is called "resonant condition": oscillations in matter become maximal independently to the value of the vacuum mixing angle θ.

For density varying matter $N_e(x)$ the MSW equation is non trivial. In fact the matrix U^m is in turn function of x and makes the equation for the states ν^m non diagonal.

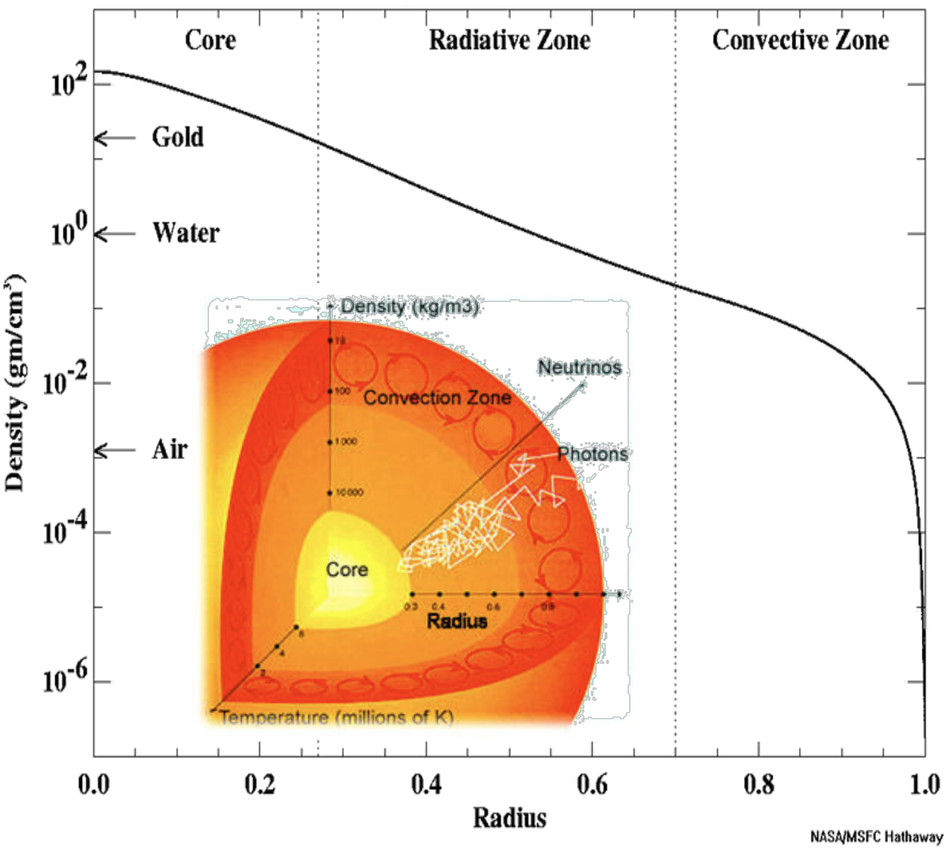
$$i\frac{d}{dx}\nu^m = \left[\frac{1}{2E}\mathrm{diag}\{M_a^2\} - iU^{m\dagger}\frac{d}{dx}U^m\right]\nu^m$$

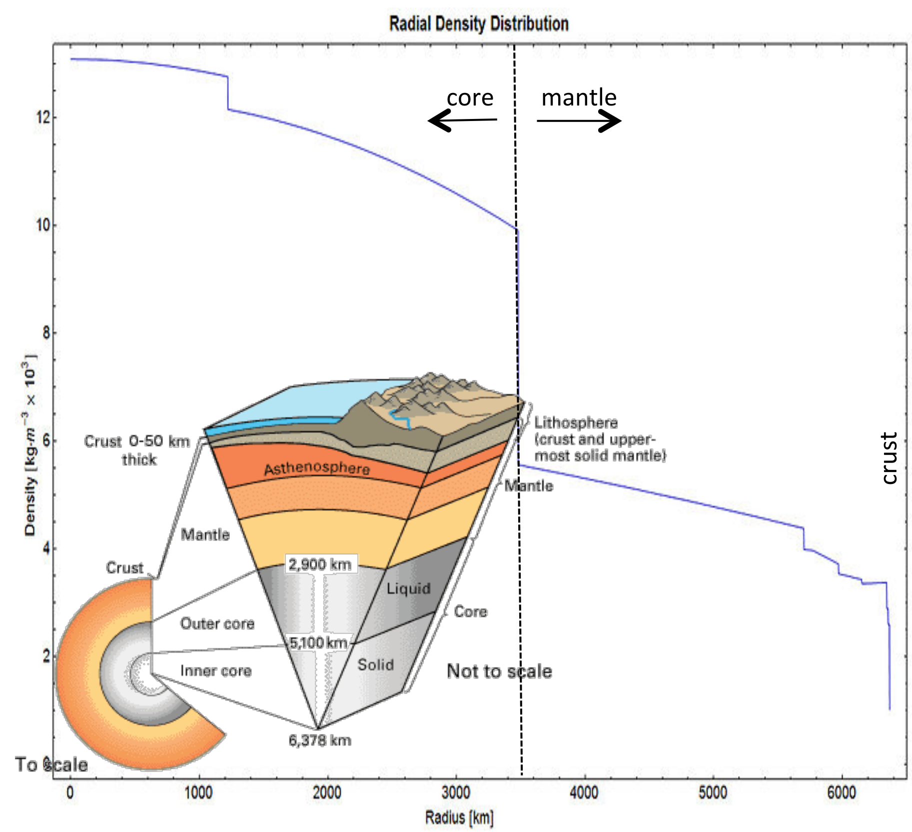
In general the last term in the Hamiltonian is non diagonal

For varying density the MSW equation must be solved in general by numerical integration (except for very special profiles $N_e(x)$). However, we have two practical situations

1) Slowly varying potentials (such as into Sun or in Supernovae)

2) Slabs of potentials with (almost) constant density (such as inside the Earth)





In the second case we can write the total propagation operator as the product of the propagation operators for each i-th slab.

$$\mathcal{T} = \mathcal{T}_n \cdot \mathcal{T}_{n-1} \cdots \mathcal{T}_0$$

Each $\mathcal{T}_i$ is calculated considering a constant (average) density in the slab. The deviation from constancy is taken in account with perturbative methods.

The first case is more tricky. The idea is based on the fact that the term $U^{m\dagger} dU^m/dx$ is always small except near resonance point(s). Far from resonance points the Hamiltonian for mass eigenstates in matter is almost diagonal (<u>adiabatic approximation</u>), while in the resonance point there is a nonzero probability of "jumping" between mass eigenstates.

This can be better seen in 2 generation case

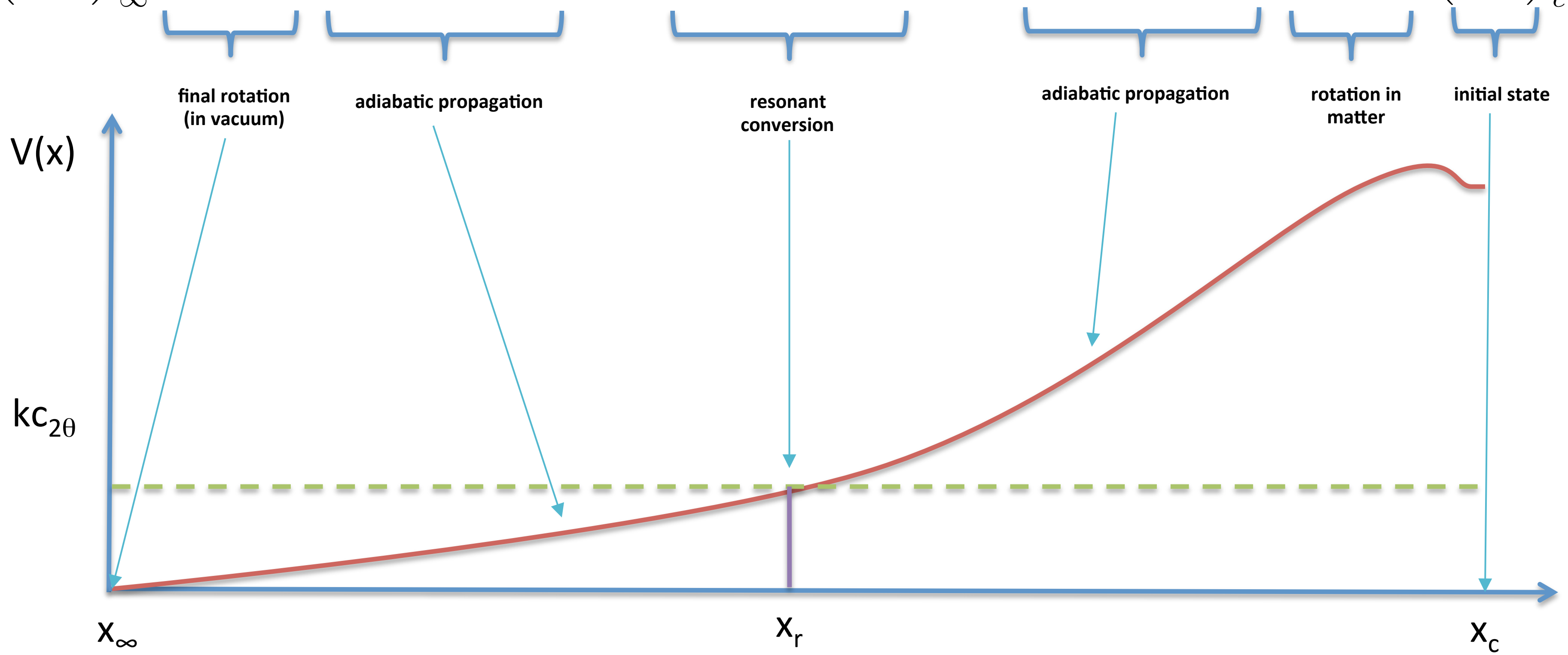
$$i\frac{d}{dx}\begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} = \left[-\frac{k_m}{2}\sigma_3 + \theta'_m \sigma_2\right]\begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

with $$k^m(x) = \sqrt{(kc_{2\theta} - V(x))^2 + k^2 s_{2\theta}^2}, \quad k = \delta m^2/2E$$

$$\theta'_m = \frac{k s_{2\theta}}{2k_m^2} V'(x)$$ → θ_m' is maximum when k_m is minimum (resonance)

If we neglect θ_m' the hamiltonian is diagonal and the propagation of ν_a^m is trivial
Suppose for example that an electron neutrino is produced in the center of the Sun.

$$\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}_\infty = U(\theta)\cdot \mathrm{diag}\{e^{-i\int_{x_r}^{x_\infty} \pm\frac{k_m(x)}{2}dx}\}\cdot \begin{pmatrix} T_{\nu_1^m\to\nu_1^m} & T_{\nu_2^m\to\nu_1^m}\\ T_{\nu_1^m\to\nu_2^m} & T_{\nu_2^m\to\nu_2^m}\end{pmatrix}\cdot \mathrm{diag}\{e^{-i\int_{x_c}^{x_r}\pm\frac{k_m(x)}{2}dx}\}\cdot U(\theta_0^m)\cdot \begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}_c$$



Averaging on the (fast) oscillating terms we get the oscillation probability

$$P(\nu_e \to \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - P_J\right)\cos 2\theta_0^m \cos 2\theta, \quad P_J = |T_{\nu_2^m\to\nu_1^m}|^2$$

The average on the oscillating terms can be done for all practical cases (when integration in energy, time and production zone must be done). In some special regimes this approximation can drop and corrections to the previous formula are needed ("quasi-vacuum" oscillations). The discussion of these cases is beyond our purposes.

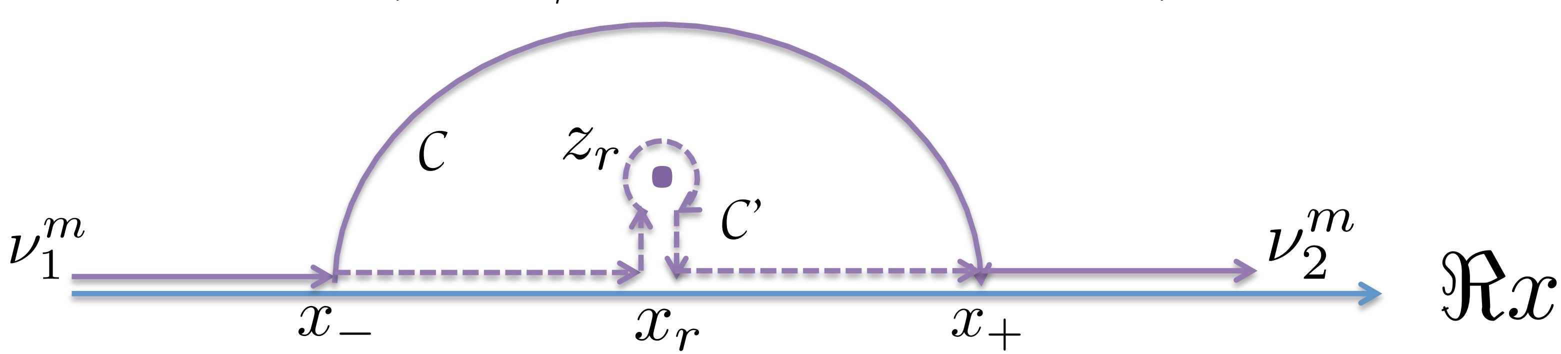
The "jumping probability" can be calculated with the semiclassical theory. A sketch of the calculation is the following:

1. Continue analytically the equation in the complex field
2. "Turn away" from the branch point z_r defined by $k_m(z_r)=0$. Since we are "far" from the resonance point the solution on the curve C is still "adiabatic". However z_r is a branch point and thus $k_m(z)$ evolved on C has opposite sign respect to those evolved on the real axis: this means that at the end the final mass state has opposite eigenvalue and thus is "switched"

$$\nu_2^m(x_+) = e^{-\frac{i}{2}\int_C k_m(z)dz}\nu_1^m(x_-)$$

3. Deform the curve C to the curve C'. The final result is

$$P_J = \exp\left(-2\Im\int_{x_r}^{z_r} dz\sqrt{[kc_{2\theta} - V(z)]^2 + k^2 s_{2\theta}^2}\right)$$



The previous integral can be calculated only if an analytic form for $V(z) \propto N_e(z)$ is known. The result is

$$P_J = \exp\left[-\frac{\pi}{2}\gamma F(\theta)\right]$$

where the *adiabaticity parameter* γ is given by

$$\gamma = \frac{\delta m^2}{2E}\frac{s_{2\theta}^2}{c_{2\theta}}\left(\left|\frac{d\log N_e}{dx}\right|_{x=x_r}\right)^{-1}$$

which depends on the the gradient of N_e at the resonant point, ad $F(\theta)$ depends on the profile of $N_e(x)$. In practice the profile $N_e(x)$ comes from simulations and is thus tabulated. However, in most cases $N_e(x)$ is not far from an exponential profile. In this case

$$F(\theta) = 1 - \tan^2\theta$$

Previous formulae reproduce with high accuracy numerical solutions but are extremely handy and physically transparent. Corrections for the previous formulae are known, but are beyond our purposes.

For solar neutrinos with the actual parameters (large θ) the adiabatic approximation is good enough ($P_J \simeq 0$). Jumping probability nowadays have practical interest essentially for supernovae.

Back to ν masses: The see-saw

As anticipated, the usual standard model Higgs mechanism cannot naturally explain why $m_\nu \lesssim O(\mathrm{eV})$. Extensions of the Standard Model are invoked for account the smallness of neutrino masses. Among other possible mechanisms, the most reliable is the so called see-saw model. In this model new massive d.o.f. are included.

The simplest example of see-saw (called type I see saw) is to include three new heavy right handed neutrinos with a Majorana mass term (since RH neutrinos are singlets, an explicit mass term does not break symmetry)

$$L_Y = \bar{E}_L Y \phi l_R + \bar{E}_L Y' \hat{\phi} \nu_R + \frac{1}{2} \bar{\nu}_R^c m_R \nu_R + h.c.$$

In this case **Y** and **Y'** can be of the same order of magnitude. After SSB neutrinos acquire both a Dirac and a Majorana mass term

$$L_{M,\nu} = m_D \bar{\nu}_L \nu_R + \frac{1}{2} m_R \bar{\nu}_R^c \nu_R + h.c.$$

$$m_D = Y' v / \sqrt{2}$$

Let us consider one generation for the moment…

The previous mass Lagrangian can be rewritten as

$$L_M = \frac{1}{2}(N_R, N_L)\begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} N_R \\ N_L \end{pmatrix}$$

where we have defined the Majorana fields

$$N_{L,R} = \nu_{L,R} + \nu^c_{L,R}$$

We now can suppose that m_D is of the order of the SM scale, while m_R is generated by new physics beyond SM. In general m_D is protected by SM symmetries while m_R is not (because the RH states are singlet of the SM). We can thus suppose that $m_R \gg m_D$.

Diagonalizing the mass matrix we have two Majorana mass eigenstates

$$\nu'_L \simeq i\left(N_L - \frac{m_D}{m_R}N_R\right) \quad \text{with mass} \quad \frac{m_D^2}{m_R} \ll m_D$$

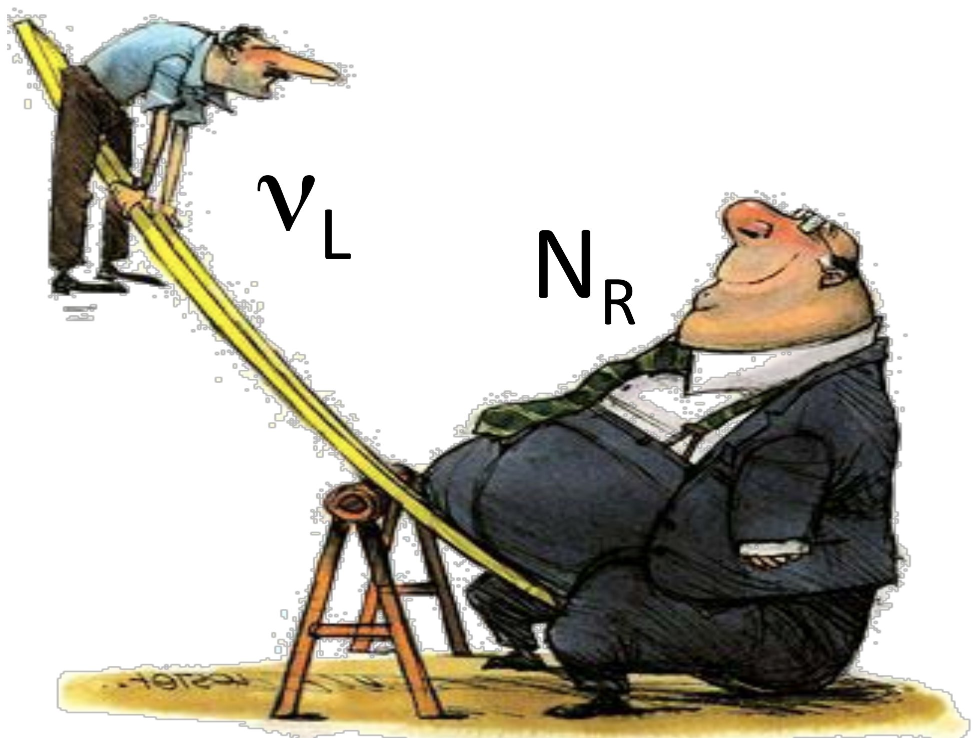
$$N'_R \simeq N_R + \frac{m_D}{m_R}N_L \quad \text{with mass} \quad m_R$$

For three generation neutrinos the neutrino mass matrix for ν'_L is $M_D \cdot M_R^{-1} \cdot M_D$

Summarizing:

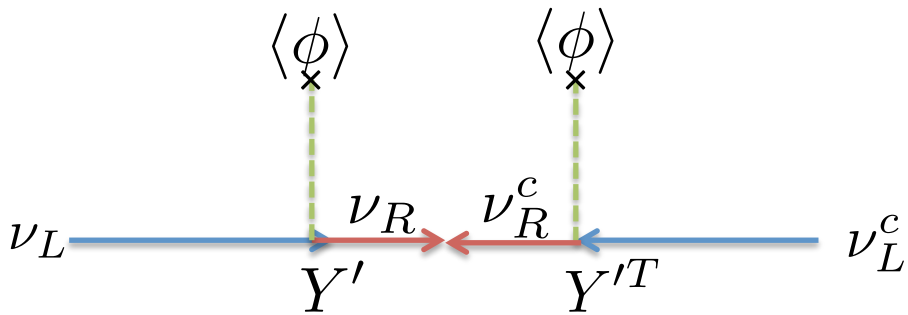
1. Neutrinos evolve a Majorana mass (or equivalently, they are Majorana particles).
2. The state ν'_L is is almost "left handed" (mostly active). Its mass is suppressed by higher scale m_R. It can be identified with the ordinary neutrino.
3. The state N'_R is almost "right handed" (mostly sterile). Its mass is of the order of the GUT scale. It cannot be observed due to its very high mass and very weak coupling with the SM sector, however can play a role in leptogenesis.

The see saw mechanism yield a simple and natural explanation for the smallness of the neutrino masses .



Kind of see saw

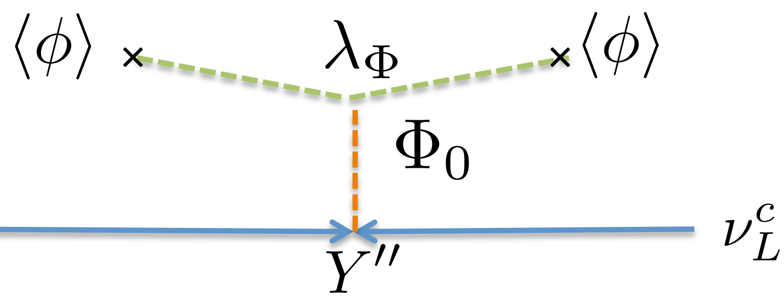
- Type I see-saw $L_Y = \bar{E}_L Y \phi l_R + \bar{E}_L Y' \hat{\phi} \nu_R + \frac{1}{2} \bar{\nu}_R^c m_R \nu_R + h.c.$



- Type II see-saw: in this case no new RH neutrinos are needed but a new SU(2) scalar triplet Φ takes the role of the massive d.o.f. The most general renormalizabe Lagrangian is

$$L_Y = \bar{E}_L Y \phi l_R + \frac{1}{2} \bar{E}_L Y'' \Phi i\sigma_2 E_L^c - \lambda_\Phi M_\Phi \phi^T i\sigma_2 \Phi \phi + h.c.$$

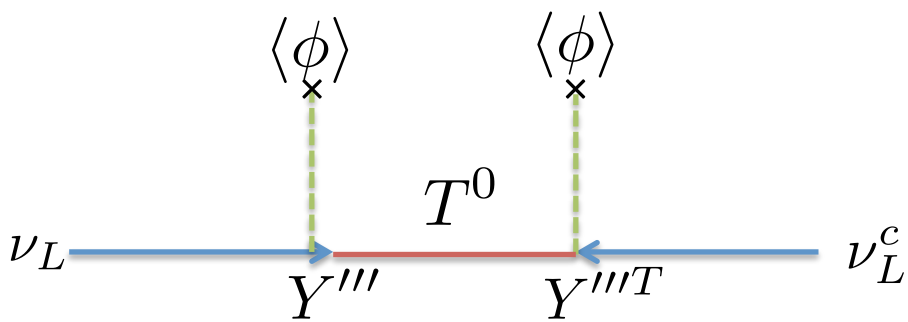
with $$\Phi = \begin{pmatrix} \Phi^- & -\sqrt{2}\Phi^0 \\ \sqrt{2}\Phi^{--} & -\Phi^- \end{pmatrix}$$



- Type III see-saw: in this case no new RH neutrinos are needed but a new SU(2) fermion triplet **T** takes the role of the massive d.o.f. The most general renormalizabe Lagrangian is

$$L_Y = \bar{E}_L Y \phi l_R + \sqrt{2}\bar{E}_L Y''' \mathbf{T}^c \phi + \frac{1}{2}\mathrm{Tr}\left(\bar{\mathbf{T}} M_T \mathbf{T}^c\right) + h.c.$$

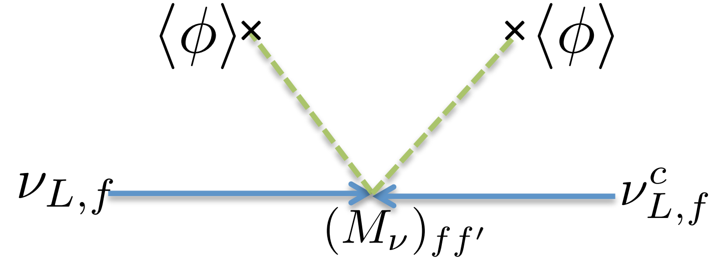
with $\mathbf{T} = \begin{pmatrix} T^0/\sqrt{2} & T^+ \\ T^- & -T^0/\sqrt{2} \end{pmatrix}$



All these models (or combination of them) can be justified in the context of SUSY and/or GUT theories.

“Integrating away” the massive d.o.f. we obtain the most general dimension 5 (effective) operator that can generate a Majorana neutrino mass (“Weimberg operator”)

$$\frac{(M_\nu)_{ff'}}{v^2}\bar{E}_{L,f}\hat{\phi}\cdot\hat{\phi}^T E_{L,f'}$$



$$M_\nu = \begin{cases} -\frac{v^2}{2}Y' M_R^{-1} Y'^T & \text{Type I} \\ \lambda_\Phi \frac{v^2}{M_\phi} Y'' & \text{Type II} \\ -\frac{v^2}{2}Y''' M_T^{-1} Y'''^T & \text{Type III} \end{cases}$$

The matrix M_ν is suppressed by the scale of new physics (as expected). This again explains the smallness of the neutrino masses. Moreover with a Majorana mass term, the total lepton number is no longer conserved ($\Delta L=2$).

The proof that neutrinos have a small Majorana mass is a strong indication of new Physics beyond Standard Model!

Majorana mixing matrix

To cancel out non physical phases in the mixing matrix we have used the fact that with Dirac fields the Lagrangian is invariant under the transformation

$$\nu_a^m \to \exp(i\alpha_a)\nu_a^m,\ l_f \to \exp(i\beta_f)l_f$$

This is no longer true with Majorana fields since the Majorana mass term

$$\sum_a m_a \bar{\nu}_a^c \nu_a + h.c.$$

This means that the angles α_a are physical and can be measured (in principle). However, a global phase can be neglected since all physical quantities always contain the modulus of a given linear combination of the U_{fa}'s. In conclusion, for Majorana neutrinos the Mixing matrix can be written without loss of generality as

$$U^M = U^D(\theta_{23}, \theta_{13}, \theta_{12}, \delta_{CP}) \cdot \Gamma, \ \ \Gamma = \mathrm{diag}\{1, e^{i\alpha_2}, e^{i\alpha_3}\}$$

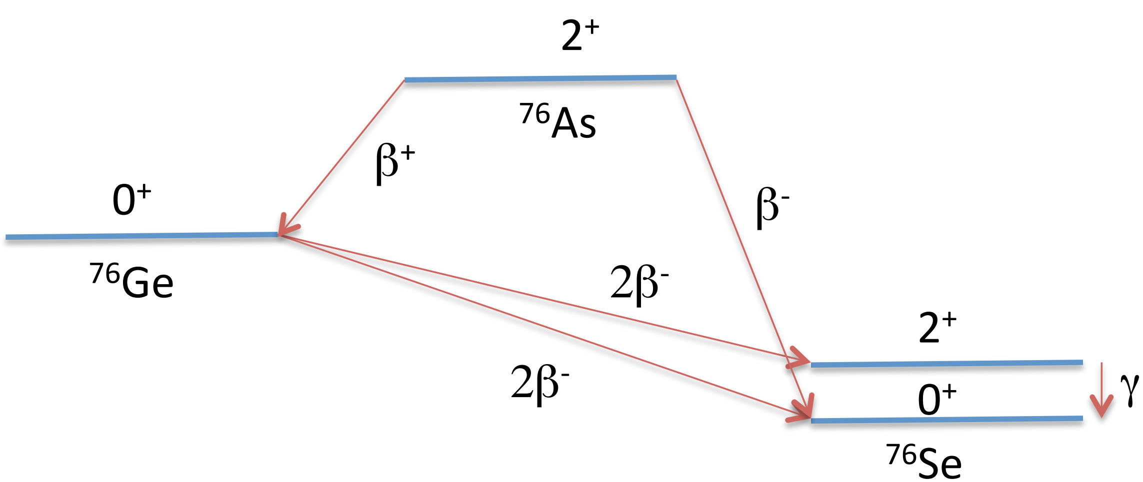
where U^D is the mixing matrix already introduced for Dirac neutrinos and without loss of generality we have chosen α_1=0. The phases α_a are called Majorana Phases. They cannot be observed neither in neutrino oscillations nor in beta decay experiment.

Neutrinoless double beta decay

It is well known that some isotopes decay through a double beta process

$$(A, Z) \to (A, Z+2) + 2e^- + 2\bar{\nu}_e$$
$$(A, Z) \to (A, Z-2) + 2e^+ + 2\nu_e$$

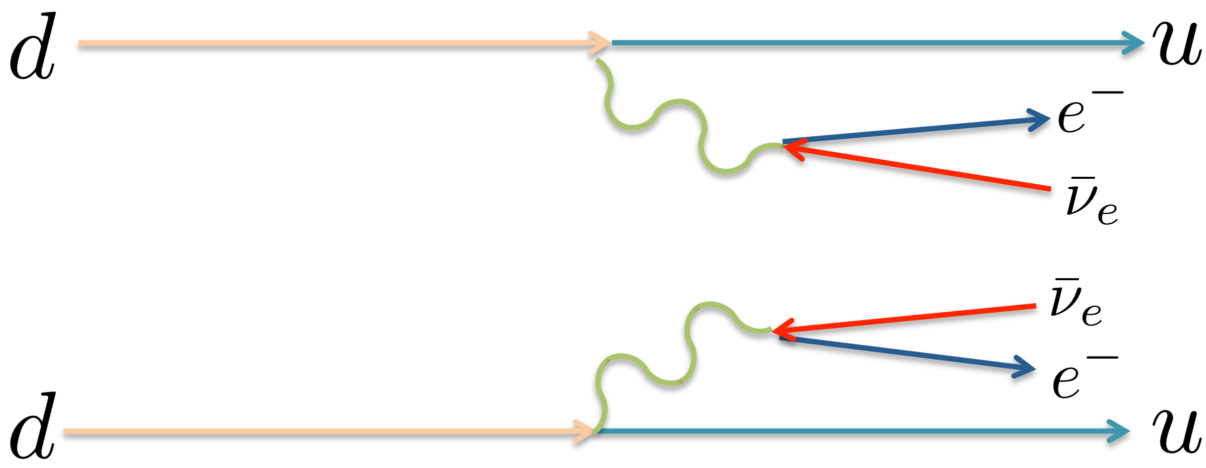
because the single beta process is kinematically forbidden or (more rarely) suppressed respect to the 2β process. Example: the decay ^{76}Ge in ^{76}As is forbidden because the mass of ^{76}As is higher (^{76}As is a 2^+ nuclei while ^{76}Ge is 0^+)



$2\beta^-$-decay	$Q_{2\beta}$ [keV]	$T^{2\nu}_{1/2}$ [y]	$T^{0\nu}_{1/2}$ [y]
$^{46}Ca \to ^{46}Ti$	990.4 ± 2.4		$> 1.0 \times 10^{17}$ (90%)
$^{48}Ca \to ^{48}Ti^a$	4272 ± 4	$4.2^{+3.3}_{-1.3} \times 10^{19}$	$> 1.5 \times 10^{21}$ (90%)
$^{70}Zn \to ^{70}Ge$	1000.9 ± 3.4		$> 4.8 \times 10^{14}$
$^{76}Ge \to ^{76}Se$	2039.006 ± 0.050	$(1.8 \pm 0.1) \times 10^{21}$	$> 1.9 \times 10^{25}$ (90%)
$^{80}Se \to ^{80}Kr$	133.9 ± 3.7		
$^{82}Se \to ^{82}Kr$	2995.1 ± 2.0	$(8.3 \pm 1.2) \times 10^{19}$	$> 2.7 \times 10^{22}$ (68%)
$^{86}Kr \to ^{86}Sr$	1255.6 ± 2.4		
$^{94}Zr \to ^{94}Mo$	1144.1 ± 2.0	$> 1.1 \times 10^{17}$ (90%)	$> 1.9 \times 10^{19}$ (90%)
$^{96}Zr \to ^{96}Mo^a$	3350.4 ± 2.9	$2.1^{+0.8}_{-0.4} \times 10^{19}$	$> 1.0 \times 10^{21}$ (90%)
$^{98}Mo \to ^{98}Ru$	112 ± 6		$> 1.0 \times 10^{14}$
$^{100}Mo \to ^{100}Ru$	3034 ± 6	$6.8^{+0.8}_{-0.9} \times 10^{18}$	$> 5.5 \times 10^{22}$ (90%)
$^{104}Ru \to ^{104}Pd$	1300 ± 4		
$^{110}Pd \to ^{110}Cd$	2000 ± 11	$> 6.0 \times 10^{16}$	$> 6.0 \times 10^{16}$
$^{114}Cd \to ^{114}Sn$	536.8 ± 3.3	$> 9.2 \times 10^{16}$ (99%)	$> 2.0 \times 10^{20}$ (90%)
$^{116}Cd \to ^{116}Sn$	2805.0 ± 3.8	$2.6^{+0.7}_{-0.4} \times 10^{19}$	$> 7.0 \times 10^{22}$ (90%)
$^{122}Sn \to ^{122}Te$	366.2 ± 2.8		$> 5.8 \times 10^{13}$
$^{124}Sn \to ^{124}Te$	2287.0 ± 1.5	$> 1.0 \times 10^{17}$	$> 2.4 \times 10^{17}$ (95%)
$^{128}Te \to ^{128}Xe$	867.2 ± 1.0	$(2.2 \pm 0.3) \times 10^{24}$ (G)	$> 8.6 \times 10^{22}$ (90%)
$^{130}Te \to ^{130}Xe$	2528.8 ± 1.3	$(7.9 \pm 1.0) \times 10^{20}$ (G)	$> 1.4 \times 10^{23}$ (90%)
$^{134}Xe \to ^{134}Ba$	830.1 ± 3.0	$> 1.1 \times 10^{16}$	$> 8.2 \times 10^{19}$ (68%)
$^{136}Xe \to ^{136}Ba$	2468 ± 7	$> 8.1 \times 10^{20}$ (90%)	$> 4.4 \times 10^{23}$ (90%)
$^{142}Ce \to ^{142}Nd$	1416.9 ± 2.1	$> 1.6 \times 10^{17}$ (90%)	$> 1.5 \times 10^{19}$ (68%)
$^{146}Nd \to ^{146}Sm$	70.2 ± 2.9		
$^{148}Nd \to ^{148}Sm$	1928.8 ± 1.9	$> 3.0 \times 10^{18}$ (90%)	$> 3.0 \times 10^{18}$ (90%)
$^{150}Nd \to ^{150}Sm$	3367.5 ± 2.2	$(6.8 \pm 0.8) \times 10^{18}$	$> 1.2 \times 10^{21}$ (90%)
$^{154}Sm \to ^{154}Gd$	1251.0 ± 1.3	$> 2.3 \times 10^{18}$ (68%)	$> 2.3 \times 10^{18}$ (68%)
$^{160}Gd \to ^{160}Dy$	1729.7 ± 1.3	$> 1.9 \times 10^{19}$ (90%)	$> 1.3 \times 10^{21}$ (90%)
$^{170}Er \to ^{170}Yb$	653.6 ± 1.7	$> 3.2 \times 10^{17}$ (68%)	$> 3.2 \times 10^{17}$ (68%)
$^{176}Yb \to ^{176}Hf$	1086.7 ± 1.9	$> 1.6 \times 10^{17}$ (68%)	$> 1.6 \times 10^{17}$ (68%)
$^{186}W \to ^{186}Os$	488.0 ± 1.7	$> 5.9 \times 10^{17}$ (90%)	$> 2.7 \times 10^{20}$ (90%)
$^{192}Os \to ^{192}Pt$	413.5 ± 3.0		$> 9.8 \times 10^{12}$
$^{198}Pt \to ^{198}Hg$	1047 ± 3		$> 3.2 \times 10^{14}$
$^{204}Hg \to ^{204}Pb$	416.3 ± 1.5		
$^{232}Th \to ^{232}U$	842.2 ± 2.5		
$^{238}U \to ^{238}Pu$	1145.0 ± 1.3	$(2.0 \pm 0.6) \times 10^{21}$ (R)	

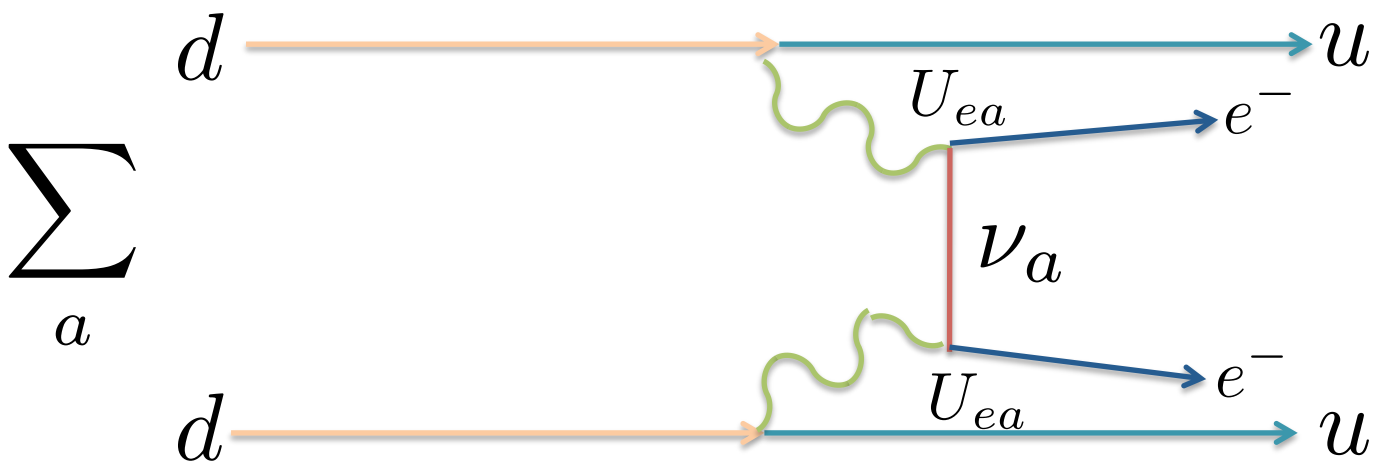
[a] β^--decay energetically allowed but enormously suppressed.

Table of $2\beta^-$ decaying nuclei (taken from the Giunti & Kim book)



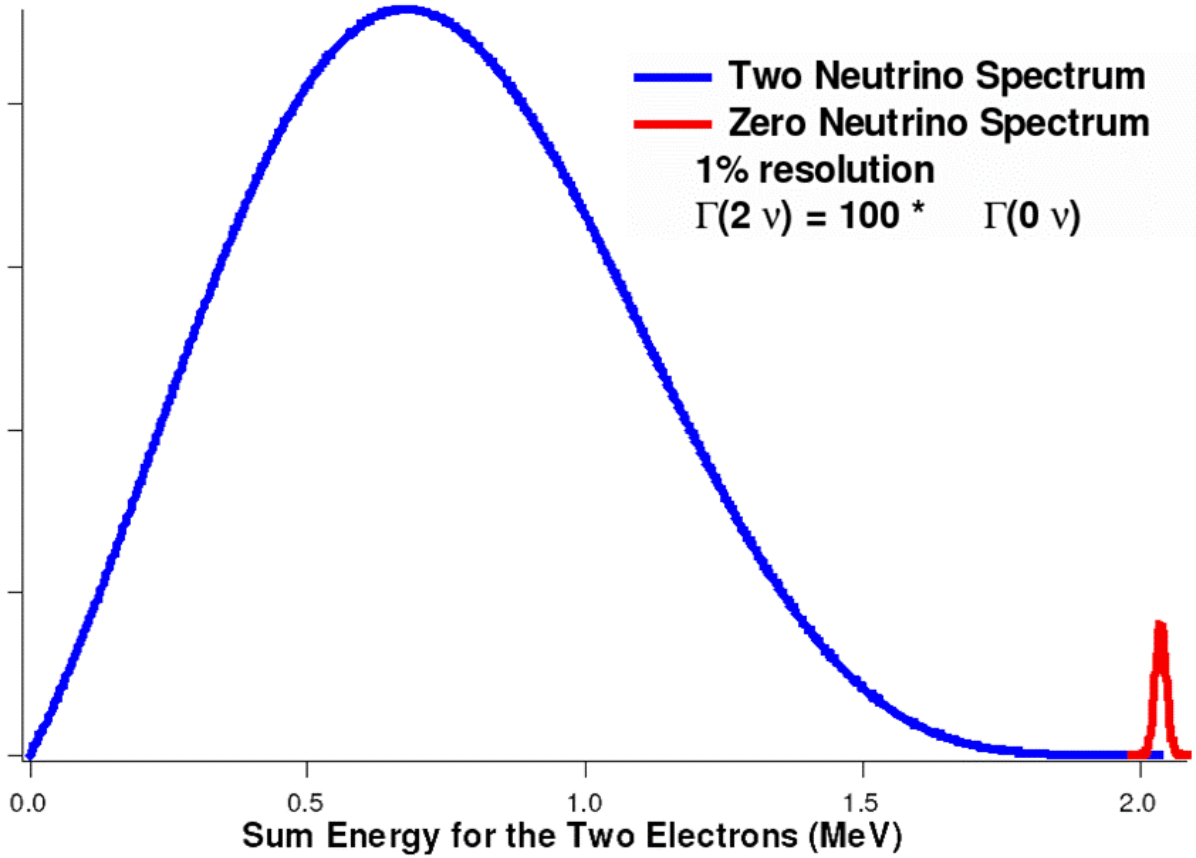
If neutrinos have Majorana masses, (lepton violating) double beta decay without neutrinos (0ν2β) is also possible

$$(A, Z) \to (A, Z+2) + 2e^-$$



This process is subleading to the main 2ν2β process.

The 0ν2β process can be observed as a small "peak" at the end of the spectrum of the two electrons for the 2ν2β process.



The decay amplitude for this process is proportional to the propagator describing the internal neutrino line

$$G_\nu(x_2 - x_1) = \sum_a U_{ea}^2 \langle 0|\mathcal{T}[\nu_{a,L}(x_1)\nu_{a,L}^T(x_2)]|0\rangle$$

Remembering that $\nu_a^T = -\bar{\nu}_a C$

we obtain, after some straightforward calculations

$$G_\nu(x_2 - x_1) = -i\sum_a U_{ea}^2 \int \frac{d^4p}{(2\pi)^4}\frac{m_a}{p^2 - m_a^2} e^{ip\cdot(x_2 - x_1)}\frac{1-\gamma^5}{2}C$$

The typical momentum inside nuclei is

$$\langle p^0\rangle \sim \langle|\mathbf{p}|\rangle \sim 1/R \sim O(10-100)\mathrm{MeV}$$

which is of course much greater than the typical neutrino mass. This means that m_a^2 can be neglected at the denominator and thus

$$G_\nu(x_2 - x_1) = -i\left(\sum_a U_{eq}^2 m_a\right)\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip\cdot(x_2 - x_1)}}{p^2}\frac{1-\gamma^5}{2}C$$

The decay rate of the process is thus proportional to the term

$$m_{\beta\beta} = \left| \sum_a U_{ea}^2 m_a \right|$$

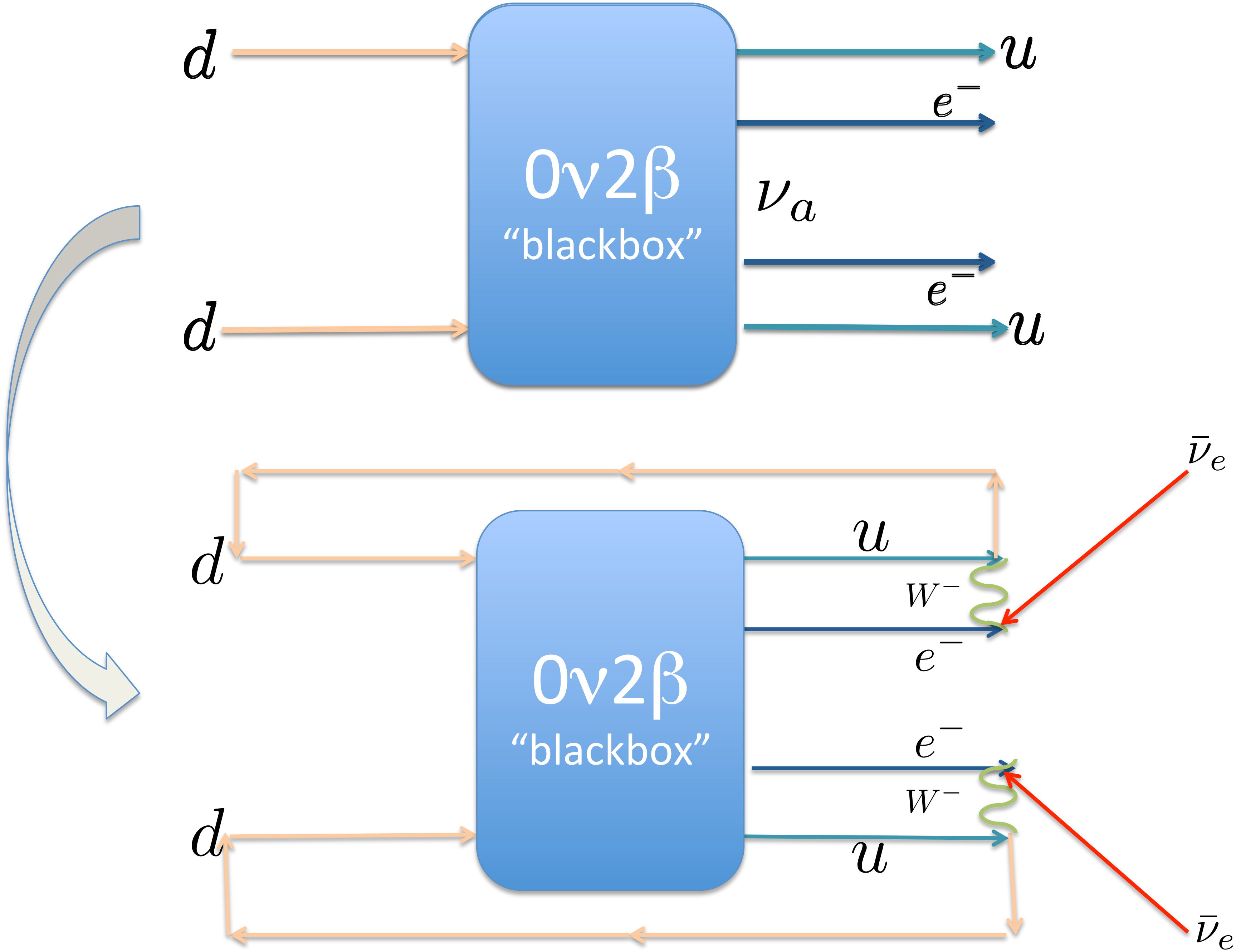
$$= \left| c_{13}^2 (c_{12}^2 m_1 + s_{12}^2 m_2 e^{2i\alpha_2}) + s_{13}^2 m_3 e^{2i(\alpha_3 - \delta_{CP})} \right|$$

(notice that the Majorana phases now enter in a physical quantity). In fact the half life is given by

$$(T_{1/2}^{0\nu})^{-1} = G^{0\nu} |M^{0\nu}|^2 \frac{m_{\beta\beta}^2}{m_e^2}$$

where $G^{0\nu}$ is the space factor and $M^{0\nu}$ the nuclear matrix element . While the first is relatively easy to calculate, the second is very difficult and constitutes the major source of uncertainty of the measure.

Notice that the $0\nu2\beta$ decay may take contributions also (or mainly) by other non standard interactions (for example leptoquark or SUSY particles). However, the mere existence of $0\nu2\beta$ is a proof of a Majorana neutrino mass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Fundamental physics is not exclusively a matter of high energy accelerator experiments but can be probed also by low energy nuclear processes!