Evgeny Epelbaum, RUB

Nuclear Physics School 2013, Otranto, Italy, May 27-31, 2013

Modern Theory of nuclear forces

Lectures 1+2: Foundations

- History
- Introduction
- Chiral Perturbation Theory
- Pionless EFT for two nucleons
- NN beyond effective range expansion
- KSW vs Weinberg
- From effective Lagrangian to nuclear forces

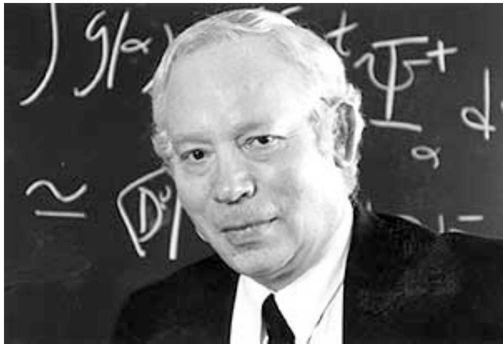
Lecture 3: Chiral nuclear forces: State of the art and applications

Lecture 4: Nuclear lattice simulations

Historical overview

Yukawa's theory	Proca Kemmer Moller Rosenfeld Schwinger Pauli …	discovery of pions	two-pion exchange, meson theory…	discovery of heavy mesons
1930	**1940**	**1950**	**1960**	**1970**

BE models inverse scattering dispersion theory quark cluster models phenomenology …	AV18 CD Bonn Nijm I,II Reid93 …	(Chiral) Effective Field Theory Lattice QCD $V_{\text{low-k}}$ …	
1980	**1990**	**2000**	**2010**



Effective Theories

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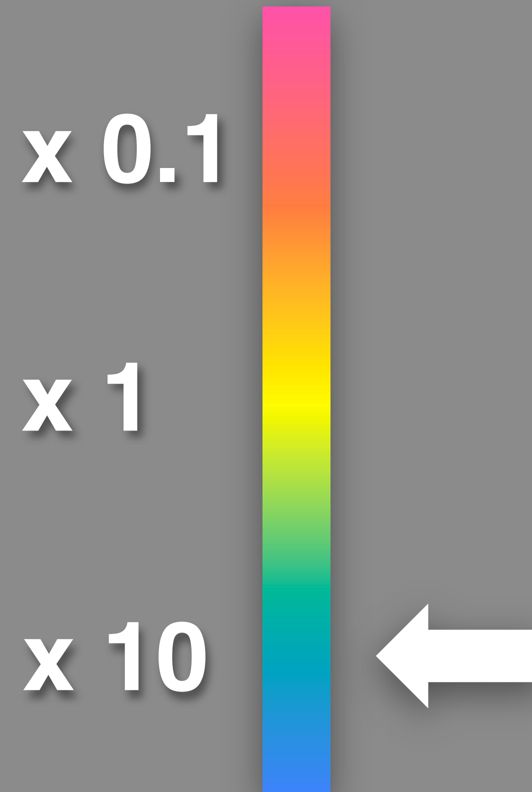

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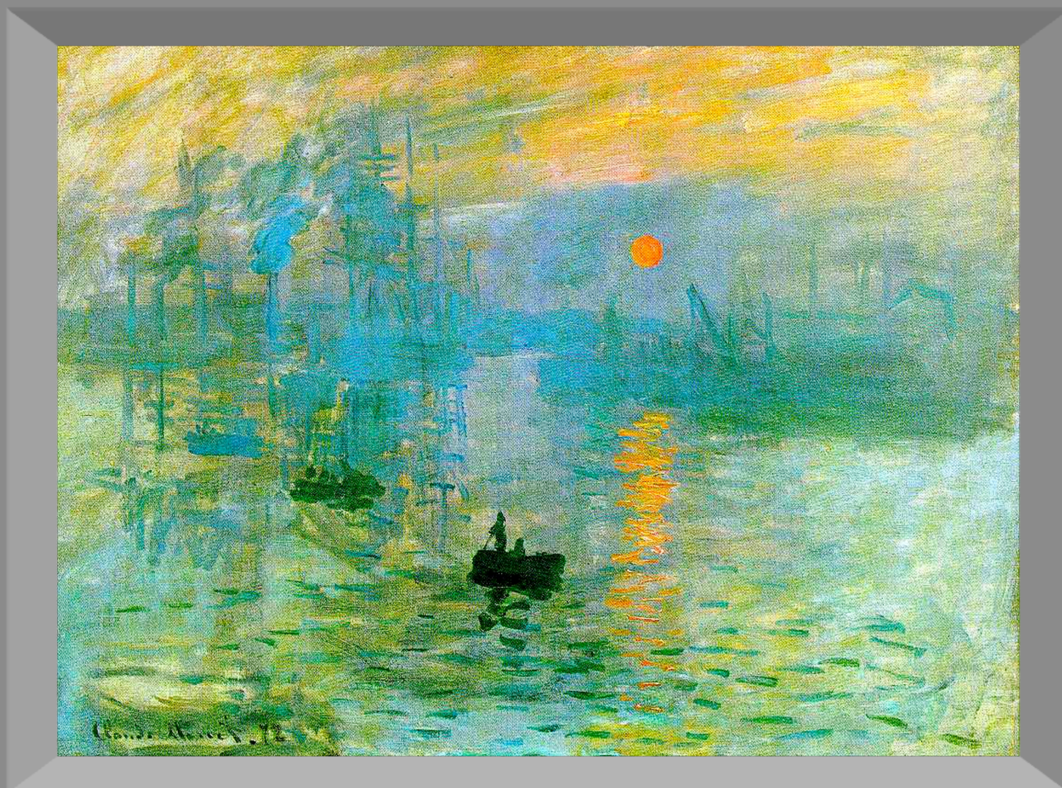
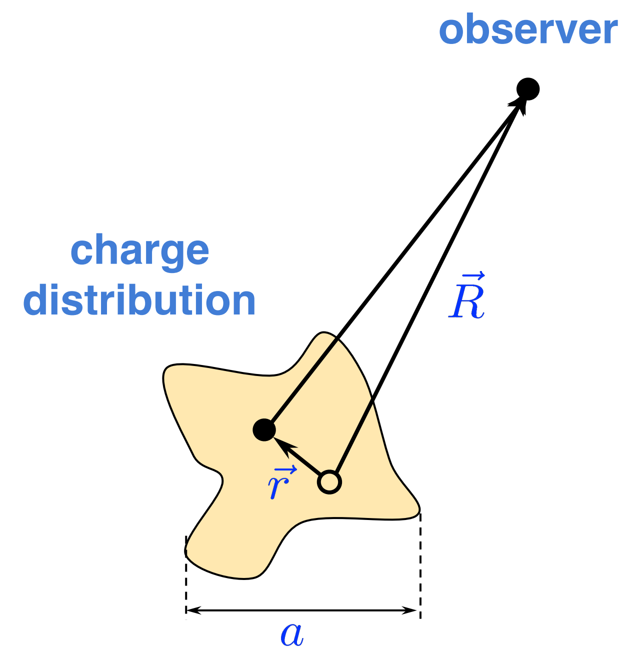


→ it is crucial to choose a proper resolution !

What is an effective theory?

Example from electrostatics

The goal: compute electric potential generated by a localized charge distribution $\rho(\vec{r}\,)$



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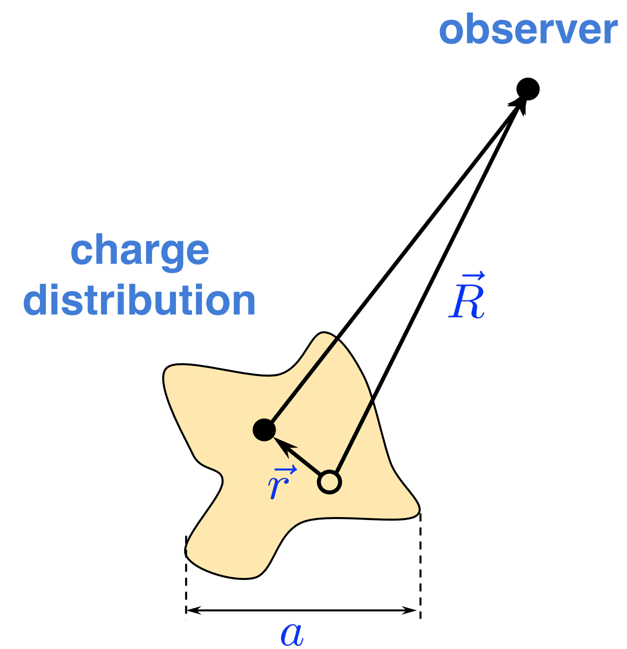
- The ultimate answer: $V(\vec{R}) \propto \int d^3r \, \frac{\rho(\vec{r})}{|\vec{R}-\vec{r}|}$



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- For $R \gg a$, only moments of $\rho(\vec{r})$ are needed:

$$V(\vec{R}) = \frac{\boldsymbol{q}}{R} + \frac{1}{R^3}\sum_i R_i \boldsymbol{P}_i + \frac{1}{6R^5}\sum_{ij}(3R_iR_j - \delta_{ij}R^2)\boldsymbol{Q}_{ij} + \ldots$$

with multipole moments („low-energy constants“):

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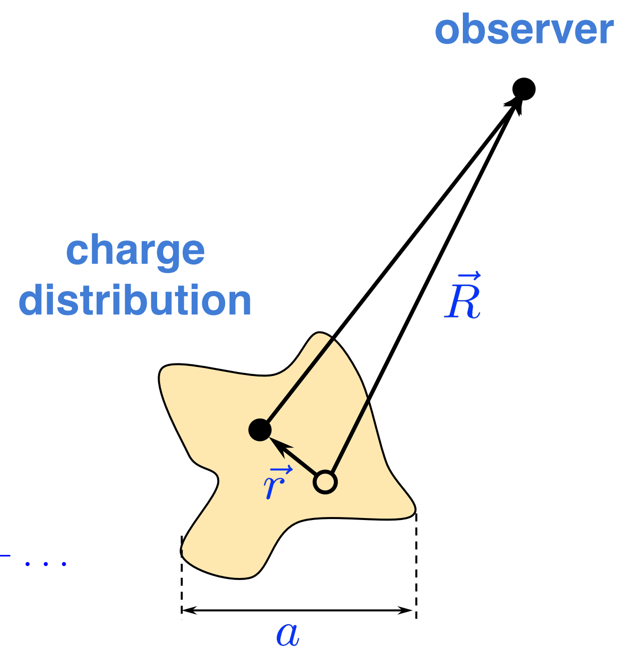

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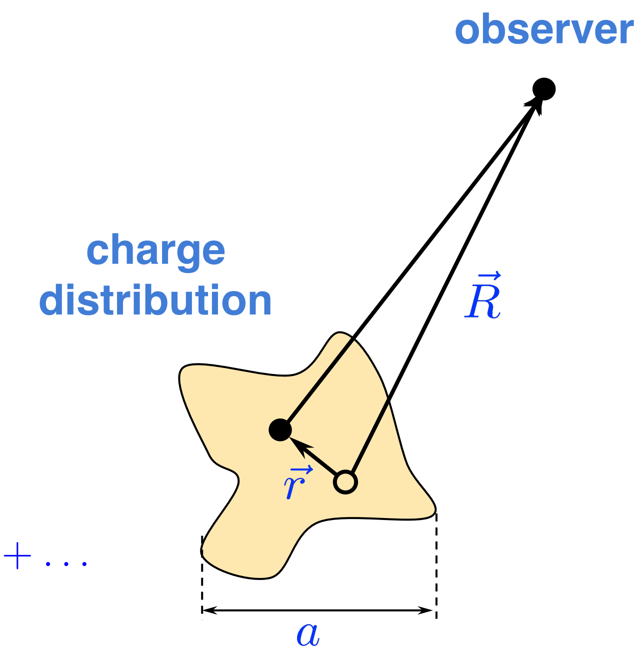

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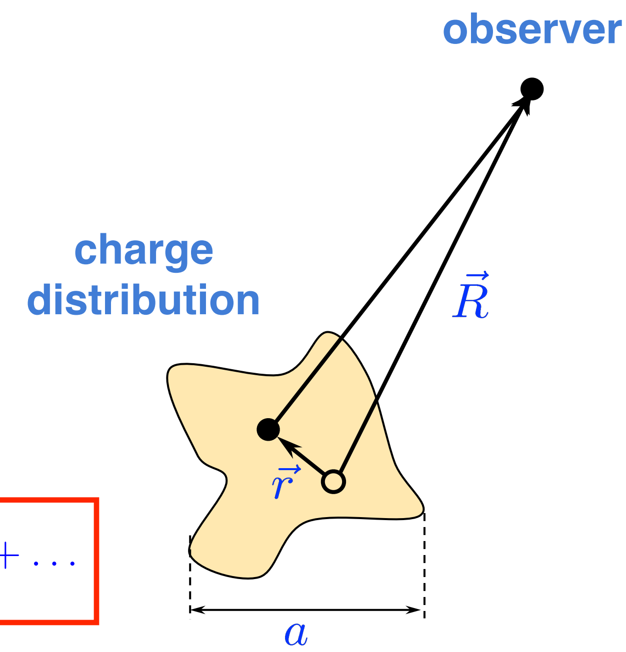

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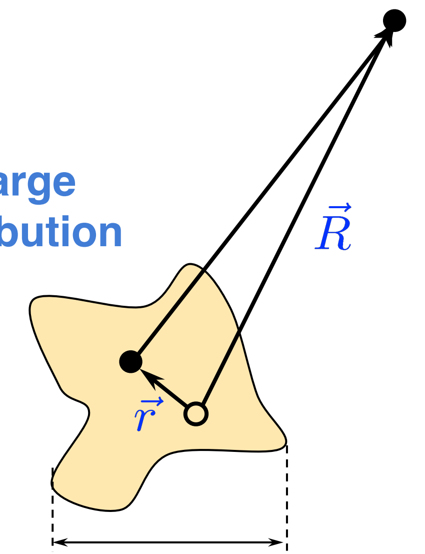

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  - write down the most general rotationally invariant (symmetry!) expression for $V(\vec{R})$
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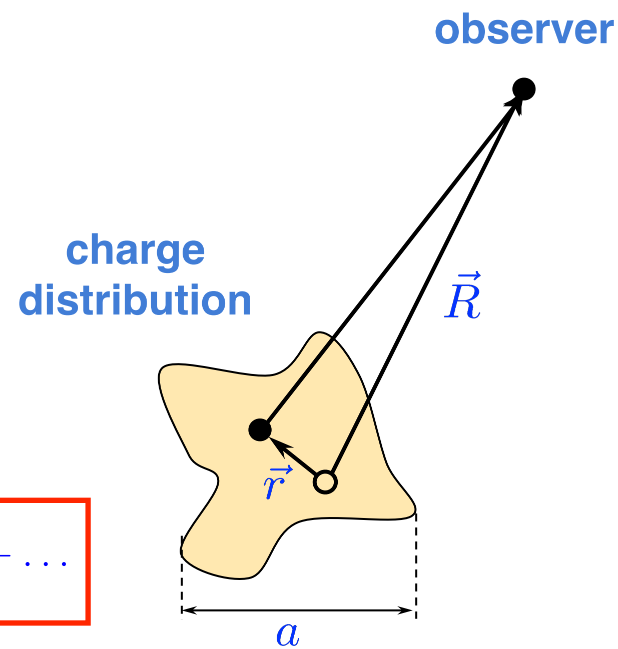

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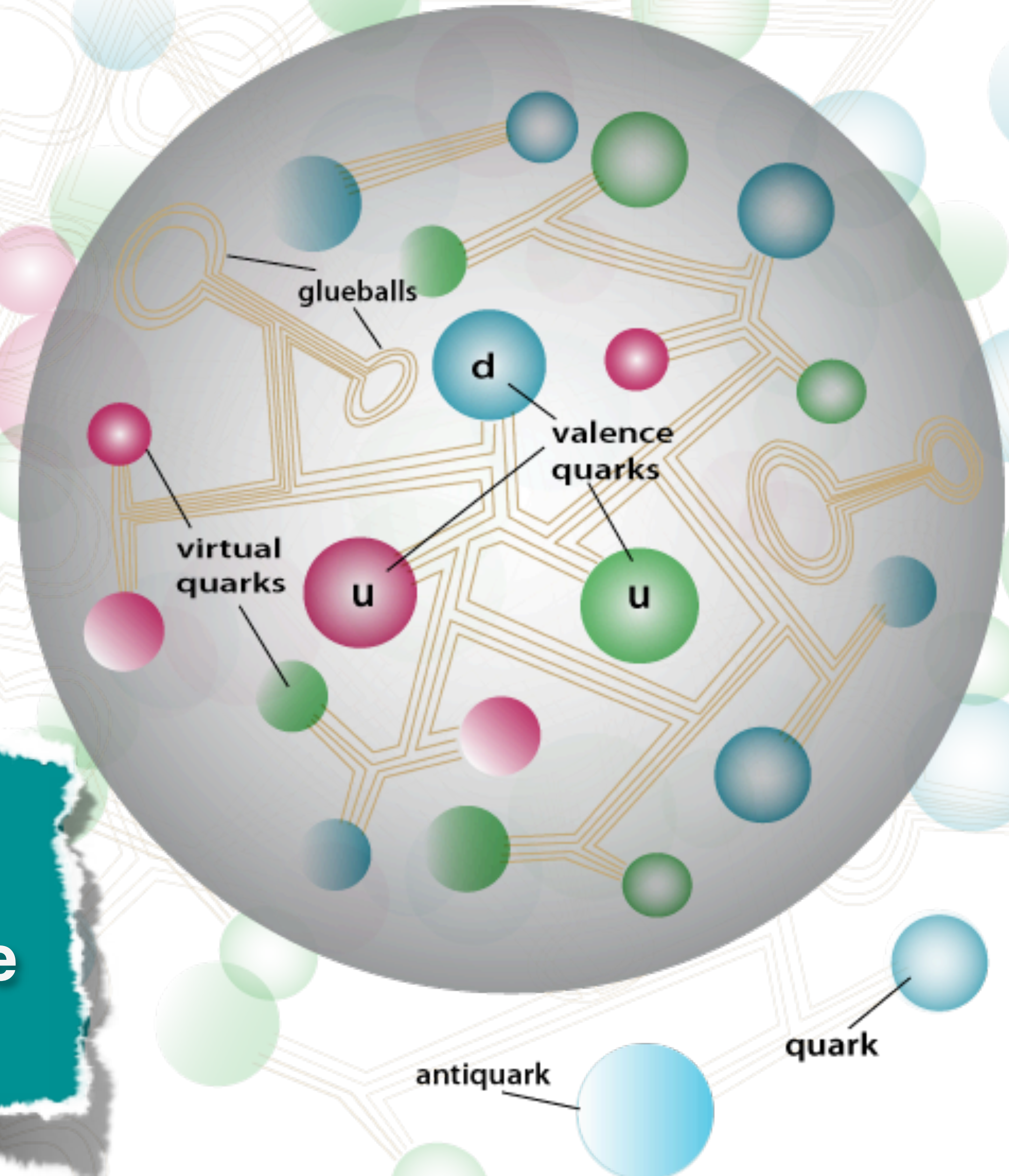
  - measure LECs & compute $V(\vec{R})$ via expansion in $\frac{a}{R}$ (power counting, separation of scales)

NN interaction at different resolutions



Resolution scale << 1 fm: probing the structure of the nucleons…

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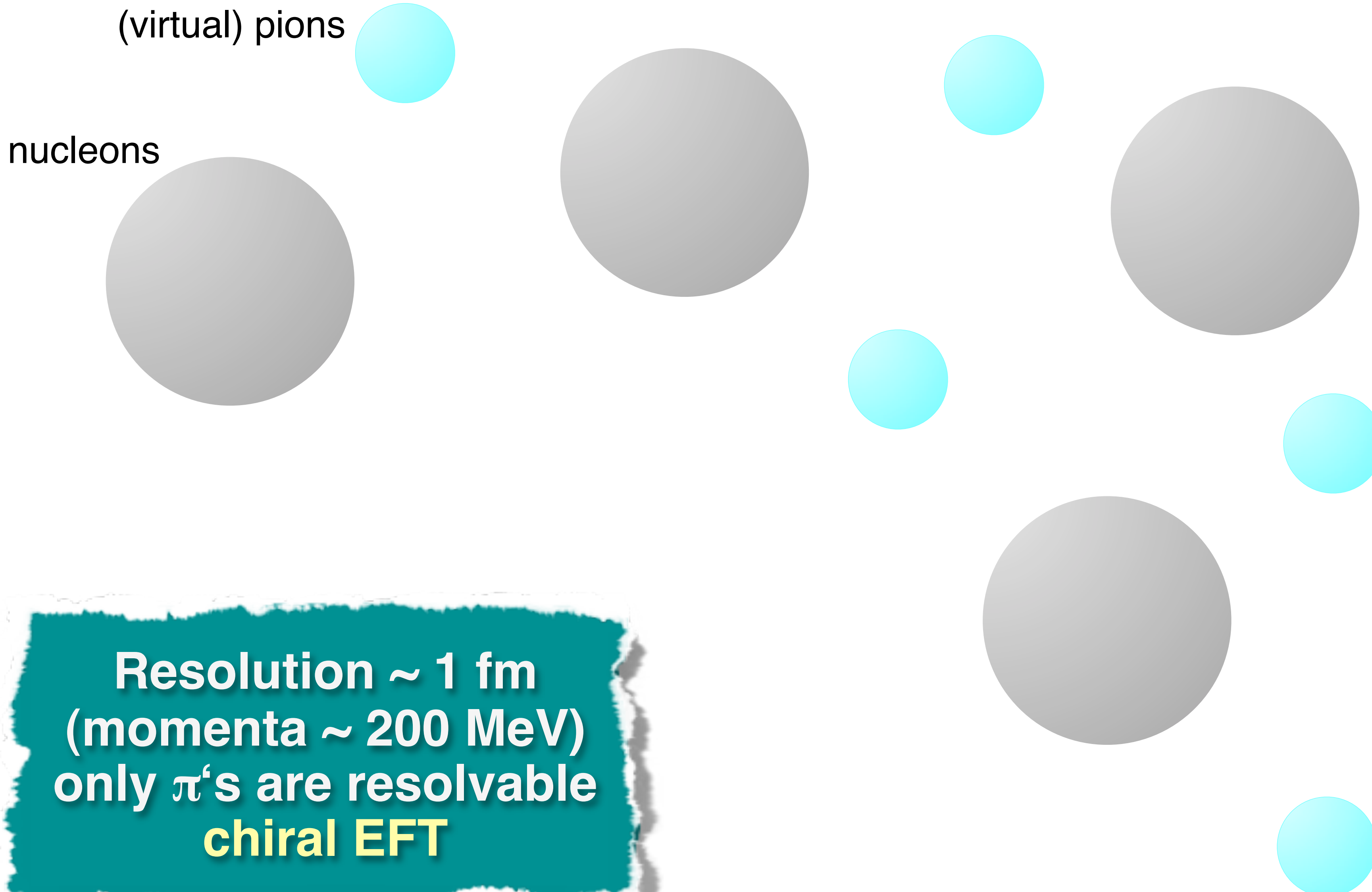
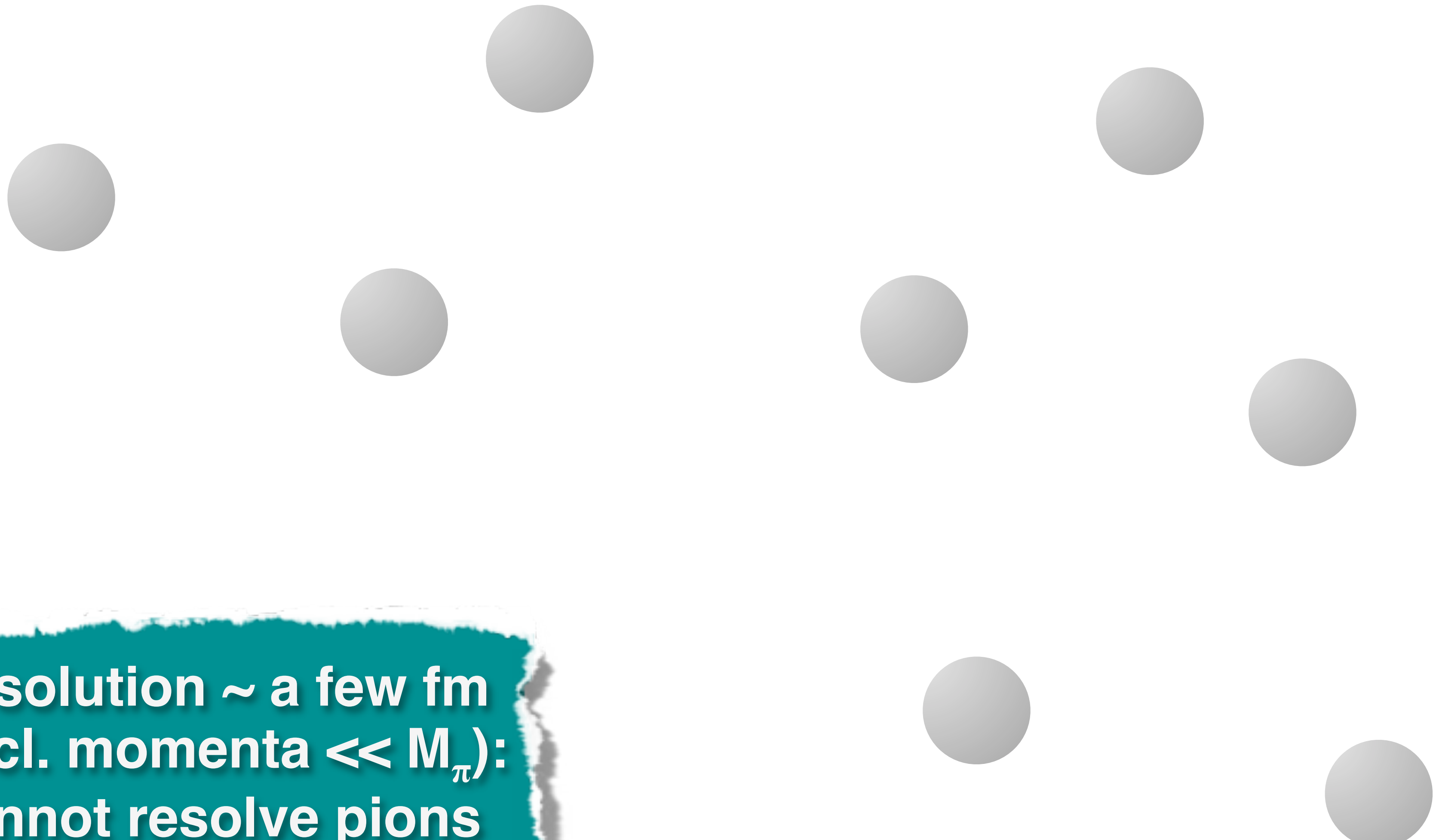


Resolution ~ 1 fm
(momenta ~ 200 MeV)
only π's are resolvable
chiral EFT

NN interaction at different resolutions



Resolution ~ a few fm
(nucl. momenta << M_π):
cannot resolve pions
π-less EFT

Chiral Perturbation Theory

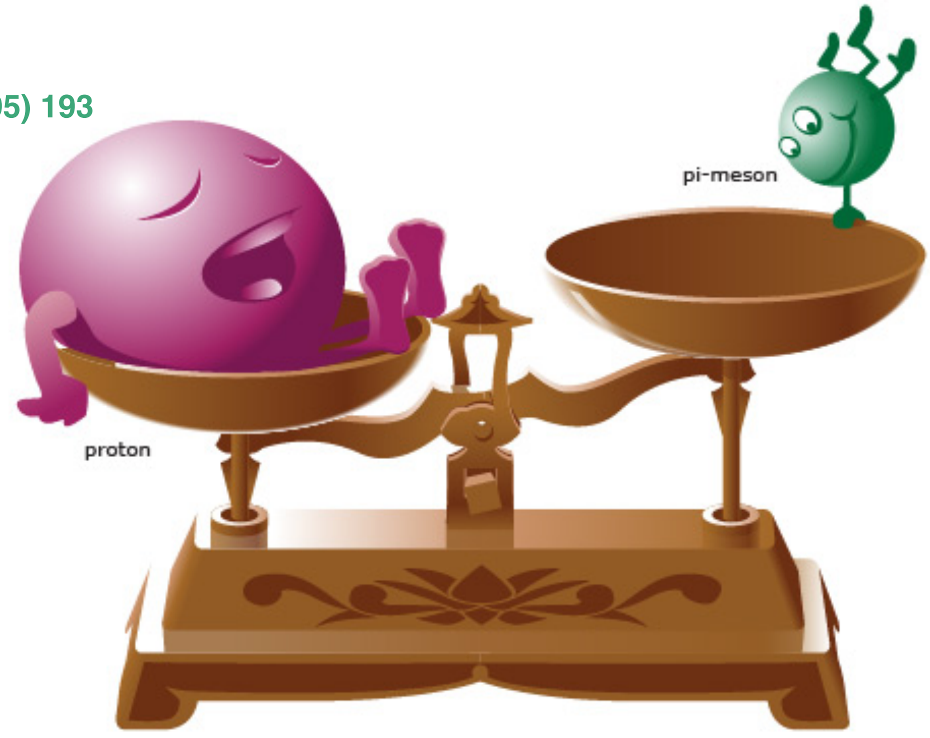
Weinberg, Gasser, Leutwyler, Bernard, Kaiser, Meißner, Bijnens, ...

Some recent review articles

- Bernard, Kaiser, Meißner, Int. J. Mod. Phys. E4 (1995) 193
- Pich, Rep. Prog. Phys. 58 (1995) 563
- Bernard, Prog. Part. Nucl. Phys. 60 (2007) 82
- Scherer, Prog. Part. Nucl. Phys. 64 (2010) 1

Lecture notes

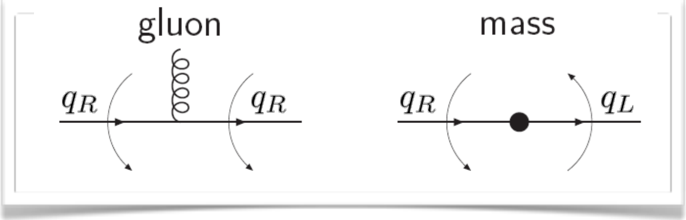
- Scherer, Adv. Nucl. Phys. 27 (2003) 277
- Gasser, Lect. Notes Phys. 629 (2004) 1



Chiral symmetry of QCD

- **QCD and chiral symmetry**

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \underbrace{\bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R}_{\text{SU(2)}_L \text{ x SU(2)}_R \text{ invariant}} - \underbrace{\bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M} q_L}_{\text{break chiral symmetry}}$$



Light quark masses ($\overline{\rm MS},\ \mu = 2$ GeV):

$$\boxed{\begin{array}{rcl} m_u &=& 1.5\ldots 3.3 \text{ MeV} \\ m_d &=& 3.5\ldots 6.0 \text{ MeV} \end{array} \ll \Lambda_{QCD} \sim 220 \text{ MeV}}$$

→ $\mathcal{L}_{\rm QCD}$ is approx. $SU(2)_L$ x $SU(2)_R$ invariant

spontaneous breakdown to $SU(2)_V \subset SU(2)_L$ x $SU(2)_R$ → Goldston Bosons (pions)

- **Chiral perturbation theory**
  - Ideal world [$m_u = m_d = 0$], zero-energy limit: non-interacting massless GBs (+ strongly interacting massive hadrons)
  - Real world [$m_u,\ m_d \ll \Lambda_{QCD}$], low energy: weakly interacting light GBs (+ strongly interacting massive hadrons)

→ expand about the ideal world (ChPT)

Effective Lagrangian for pions

Pions transform linearly under isospin (isotriplet): $|\pi_1\rangle = \frac{|\pi^+\rangle - |\pi^-\rangle}{\sqrt{2}}\,, \quad |\pi_2\rangle = \frac{|\pi^+\rangle + |\pi^-\rangle}{\sqrt{2}i}\,, \quad |\pi^3\rangle = |\pi^0\rangle$

Pions have to transform nonlinearly under chiral rotations
($SU(2)_L$ x $SU(2)_R$ ~ SO(4) $\longrightarrow$ pion fields as coordinates on a 4-dimentional sphere)

Nonlinear field redefinitions of the kind $\vec{\pi} \to \vec{\pi}' = \vec{\pi}\, F[\vec{\pi}\,],\;\; F[0] = 1$ do not change physics
$\longrightarrow$ all nonlinear realizations of χ symmetry are equivalent $\longrightarrow$ use most convenient one!

Haag '58; Coleman, Callan, Wess, Zumino '69

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Example of an explicit construction:

Infinitesimal SO(4) rotation of the 4-vector $(\pi_1,\, \pi_2,\, \pi_3,\, \sigma)$:

$$\begin{pmatrix} \boldsymbol{\pi} \\ \sigma \end{pmatrix} \xrightarrow{SO(4)} \begin{pmatrix} \boldsymbol{\pi}' \\ \sigma' \end{pmatrix} = \left[\mathbf{1}_{4\times 4} + \sum_{i=1}^{3} \theta_i^V V_i + \sum_{i=1}^{3} \theta_i^A A_i \right] \begin{pmatrix} \boldsymbol{\pi} \\ \sigma \end{pmatrix}$$

where:

$$\sum_{i=1}^{3} \theta_i^V V_i = \begin{pmatrix} 0 & -\theta_3^V & \theta_2^V & 0 \\ \theta_3^V & 0 & -\theta_1^V & 0 \\ -\theta_2^V & \theta_1^V & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \sum_{i=1}^{3} \theta_i^A A_i = \begin{pmatrix} 0 & 0 & 0 & \theta_1^A \\ 0 & 0 & 0 & \theta_2^A \\ 0 & 0 & 0 & \theta_3^A \\ -\theta_1^A & -\theta_2^A & -\theta_3^A & 0 \end{pmatrix}$$

Switch to nonlinear realization: only 3 out of 4 components of the vector $(\boldsymbol{\pi},\, \sigma)$ are independent, i.e. $\boldsymbol{\pi}^2 + \sigma^2 = F^2$

$$\boldsymbol{\pi} \xrightarrow{\theta^V} \boldsymbol{\pi}' = \boldsymbol{\pi} + \boldsymbol{\theta}^V \times \boldsymbol{\pi}\,, \qquad \longleftarrow \quad \textit{linear under } \vec{\theta}^{\,V}$$

$$\boldsymbol{\pi} \xrightarrow{\theta^A} \boldsymbol{\pi}' = \boldsymbol{\pi} + \boldsymbol{\theta}^A \sqrt{F^2 - \boldsymbol{\pi}^2} \qquad \longleftarrow \quad \textit{nonlinear under } \vec{\theta}^{\,A}$$

Effective Lagrangian for pions

Can be rewritten in terms of a 2 x 2 matrix:

$$U = \frac{1}{F}\left(\sigma\, \mathbf{1}_{2\times 2} + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right) \xrightarrow{\textit{nonlinear realization}} U = \frac{1}{F}\left(\sqrt{F^2-\boldsymbol{\pi}^2}\,\mathbf{1}_{2\times 2} + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right)$$

Chiral rotations: $U \longrightarrow U' = LUR^\dagger$ with $L = \exp[-i(\boldsymbol{\theta}^V - \boldsymbol{\theta}^A)\cdot\boldsymbol{\tau}/2], \quad R = \exp[-i(\boldsymbol{\theta}^V + \boldsymbol{\theta}^A)\cdot\boldsymbol{\tau}/2]$

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Derivative expansion for the effective Lagrangian $\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$

0 derivatives: $UU^\dagger = U^\dagger U = 1$ - irrelevant $\longleftarrow$ only derivative couplings of GBs

2 derivatives: $\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \xrightarrow{g\in G} \mathrm{Tr}(L\partial_\mu U R^\dagger R\,\partial^\mu U^\dagger L^\dagger) = \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$

$$\longrightarrow \quad \mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$$

derivatives act only on the next U

4 derivatives: $[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)]^2\,,\quad \mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger)\mathrm{Tr}(\partial^\mu U \partial^\nu U^\dagger)\,,\quad \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger)$

(terms with $\partial_\mu\partial_\nu U,\ \partial_\mu\partial_\nu\partial_\rho U,\ \partial_\mu\partial_\nu\partial_\rho\partial_\sigma U$ can be eliminated via EOM/partial integration)

…

Chiral symmetry breaking terms

$\delta\mathcal{L}_{\rm QCD} = -\bar q_L \mathcal{M} q_R - \bar q_R \mathcal{M}^\dagger q_L$ can be made χ-invariant by requiring: $\mathcal{M} \to L\mathcal{M}R^\dagger$

$\longrightarrow$ construct all possible χ-invariant terms involving $\mathcal{M}$ and freeze out $\mathcal{M}$ at the end

LO term: $\delta\mathcal{L}_{\rm SB} = \dfrac{BF^2}{2}\mathrm{Tr}[\mathcal{M}U^\dagger + U\mathcal{M}^\dagger] = 2BF^2 m_q - Bm_q\vec{\pi}^2 + \ldots \ \longrightarrow\ M_\pi^2 = 2m_q B + \mathcal{O}(m_q^2)$

Effective Lagrangian for pions

The leading and subleading effective Lagrangians for pions

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + 2B(\mathcal{M}U + \mathcal{M}U^\dagger)\rangle\,,$$

terms involving external fields

$$\mathcal{L}_\pi^{(4)} = \frac{l_1}{4}\langle \partial_\mu U \partial^\mu U^\dagger\rangle^2 + \frac{l_2}{4}\langle \partial_\mu U \partial_\nu U^\dagger\rangle\langle \partial^\mu U \partial^\nu U^\dagger\rangle + \frac{l_3}{16}\langle 2B\mathcal{M}(U+U^\dagger)\rangle^2 + \ldots - \frac{l_7}{16}\langle 2B\mathcal{M}(U-U^\dagger)\rangle^2$$

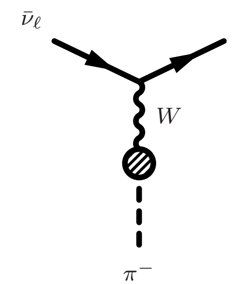
Gasser, Leutwyler '84

Low-energy constants of $\mathcal{L}_\pi^{(2)}$

- F is related to the pion decay constant F_π: $\langle 0|J^i_{A_\mu}(0)|\pi^j(\vec p)\rangle = i p_\mu F_\pi \delta^{ij}$

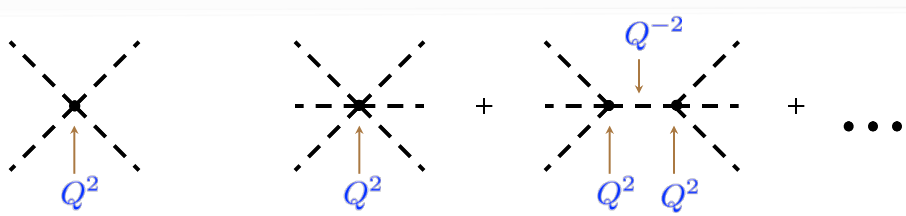
  axial current from $\mathcal{L}_\pi^{(2)}$: $J^i_{A\mu} = i\mathrm{Tr}[\tau^i(U^\dagger\partial_\mu U - U\partial_\mu U^\dagger)] = -F\partial_\mu\pi^i + \ldots$

  $\longrightarrow$ F is F_π in the chiral limit: $F_\pi = F + \mathcal{O}(m_q) \simeq 92.4\ \mathrm{MeV}$
- B is related to the chiral quark condensate



Tree-level multi-pion connected diagrams from $\mathcal{L}_\pi^{(2)}$

$$U(\boldsymbol{\pi}) = \mathbf{1}_{2\times 2} + i\frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{F} - \frac{\boldsymbol{\pi}^2}{2F^2} - i\alpha\frac{\boldsymbol{\pi}^2\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{F^3} + \mathcal{O}(\boldsymbol{\pi}^4) \longrightarrow \mathcal{L}_\pi^{(2)} = \frac{\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}}{2} - \frac{M^2\boldsymbol{\pi}^2}{2} + \frac{(\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\pi})^2}{2F^2} - \frac{M^2\boldsymbol{\pi}^4}{8F^2} + \ldots$$



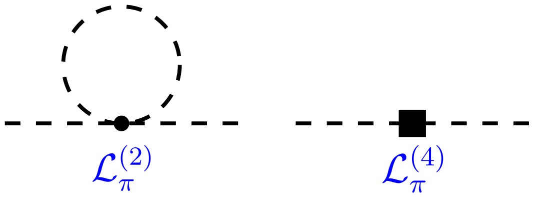
- all diagrams scale as Q^2
- insertions from $\mathcal{L}_\pi^{(4)}$, $\mathcal{L}_\pi^{(6)}$, ... suppressed by powers of Q^2
- remarkable predictive power!

From effective Lagrangian to S-matrix

Tree-level diagrams with higher-order vertices are suppressed at low energy.

The argument can be generalized to quantum corrections (loops) $\longrightarrow$ ChPT Weinberg '79

Typical example of a loop integral:



$$I = \int \frac{d^4 l}{(2\pi)^4} \frac{i}{l^2 - M^2 + i\epsilon} \xrightarrow{\text{DR}} \mu^{4-d} \int \frac{d^d l}{(2\pi)^d} \frac{i}{l^2 - M^2 + i\epsilon}$$

$$= \frac{M^2}{16\pi^2} \ln\left(\frac{M^2}{\mu^2}\right) + 2M^2 L(\mu) + \ldots$$

terms vanishing in d=4

The infinite quantity $L(\mu) = \frac{\mu^{d-4}}{16\pi^2}\left(\frac{1}{d-4} + \text{const}\right)$ can be absorbed into l_i's of $\mathcal{L}_\pi^{(4)}$: $l_i \to l_i^{\mathrm{r}}(\mu)$

The bottom line: after renormalization, all momenta flowing through loop graphs are soft, $\sim Q$

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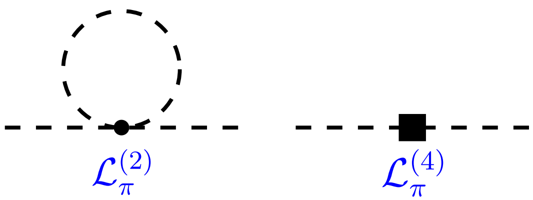

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Power counting (Naive Dimensional Analysis)

- pion propagators: $1/(p^2 - M_\pi^2) \sim 1/Q^2$
- momentum integrations: $d^4 l \sim Q^4$
- delta functions: $\delta^4(p - p') \sim 1/Q^4$
- derivatives: $\partial_\mu \sim Q$

# of loops

# of vertices with d derivatives

$$D = 2 + 2L + \sum_d N_d (d-2)$$

power of the soft scale Q for a given diagram

From effective Lagrangian to S-matrix

Examples:

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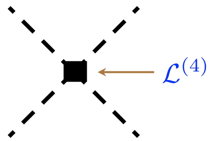

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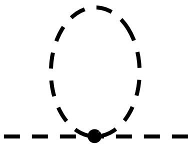

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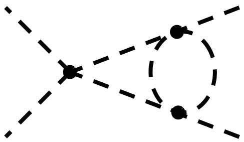

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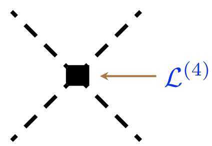

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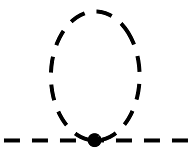

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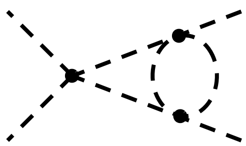

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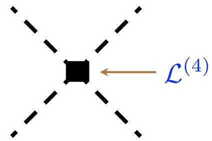

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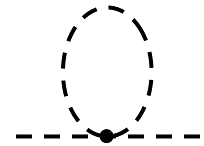

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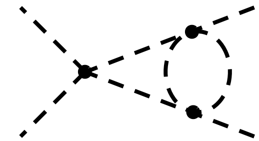

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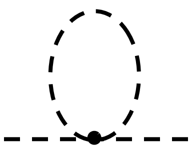

$$\frac{M^2}{F^2}\int \frac{d^4 l}{(2\pi)^4}\frac{i}{l^2 - M^2 + i\epsilon} \xrightarrow{\text{DR}} M^2 \frac{M^2}{(4\pi F)^2}\left[\ln\frac{M^2}{\mu^2} + 2\mu^{d-4}\left(\frac{1}{d-4} + \text{const}\right)\right]$$

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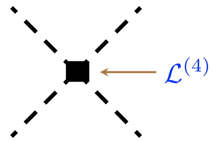

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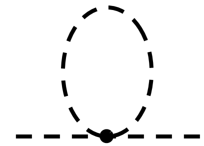

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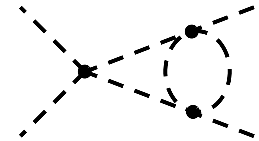

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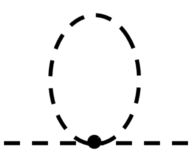

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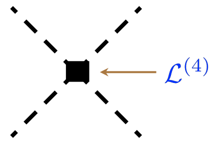

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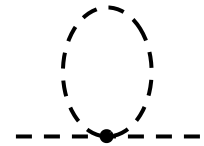

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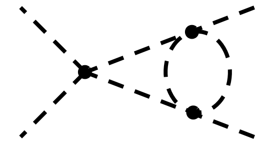

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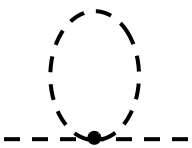

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angular integration in 4 dimensions

dimensional arguments

$$\int \frac{d^d l}{(2\pi)^d} = \int \frac{d\Omega_d}{(2\pi)^d} \int l^{d-1} dl = \frac{1}{2^{d-1} \pi^{d/2} \Gamma(d/2)} \int l^{d-1} dl \xrightarrow{d \to 4} \frac{2}{(4\pi)^2} \int l^3 dl$$

ChPT vs Multipole Expansion

Chiral Perturbation Theory

- Most general effective Lagrangian for pions [and matter fields], chiral symmetry!

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + 2B(\mathcal{M}U + \mathcal{M}U^\dagger)\rangle\,,$$
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Electric potential

Most general expression for the electric potential (rotational invariance)

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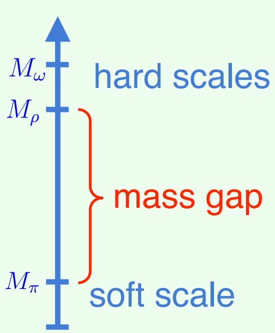
- Separation of scales: [soft] $Q \sim M_\pi \ll \Lambda_\chi \sim M_\rho$ [hard]



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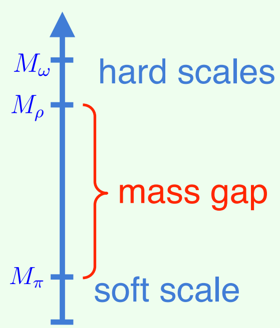
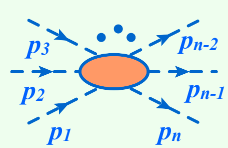


- Chiral expansion of S-matrix elements (Feynman graphs, power counting, renorm.)



$$= E^D f\left(\frac{E}{\mu},\, g^r\right)$$

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Multipole expansion for $V(\vec{R})$ in powers of a/R

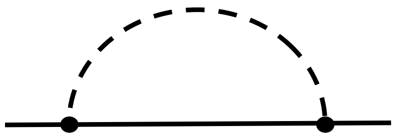
Inclusion of the nucleons

Lowest-order ($\mathcal{O}(|\vec{q}\,|) = \mathcal{O}(M_\pi)$) effective Lagrangian for a single nucleon:

known functions of the pion fields

$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}\left(i\gamma^\mu D_\mu - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)N$$

Problem (?): new hard mass scale m $\longrightarrow$ power counting ??



$$\delta m \xrightarrow{\mathcal{M}\to 0} -m\frac{3g_A^2 m^2}{(4\pi F)^2}\left[\log\frac{m}{\mu} + \mu^{d-4}\left(\frac{1}{d-4} + \text{const}\right)\right]$$

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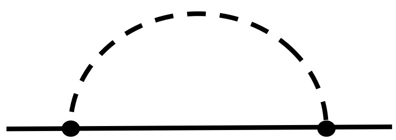

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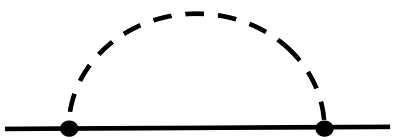

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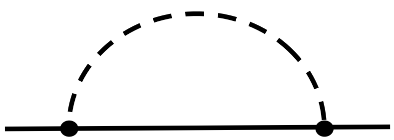


$$\longrightarrow \quad \delta m \xrightarrow{M\to 0} -m\frac{3g_A^2 m^2}{(4\pi F)^2}\left[\log\frac{m}{\mu} + \mu^{d-4}\left(\frac{1}{d-4} + \text{const}\right)\right]$$

Making power counting manifest: The heavy-baryon framework

Jenkins & Manohar '91; Bernard et al. '92

$P_v^\pm = (1 \pm \gamma \cdot v)/2 \longleftarrow$ projection operators

$$\underbrace{N' = e^{imv\cdot x} P_v^+ N,}_{\text{large component (massless)}} \quad \underbrace{h = e^{imv\cdot x} P_v^- N}_{\text{small component (heavy) — integrated out}} \quad \longrightarrow \quad \mathcal{L}_{\pi N}^{(1)} = N'^\dagger \left(iD_0 + \frac{g_A}{2}\vec{\sigma}\cdot\vec{u}\right) N' + \mathcal{O}(1/m)$$

m disappeared from $\mathcal{L}_{\pi N}^{(1)}$ $\longrightarrow$ power counting manifest! For example: $(\delta m)^{\text{HB}} = -\frac{3g_A^2 M_\pi^3}{32\pi F^2}$

(Some) Current topics in and beyond ChPT

- **Resumming leading Log's**

  Weinberg, Bijnens, Colangelo, Bissiger, Fuhrer, Kivel, Polyakov, Vladimirov, …

  Leading logs can be computed for higher loops, all orders possible in certain cases

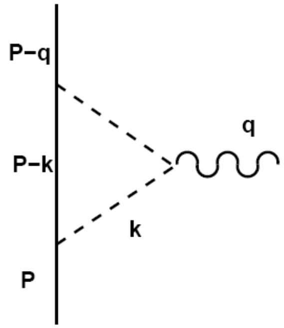
- **Combining ChPT and dispersion theory**

  Colangelo, Gasser, Leutwyler, Bernard, Meißner, Descotes Genon, Knecht, Stern, Pelaez, Lutz, …

- **Covariant baryon ChPT**

  Becher, Leutwyler, Bernard, Meißner, Kubis, Gegelia, Scherer, Higa, Robilotta, …

  HB expansion has a very limited convergence range for some types of diagrams → better to resum 1/m recoil corrections up to infinite order (IR-ChPT). Alternatively, use manifestly covariant framework + appropriate subtraction (EOMS) to enforce power counting



- **ChPT with explicit spin-3/2 degrees of freedom**

  Hemmert, Bernard, Fettes, Meißner, Pascalutsa, Vanderhaeghen, Kaiser, Weise, Gegelia, Scherer, EE, Krebs, …

  Δ(1232) has low excitation energy ~ 300 MeV → better to include as an explicit DOF…

- **ChPT and/for lattice QCD**

  Colangelo, Beane, Savage, Jiang, Tiburzi, Procura, Weise, Walker Loud, Bernard, Meißner, Rusetsky, Hemmert, …

  Chiral extrapolations, finite volume corrections, quenched ChPT, …

- **Unitarized ChPT and resonance physics**

  Oeller, Meißner, Dobado, Pelaez, Oset, Hanhart, Llanes-Estrada, Kaiser, Weise, ,…

From one nucleon to few: Not so easy…

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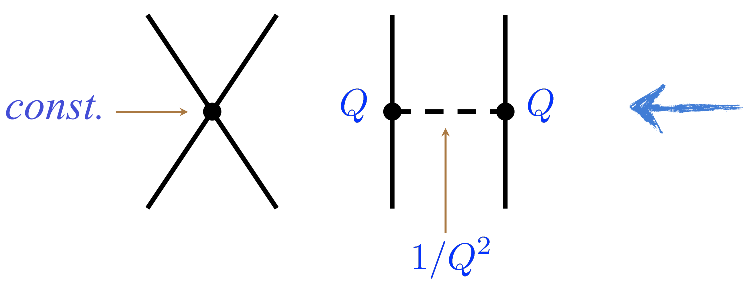
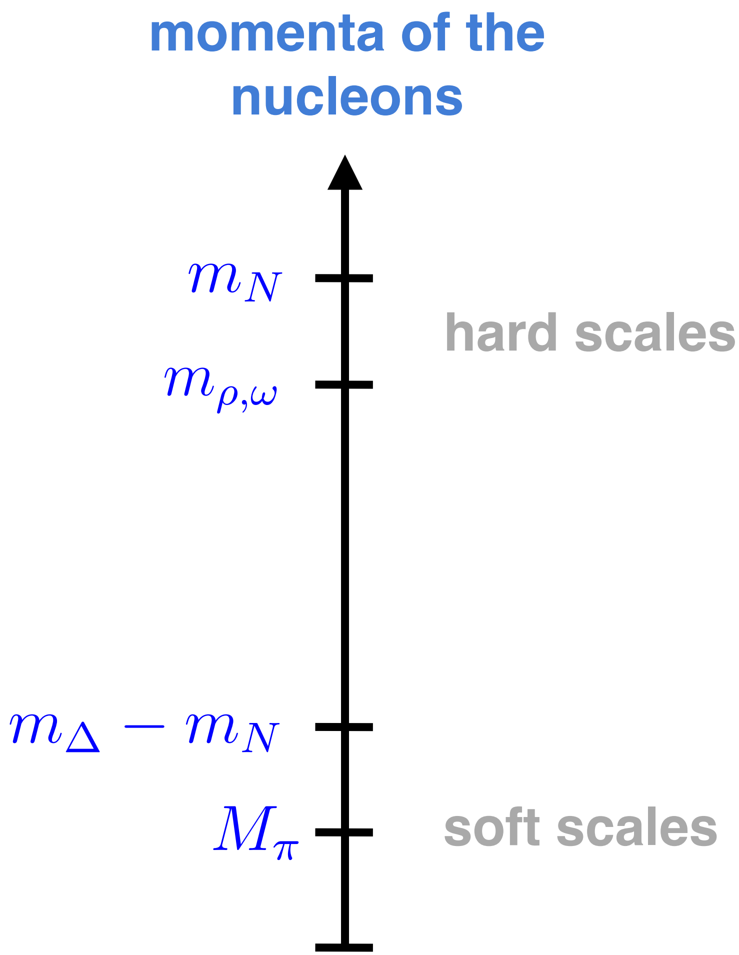


do not vanish for
$E = 0,\ m_q = 0$

The presence of shallow bound states (^{2}H, ^{3}H, ^{3}He, ^{4}He, …) indicates breakdown of perturbation theory even at very low energy!

How to organize EFT in the non-perturbative regime?

Hierarchy of scales in nuclear physics



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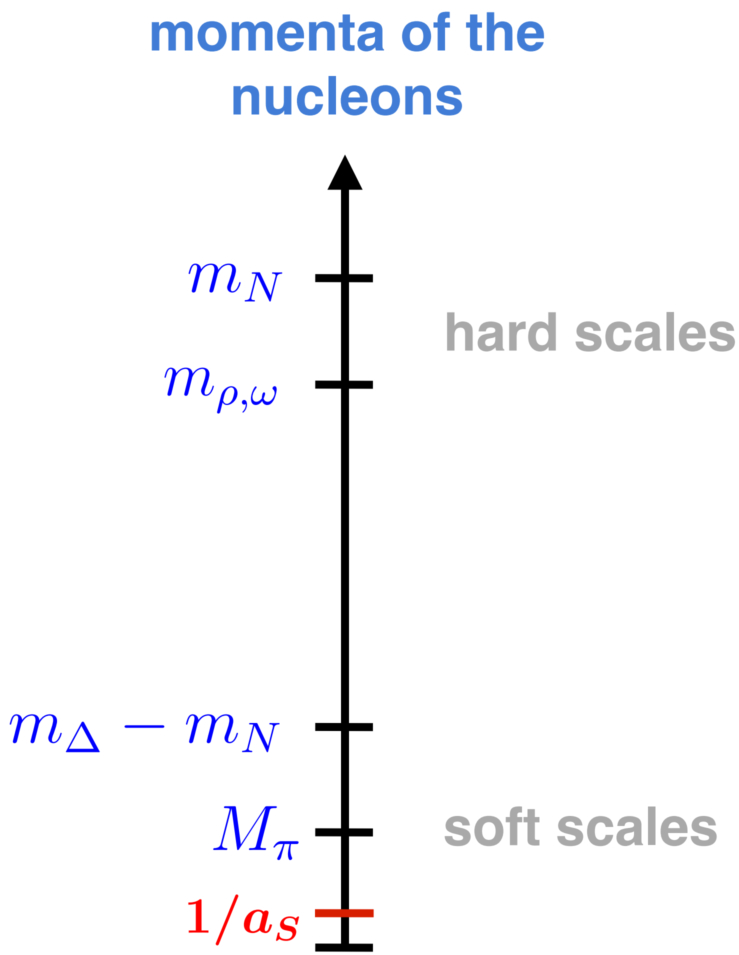

A new, soft scale associated with nuclear binding
$Q \sim 1/a_S \simeq 8.5\ \mathrm{MeV}(36\ \mathrm{MeV})$ in $^1S_0$ ($^3S_1$)
has to be generated dynamically
(need resummations...)

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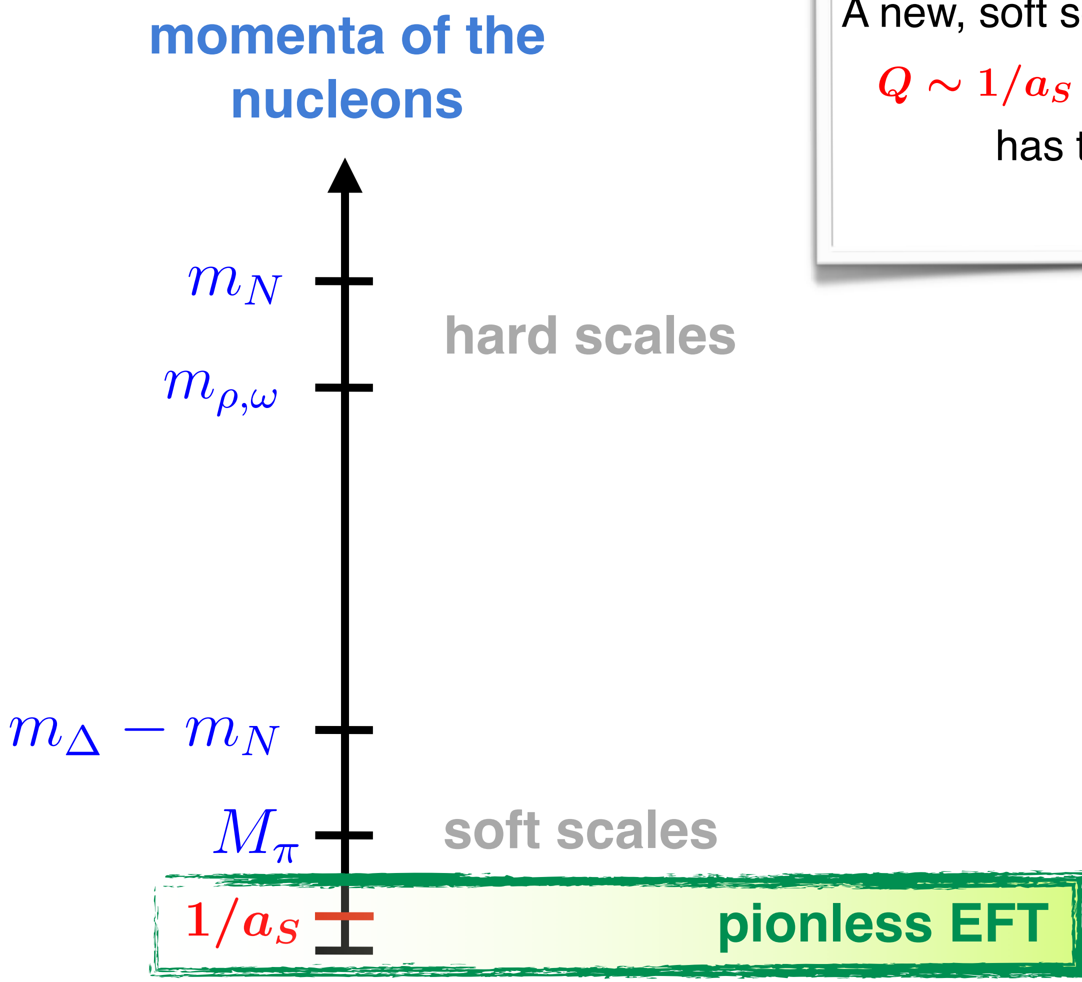

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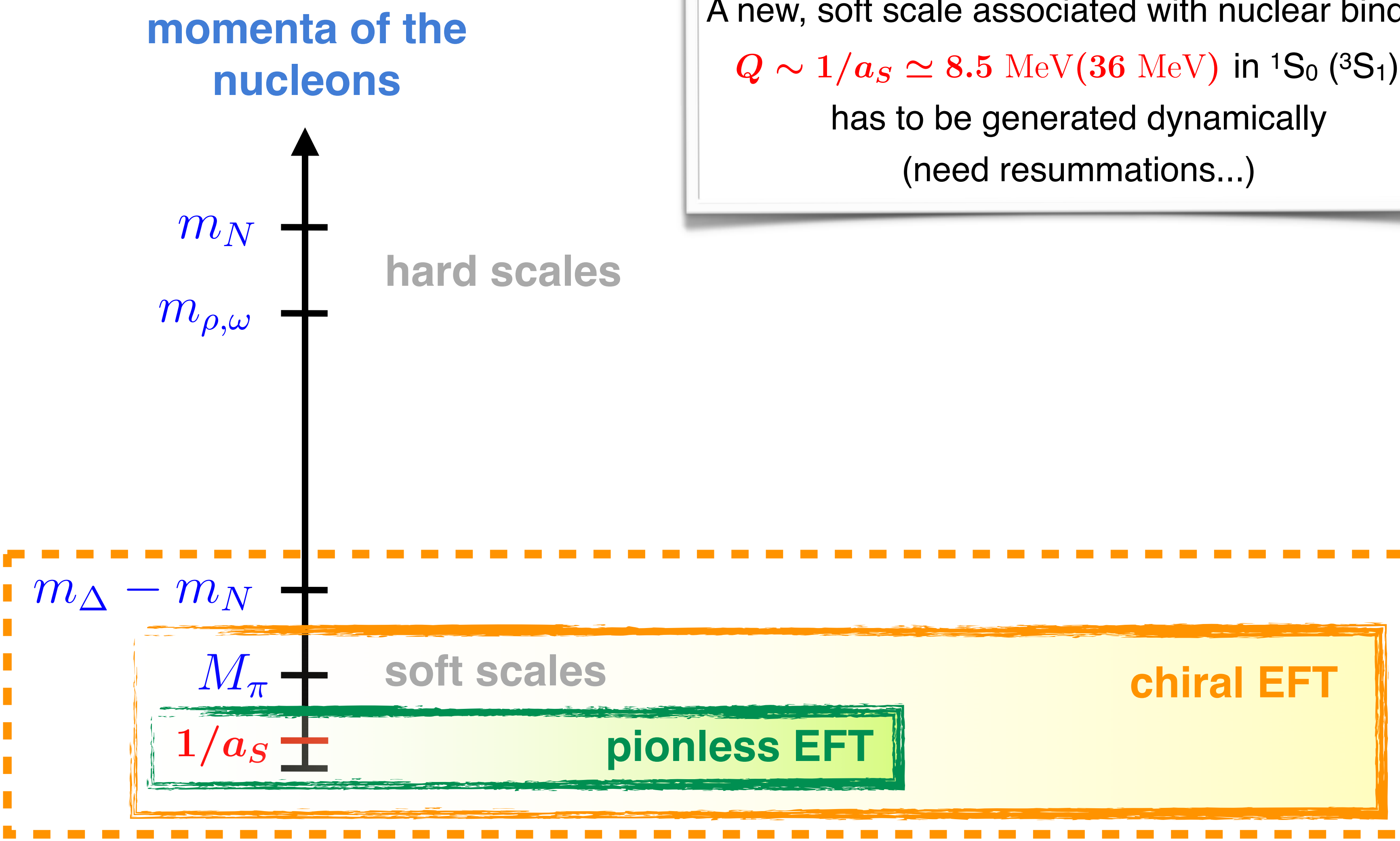

Pionless effective field theory

Goal: EFT for NN scattering at typical momenta $Q \ll M_\pi$

Formulation

- Kaplan, Savage, Wise, Phys. Lett. B424 (98) 390; Nucl. Phys. B534 (98) 329
- Bedaque, Hammer, van Kolck, Phys. Rev. Lett. 92 (99) 463; Nucl. Phys. A646 (99) 444

(Some) recent review articles

- Beane et al., arXiv:nucl-th/0008064, in Boris Ioffe Festschrift, ed. By M. Shifman, World Scientific
- Bedaque, van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (02) 339
- Braaten, Hammer, Phys. Rept. 428 (06) 259
- Hammer, Platter, arXiv:1102.3789

Effective Range Expansion

Blatt, Jackson '49; Bethe '49

Nonrelativistic nucleon-nucleon scattering (uncoupled case):

effective-range function

$$S_l(k) = e^{2i\delta_l(k)} = 1 + i\frac{mk}{2\pi}T_l(k) \quad \text{where} \quad T_l(k) = \frac{4\pi}{m}\frac{k^{2l}}{F_l(k) - ik^{2l+1}} \quad \text{and} \quad F_l(k) \equiv k^{2l+1}\cot\delta_l(k)$$

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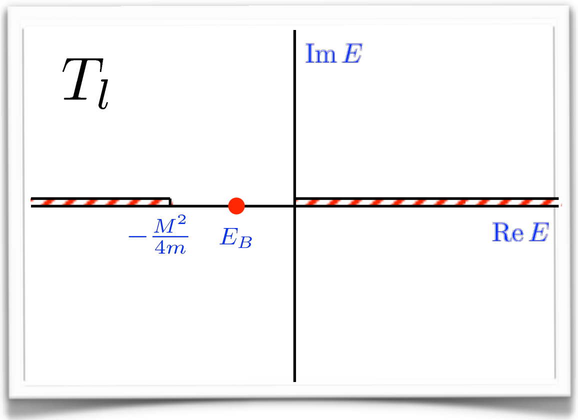
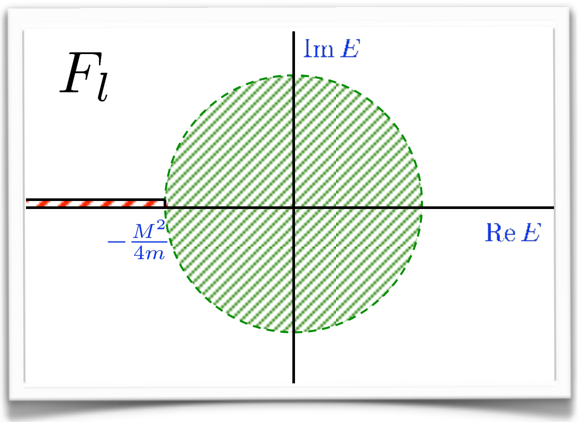
If $V(r)$ satisfies certain conditions, F_l is a meromorphic function of k^2 near the origin





→ effective range expansion (ERE):

$$\boxed{F_l(k^2) = -\frac{1}{a} + \frac{1}{2}rk^2 + v_2k^4 + v_3k^6 + v_4k^8 + \ldots}$$

The analyticity domain depends on the range M^{-1} of $V(r)$ defined as $M = \min(\mu)$

such that $\int_{R>0}^{\infty} |V(r)|\, e^{\mu r} dr = \infty$ (for strongly interacting nucleons $M = M_\pi$)

Pionless EFT: natural scattering length

Effective Lagrangian: for $Q \ll M_\pi$ only point-like interactions

$$\mathcal{L}_{\rm eff} = N^\dagger\left(i\partial_0 + \frac{\vec{\nabla}^2}{2m}\right)N - \frac{1}{2}C_1^0(N^\dagger N)^2 - \frac{1}{2}C_2^0(N^\dagger\vec{\sigma}N)^2 - \frac{1}{4}C_1^2(N^\dagger\vec{\nabla}^2 N)(N^\dagger N) + \text{h.c.} + \ldots$$

Scattering amplitude (S-waves):

$$S = e^{2i\delta} = 1 - i\left(\frac{km}{2\pi}\right)T\,, \qquad T = -\frac{4\pi}{m}\,\frac{1}{k\cot\delta - ik} = -\frac{4\pi}{m}\,\frac{1}{\left(-\frac{1}{a} + \frac{1}{2}r_0k^2 + v_2k^4 + v_3k^6 + \ldots\right) - ik}$$

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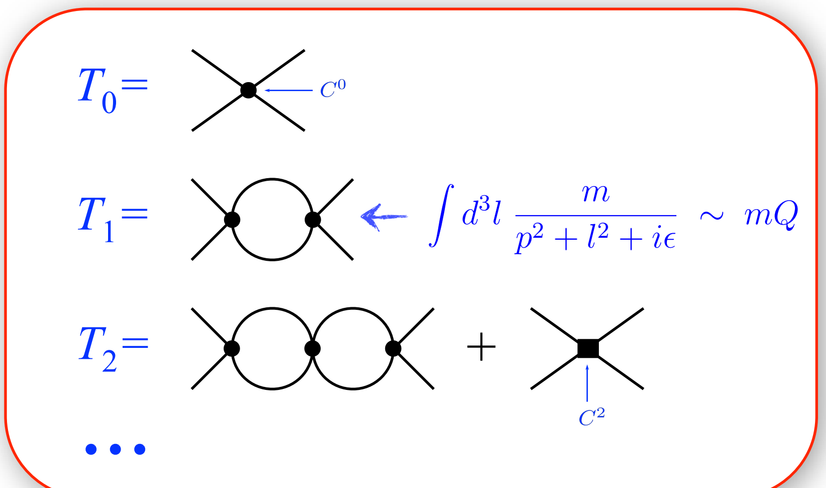
- **Natural case**

$$|a| \sim M_\pi^{-1}\,,\ |r| \sim M_\pi^{-1}\,,\ \ldots \quad \rightarrow \quad T = T_0 + T_1 + T_2 + \ldots = \frac{4\pi a}{m} \Big[\underbrace{1}_{\sim Q^0} - \underbrace{iak}_{\sim Q^1} + \underbrace{\left(\frac{a r_0}{2} - a^2 \right) k^2}_{\sim Q^2} + \ldots \Big]$$



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Using e.g. dimensional or subtractive ragularization yields:

- perturbative expansion for *T*;
- scaling of the LECs: $C^i \sim Q^0$

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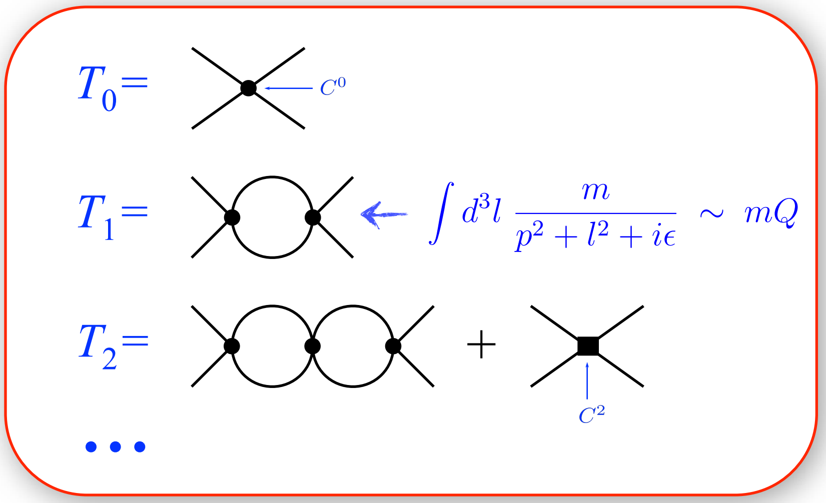


The EFT expansion can be arranged to match the above expansion for *T*.

Using e.g. dimensional or subtractive ragularization yields:

- perturbative expansion for *T*;
- scaling of the LECs: $C^i \sim Q^0$

In reality: $a_{^1S_0} = -23.741 \text{ fm} = -16.6\, M_\pi^{-1} \qquad a_{^3S_1} = -5.42 \text{ fm} = 3.8\, M_\pi^{-1}$

Pionless EFT: large scattering length

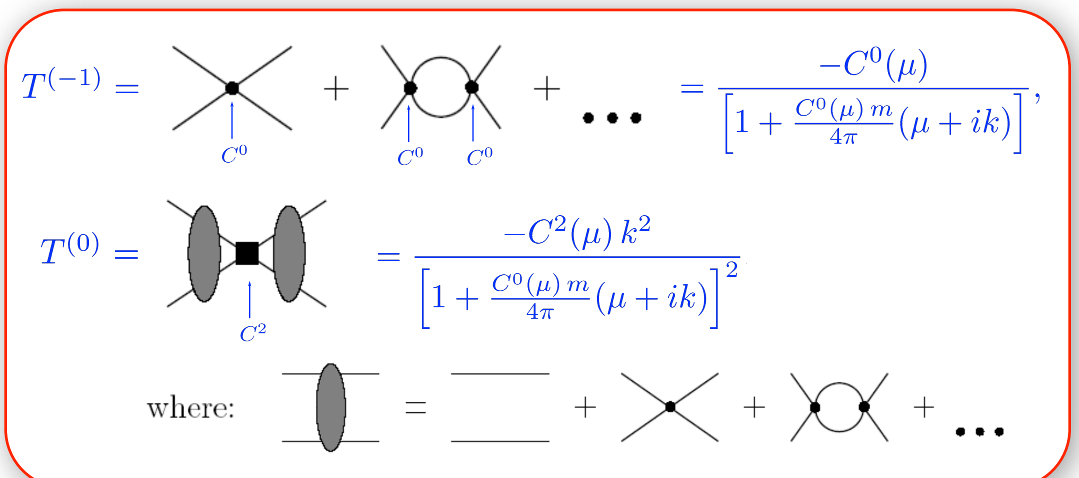
- **Large scattering length:** $|a| \gg M_\pi^{-1}$ Kaplan, Savage, Wise '97

Keep ak fixed, i.e. count $a \sim Q^{-1}$:

$$T = -\frac{4\pi}{m}\frac{1}{\left(-\frac{1}{a}+\frac{1}{2}r_0k^2+v_2k^4+v_3k^6+\ldots\right)-ik} = \frac{4\pi}{m}\frac{1}{(1+iak)}\Bigg[\underset{\sim Q^{-1}}{a} + \underbrace{\frac{ar_0}{2(a^{-1}+ik)}}_{\sim Q^{0}}k^2 + \underset{\sim Q^{1}}{\ldots}\Bigg].$$

Notice: perturbation theory for T breaks down as it has a pole at $|k| \sim |a|^{-1} \ll M_\pi$

KSW expansion (DR+PDS or subtractive renormalization $C^0 \sim 1/Q,\ C^2 \sim 1/Q^2,\ \ldots$ **)**



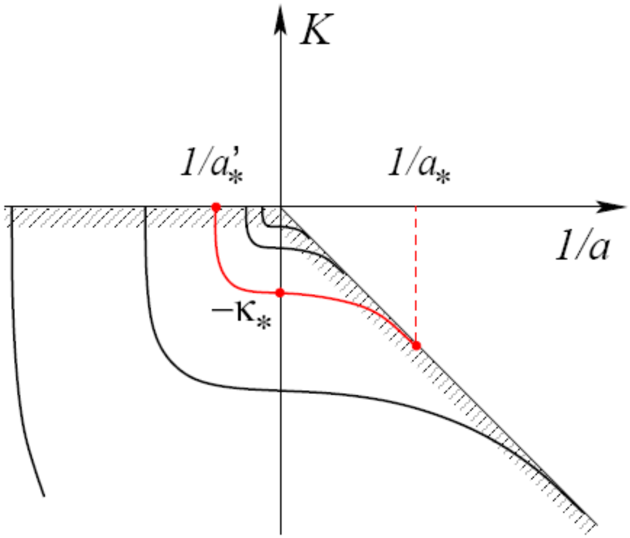
$$T^{(-1)} = \ldots = \frac{-C^0(\mu)}{\left[1+\frac{C^0(\mu)\,m}{4\pi}(\mu+ik)\right]},$$

$$T^{(0)} = \ldots = \frac{-C^2(\mu)\,k^2}{\left[1+\frac{C^0(\mu)\,m}{4\pi}(\mu+ik)\right]^2}$$

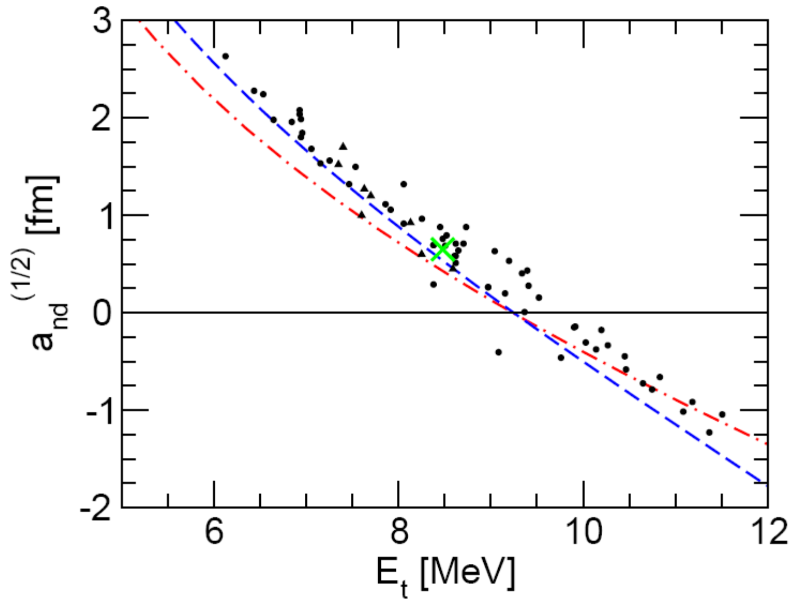
Pionless EFT: (some) applications

- Astrophysical reactions Butler, Chen, Kong, Ravndal, Rupak, Savage, …
- Efimov physics and universality in few-body systems with large 2-body scatt. length (e.g. Phillips/Tjon „lines“) Braaten, Hammer, Meißner, Platter, von Stecher, Schmidt, Moroz, …
- Halo-nuclei Bedaque, Bertulani, Hammer, Higa, van Kolck, Phillips, …
- Many other topics…

Efimov effect (3-body spectrum)



Phillips line



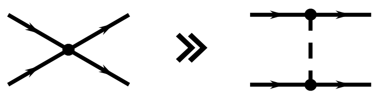
Braaten, Hammer, Phys. Rept 428 (06) 259

Two nucleons beyond ERE

Goal: EFT for NN scattering at typical momenta Q ~ M_π

From pion-less to pion-full: possible scenarios

KSW: treat pion exchange in perturbation theory
straightforward, analytical calculations, but poor convergence...



Weinberg: both LO contacts & OPEP must be resummed
numerical results, phenomenologically successful,
but renormalization rather intransparent...

How to judge whether pion dynamics is properly included?

Modified Effective Range Expansion (MERE)

Both ERE & $\not\pi$-EFT provide an expansion of NN observables in powers of k/M_π , have the same validity range and incorporate the same physics

$\longrightarrow$ **ERE ~ $\not\pi$-EFT**

Beyond π-less EFT: higher energies, <u>LETs</u>...

Two-range potential $V(r) = V_L(r) + V_S(r)$, $M_L^{-1} \gg M_H^{-1}$

- $F_l(k^2)$ is meromorphic in $|k| < M_L/2$

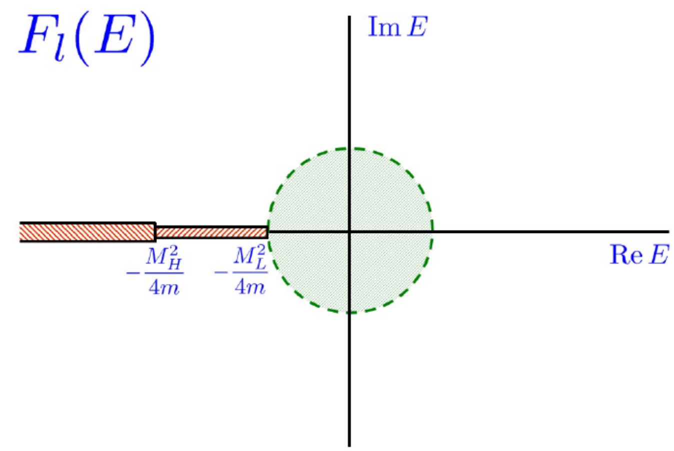
- $$\boxed{F_l^M(k^2) \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2} \cot\left[\delta_l(k) - \delta_l^L(k)\right]}$$

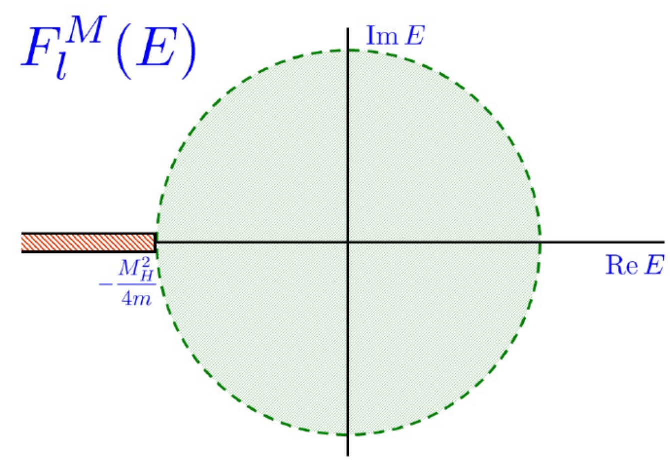
$\longleftarrow$ modified effective range function
Haeringen, Kok '82

$$\underbrace{f_l^L(k)}_{\text{Jost function for } V_L(r)} = \lim_{r\to 0}\left(\frac{l!}{(2l)!}(-2ikr)^l \underbrace{f_l^L(k,r)}_{\text{Jost solution for } V_L(r)}\right)$$

$$M_l^L(k) = Re\left[\frac{(-ik/2)^l}{l!} \lim_{r\to 0}\left(\frac{d^{2l+1}}{dr^{2l+1}} \frac{r^l f_l^L(k,r)}{f_l^L(k)}\right)\right]$$

Per construction, F_l^M reduces to F_l for $V_L = 0$ and is meromorphic in $|k| < M_H/2$





MERE and Low-Energy Theorems

Example: proton-proton scattering

$$F_C(k^2) = C_0^2(\eta)\,k\,\cot[\delta(k) - \delta^C(k)] + 2k\,\eta\,h(\eta) = -\frac{1}{a^M} + \frac{1}{2}r^M k^2 + v_2^M k^4 + \ldots$$

where $\underbrace{\delta^C \equiv \arg\Gamma(1+i\eta)}_{\text{Coulomb phase shift}}$, $\eta = \frac{m}{2k}\alpha$, $\underbrace{C_0^2(\eta) = \frac{2\pi\eta}{e^{2\pi\eta}-1}}_{\text{Sommerfeld factor}}$, $h(\eta) = \mathrm{Re}\Big[\underbrace{\Psi(i\eta)}_{\text{Digamma function}}\Big] - \ln(\eta)$

$\Psi(z) \equiv \Gamma'(z)/\Gamma(z)$

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MERE and low-energy theorems

Long-range forces impose correlations between the ER coefficients (low-energy theorems)

Cohen, Hansen '99; Steele, Furnstahl '00

The emergence of the LETs can be understood in the framework of MERE:

$$\underbrace{F_l^M(k^2)}_{\text{meromorphic for } k^2 < (M_H/2)^2} \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2} \cot\left[\delta_l(k) - \delta_l^L(k)\right]$$

can be computed if the long-range force is known

– approximate $F_l^M(k^2)$ by first 1,2,3,… terms in the Taylor expansion in k^2

– calculate all "light" quantities

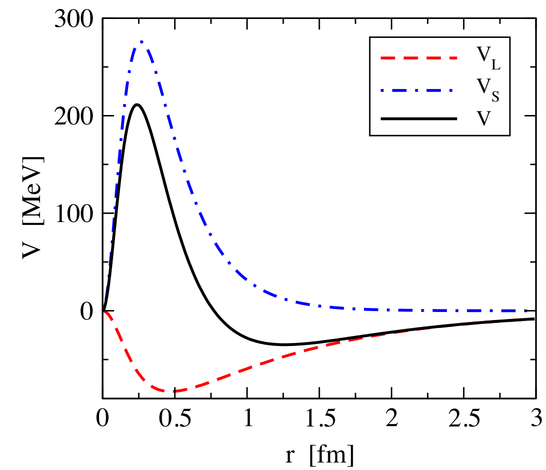
– reconstruct $\delta_l^L(k)$ and predict all coefficients in the ERE

Toy model: Low-energy theorems

$$V(r) = \underbrace{v_L e^{-M_L r} f(r)}_{V_L} + \underbrace{v_H e^{-M_H r} f(r)}_{V_H}$$

where $f(r) = \dfrac{(M_H r)^2}{1 + (M_H r)^2}$

and $M_L = 1.0\,,\ v_L = -0.875\,,\ M_H = 3.75\,,\ v_H = 7.5$ (all in fm^{-1})



ERE and MERE

	a	r	v_2	v_3	v_4
F_0 [fm^n]	5.458	2.432	0.113	0.515	-0.993
F_0^M [fm^n]	6.413	-3.986	-2.289×10^1	-5.043×10^2	2.736×10^4
F_0^M $[M_S^{-n}]$	1.710	-1.063	-0.434	-0.680	2.624

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Low-Energy Theorems

	LO	NLO	NNLO	"Exp"
r	2.447(38)	2.432197161	2.432197161	2.432197161
v_2	0.12(11)	0.1132(29)	0.112815751	0.112815751
v_3	0.61(12)	0.517(16)	0.51533(20)	0.51529
v_4	$-0.95(5)$	$-0.991(14)$	$-0.9925(11)$	-0.9928

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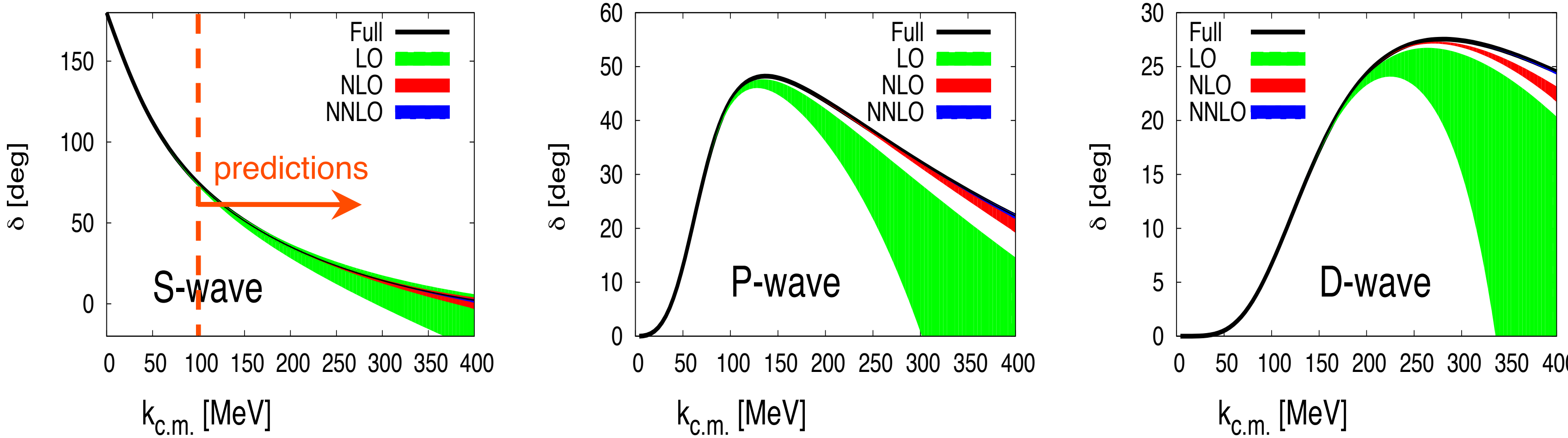
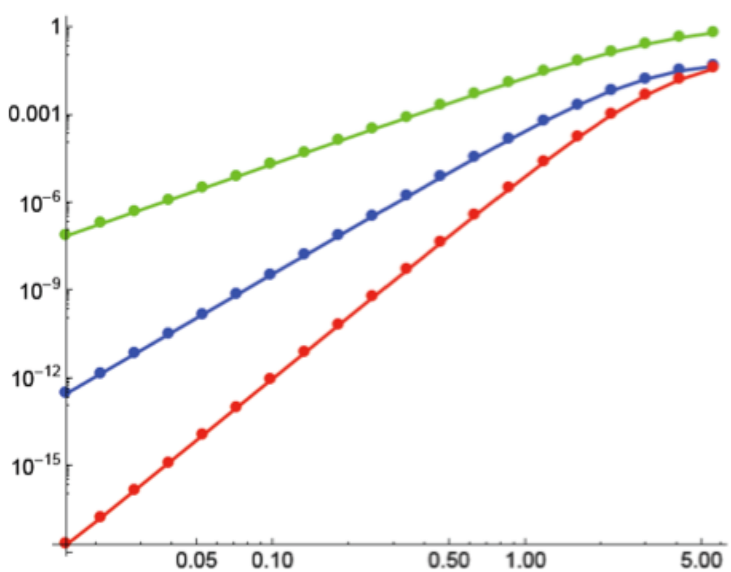
Toy model: phase shifts & error plots



Error plots for $\delta^M(k)$



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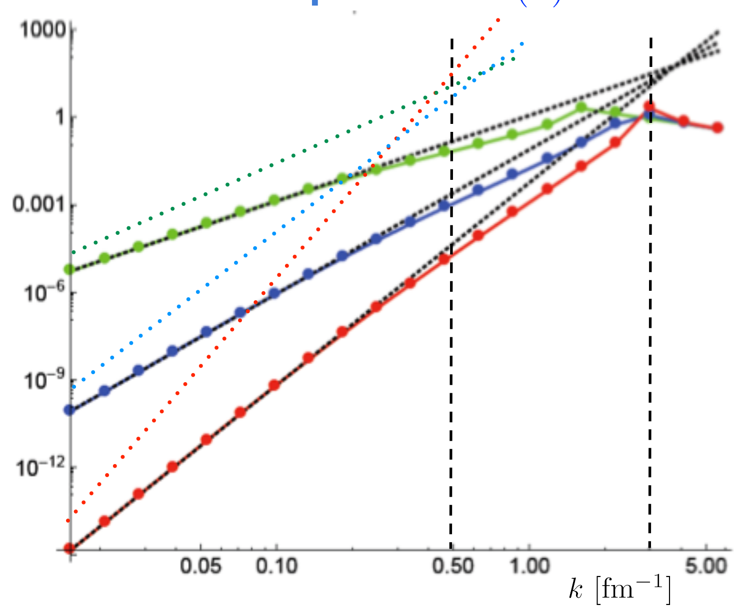
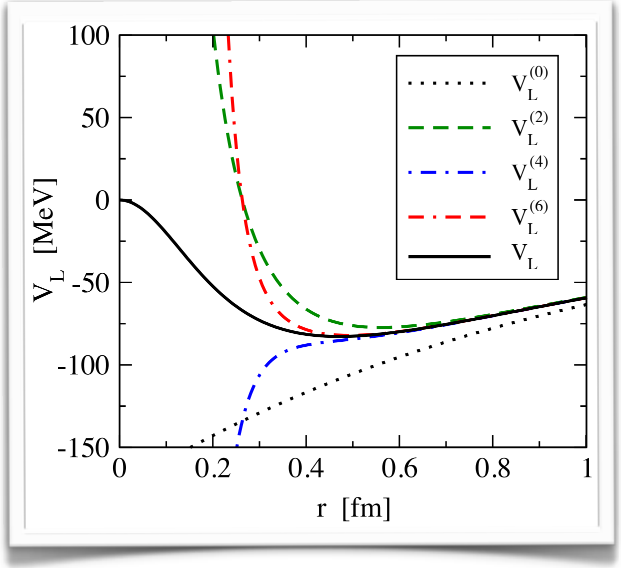


Toy model: The „chiral expansion“

Expansion of the long-range potential:

$$V_L = v_L e^{-M_L r} \frac{(M_H r)^2}{1+(M_H r)^2} = v_L e^{-M_L r} \left[1 - \frac{1}{M_H^2 r^2} - \frac{1}{M_H^4 r^4} - \dots \right]$$



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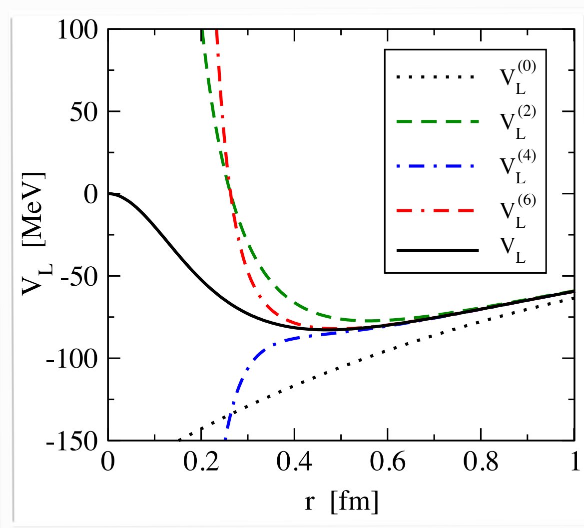

Low-energy theorems (long-range@NNLO, R=0.5fm)

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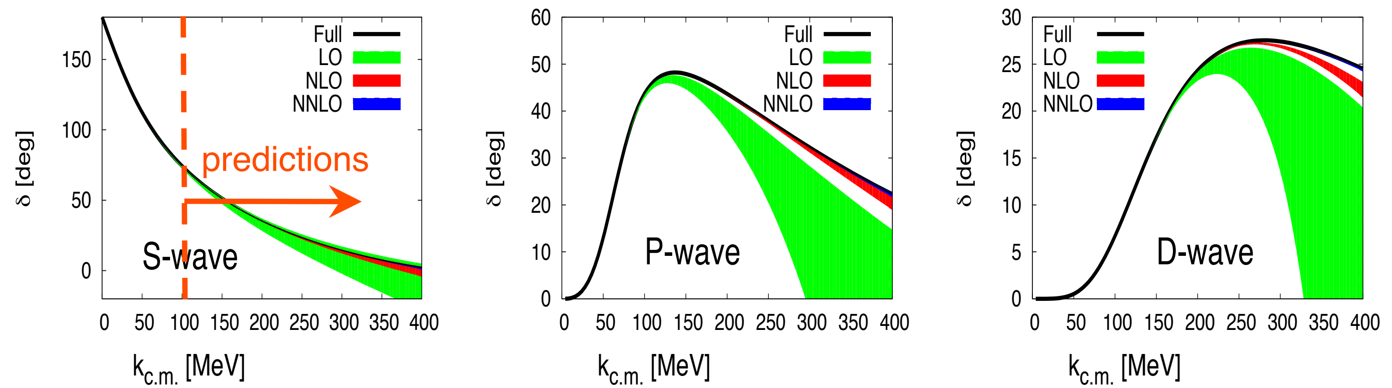
v_4	−0.93(5)	−0.987(13)	−0.9925(12)	−0.9928

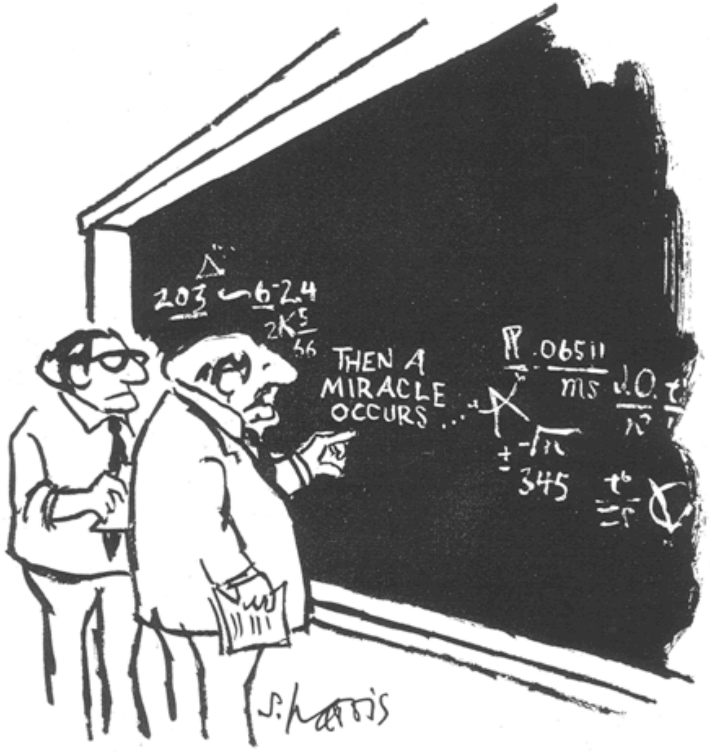
Predictions for phase shifts (long-range@NNLO, R=0.5fm)



For an analytical model see: EE, Gegelia, EPJA 41 (2009) 341

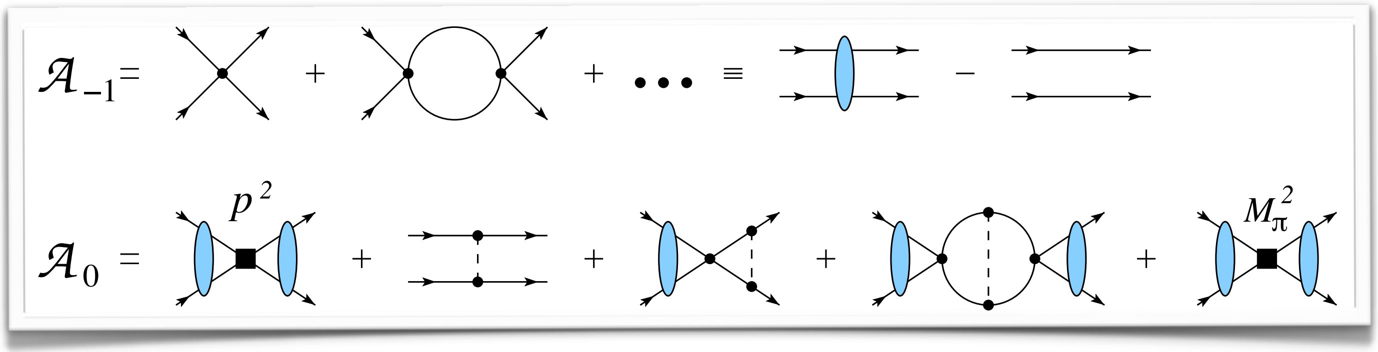
One-pion exchange:
perturbative or nonperturbative?

Equipped with these tools, one can rigorously test the proper inclusion of the long-range force in various pion-full formulations (Trust but verify… ☺)



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

KSW approach (perturbative pions)



Low Energy Theorems at NLO   Cohen, Hansen '99

$$k\ \cot\delta = -a^{-1} + \tfrac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \ldots$$

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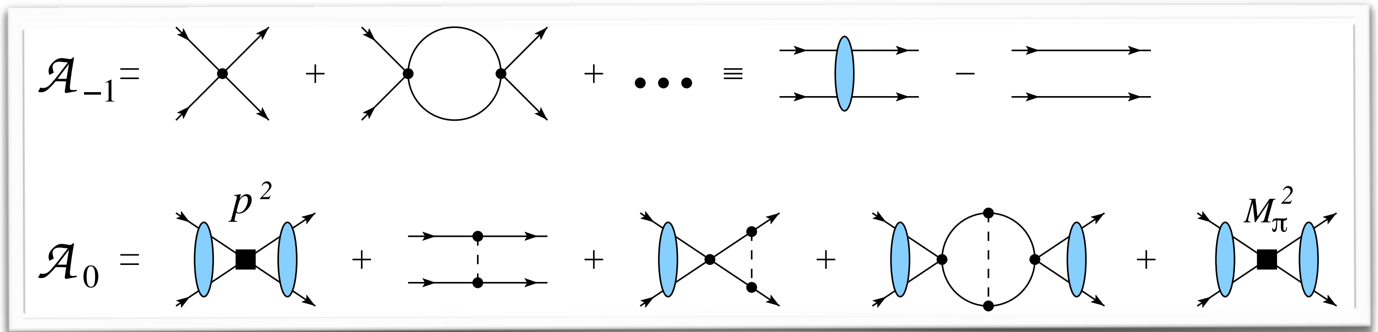

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$$k \cot\delta = -a^{-1} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \ldots$$

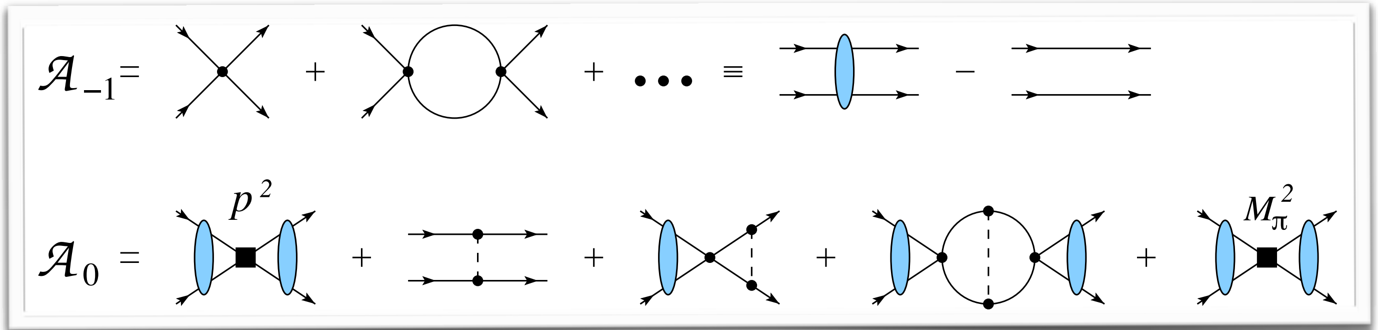
$$v_2 = \frac{g_A^2 m}{16\pi F_\pi^2}\left(-\frac{16}{3a^2 M_\pi^4} + \frac{32}{5a M_\pi^3} - \frac{2}{M_\pi^2}\right)$$

$$v_3 = \frac{g_A^2 m}{16\pi F_\pi^2}\left(-\frac{16}{3a^2 M_\pi^6} - \frac{128}{7a M_\pi^5} + \frac{16}{3 M_\pi^4}\right)$$

	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)
theory	-3.3	18.	-108.	-0.95	4.6	-25.
NPWA	-0.5	4.0	-20.	0.04	0.67	-4.0

spin-singlet | spin-triplet

KSW approach (perturbative pions)



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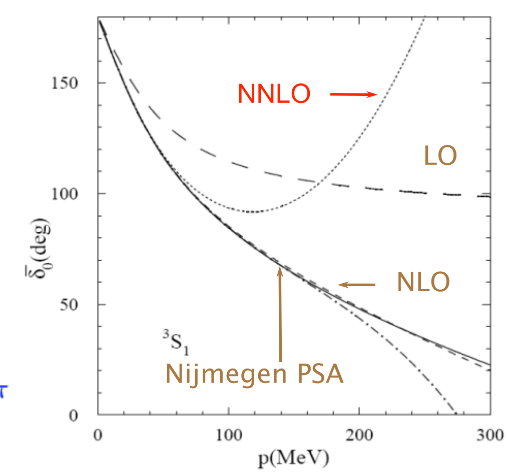
$$v_2 = \frac{g_A^2 m}{16\pi F_\pi^2}\left(-\frac{16}{3a^2M_\pi^4} + \frac{32}{5aM_\pi^3} - \frac{2}{M_\pi^2}\right)$$

$$v_3 = \frac{g_A^2 m}{16\pi F_\pi^2}\left(-\frac{16}{3a^2M_\pi^6} - \frac{128}{7aM_\pi^5} + \frac{16}{3M_\pi^4}\right)$$

	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)
theory	-3.3	18.	-108.	-0.95	4.6	-25.
NPWA	-0.5	4.0	-20.	0.04	0.67	-4.0

spin-singlet   spin-triplet

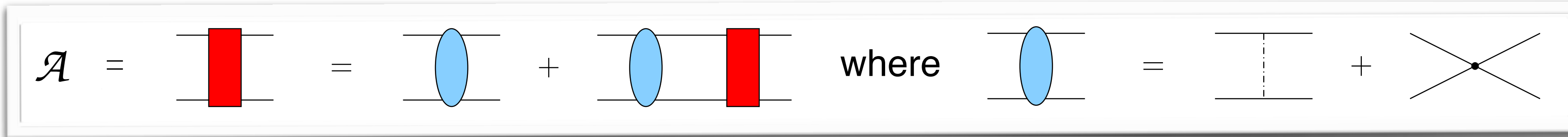


Higher-order calculations also show problems in S=1 channels

Mehen, Stewart '00

→ it seems necessary to treat pions non-perturbatively at $p \sim M_\pi$

W. approach (non-perturbative pions)



Low Energy Theorems: perturbative vs nonperturbative OPE

(cutoff-independent results from EE, Gegelia PLB (2012))

$^1S_0$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
KSW	fit	fit	-3.3	18	-108
Weinberg	fit	1.50	-1.9	8.6(8)	$-37(10)$
Nijmegen PWA	-23.7	2.67	-0.5	4.0	-20

$^3S_1$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
KSW	fit	fit	-0.95	4.6	-25
Weinberg	fit	1.60	-0.05	0.8(1)	$-4(1)$
Nijmegen PWA	5.42	1.75	0.04	0.67	-4.0

Notice: Lippmann-Schwinger eq. with OPE potential is non-renormalizable $\longrightarrow$ calculations are to be done using a finite cutoff. Cutoff-independent results can be achieved in a semi-relativistic version of LS eq.

Few-N in χ EFT: W approach in a nutshell

- Write down the most general effective Lagrangian for pions and nucleons

$$\mathcal{L}^{(1)}_{\pi N} = N^\dagger \left[i\partial_0 - \frac{g_A}{2F}\boldsymbol{\tau}\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi} - \frac{1}{4F^2}\boldsymbol{\tau}\times\boldsymbol{\pi}\cdot\dot{\boldsymbol{\pi}} + \frac{g_A}{4F^3}\Big((4\alpha-1)\boldsymbol{\tau}\cdot\boldsymbol{\pi}(\boldsymbol{\pi}\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi}) + 2\alpha\pi^2(\boldsymbol{\tau}\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi})\Big) + \ldots \right] N$$

$$\mathcal{L}^{(2)}_{\pi N} = N^\dagger \left[4M^2 c_1 - \frac{2c_1}{F^2}M^2\pi^2 + \frac{c_2}{F^2}\dot{\pi}^2 + \frac{c_3}{F^2}(\partial_\mu\boldsymbol{\pi})\cdot(\partial^\mu\boldsymbol{\pi}) - \frac{c_4}{4F^2}(\boldsymbol{\tau}\vec{\sigma}\times\vec{\nabla}\boldsymbol{\pi})\cdot\vec{\nabla}\boldsymbol{\pi} + \ldots \right] N$$

$$\mathcal{L}^{(0)}_{NN} = \frac{1}{2}C_S N^\dagger N\, N^\dagger N + \frac{1}{2}C_S N^\dagger\vec{\sigma}N\cdot N^\dagger\vec{\sigma}N$$

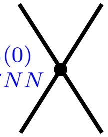
...

- Naively, power counting for a N-nucleon connected Feynman graph is:
  Weinberg '90

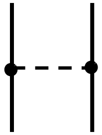
$$\nu = 2 - N + 2L + \sum_i V_i \Delta_i \quad \text{where} \quad \Delta_i = -2 + \frac{1}{2}n_i + d_i$$

power of Q *# of loops* *# of vertices of type Δ_i*

Examples:

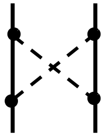


$\mathcal{L}^{(0)}_{NN}$ $\sim Q^0$



$\mathcal{L}^{(1)}_{\pi N}$ $\sim Q^0$

ν = 2 [derivatives]
– 2 [π-propagator]

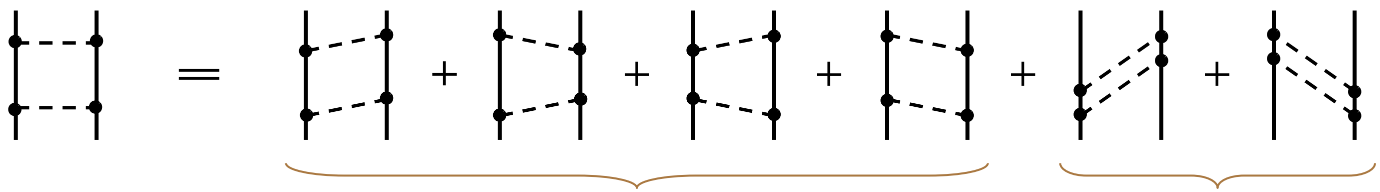


$\mathcal{L}^{(1)}_{\pi N}$ $\sim Q^2$

ν = 4 [loop int.]
+ 4 [derivatives]
– 4 [2 π-propagators]
– 2 [2 HB nucl. propagators]

Few-N in χEFT: W approach in a nutshell

- But... If true, NN scattering would be perturbative!
  Diagrams involving NN cuts (i.e. reducible) are enhanced (IR divergent in the $m_N \to \infty$ limit)



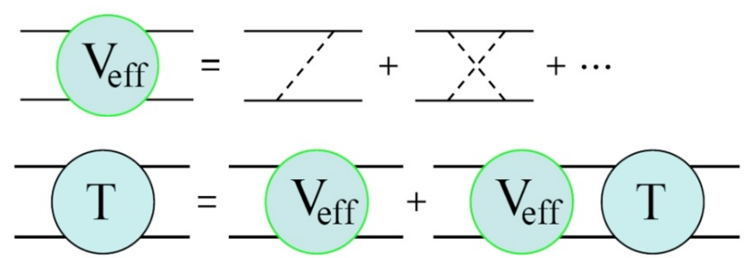
reducible, enhanced:

$$\frac{1}{E_{NN}-E_\Psi} = \frac{m_N}{\vec{p}^{\,2} - \vec{q}^{\,2}} \sim \frac{m_N}{Q^2} \gg \frac{1}{Q}$$

irreducible:

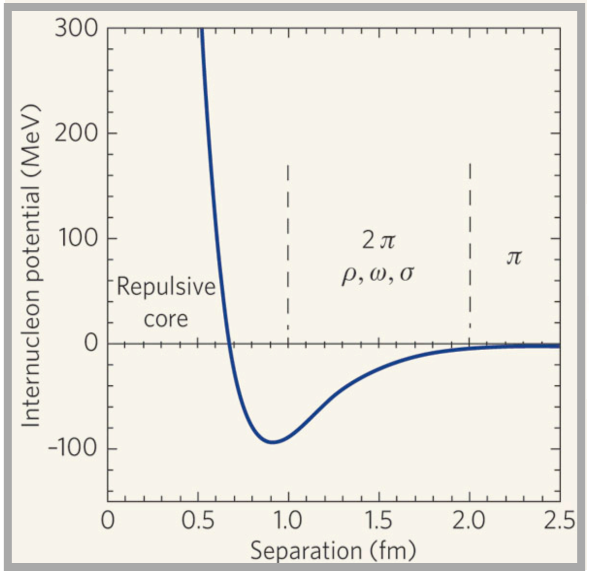
$$\frac{1}{E_{NN}-E_\Psi} \sim \frac{1}{M_\pi} \sim \frac{1}{Q}$$

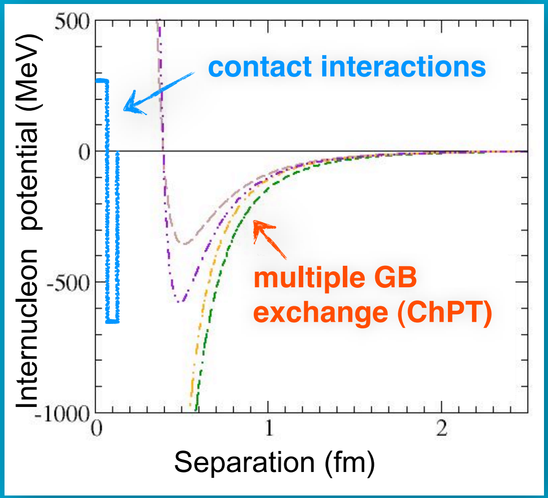
- **Weinberg's approach**
  - Use ChPT to compute irreducible graphs = nuclear forces/currents
  - Resum enhanced reducible graphs by solving the Schrödinger/LS eq.



$$\boxed{\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle}$$

From effective Lagrangian to nuclear forces





- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei