Scuola
di Fisica Nucleare
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RUB

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Modern Theory of nuclear forces

Lectures 1+2: Foundations

Lecture 3: Foundations (cont.) + state of the art for NN force

Lecture 4: Many-body forces & nuclear lattice simulations

Summary of parts I + II

- Effective field theories aim to describe phenomena in a certain energy range/distance scale. Crucial: use the proper degrees of freedom and exploit the symmetries.

- Low-energy interactions of pions can be systematically described in Chiral Perturbation Theory (the EFT of QCD).

- NN interaction is strong, need some resummation beyond perturbation theory.

- NN at very low momenta $Q \ll M_\pi$ can be described by pionless EFT (~ ERE).

- To go to higher energies one needs to include pions. There is evidence that OPEP is nonperturbative in certain spin-triplet channels.

Today:

- Few-N in chiral EFT: Weinberg‘s approach in a nutshell
- From effective Lagrangians to nuclear forces: Method of Unitary Transformation
- Chiral expansion for the 2N force: State of the art

Few-N in χEFT: W approach in a nutshell

- Write down the most general effective Lagrangian for pions and nucleons

$$\mathcal{L}_{\pi N}^{(1)} = N^\dagger \Big[i\partial_0 - \frac{g_A}{2F}\boldsymbol{\tau}\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi} - \frac{1}{4F^2}\boldsymbol{\tau}\times\boldsymbol{\pi}\cdot\dot{\boldsymbol{\pi}} + \frac{g_A}{4F^3}\Big((4\alpha-1)\boldsymbol{\tau}\cdot\boldsymbol{\pi}(\boldsymbol{\pi}\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi}) + 2\alpha\pi^2(\boldsymbol{\tau}\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi})\Big) + \ldots \Big] N$$

$$\mathcal{L}_{\pi N}^{(2)} = N^\dagger \Big[4M^2 c_1 - \frac{2c_1}{F^2}M^2\pi^2 + \frac{c_2}{F^2}\dot{\pi}^2 + \frac{c_3}{F^2}(\partial_\mu\boldsymbol{\pi})\cdot(\partial^\mu\boldsymbol{\pi}) - \frac{c_4}{4F^2}(\boldsymbol{\tau}\vec{\sigma}\times\vec{\nabla}\boldsymbol{\pi})\cdot\vec{\nabla}\boldsymbol{\pi} + \ldots \Big] N$$

$$\mathcal{L}_{NN}^{(0)} = \frac{1}{2}C_S N^\dagger N\, N^\dagger N + \frac{1}{2}C_S N^\dagger\vec{\sigma}N\cdot N^\dagger\vec{\sigma}N$$

...

Few-N in χEFT: W approach in a nutshell

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...

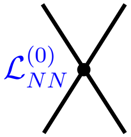
- Naively, power counting for a N-nucleon connected Feynman graph is:
  Weinberg '90

# of derivatives

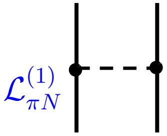
$$\nu = 2 - N + 2L + \sum_i V_i\Delta_i \quad \text{where} \quad \Delta_i = -2 + \frac{1}{2}n_i + d_i$$

power of Q *# of loops* *# of vertices of type Δᵢ* *# of nucleon field operators*

Examples:

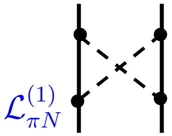


$\sim Q^0$



$\sim Q^0$

ν = 2 [derivatives]
− 2 [π-propagator]

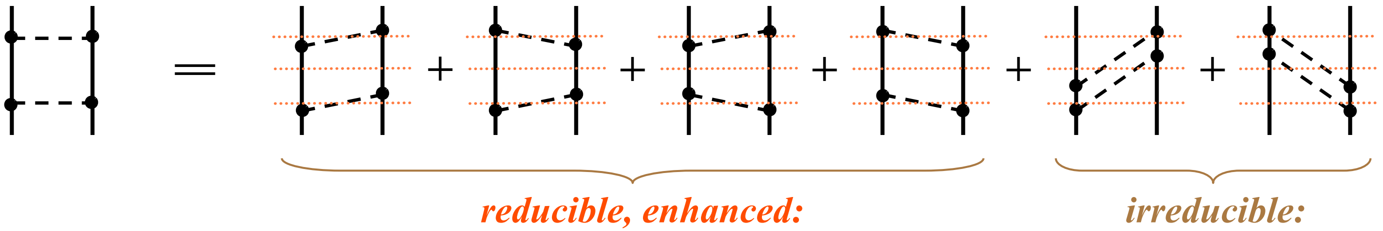


$\sim Q^2$

ν = 4 [loop int.]
+ 4 [derivatives]
− 4 [2 π-propagators]
− 2 [2 HB nucl. propagators]

Few-N in χEFT: W approach in a nutshell

- But... If true, NN scattering would be perturbative!
  Diagrams involving NN cuts (i.e. reducible) are enhanced (IR divergent in the $m_N \to \infty$ limit)



$$\frac{1}{E_{NN} - E_{\Psi}} = \frac{m_N}{\vec{p}^{\,2} - \vec{q}^{\,2}} \sim \frac{m_N}{Q^2} \gg \frac{1}{Q}$$

$$\frac{1}{E_{NN} - E_{\Psi}} \sim \frac{1}{M_\pi} \sim \frac{1}{Q}$$

Few-N in χEFT: W approach in a nutshell

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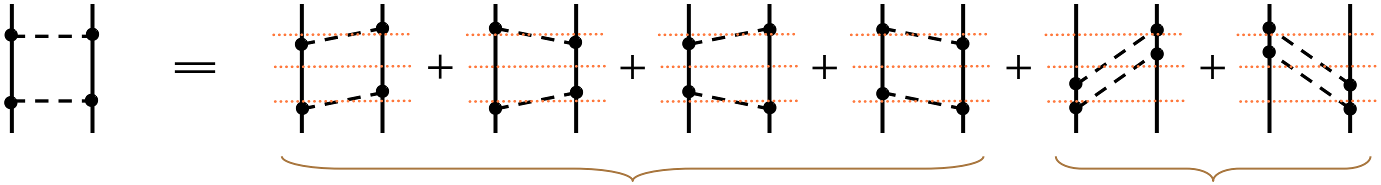
  Diagrams involving NN cuts (i.e. reducible) are enhanced (IR divergent in the $m_N \to \infty$ limit)

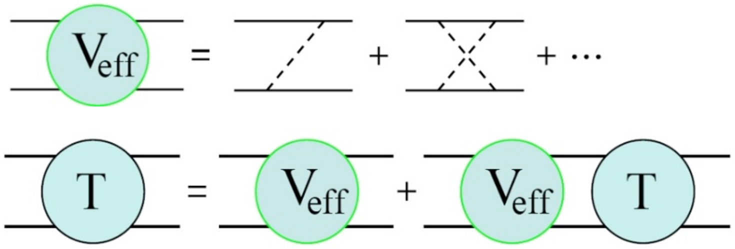


reducible, enhanced:

irreducible:

$$\frac{1}{E_{NN}-E_\Psi} = \frac{m_N}{\vec{p}^{\,2}-\vec{q}^{\,2}} \sim \frac{m_N}{Q^2} \gg \frac{1}{Q} \qquad \frac{1}{E_{NN}-E_\Psi} \sim \frac{1}{M_\pi} \sim \frac{1}{Q}$$

- **Weinberg's approach**
  - Use ChPT to compute irreducible graphs = nuclear forces/currents
  - Resum enhanced reducible graphs by solving the Schrödinger/LS eq.



$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

From effective Lagrangian to nuclear forces

see also lectures by Rocco

From $\mathcal{L}_{\mathrm{eff}}$ to nuclear forces

Complication: nuclear forces $\neq$ scattering amplitude

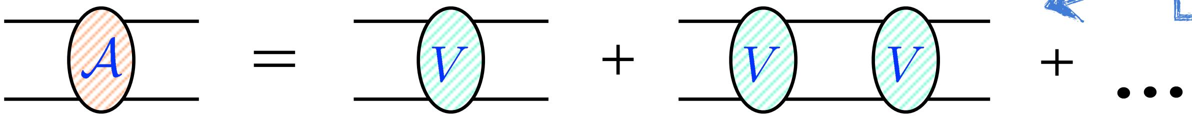
→ **scheme-dependent, renormalizable ??**

From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Complication: nuclear forces $\neq$ scattering amplitude

→ scheme-dependent, renormalizable ??

calculate in ChPT →

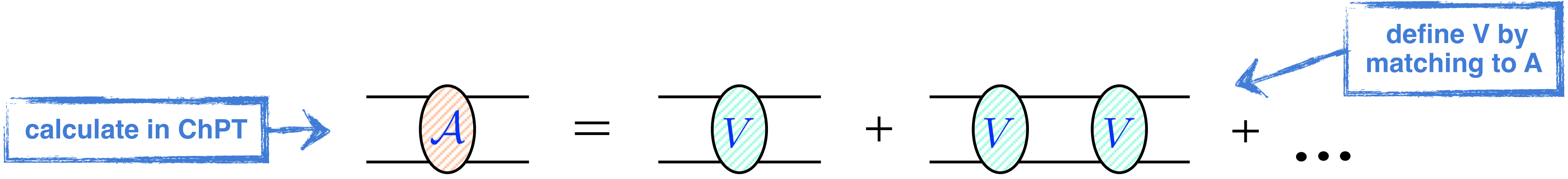


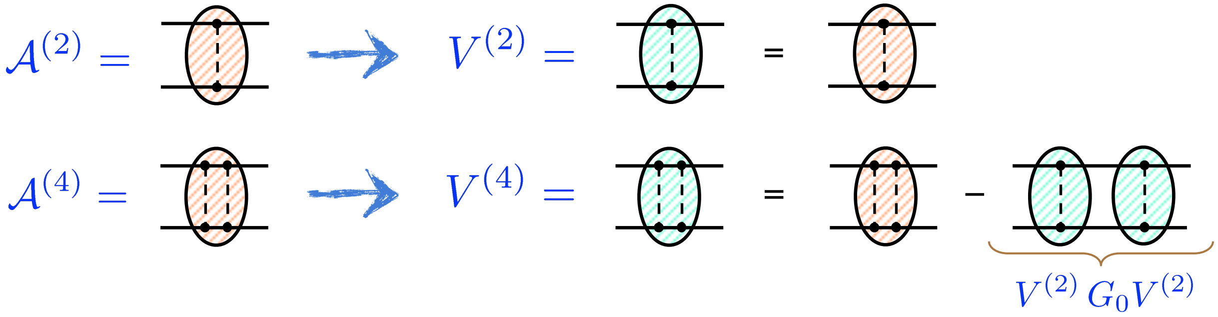
define V by matching to A

From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Complication: nuclear forces $\neq$ scattering amplitude

→ scheme-dependent, renormalizable ??

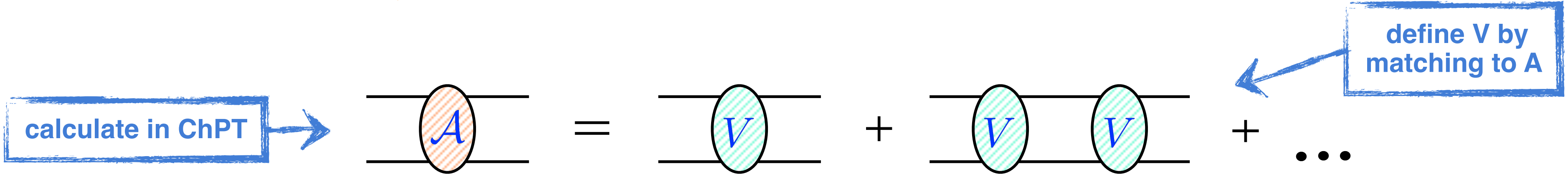




From $\mathcal{L}_{\text{eff}}$ to nuclear forces

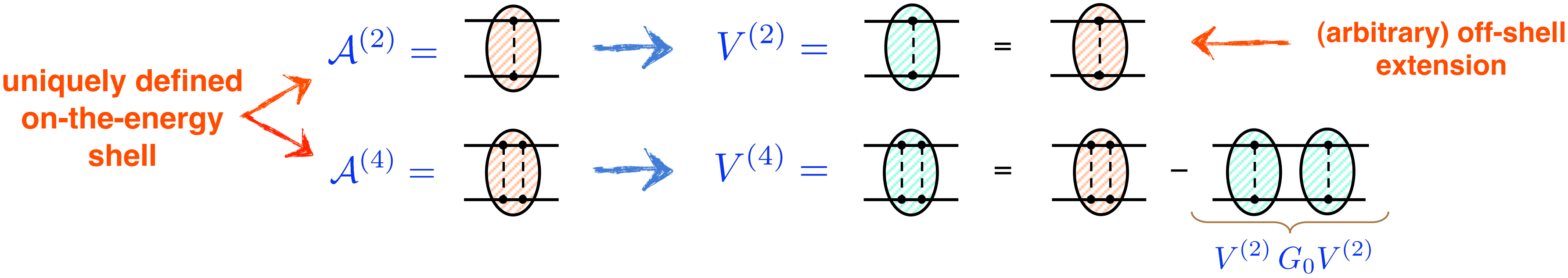
Complication: nuclear forces ≠ scattering amplitude

→ **scheme-dependent, renormalizable ??**



calculate in ChPT

define V by matching to A



uniquely defined on-the-energy shell

(arbitrary) off-shell extension

From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Complication: nuclear forces ≠ scattering amplitude

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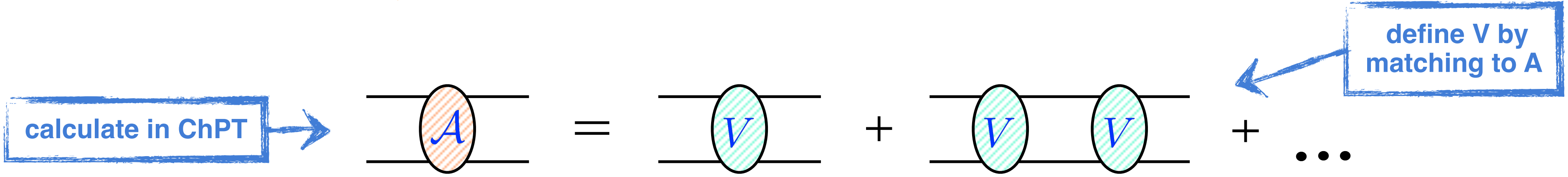

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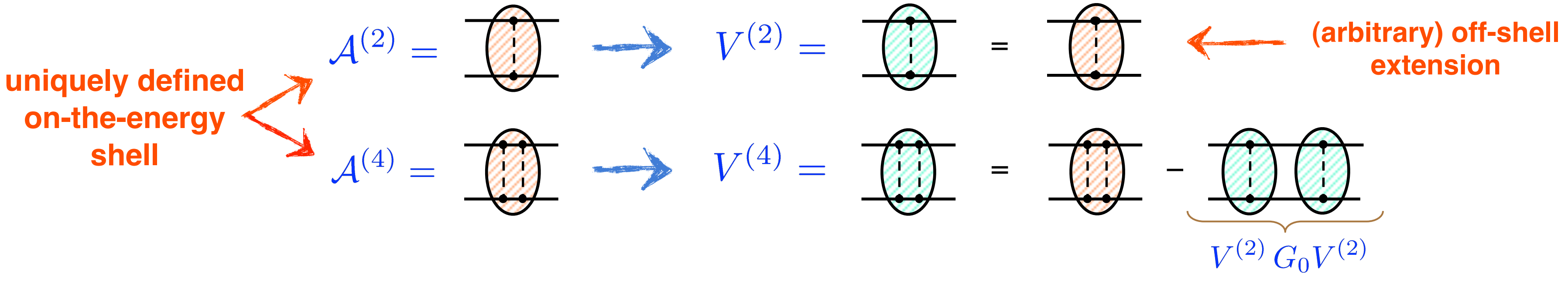
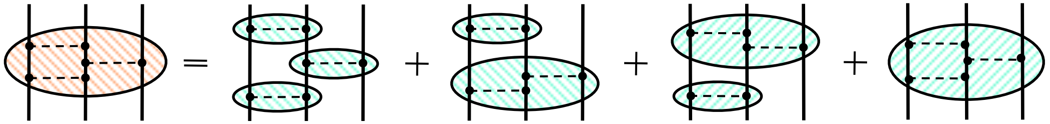


uniquely defined on-the-energy shell

(arbitrary) off-shell extension

- Higher-order terms in the Hamiltonian „know“ about the choice made for the off-shell extension (consistency...)
- Finite (=renormalized) matrix elements only for specific choices possible...

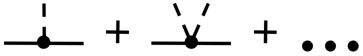


From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Method of unitary transformation

Taketani, Mashida, Ohnuma’52; Okubo ’54; EE, Glöckle, Meißner, Krebs, Kölling, ...

1. Canonical transformation & quantization: $\mathcal{L}_{\pi \mathrm{N}} \longrightarrow \mathcal{H}_{\pi \mathrm{N}} =$



nucleonic states $|N\rangle,\ |NN\rangle,\ \ldots$

EOM: $$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$$

projectors

states with mesons $|N\pi\rangle,\ |N\pi\pi\rangle,\ \ldots$

← can not solve
(infinite-dimensional eq.)

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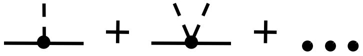

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projectors

states with mesons $|N\pi\rangle, \; |N\pi\pi\rangle, \; \ldots$

⟵ can not solve (infinite-dimensional eq.)

2. Decouple pions via a suitable UT: $\tilde H \equiv U^\dagger \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} U = \begin{pmatrix} \eta \tilde H \eta & 0 \\ 0 & \lambda \tilde H \lambda \end{pmatrix}$

A minimal parametrization of U: $U = \begin{pmatrix} \eta (1 + A^\dagger A)^{-1/2} & -A^\dagger (1 + A A^\dagger)^{-1/2} \\ A (1 + A^\dagger A)^{-1/2} & \lambda (1 + A A^\dagger)^{-1/2} \end{pmatrix}, \quad A = \lambda A \eta$

Okubo '54

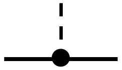
Require: $\eta \tilde H \lambda = \lambda \tilde H \eta = 0 \longrightarrow \boxed{\lambda \left(H - [A, \; H] - A H A \right) \eta = 0}$

The major problem is to solve the nonlinear decoupling equation.

Notice: similar methods widely used in nuclear & many-body physics (Lee-Suzuki)

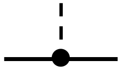
From L_{eff} to nuclear forces

Example: expansion in powers of the coupling constant

$H_I =$  $\propto g$ → ansatz: $A = A^{(1)} + A^{(2)} + A^{(3)} + \ldots$

From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Example: expansion in powers of the coupling constant

$H_I =$  $\propto g \quad \longrightarrow$ ansatz: $A = A^{(1)} + A^{(2)} + A^{(3)} + \ldots$

Recursive solution of the decoupling equation $\lambda \, (H - [A, \; H] - A H A) \, \eta = 0$

$$g^1 : \quad \lambda(H_I - [A^{(1)}, \; H_0])\eta = 0 \qquad \longrightarrow \qquad A^{(1)} = -\lambda \frac{H_I}{E_\eta - E_\lambda} \eta$$

$$g^2 : \quad \lambda(H_I \, A^{(1)} - [A^{(2)}, \; H_0])\eta = 0 \qquad \longrightarrow \qquad A^{(2)} = -\lambda \frac{H_I \, A^{(1)}}{E_\eta - E_\lambda} \eta$$

$$\ldots$$

In the static approximation, i.e. in the limit $m \to \infty$, one has: $E_\eta - E_\lambda \sim E_\pi$.

From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Example: expansion in powers of the coupling constant

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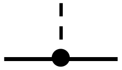

$\propto g \longrightarrow$ ansatz: $A = A^{(1)} + A^{(2)} + A^{(3)} + \ldots$

Recursive solution of the decoupling equation $\lambda \left(H - [A,\ H] - AHA \right) \eta = 0$

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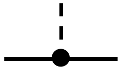
- **LO:** $V_{\text{eff}}^{(2)} \;=\; -\,\eta H_I \frac{\lambda}{E_\pi} H_I \eta$

From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Example: expansion in powers of the coupling constant

$H_I =$  $\propto g \longrightarrow$ ansatz: $A = A^{(1)} + A^{(2)} + A^{(3)} + \ldots$

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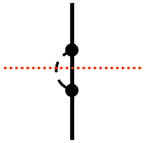
$$\cdots$$

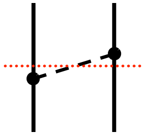
In the static approximation, i.e. in the limit $m \to \infty$, one has: $E_\eta - E_\lambda \sim E_\pi$.

- **LO:** $V_{\text{eff}}^{(2)} \; = \; - \eta H_I \frac{\lambda}{E_\pi} H_I \eta$ Taking the LO πN vertex from $\mathcal{L}^{(1)}_{\pi \text{N}}$, $\frac{g_A}{2F_\pi} \tau_i \vec{\sigma} \cdot \vec{q}$, one gets:

1-nucleon operator
(renormalization of m_N)



2-nucleon operator
(one-pion exchange potential)



$$V_{1\pi} = - \left(\frac{g_A}{2F_\pi} \right)^2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

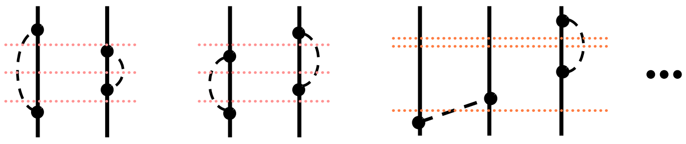
From L_{eff} to nuclear forces

- **NLO:** $V_{\text{eff}}^{(2)} \;=\; -\,\eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta \;+\; \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta \;+\; \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta$

From $\mathcal{L}_{\text{eff}}$ to nuclear forces

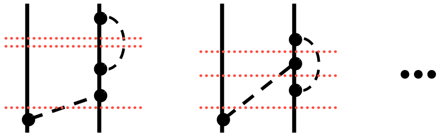
- **NLO:** $V_{\text{eff}}^{(2)} = -\,\eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta + \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta + \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta$

1. All disconnected contributions to 2N, 3N and 4N operators dissapear (general feature in the method of UT; not automatically the case in TOPT)

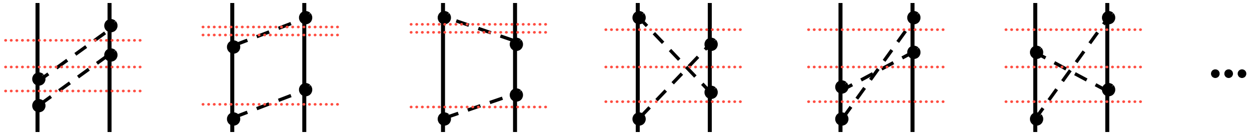


2. 1N contribution again only leads to renormalization of the nucleon mass

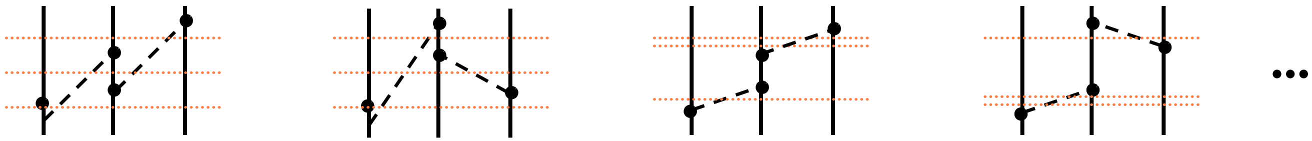
3. 1-loop contributions to the OPE 2N potential do not produce any new structures (renormalization of $m_N,\ g_A,\ F_\pi$) EE, Glöckle, Meißner '02



4. Two-pion exchange 2N potential

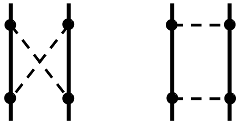


5. Two-pion exchange 3N potential vanishes



From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Example: chiral 2π-exchange potential proportional to $g_A{}^4$:



$$V_{2\pi}^{(2)}(q) = \; -\eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta \; + \; \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta \; + \; \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta \; .$$

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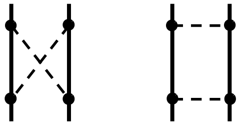

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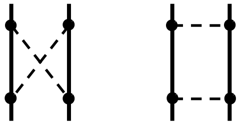
$$= - \frac{g_A^4}{2(2F_\pi)^4} \int \frac{d^3 l}{(2\pi)^3} \frac{\omega_+^2 + \omega_+ \omega_- + \omega_-^2}{\omega_+^3 \omega_-^3 (\omega_+ + \omega_-)} \left\{ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left(\vec{l}^{\,2} - \vec{q}^{\,2} \right)^2 + 6 (\vec{\sigma}_2 \cdot [\vec{q} \times \vec{l}\,]) (\vec{\sigma}_1 \cdot [\vec{q} \times \vec{l}\,]) \right\}$$

$$\omega_\pm = \sqrt{(\vec{q} \pm \vec{l}\,) + 4 M_\pi^2}$$

From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Example: chiral 2π-exchange potential proportional to $g_A{}^4$:



$$V^{(2)}_{2\pi}(q) = -\eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta + \frac{1}{2}\eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta + \frac{1}{2}\eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta \, .$$

$$= -\frac{g_A^4}{2(2F_\pi)^4}\int \frac{d^3l}{(2\pi)^3}\frac{\omega_+^2+\omega_+\omega_-+\omega_-^2}{\omega_+^3\omega_-^3(\omega_++\omega_-)}\left\{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(\vec{l}^{\,2}-\vec{q}^{\,2}\right)^2+6(\vec{\sigma}_2\cdot[\vec{q}\times\vec{l}\,])(\vec{\sigma}_1\cdot[\vec{q}\times\vec{l}\,])\right\}$$

$$\omega_\pm = \sqrt{(\vec{q}\pm\vec{l}\,)+4M_\pi^2}$$

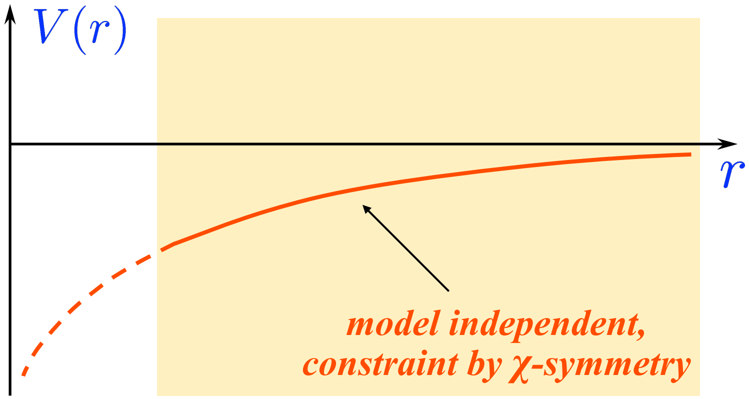
$$= -\frac{g_A^4}{384\pi^2F_\pi^4}\left[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(20M_\pi^2+23q^2+\frac{48M_\pi^4}{4M_\pi^2+q^2}\right)-18\left(\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}-q^2\,\vec{\sigma}_1\cdot\vec{\sigma}_2\right)\right]L(q)+\ldots$$

where the loop function is given by (in DR):

$$L(q)=\frac{1}{q}\sqrt{4M_\pi^2+q^2}\,\ln\frac{\sqrt{4M_\pi^2+q^2}+q}{2M_\pi}$$

The integral has logarithmic and quadratic divergences can be absorbed into short-range terms:

$$V_{\rm cont} = (\alpha_1+\alpha_2\,q^2)\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+\alpha_3(\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}\,)\,\cdot$$
$$+\,\alpha_4(\vec{\sigma}_1\cdot\vec{\sigma}_2)\,q^2$$



From $\mathcal{L}_{\mathrm{eff}}$ to nuclear forces

So far, we assumed an expansion in powers of the coupling constant. In chiral EFT, we are doing an expansion in powers of the soft scales ($Q \sim M_\pi$).

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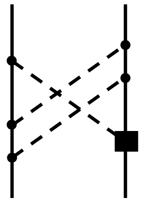
Recall: chiral power counting for N-nucleon connected irreducible diagrams:



$\sim \left(\frac{Q}{\Lambda}\right)^\nu$ where $\boxed{\nu = 2 - N + 2L + \sum_i V_i \Delta_i}$ and $\Delta_i = -2 + \frac{1}{2} n_i + d_i$

# of derivatives

# of loops

# of vertices of type Δ_i

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Perfect for diagrams, but inconvenient for solving $\lambda \left(H - [A,\ H] - A H A\right) \eta = 0$

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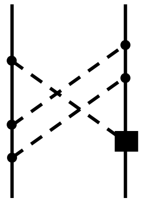

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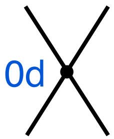
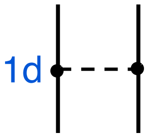
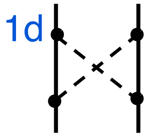
Let's rewrite the power counting in a more suitable way. Trick: count the powers of the *hard* scale Λ rather than the soft scale Q. Given that the only way for Λ to emerge is through the LECs of the effective Lagrangian, the power ν is given by:

$$\boxed{\nu = -2 + \textstyle\sum_i V_i \kappa_i}$$

where κ is an inverse mass dimension of the coupling constant of a vertex i.

$$\mathcal{L}_i = c_i \, (N^\dagger (\ldots) N)^{\frac{n_i}{2}} \, \pi^{p_i} \, (\partial_\mu, M_\pi)^{d_i} \longrightarrow [c_i] = (mass)^{-\kappa_i} \quad \text{with} \quad \kappa_i = d_i + \tfrac{3}{2} n_i + p_i - 4$$

From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Examples:	 $\sim Q^0$	 $\sim Q^0$ ν = 2 [derivatives] − 2 [π-propagator]	 $\sim Q^2$ ν = 4 [loop int.] + 4 [derivatives] − 4 [2 π-propagators] − 2 [2 HB nucl. prop.]
$\Delta_i = -2 + \frac{1}{2} n_i + d_i$ $\nu = 2 - N + 2L + \sum_i V_i \Delta_i$	Δ = -2 + 2 + 0 = 0 ν = 2 - 2 + 0 + 0 = 0	Δ = -2 + 1 + 1 = 0 ν = 2 - 2 + 0 + 2*0 = 0	Δ = -2 + 1 + 1 = 0 ν = 2 - 2 + 2 + 4*0 = 2
$\kappa_i = d_i + \frac{3}{2} n_i + p_i - 4$ $\nu = -2 + \sum_i V_i \kappa_i$	κ = 0 + 6 + 0 - 4 = 2 ν = -2 + 2 = 0	κ = 1 + 3 + 1 - 4 = 1 ν = -2 + 2*1 = 0	κ = 1 + 3 + 1 - 4 = 1 ν = -2 + 4*1 = 2

Notice: chiral symmetry guarantees that only non-renormalizable interactions with $\kappa > 0$, i.e. the so called irrelevant interactions, appear in $\mathcal{L}_{\text{eff}}$ $\longrightarrow$ perturbative expansion for nuclear forces

From $\mathcal{L}_{\text{eff}}$ to nuclear forces

The new form of the power counting is ideally suited for derivation of the potential using the method of UT.

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We are looking for a unitary operator

$$U = \begin{pmatrix} \eta(1+A^\dagger A)^{-1/2} & -A^\dagger(1+AA^\dagger)^{-1/2} \\ A(1+A^\dagger A)^{-1/2} & \lambda(1+AA^\dagger)^{-1/2} \end{pmatrix} \quad \text{such that} \quad \tilde{H} \equiv U^\dagger H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$$

This leads to the decoupling equation: $\lambda\,(H - [A,\ H] - AHA)\,\eta = 0$

Once this equation is solved, the effective potential can be calculated via:

$$\tilde{V}_{\text{eff}}^{\text{UT}} = \eta(\tilde{H} - H_0) = \eta\Big[(1+A^\dagger A)^{-1/2}(H + A^\dagger H + HA + A^\dagger HA)(1+A^\dagger A)^{-1/2} - H_0\Big]\eta$$

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These expressions can be computed in parturbation theory by making expansion in inverse mass dimension of coupling constants in the effective pion-nucleon Hamiltonian:

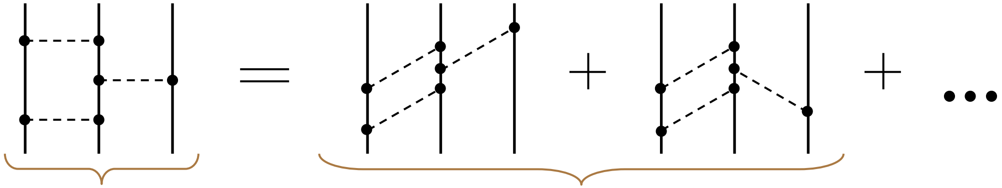
$$H_I = \sum_{\kappa=1}^{\infty} H^{(\kappa)} \quad \longrightarrow \quad \text{ansatz:} \quad A = \sum_{\alpha=1}^{\infty} A^{(\alpha)}$$

Recursive solution of the decoupling equation:

$$A^{(\alpha)} = -\frac{1}{E_\lambda} \lambda \Big[H^{(\alpha)} + \sum_{i=1}^{\alpha-1} H^{(i)} A^{(\alpha-i)} - \sum_{i=1}^{\alpha-1} A^{(\alpha-i)} H^{(i)} - \sum_{i=1}^{\alpha-2} \sum_{j=1}^{\alpha-j-1} A^{(i)} H^{(j)} A^{(\alpha-i-j)} \Big] \eta$$

$\longrightarrow \quad \tilde{V}_{\text{eff}}^{\text{UT}} = \ldots$ (can be straightforwardly implemented in e.g. FORM, MATHEMATICA, ...)

A note on renormalization...



1π-exchange should factorize out

time-ordered graphs

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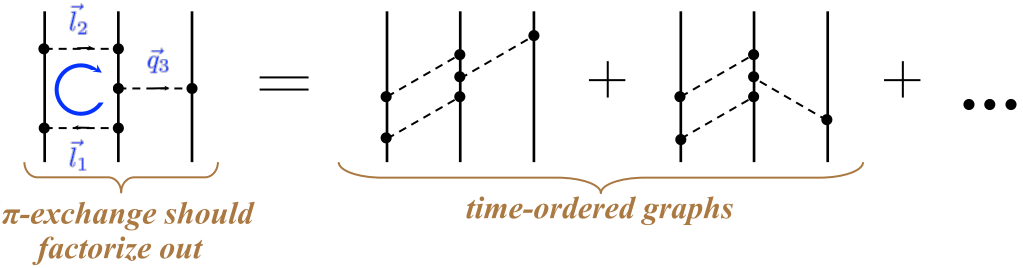

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time-ordered graphs

$$V = \ldots = \int d^3 l_1\, d^3 l_2\, \delta(\vec l_1 - \vec l_2 - \vec q_1)\, \Big[\ldots\Big]$$

$$\times \left[2\frac{\omega_1^2+\omega_2^2}{\omega_1^4\,\omega_2^4\,\omega_3^2} + \frac{8}{\omega_1^2\,\omega_2^2\,\omega_3^4} - \frac{\omega_1+\omega_2}{\omega_1^3\,\omega_2^3\,\omega_3^3} - \frac{2}{\omega_1^4\,\omega_2^2\,\omega_3\,(\omega_1+\omega_3)} - \frac{2}{\omega_1^2\,\omega_2^4\,\omega_3\,(\omega_2+\omega_3)} \right]$$

$$\sqrt{\vec l_{1,2}^{\,2} + M_\pi^2}$$

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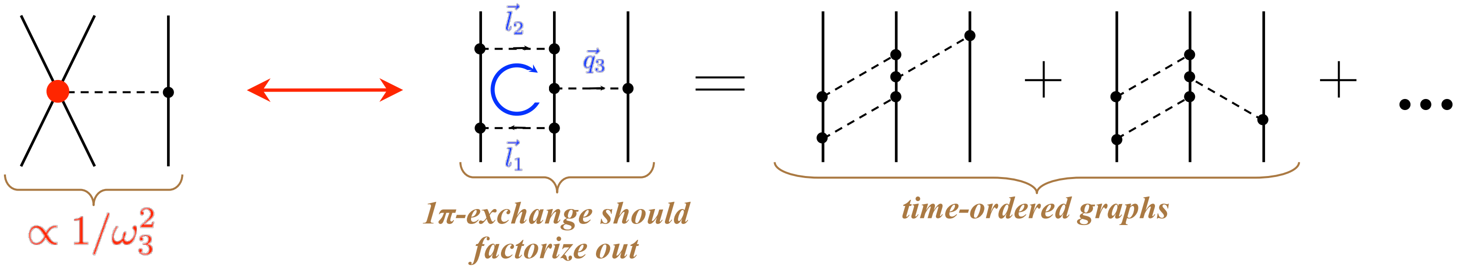

$\propto 1/\omega_3^2$

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⇨ cannot renormalize the potential !

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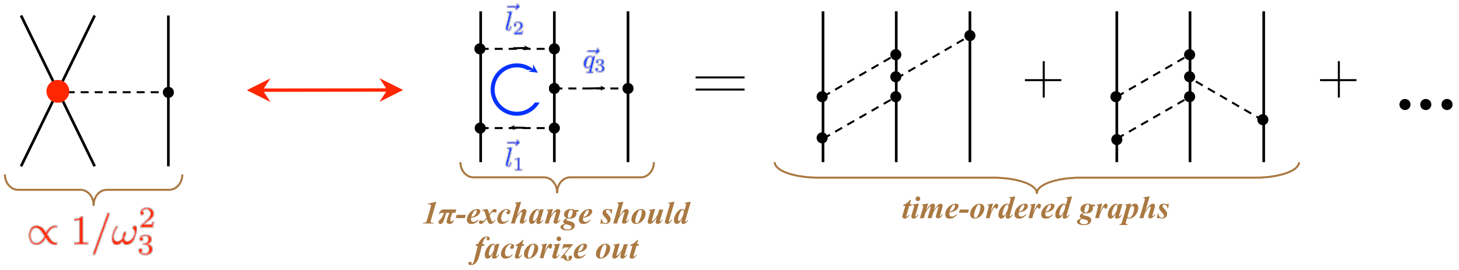

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$\sqrt{\vec{l}_{1,2}^{\,2} + M_\pi^2}$

⇒ cannot renormalize the potential !

Solution EE ’06

Nuclear potentials are not uniquely defined. Employing additional UTs in Fock space, it was (so far) always possible to maintain renormalizability at the level of the nuclear Hamiltonian. Same problem emerges for the current operators…

Summary of part III

- Weinberg‘s approach to nuclear chiral EFT: use ChPT to derive the potential & solve the Schrödinger eq. (nonperturbative resummations).
- Nuclear potentials can be derived from the effective chiral Lagrangian e.g. using the method of unitary transformation.

Next: (i) Chiral nuclear forces,
(ii) Nuclear lattice simulations