Evgeny Epelbaum, RUB

Nuclear Physics School 2013, Otranto, Italy, May 27-31, 2013

Modern Theory of nuclear forces

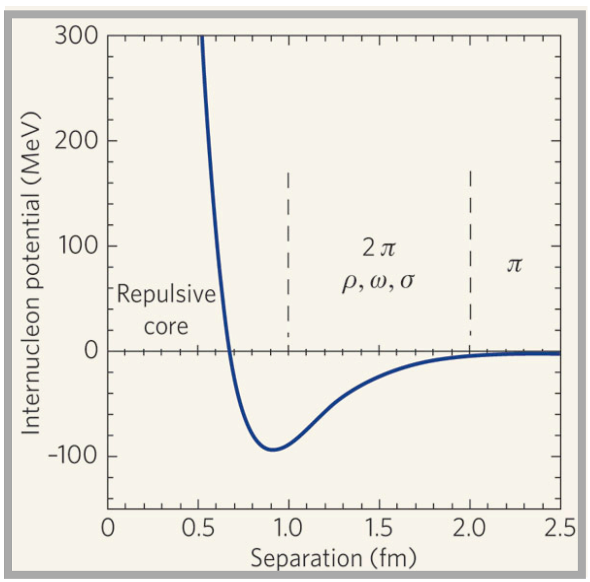
Lectures 1+2: Foundations

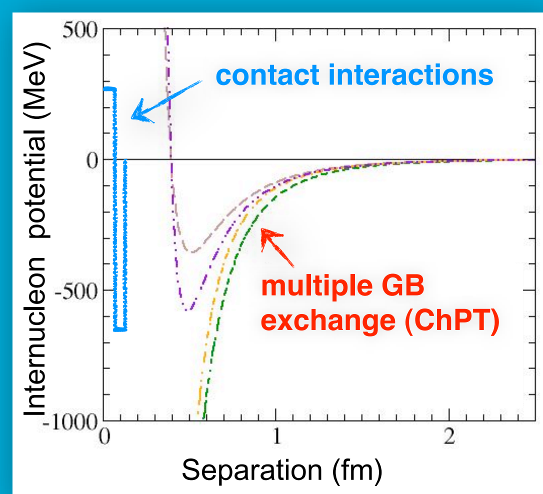
Lecture 3: Foundations (cont.) + derivation of nuclear forces

Lecture 4: (i) Chiral nuclear forces: State of the art
(ii) Nuclear lattice simulations

Intermediate summary: Nuclear chiral EFT a-la Weinberg

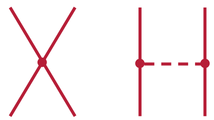
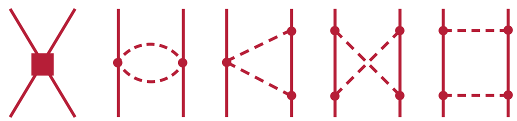
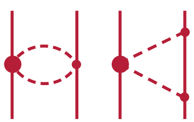
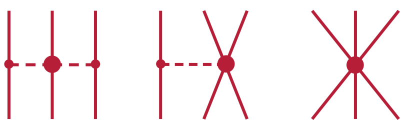
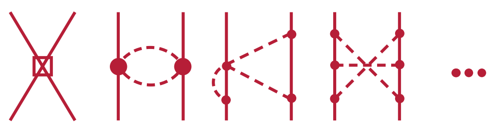
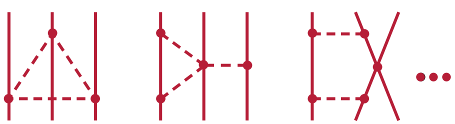
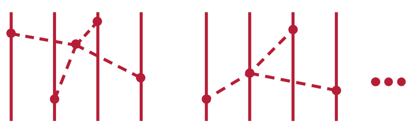
$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$





- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., …)
- precision physics with/from light nuclei

Chiral expansion of nuclear forces

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)		—	—
NLO (Q^2)		—	—
N²LO (Q^3)			—
N³LO (Q^4)			

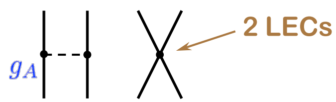
$\langle V_{2N} \rangle \sim 20\ \mathrm{MeV/pair}$ $\langle V_{3N} \rangle \sim 1\ \mathrm{MeV/triplet}$ $\langle V_{4N} \rangle \sim 0.1\ \mathrm{MeV/quartet}$

(numbers from Pudliner et al. PRL 74 (95) 4396)

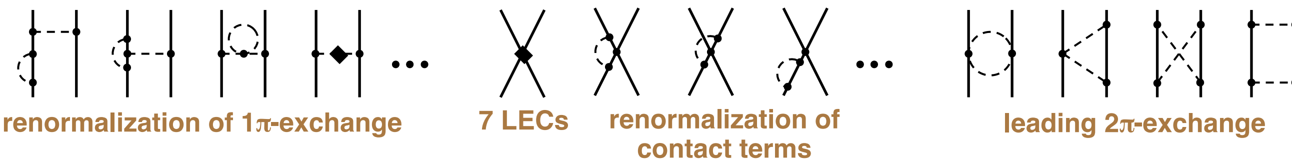
Nucleon-nucleon force up to N^{3}LO

Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; E.E. et al. ’98,‘03; Kaiser ’99-’01; Higa, Robilotta ’03; …

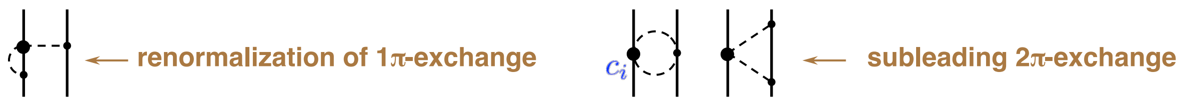
- **LO (Q^0):**



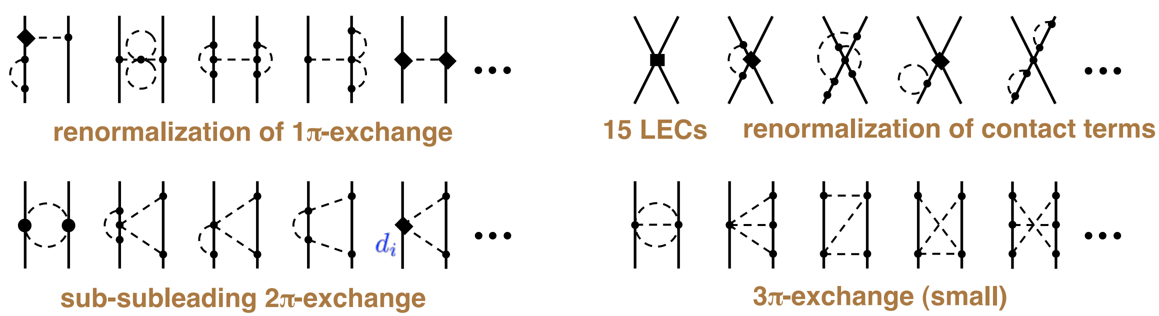
- **NLO (Q^2):**



- **N^{2}LO (Q^3):**



- **N^{3}LO (Q^4):**



+ isospin-breaking corrections…

van Kolck et al. ’93, ’96; Friar et al. ’99, ’03, ’04; Niskanen ’02; Kaiser ’06; E.E. et al. ’04,’05,’07; …

Nucleon-nucleon force up to N^{3}LO

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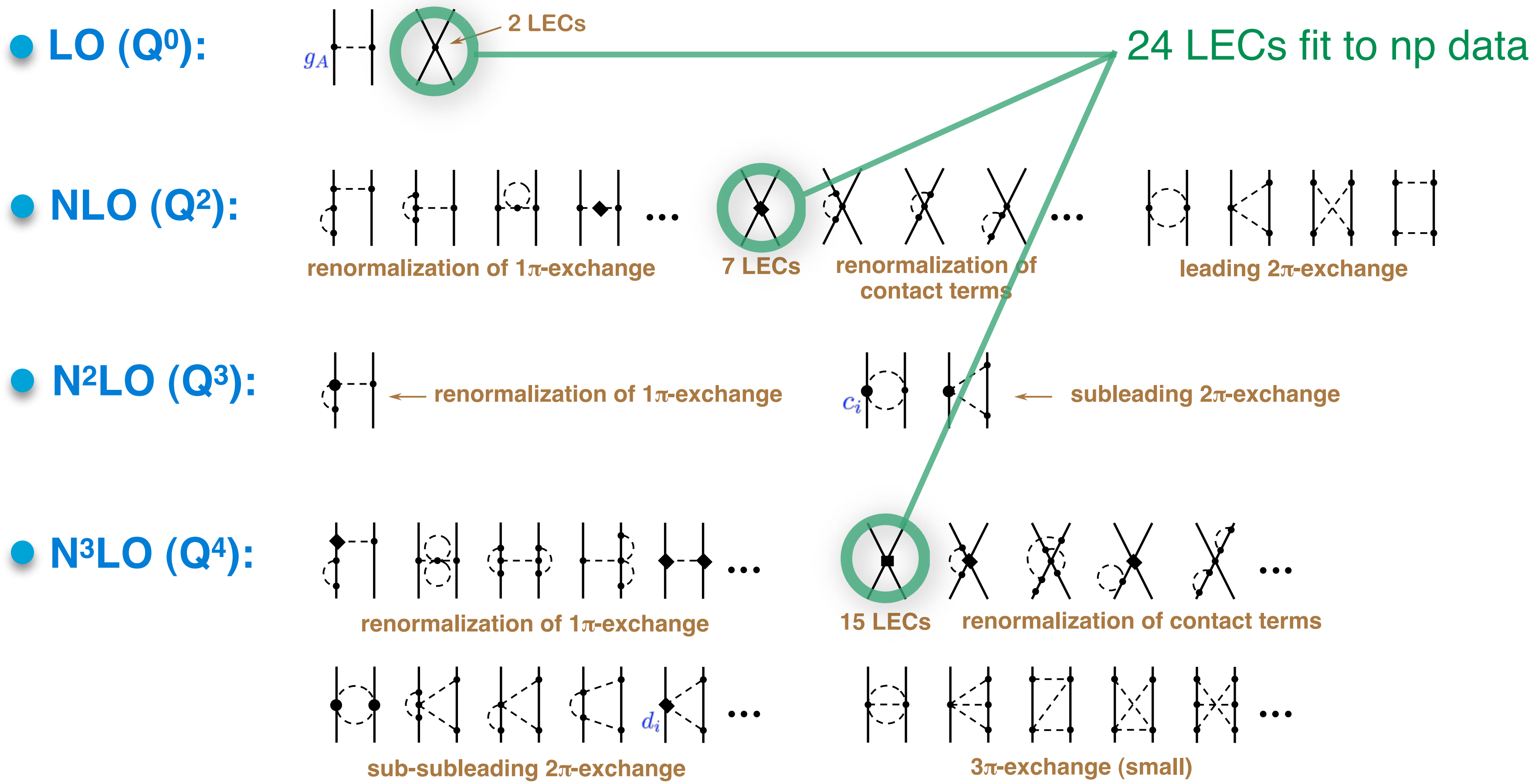

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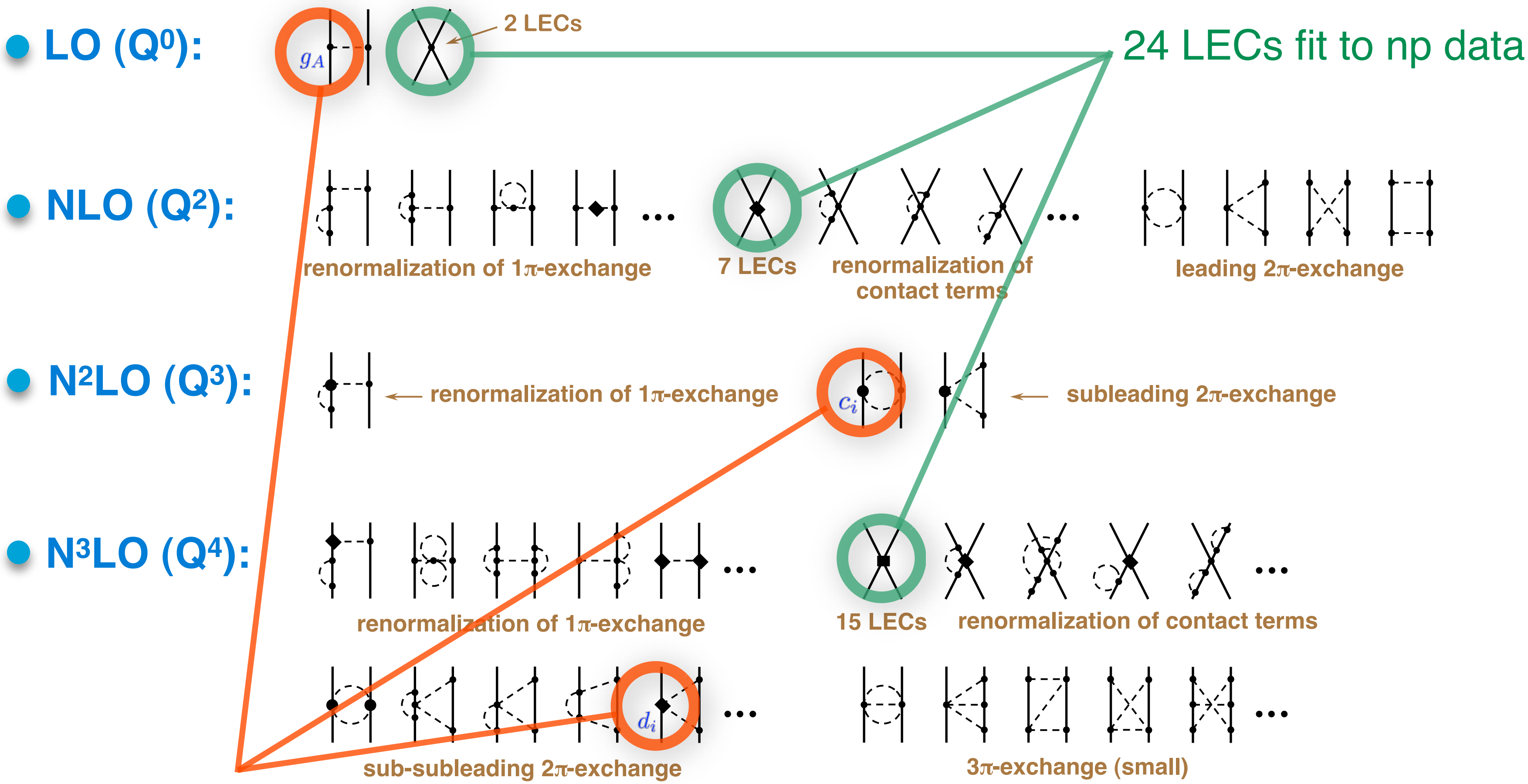


LECs fixed from πN

➙ long-range tail of the nuclear force fixed by chiral symmetry and exp. information on the πN system

\+ isospin-breaking corrections…

van Kolck et al. ’93, ’96; Friar et al. ’99, ’03, ’04; Niskanen ’02; Kaiser ’06; E.E. et al. ’04,’05,’07; …

Chiral 2π exchange (upto N²LO)

$$\mathcal{V}_{NN} = V_C(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_C(r) + \left[V_S(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_S(r)\right] \vec{\sigma}_1 \cdot \vec{\sigma}_2$$
$$+ \left[V_T(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_T(r)\right] (3\vec{\sigma}_1 \cdot \hat{r}\vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2) + \left[V_{LS}(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_{LS}(r)\right] \vec{L} \cdot \vec{S}\,,$$

Chiral 2π exchange (upto N²LO)

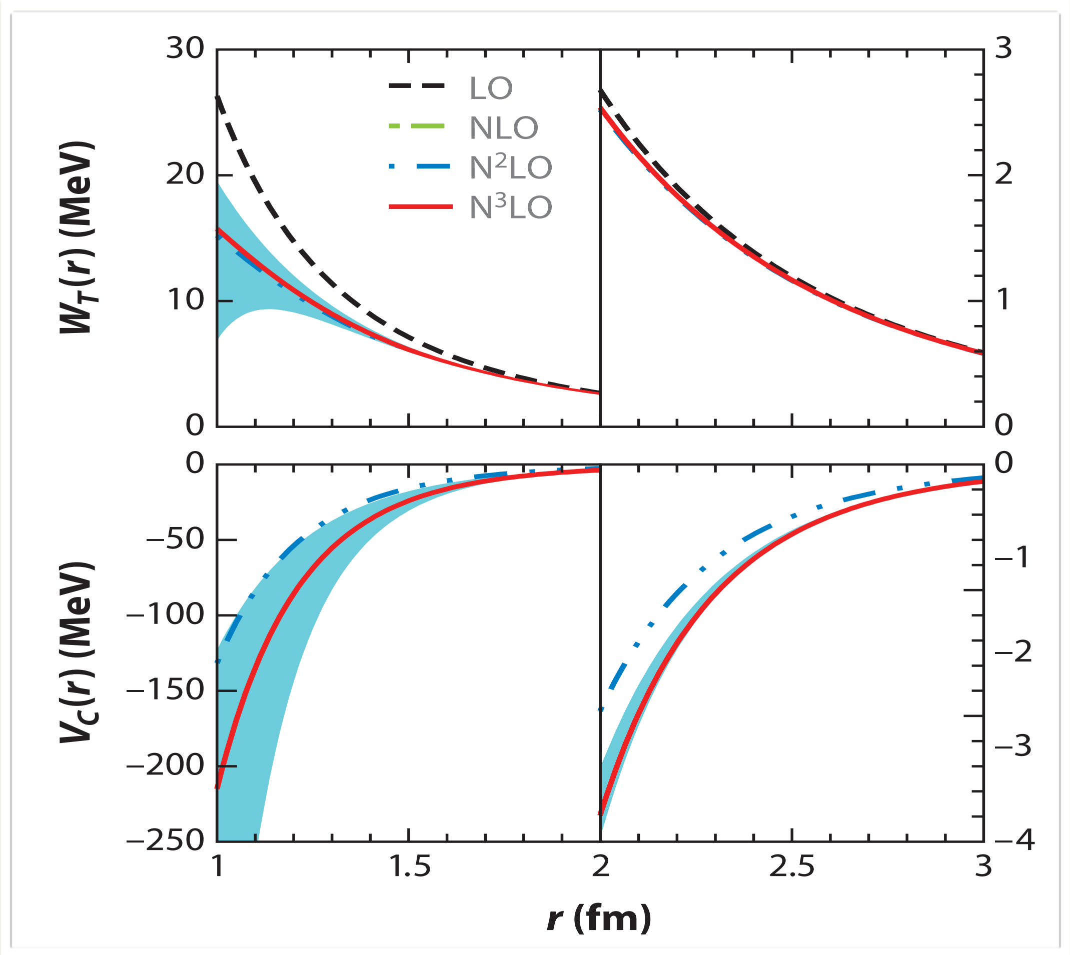
$$\begin{aligned}\mathcal{V}_{NN} &= V_C(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_C(r) \;+\; \left[V_S(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_S(r)\right]\vec{\sigma}_1\cdot\vec{\sigma}_2 \\ &+ \left[V_T(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_T(r)\right]\left(3\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2\right) \;+\; \left[V_{LS}(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_{LS}(r)\right]\vec{L}\cdot\vec{S}\,,\end{aligned}$$

The profile functions (in Dimensional Regularization)

$$\begin{aligned}
V_C^{TPE}(r) &= \frac{3g^2m^6}{32\pi^2f^4}\frac{e^{-2x}}{x^6}\left\{\left(2c_1+\frac{3g^2}{16M}\right)x^2(1+x)^2+\frac{g^5x^5}{32M}+\left(c_3+\frac{3g^2}{16M}\right)(6+12x+10x^2+4x^3+x^4)\right\}\\
W_T^{TPE}(r) &= \frac{g^2m^6}{48\pi^2f^4}\frac{e^{-2x}}{x^6}\left\{-\left(c_4+\frac{1}{4M}\right)(1+x)(3+3x+x^2)+\frac{g^2}{32M}\left(36+72x+52x^2+17x^3+2x^4\right)\right\},\\
V_T^{TPE}(r) &= \frac{g^4m^5}{128\pi^3f^4x^4}\left\{-12K_0(2x)-(15+4x^2)K_1(2x)+\frac{3\pi me^{-2x}}{8Mx}\left(12x^{-1}+24+20x+9x^2+2x^3\right)\right\},\\
W_C^{TPE}(r) &= \frac{g^4m^5}{128\pi^3f^4x^4}\left\{\left[1+2g^2(5+2x^2)-g^4(23+12x^2)\right]K_1(2x)+x\left[1+10g^2-g^4(23+4x^2)\right]K_0(2x)\,,\right.\\
&\left.+\frac{g^2m\pi e^{-2x}}{4Mx}\left[2(3g^2-2)\left(6x^{-1}+12+10x+4x^2+x^3\right)\right]+g^2x\left(2+4x+2x^2+3x^2\right)\right\},\\
V_S^{TPE}(r) &= \frac{g^4m^5}{32\pi^3f^4}\left\{3xK_0(2x)+(3+2x^2)K_1(2x)-\frac{3\pi me^{-2x}}{16Mx}\left(6x^{-1}+12+11x+6x^2+2x^3\right)\right\},\\
W_S^{TPE}(r) &= \frac{g^2m^6}{48\pi^2f^4}\frac{e^{-2x}}{x^6}\left\{\left(c_4+\frac{1}{4M}\right)(1+x)(3+3x+2x^2)-\frac{g^2}{16M}\left(18+36x+31x^2+14x^3+2x^4\right)\right\},\\
V_{LS}^{TPE}(r) &= -\frac{3g^4m^6}{64\pi^2Mf^4}\frac{e^{-2x}}{x^6}(1+x)\left(2+2x+x^2\right)\,,\\
W_{LS}^{TPE}(r) &= \frac{g^2(g^2-1)m^6}{32\pi^2Mf^4}\frac{e^{-2x}}{x^6}(1+x)^2\,,
\end{aligned}$$

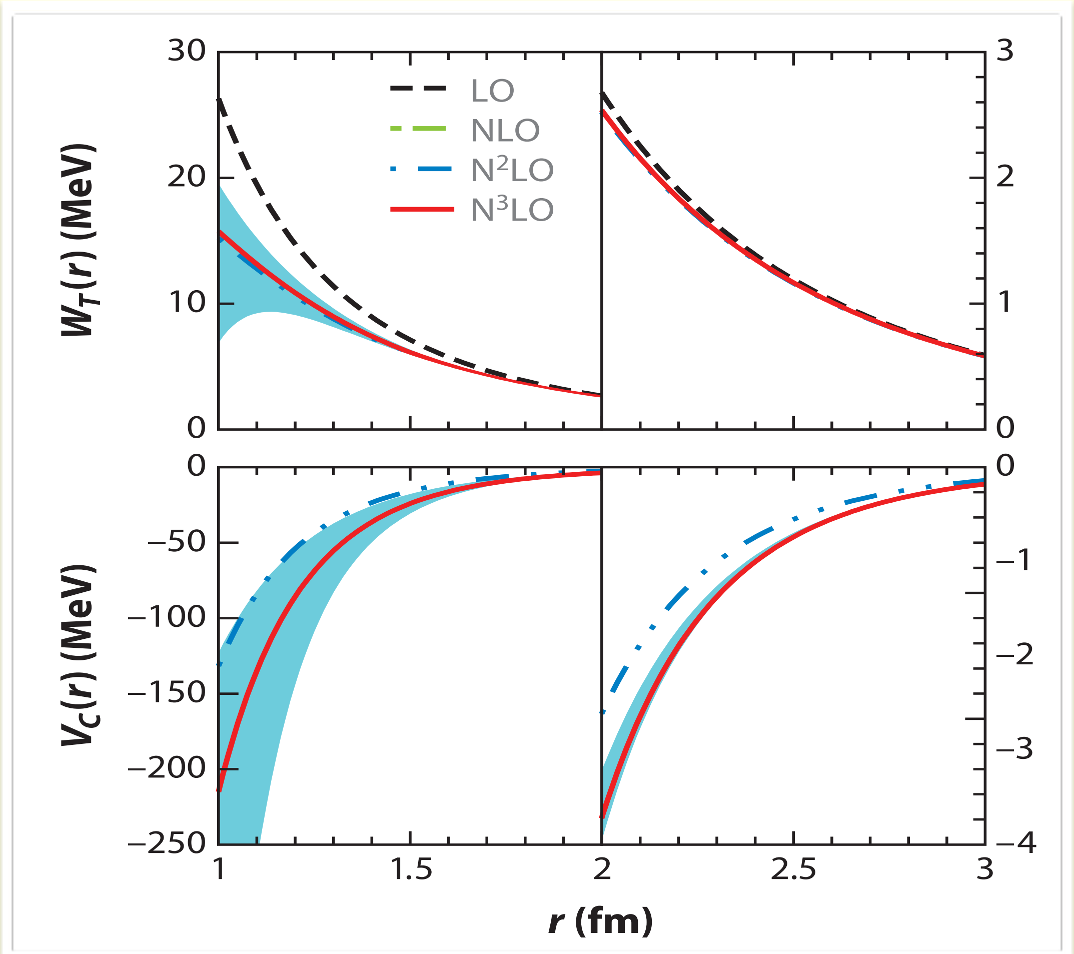
Chiral 2π exchange (upto N²LO)

$$\mathcal{V}_{NN} = V_C(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_C(r) + \left[V_S(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_S(r)\right]\vec{\sigma}_1\cdot\vec{\sigma}_2$$
$$+ \left[V_T(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_T(r)\right](3\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2) + \left[V_{LS}(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_{LS}(r)\right]\vec{L}\cdot\vec{S},$$



Chiral 2π exchange (upto N²LO)

$$\mathcal{V}_{NN} = V_C(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_C(r) + [V_S(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_S(r)]\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$
$$+ [V_T(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_T(r)]\, (3\vec{\sigma}_1 \cdot \hat{r}\vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2) + [V_{LS}(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_{LS}(r)]\, \vec{L} \cdot \vec{S}\,,$$



Is there any evidence from NN data?

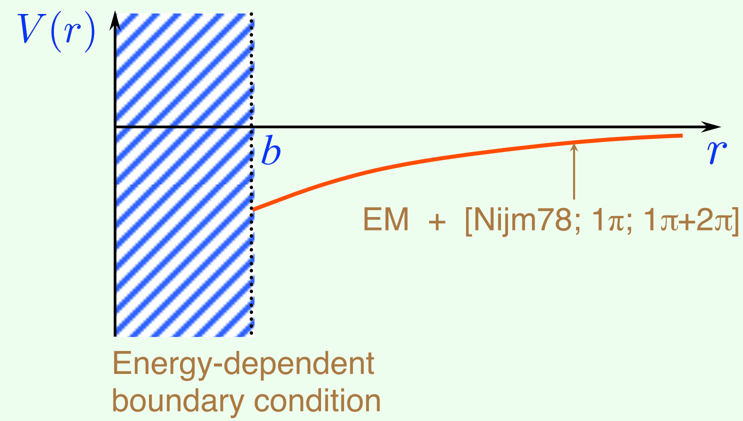
Chiral two-pion exchange and NN data

Nijmegen Partial Wave Analysis

Rentmeester et al.'99,'03

Number of BC parameters needed to achieve $\chi^2_{\text{datum}} \sim 1$ for a given long-range part (input)

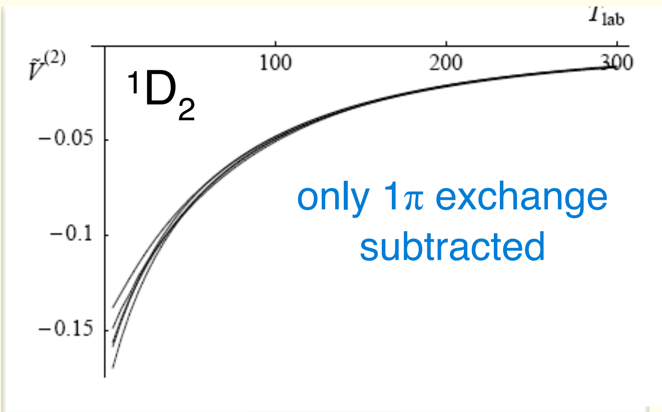
31 (1π) → 28 ($1\pi + 2\pi$ [NLO]) → 23 ($1\pi + 2\pi$ [N^2LO])

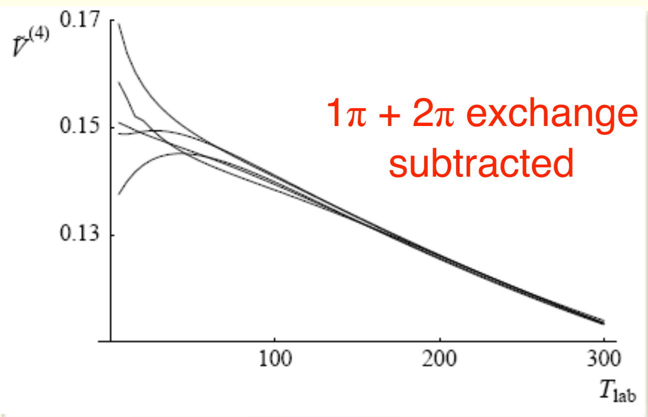


„Deconstructing" neutron-proton phase shufts

Birse, McGovern '06

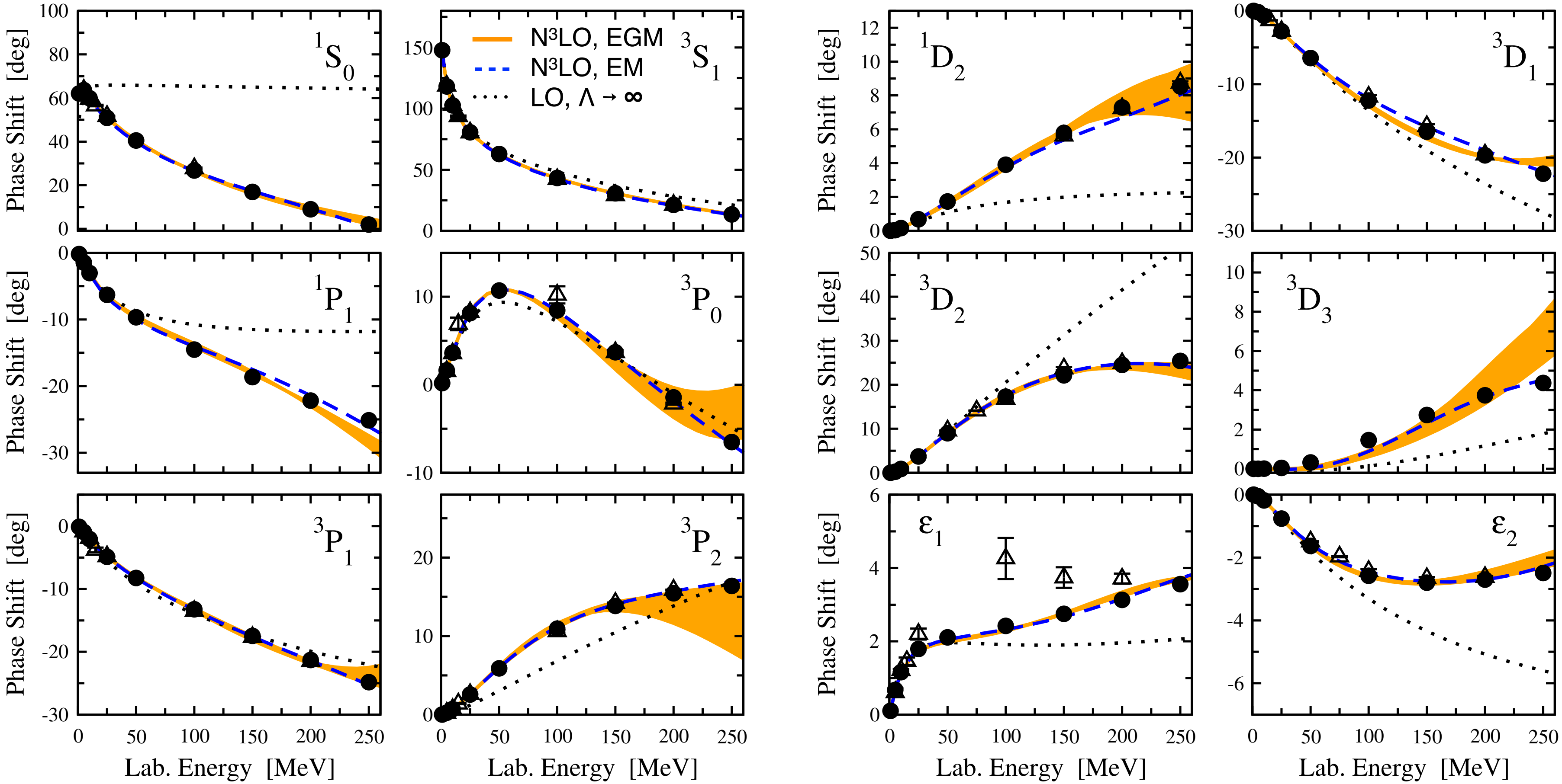
Idea: Subtract effects of the long-range intersction from phase shufts (DWBA) and look at the residual energy dependence





Neutron-proton phase shifts at N³LO

Entem, Machleidt '04; E.E., Glöckle, Meißner '05

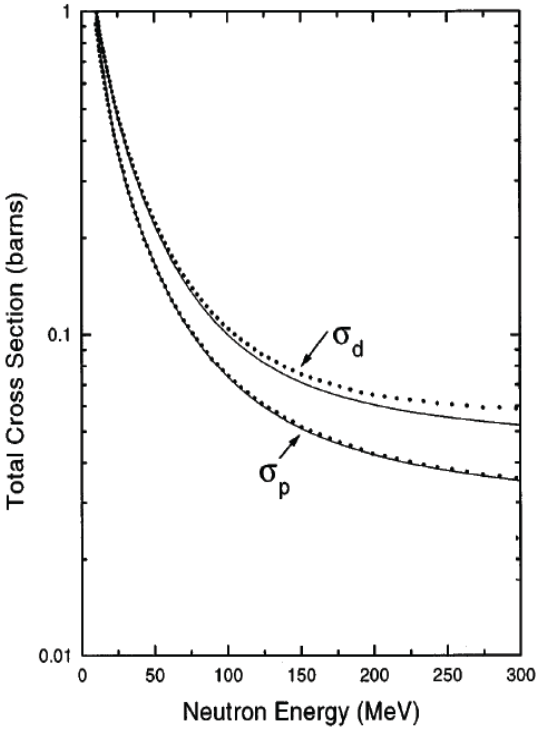


The challenge: Understanding the 3N force

- Today's few- and many-body calculations have reached the level of accuracy at which it is necessary to include 3NFs

- Inspite of decades of efforts, the structure of the 3NF is still poorly understood

  Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rev. Mod. Phys. 75 (2012) 016301

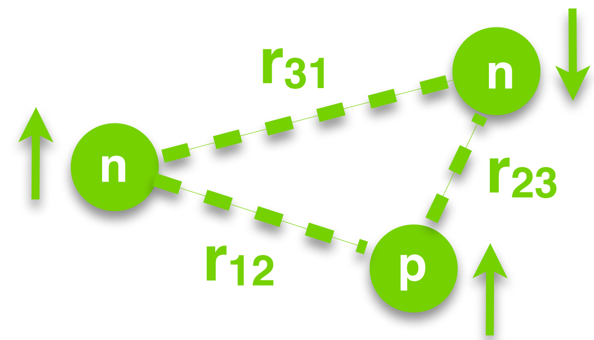


Most general structure of a local 3NF

Krebs, Gasparyan, EE, arXiv:1302.2872 [nucl-th]

22 independent operators (coord. space)

$$
\begin{aligned}
\tilde{\mathcal{G}}_1 &= 1\,, \\
\tilde{\mathcal{G}}_2 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\,, \\
\tilde{\mathcal{G}}_3 &= \vec{\sigma}_1 \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_4 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_5 &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_6 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)\,, \\
\tilde{\mathcal{G}}_7 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_8 &= \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_9 &= \hat{r}_{23} \cdot \vec{\sigma}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\,, \\
\tilde{\mathcal{G}}_{10} &= \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{11} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{12} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{13} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{14} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{15} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{13} \cdot \vec{\sigma}_1\, \hat{r}_{13} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{16} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_2\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{17} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{18} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{19} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_3 \cdot \hat{r}_{23}\, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\,, \\
\tilde{\mathcal{G}}_{20} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_2 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{21} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{13}\, \vec{\sigma}_3 \cdot \hat{r}_{13}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{22} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{12}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,,
\end{aligned}
$$



Building blocks:

$$\boldsymbol{\tau}_1,\ \boldsymbol{\tau}_2,\ \boldsymbol{\tau}_3,\ \vec{\sigma}_1,\ \vec{\sigma}_2,\ \vec{\sigma}_3,\ \vec{r}_{12},\ \vec{r}_{23}$$

Constraints:

- locality,
- isospin symmetry,
- parity and time-reversal invariance

$$\longrightarrow \quad V_{3\mathrm{N}} = \sum_{i=1}^{22} \mathcal{G}_i\, F_i(r_{12}, r_{23}, r_{31}) \; + \; 5 \text{ perm.}$$

Most general structure of a local 3NF

Krebs, Gasparyan, EE, arXiv:1302.2872 [nucl-th]

22 independent operators (coord. space)

$$
\begin{aligned}
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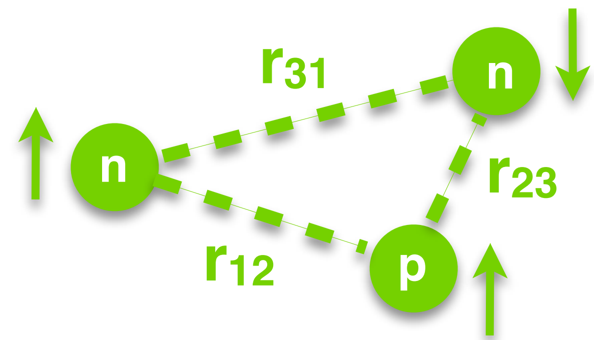
\tilde{\mathcal{G}}_7 &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_2\cdot(\hat{r}_{12}\times\hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_8 &= \hat{r}_{23}\cdot\vec{\sigma}_1\,\hat{r}_{23}\cdot\vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_9 &= \hat{r}_{23}\cdot\vec{\sigma}_3\,\hat{r}_{12}\cdot\vec{\sigma}_1\,, \\
\tilde{\mathcal{G}}_{10} &= \hat{r}_{23}\cdot\vec{\sigma}_1\,\hat{r}_{12}\cdot\vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{11} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat{r}_{23}\cdot\vec{\sigma}_1\,\hat{r}_{23}\cdot\vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{12} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat{r}_{23}\cdot\vec{\sigma}_1\,\hat{r}_{12}\cdot\vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{13} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat{r}_{12}\cdot\vec{\sigma}_1\,\hat{r}_{23}\cdot\vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{14} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat{r}_{12}\cdot\vec{\sigma}_1\,\hat{r}_{12}\cdot\vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{15} &= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\hat{r}_{13}\cdot\vec{\sigma}_1\,\hat{r}_{13}\cdot\vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{16} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat{r}_{12}\cdot\vec{\sigma}_2\,\hat{r}_{12}\cdot\vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{17} &= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\hat{r}_{23}\cdot\vec{\sigma}_1\,\hat{r}_{12}\cdot\vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{18} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_1\cdot\vec{\sigma}_3\,\vec{\sigma}_2\cdot(\hat{r}_{12}\times\hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{19} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_3\cdot\hat{r}_{23}\,\hat{r}_{23}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)\,, \\
\tilde{\mathcal{G}}_{20} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_1\cdot\hat{r}_{23}\,\vec{\sigma}_2\cdot\hat{r}_{23}\,\vec{\sigma}_3\cdot(\hat{r}_{12}\times\hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{21} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_1\cdot\hat{r}_{13}\,\vec{\sigma}_3\cdot\hat{r}_{13}\,\vec{\sigma}_2\cdot(\hat{r}_{12}\times\hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{22} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_1\cdot\hat{r}_{23}\,\vec{\sigma}_3\cdot\hat{r}_{12}\,\vec{\sigma}_2\cdot(\hat{r}_{12}\times\hat{r}_{23})\,,
\end{aligned}
$$



Building blocks:

$$\boldsymbol{\tau}_1,\ \boldsymbol{\tau}_2,\ \boldsymbol{\tau}_3,\ \vec{\sigma}_1,\ \vec{\sigma}_2,\ \vec{\sigma}_3,\ \vec{r}_{12},\ \vec{r}_{23}$$

Constraints:

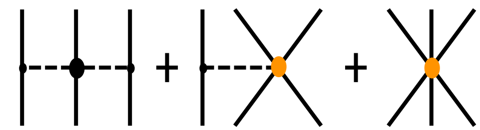
– locality,
– isospin symmetry,
– parity and time-reversal invariance

$$\longrightarrow\quad V_{3\text{N}} = \sum_{i=1}^{22} \mathcal{G}_i\, F_i(r_{12}, r_{23}, r_{31}) + 5\ \text{perm.}$$

derivable in ChPT; long-range terms parameter-free predictions

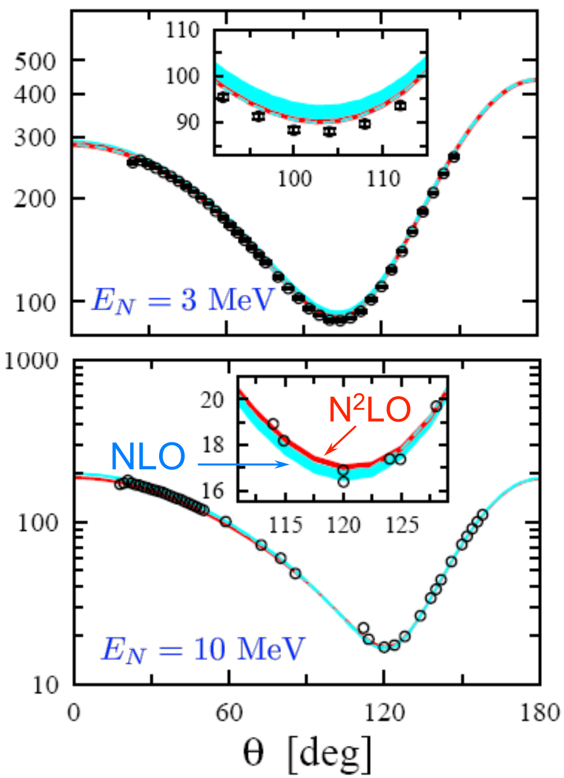
Leading chiral 3NF and 3N/4N continuum

- Nd scattering: accurate description at low energy except for A_y-puzzle (fine tuned) and some breakup configurations
- Uncertainty grows rapidly with energy (higher orders ?)
- 4N continuum: an emerging field (lectures by Michele)



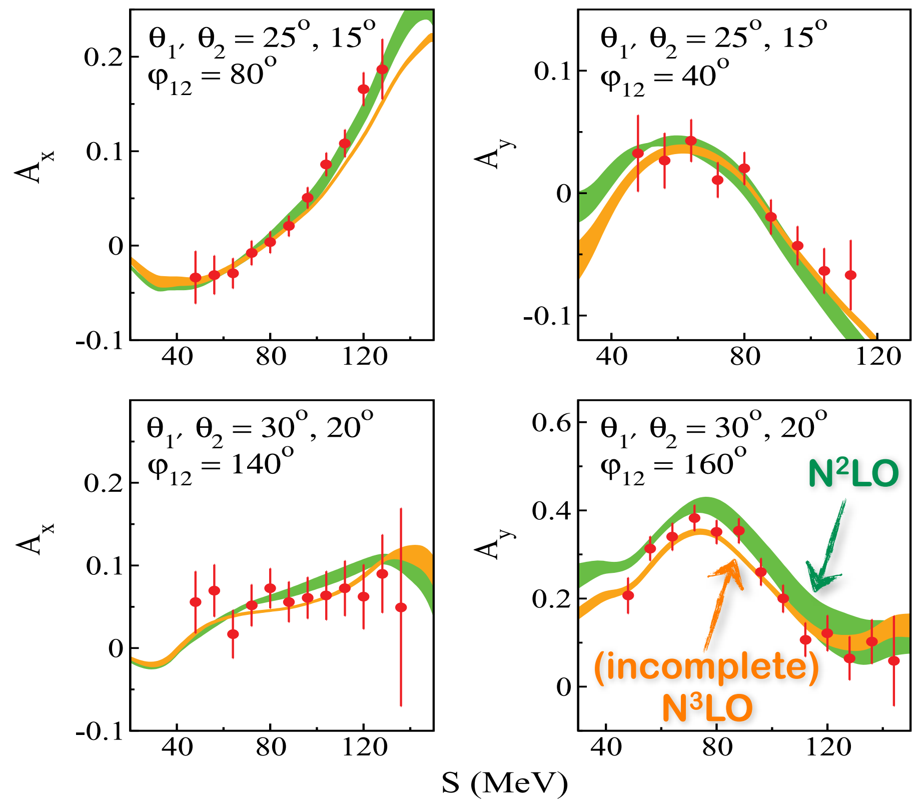
2 LECs tuned to few-N data (e.g. ^{3}H, ^{4}He BEs)

Nd elastic cross sections at low energies



Nd breakup at E_d=130 MeV

Stephan et al., PRC 82 (2010) 014003



Leading chiral 3NF and 3N/4N continuum

- Nd scattering: accurate description at low energy except for A_y-puzzle (fine tuned) and some breakup configurations
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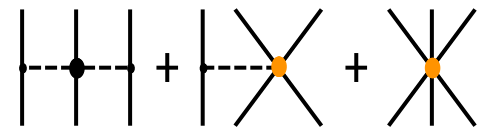

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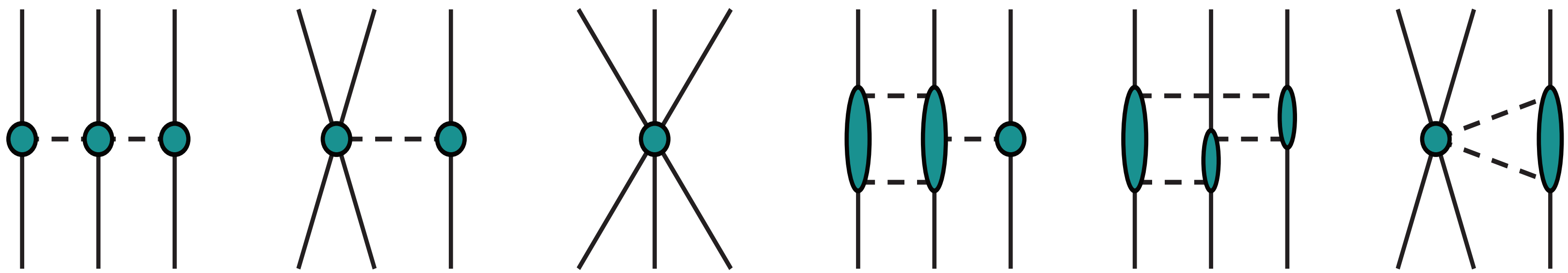
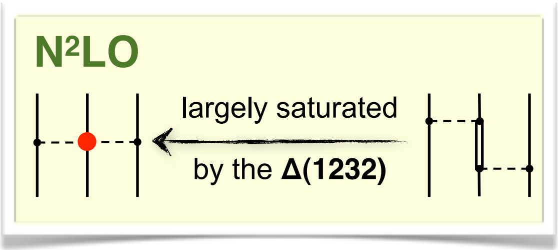
(e.g. ^{3}H, ^{4}He BEs)

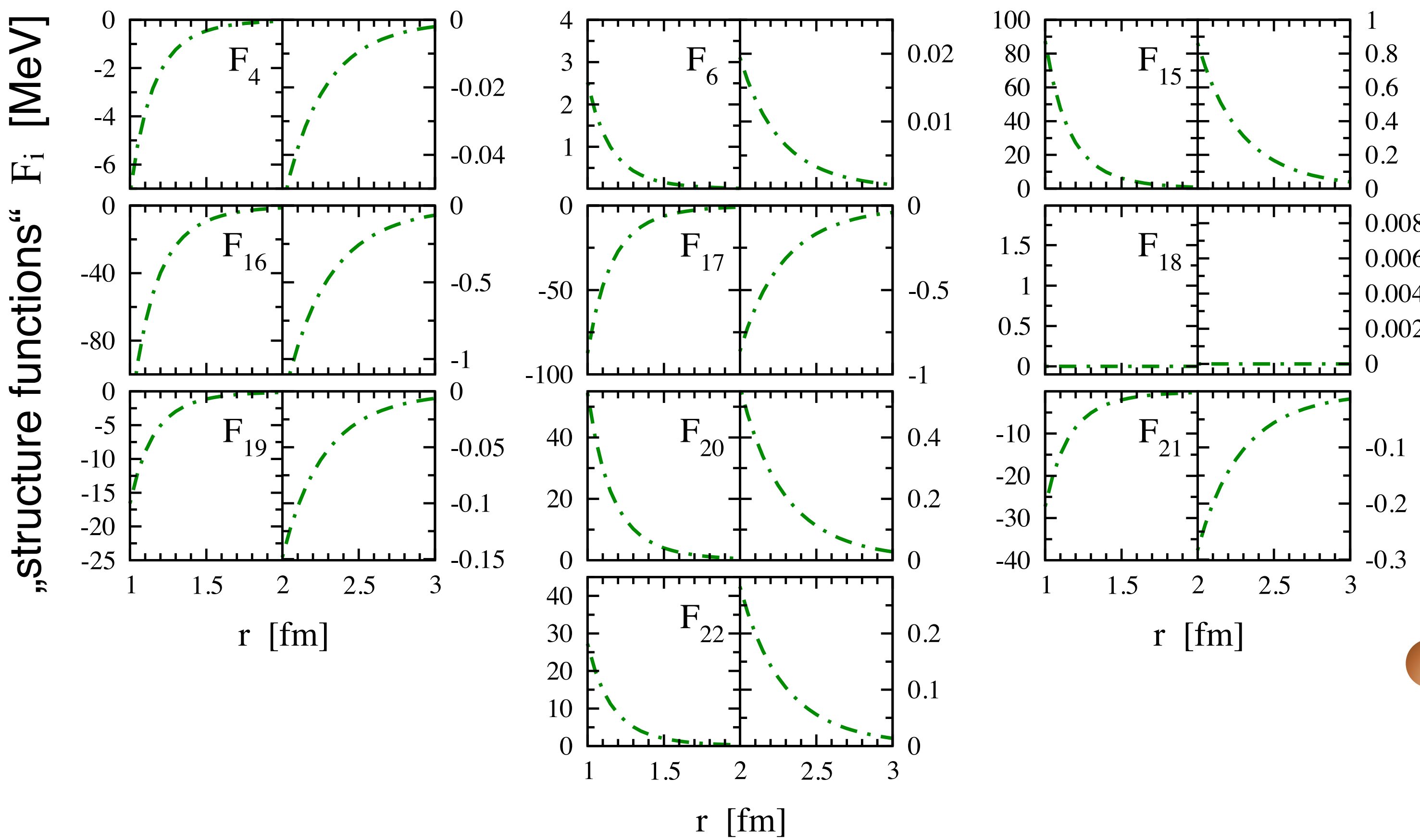


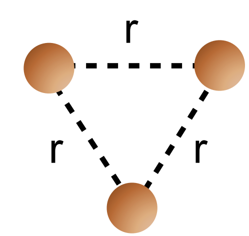
Corrections to the leading 3NF beyond N^{2}LO are being investigated

Two-pion exchange 3NF up to N^4LO

Krebs, Gasparyan, EE '12



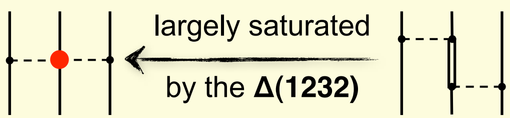




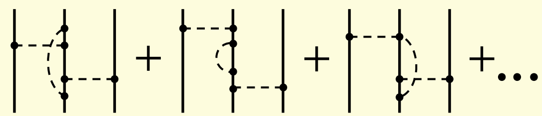
Two-pion exchange 3NF up to N⁴LO

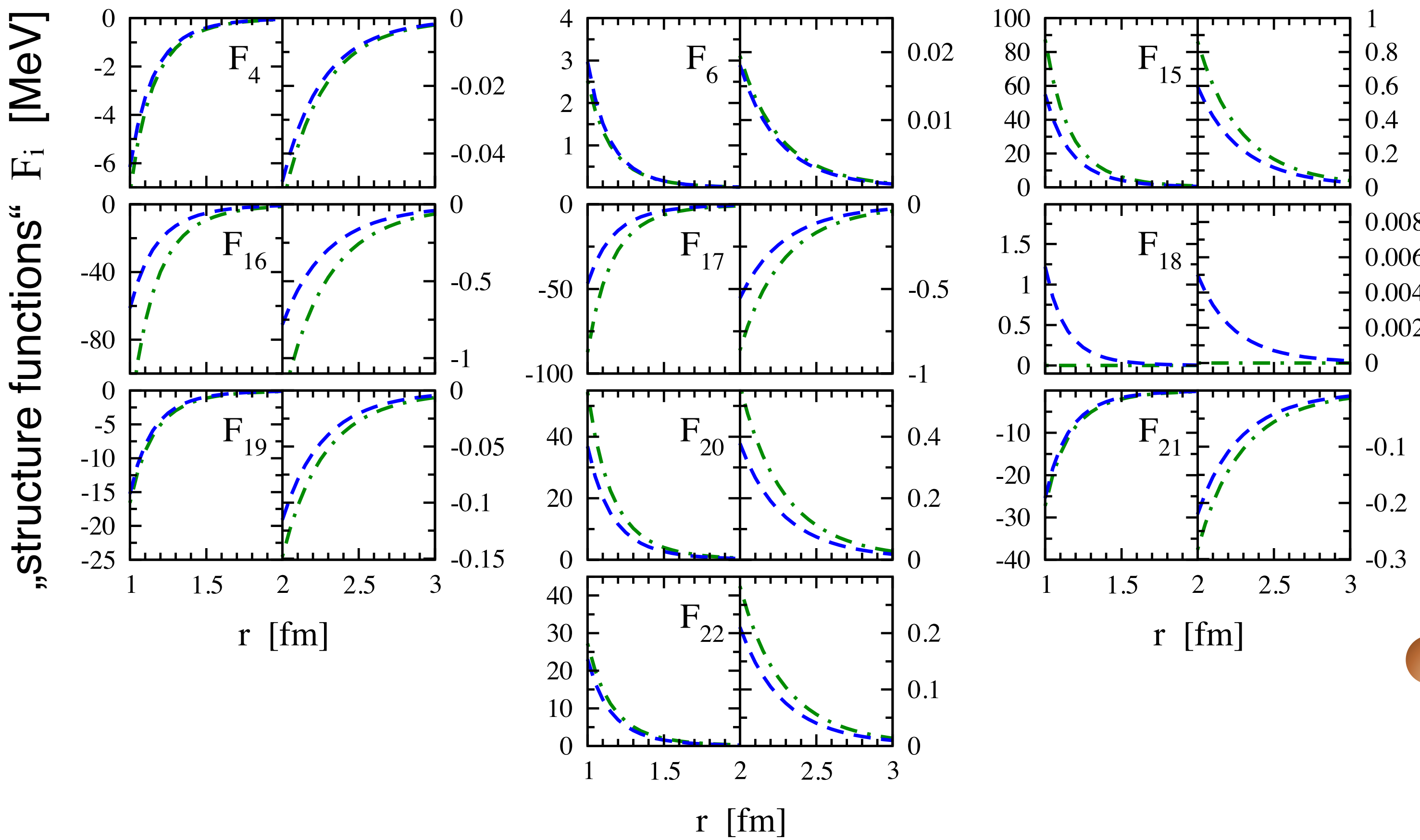
Krebs, Gasparyan, EE ’12

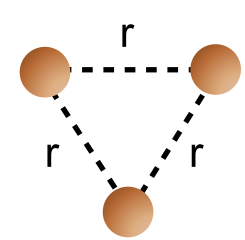
N^2LO



+ N^3LO

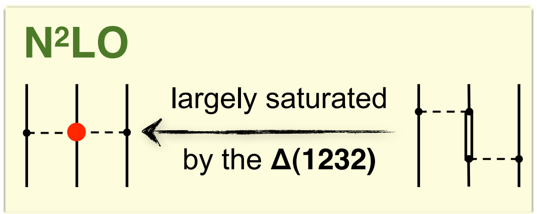


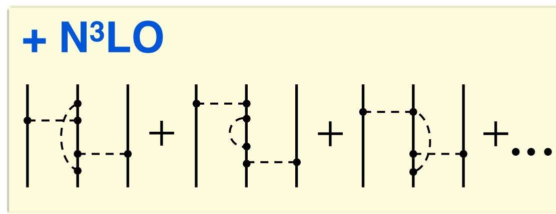


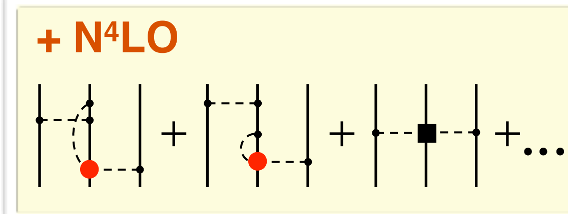


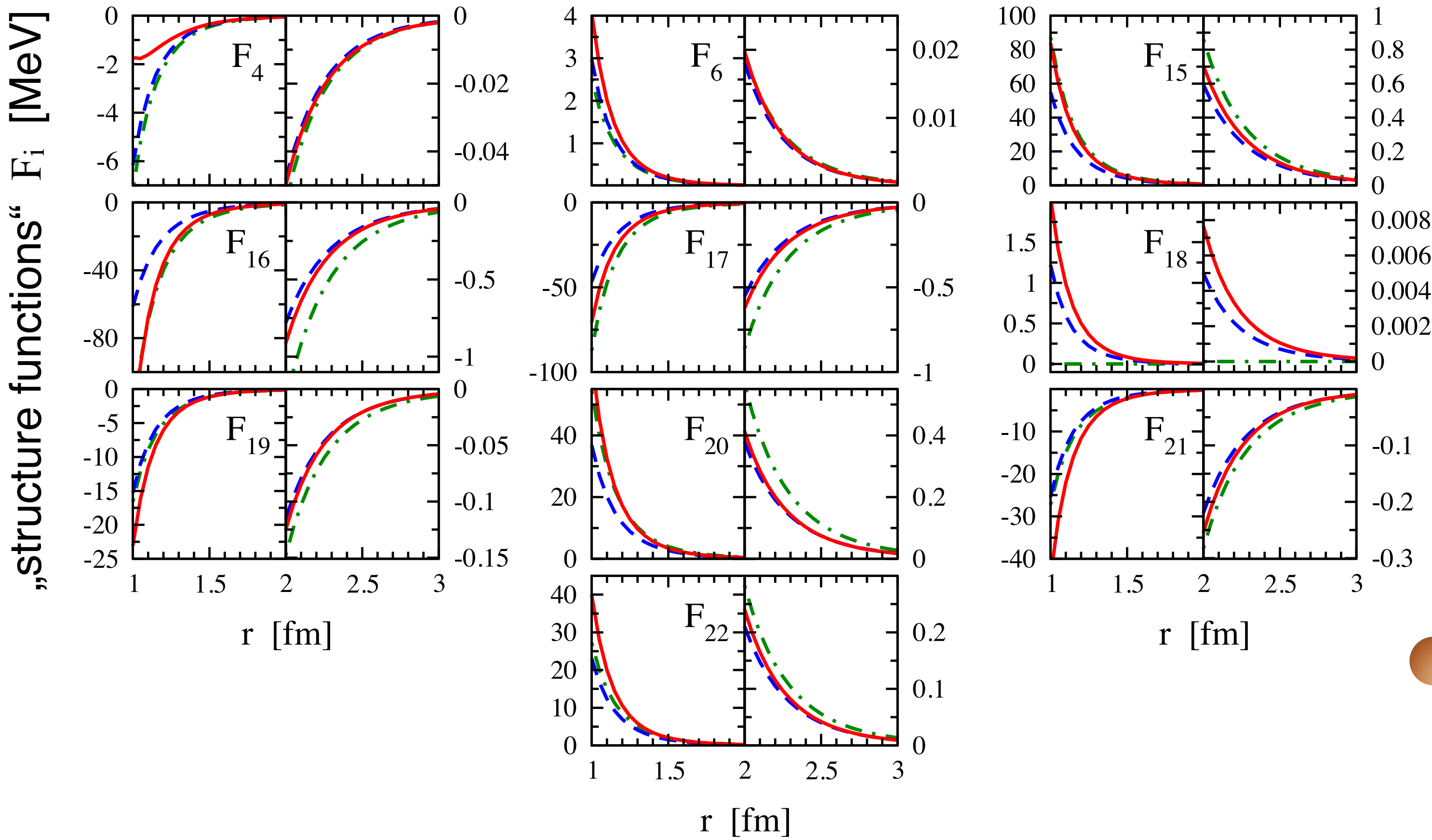
Two-pion exchange 3NF up to N^4LO

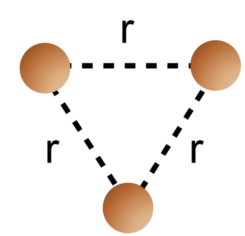
Krebs, Gasparyan, EE ’12







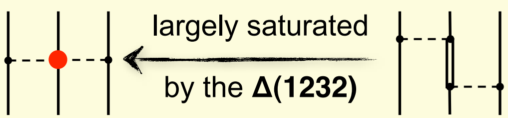




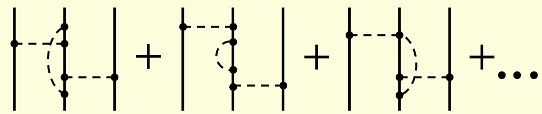
Two-pion exchange 3NF up to N^{4}LO

Krebs, Gasparyan, EE '12

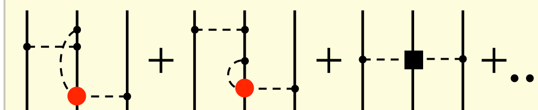
N^{2}LO



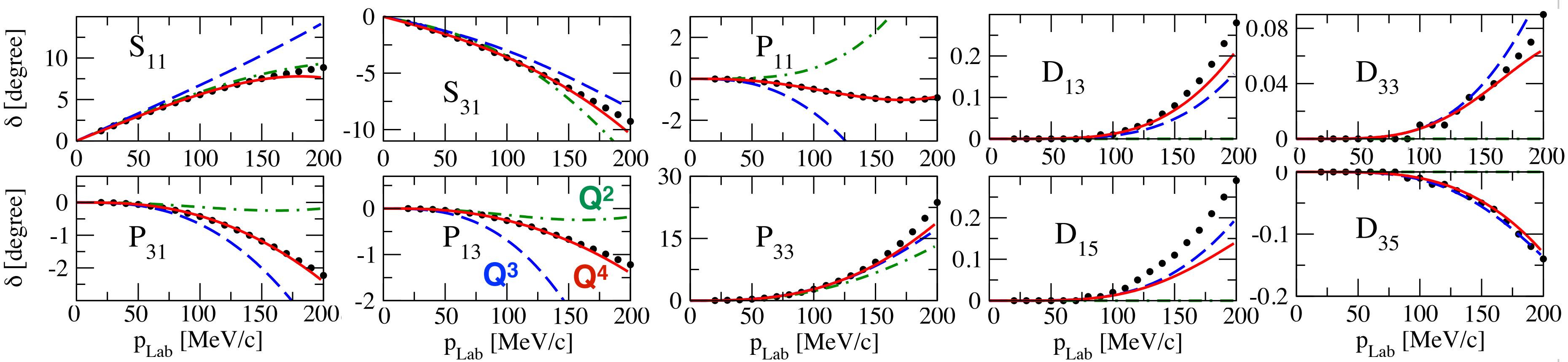
+ N^{3}LO



+ N^{4}LO



Pion-nucleon phase shifts in HB ChPT up to Q^4 (KH PWA)



Values of low-energy constants extracted at Q^4 (in powers of GeV)

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Q^4 fit to GW	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.80	1.76	-0.58	0.96
Q^4 fit to KH	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

Summary: chiral nuclear forces

- Chiral NN potentials are available at N^3LO and provide accurate description of NN scattering up to $E_{lab} \sim 200$ MeV.

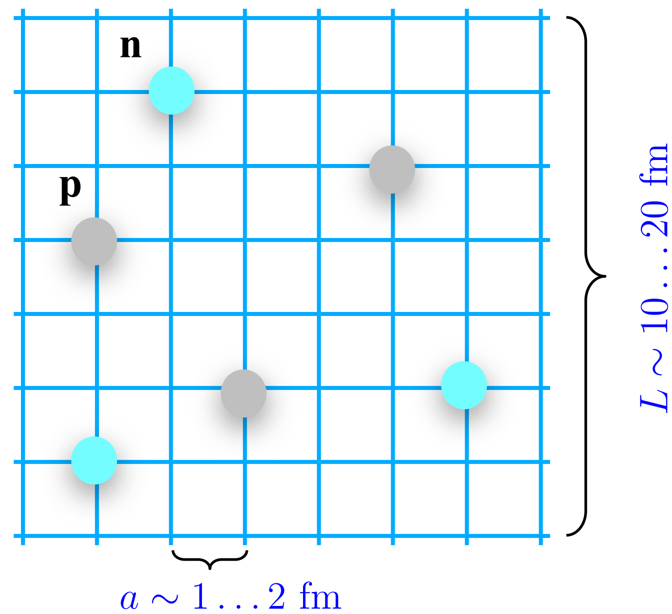
- 3NF: promising results at N^2LO; corrections are under investigation

- 4NF: starts contributing at N^3LO; probably small (expectation value for the α-particle about a few 100 keV...)

Nuclear Lattice Effective Field Theory

The Collaboration: E.E., Hermann Krebs (Bochum), Timo Lähde (Jülich), Dean Lee (NC State), Ulf-G. Meißner (Bonn/Jülich), Gautam Rupak (Mississippi State)

Borasoy, E.E., Krebs, Lee, Meißner, Eur. Phys. J. A31 (07) 105,
Eur. Phys. J. A34 (07) 185,
Eur. Phys. J. A35 (08) 343,
Eur. Phys. J. A35 (08) 357,
E.E., Krebs, Lee, Meißner, Eur. Phys. J A40 (09) 199,
Eur. Phys. J A41 (09) 125,
Phys. Rev. Lett 104 (10) 142501,
Eur. Phys. J. 45 (10) 335,
Phys. Rev. Lett. 106 (11) 192501,
E.E., Krebs, Lähde, Lee, Meißner Phys. Rev. Lett. 109 (12) 252501,
Phys. Rev. Lett. 110 (13) 112502,
arXiv:1303.4856



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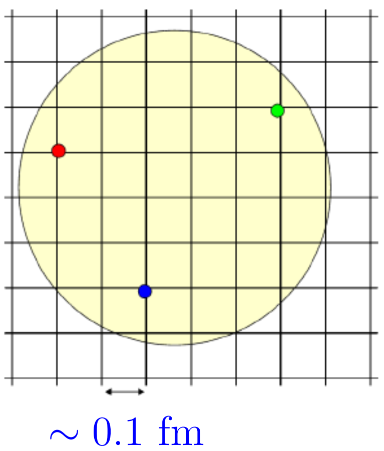
HadronPhysics2 Study of Strongly Interacting Matter
JÜLICH FORSCHUNGSZENTRUM
erc European Research Council

Nuclear lattice simulations

Discretized version of chiral EFT for nuclear dynamics

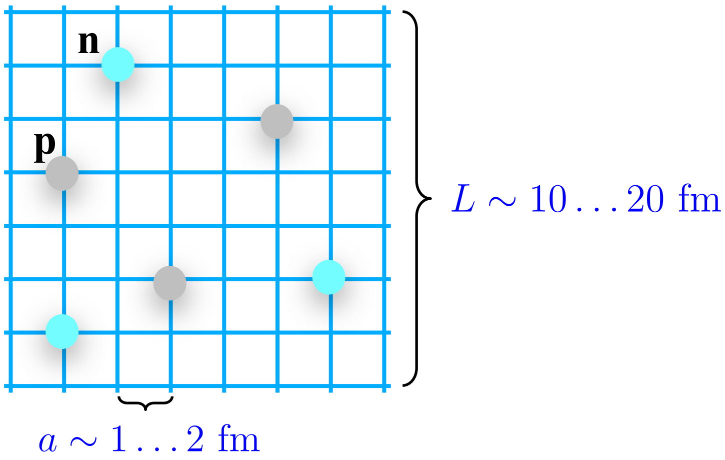
$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

Lattice QCD



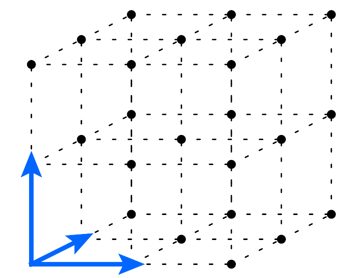
- fundamental, the only parameters are m_q, α_{strong}
- hard to go beyond 1 hadron...

Chiral EFT on the lattice



- effective hadronic description, LECs to be determined from the data/LQCD
- much more efficient for atomic nuclei

Leading-order action



- $\vec{n}$ refer to integer-valued spatial lattice vectors
- $\vec{l} = \{\hat{1}, \hat{2}, \hat{3}\}$ are unit lattice vectors in the spatial directions
- $\alpha_t = a_t/a$ is the ratio of the lattice spacings
- Derivatives (order-a^4 improved)

$$\nabla_l f(\vec{n}) = \frac{3}{4}\left[f(\vec{n}+\vec{l}) - f(\vec{n}-\vec{l})\right] - \frac{3}{20}\left[f(\vec{n}+2\vec{l}) - f(\vec{n}-2\vec{l})\right] + \frac{1}{60}\left[f(\vec{n}+3\vec{l}) - f(\vec{n}-3\vec{l})\right]$$

$$\nabla_l^2 f(\vec{n}) = -\frac{49}{18}f(\vec{n}) + \frac{3}{2}\left[f(\vec{n}+\vec{l}) + f(\vec{n}-\vec{l})\right] - \frac{3}{20}\left[f(\vec{n}+2\vec{l}) + f(\vec{n}-2\vec{l})\right] + \frac{1}{90}\left[f(\vec{n}+3\vec{l}) + f(\vec{n}-3\vec{l})\right]$$

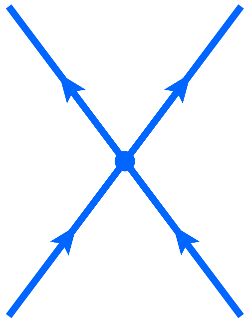
- Free Hamiltonian for non-relativistic nucleons: $H_{\text{free}} = \frac{1}{2m}\sum_{\vec{n},i,j} a^\dagger_{ij}(\vec{n})\sum_l \nabla_l^2 a_{ij}(\vec{n})$
- Nucleon local density operators $\rho(\vec{n}) = \sum_{i,j} a^\dagger_{i,j}(\vec{n})a_{i,j}(\vec{n})$, $\quad \rho_S(\vec{n}) = \sum_{i,j,k} a^\dagger_{i,j}(\vec{n})[\sigma_S]_{ik}a_{k,j}(\vec{n})$

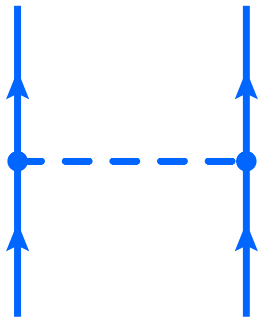
$$\rho_I(\vec{n}) = \sum_{i,j,k} a^\dagger_{i,j}(\vec{n})[\tau_I]_{jk}a_{i,k}(\vec{n}), \quad \rho_{S,I}(\vec{n}) = \sum_{i,j,k,l} a^\dagger_{i,j}(\vec{n})[\sigma_S]_{il}[\tau_I]_{jk}a_{l,k}(\vec{n})$$

Leading-order action

In the continuum (momentum space):

$$
\begin{aligned}
V_{\mathrm{LO}_3} &= C_{0,1}\, f(\vec{q}) \left(\frac{1}{4} - \frac{1}{4}\vec{\sigma}_1 \cdot \vec{\sigma}_2\right) \left(\frac{3}{4} + \frac{1}{4}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\right) \\
&+ C_{1,0}\, f(\vec{q}) \left(\frac{3}{4} + \frac{1}{4}\vec{\sigma}_1 \cdot \vec{\sigma}_2\right) \left(\frac{1}{4} - \frac{1}{4}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\right) \\
&- \left(\frac{g_A}{2F_\pi}\right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2}
\end{aligned}
$$





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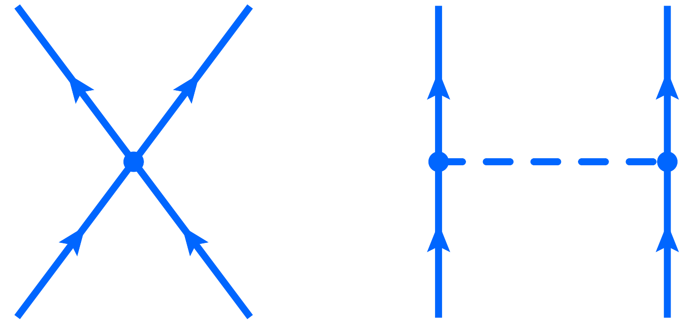

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\end{aligned}
$$

On the lattice:

$$
V_{\mathrm{LO}_3} = \frac{1}{L^3} \sum_{\vec{q}} f(\vec{q}) : \Bigg\{ C_{0,1} \Bigg[\frac{3}{32}\rho(\vec{q})\rho(-\vec{q}) - \frac{3}{32}\sum_S \rho_S(\vec{q})\rho_S(-\vec{q}) + \frac{1}{32}\sum_I \rho_I(\vec{q})\rho_I(-\vec{q}) - \frac{1}{32}\sum_{S,I} \rho_{S,I}(\vec{q})\rho_{S,I}(-\vec{q}) \Bigg]
$$

$$
+ C_{1,0}\Big[\ldots\Big]\Bigg\} : \; - \frac{g_A^2 \alpha_t}{8F_\pi^2} \sum_{S_{1,2}, I, \vec{n}_{1,2}} : \rho_{S_1,I}(\vec{n}_1)\, G_{S_1 S_2}\, \rho_{S_2,I}(\vec{n}_2) :
$$

where the two-derivative pion correlator is defined as $G_{S_1 S_2}(\vec{n}) \equiv \langle \nabla_{S_1} \pi_I(\vec{n}, n_t)\, \nabla_{S_2} \pi_I(\vec{0}, n_t) \rangle$

Leading-order action

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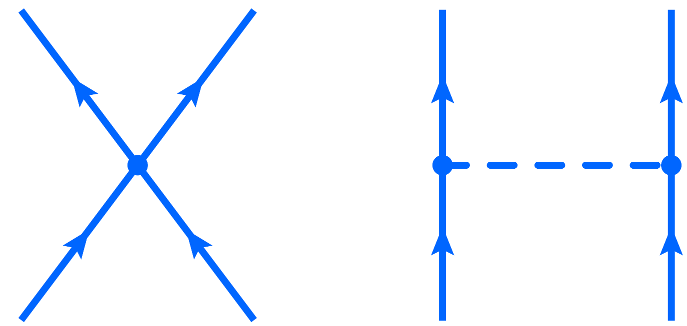

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$$- \left(\frac{g_A}{2F_\pi}\right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2}$$

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Other LO lattice actions used in the simulations:

- LO_1: no smearing, LO_2: Gaussian smearing in all waves

Two-particle phase shifts

Borasoy, E.E., Krebs, Lee, Meißner, EPJA 34 (2007) 185

Wave function in the asymptotic region (QM potential scattering):

$$\Psi(\vec{r}) = [\cos\delta_L j_L(kr) - \sin\delta_L y_L(kr)] Y_{L,m}(\theta,\phi)$$

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Borasoy, E.E., Krebs, Lee, Meißner, EPJA 34 (2007) 185

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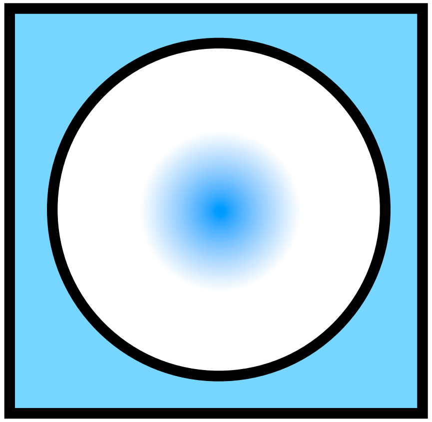

Impose a spherical-wall boundary condition $\Psi(\vec{R}_{\text{Wall}}) = 0$ (standing waves) and determine the energy spectrum

- interaction switched off: $\delta_L = 0, \;\; j_L\left(k_i^{\text{free}} R_{\text{Wall}}\right) = 0$
- interaction switched on: $\tan\left[\delta_L(k_i)\right] = \dfrac{j_L(k_i R_{\text{Wall}})}{y_L(k_i R_{\text{Wall}})}$

Generalization to coupled channels straightforward...

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Borasoy, E.E., Krebs, Lee, Meißner, EPJA 34 (2007) 185

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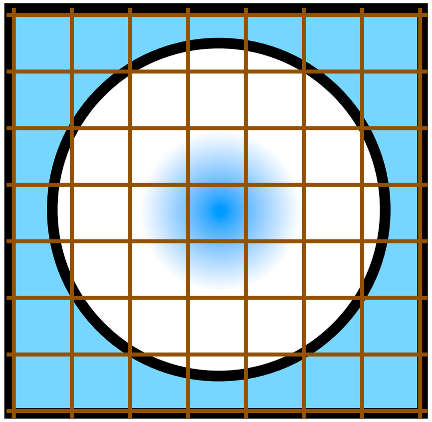

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Do the same thing on the lattice → two-particle phase shifts from the energy spectrum!

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Borasoy, E.E., Krebs, Lee, Meißner, EPJA 34 (2007) 185

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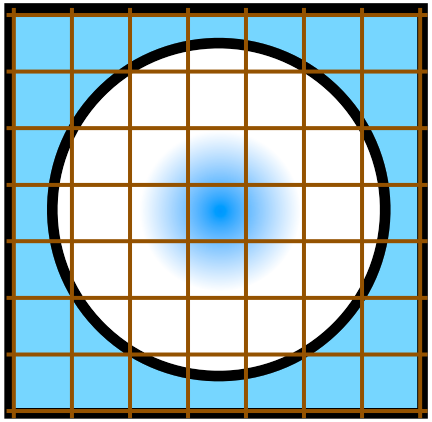

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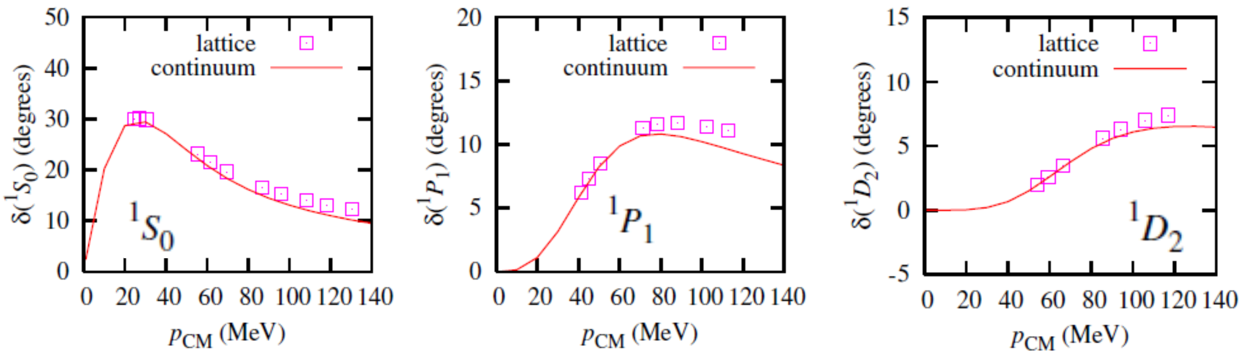
Generalization to coupled channels straightforward...

Do the same thing on the lattice → two-particle phase shifts from the energy spectrum!

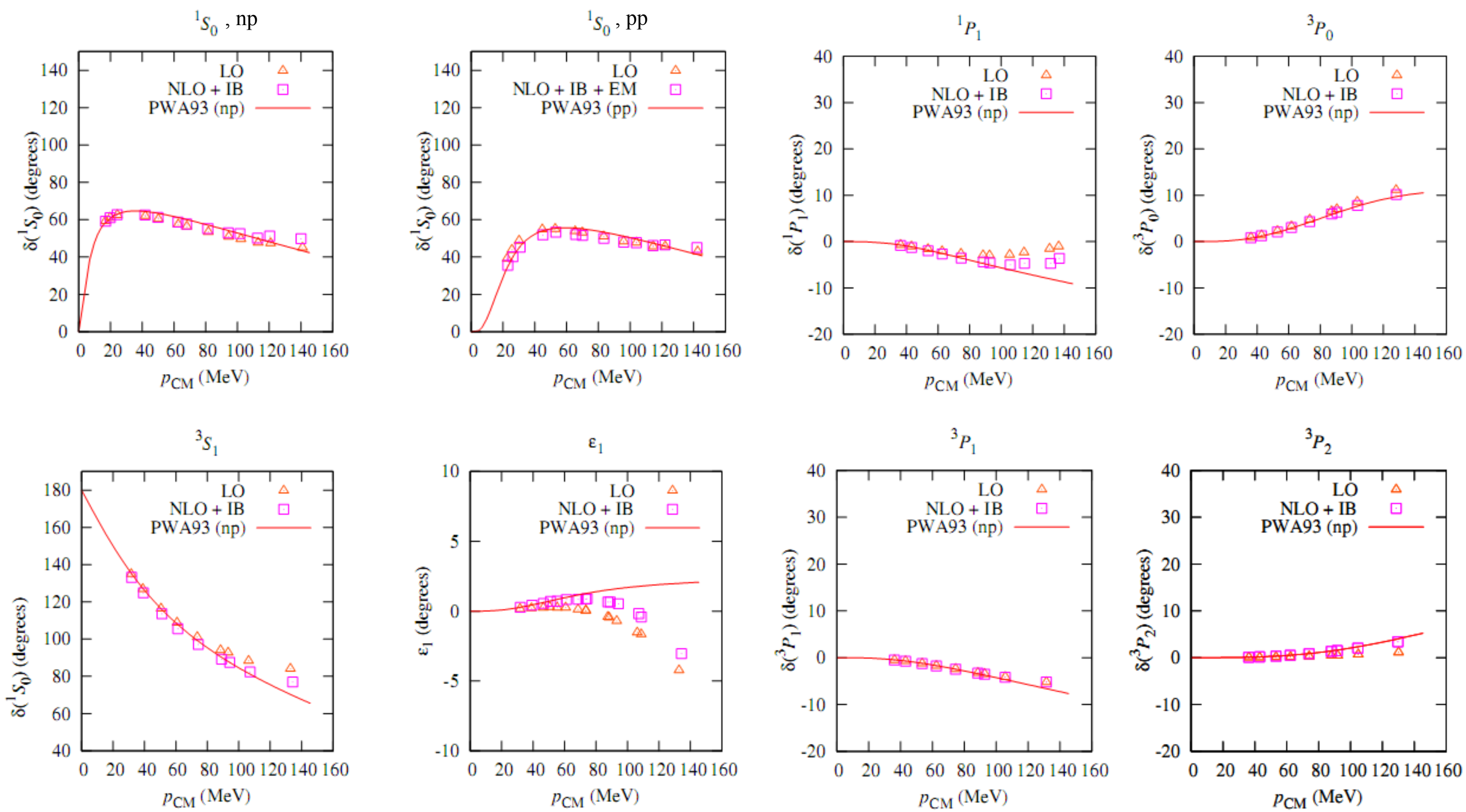
Example: a toy-model potential $V(\vec{r}) = C\left\{1 + \frac{r^2}{R_0^2}\left[3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2\right]\right\}\exp\left(-\frac{1}{2}\frac{r^2}{R_0^2}\right)$



Two-nucleon phase shifts (LO_3)

E.E., Krebs, Lee, Meißner, EPJA 45 (10) 335

- 9 LECs fitted to S- and P-waves and the deuteron quadrupole moment
- Coulomb repulsion and isospin-breaking effects taken into account
- Accurate results, deviations consistent with the expected size of higher-order terms



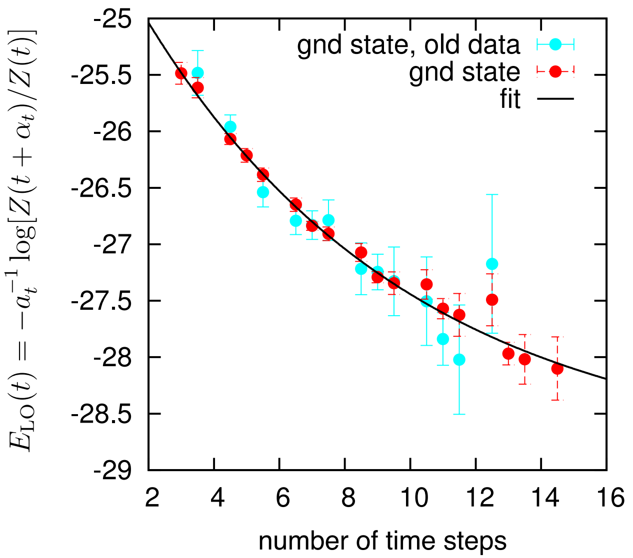
Calculation strategy

- Eucl.-time propagation of A nucleons $\longrightarrow$ transition amplitude $Z_A(t) = \langle \Psi_A | \exp(-tH) | \Psi_A \rangle$
  $\longrightarrow$ ground-state energies $E_A = -\lim_{t\to\infty} d(\ln Z_A)/dt$

- Excited state energies can be obtained from a large-t limit of a correlation matrix $Z_A^{ij}(t) = \langle \Psi_A^i | \exp(-tH) | \Psi_A^j \rangle$ between A-nucleon states Ψ_A^j with the proper quantum numb.

- Use $H_{\rm LO}$ to run the simulation, higher-order terms (incl. Coulomb, 3NF, ...) taken into account perturbatively via $Z_A^O(t) = \langle \Psi_A | \exp(-tH/2)\, O\, \exp(-tH/2) | \Psi_A \rangle$

AFQMC calculation of the ^{4}He BE



- We use temporal spacing $a_t = 1.32$ fm and vary propagation time L_t to carry out the $L_t \to \infty$ extrapolation via $E(N_t) = E(\infty) + c_E \exp(-N_t/\tau)$

Large-t extrapolated and exact results for ^{2}H (3^3, L=5.92 fm)

	^{2}H (extr.)	^{2}H (exact)
E(LO) [MeV]	$-9.070(12)$	-9.078
$\Delta E(\Delta\tilde{M}_\pi)$ [MeV]	$-0.003548(12)$	-0.003569
$\Delta E(\Delta M_\pi^{\rm IB})$ [MeV]	$-0.002372(8)$	-0.002379

Auxiliary Field QMC method

- **Transfer matrix with only nucleon fields (without smearing)**

$$M_{\rm LO} =: \exp\left(-H_{\rm free}\alpha_t - \frac{1}{2}C\alpha_t \sum_{\vec{n}} \left[\rho(\vec{n}\,)\right]^2 - \frac{1}{2}C_I\alpha_t \sum_{\vec{n},I} \left[\rho_I(\vec{n}\,)\right]^2 + \frac{g_A^2\alpha_t^2}{8F_\pi^2} \sum_{S_{1,2},I,\vec{n}_{1,2}} \rho_{S_1,I}(\vec{n}_1\,)\, G_{S_1S_2}\, \rho_{S_2,I}(\vec{n}_2\,)\right):$$

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- **Hubbard-Stratonovich transformation:** $\exp(\rho^2/2) \sim \int_{-\infty}^{\infty} ds\,\exp(-s^2/2 - s\,\rho)$

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- **Hubbard-Stratonovich transformation:** $\exp(\rho^2/2) \sim \int_{-\infty}^{\infty} ds\,\exp(-s^2/2 - s\,\rho)$

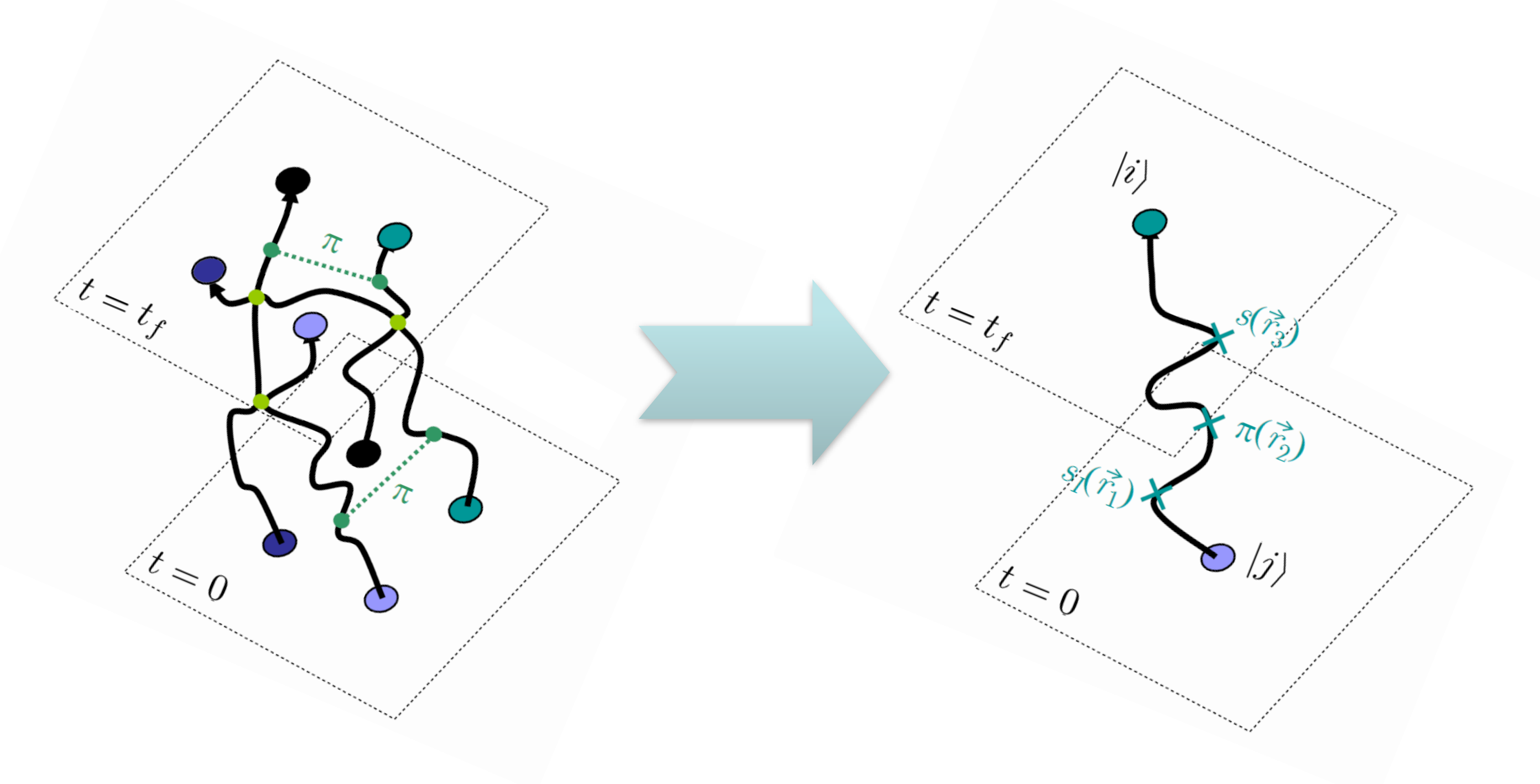
- **Transfer matrix with (instantaneous) pion and auxiliary fields**

$$M_{\rm LO} = \frac{\int D\pi_I\,Ds\,Ds_I\,e^{-S_{\pi\pi}^{(n_t)}-S_{ss}^{(n_t)}}\,M_{\rm LO}^{(n_t)}(\pi_I,\,s,\,s_I)}{\int D\pi_I\,Ds\,Ds_I\,e^{-S_{\pi\pi}^{(n_t)}-S_{ss}^{(n_t)}}}$$

with
$$\begin{aligned} M_{\rm LO}^{(n_t)}(\pi_i,\,s,\,s_I) &= :\exp\Big(-H_{\rm free}\alpha_t + \sqrt{-C\alpha_t}\sum_{\vec n}s(\vec n,n_t)\,\rho(\vec n\,) + i\sqrt{C_I\alpha_t}\sum_{\vec n,I}s_I(\vec n,n_t)\,\rho_I(\vec n\,)\\ &\quad - \frac{g_A\alpha_t}{2F_\pi}\sum_{S,I,\vec n}(\nabla_S\pi_I(\vec n,n_t))\,\rho_{S,I}(\vec n\,)\Big): \end{aligned}$$

and
$$S_{ss}^{(n_t)} = \frac{1}{2}\sum_{\vec n}(s(\vec n,n_t))^2 + \frac{1}{2}\sum_{\vec n,I}(s_I(\vec n,n_t))^2\,,\qquad S_{\pi\pi}^{(n_t)} = \frac{\alpha_t}{2}\sum_{\vec n,I}\pi_I(\vec n,n_t)\left[-\Delta + M_\pi^2\right]\pi_I(\vec n,n_t)$$

Auxiliary Field QMC method



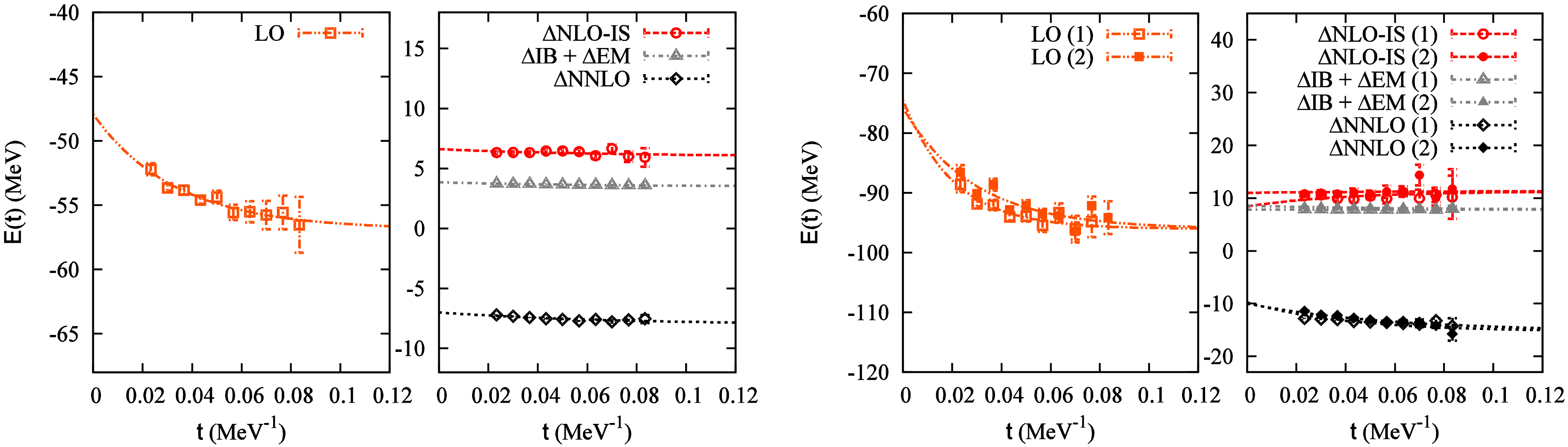
For a given configuration of the auxilliary & pion fields:

$$\langle \Psi^{\rm init} | e^{-H(s,s_I,\pi_I)t} | \Psi^{\rm init} \rangle = \det \boldsymbol{G}_{ij}(s, s_I, \pi_I) \text{ with } \boldsymbol{G}_{ij}(s, s_I, \pi_I) = \langle i | e^{-H(s,s_I,\pi_I)t} | j \rangle$$

Ground states of ^{8}Be and ^{12}C

E.E., Krebs, Lee, Meißner, PRL 106 (11) 192501

Simulations for ^{8}Be and ^{12}C, L=11.8 fm



Ground state energies (L=11.8 fm) of ^{4}He, ^{8}Be, ^{12}C & ^{16}O

	^{4}He	^{8}Be	^{12}C	^{16}O
LO [Q^0], in MeV	−28.0(3)	−57(2)	−96(2)	−144(4)
NLO [Q^2], in MeV	−24.9(5)	−47(2)	−77(3)	−116(6)
NNLO [Q^3], in MeV	−28.3(6)	−55(2)	−92(3)	−135(6)
Experiment, in MeV	−28.30	−56.5	−92.2	−127.6

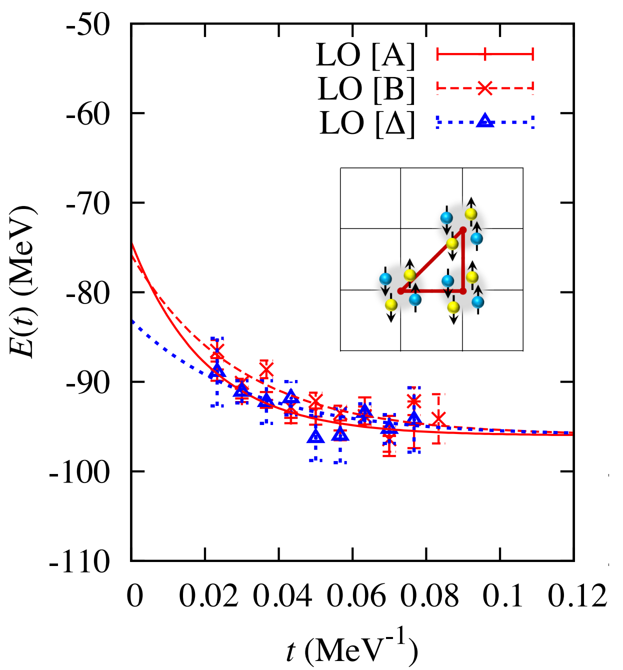
The Hoyle state

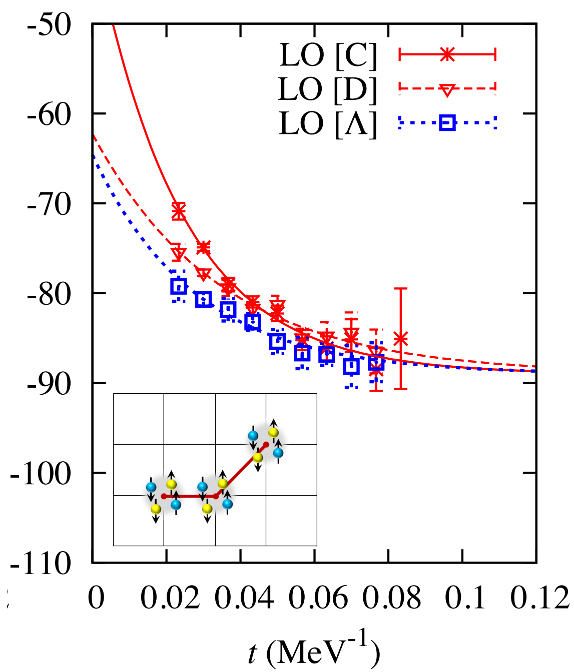
EE, Krebs, Lähde, Lee, Meißner, PRL 106 (2011) 192501; PRL 109 (2012) 252501

Lattice results for low-lying even-parity states of [12]C

	0_1^+	$2_1^+(E^+)$	0_2^+	$2_2^+(E^+)$
LO	$-96(2)$	$-94(2)$	$-89(2)$	$-88(2)$
NLO	$-77(3)$	$-74(3)$	$-72(3)$	$-70(3)$
NNLO	$-92(3)$	$-89(3)$	$-85(3)$	$-83(3)$
Exp	-92.16	-87.72	-84.51	$-82(1)$

Probing (α-cluster) structure of the 0_1^+, 0_2^+ states





RMS radii and quadrupole moments

	LO	Experiment
$r(0_1^+)$ [fm]	2.2(2)	2.47(2) [26]
$r(2_1^+)$ [fm]	2.2(2)	–
$Q(2_1^+)$ [$e\,\mathrm{fm}^2$]	6(2)	6(3) [27]
$r(0_2^+)$ [fm]	2.4(2)	–
$r(2_2^+)$ [fm]	2.4(2)	–
$Q(2_2^+)$ [$e\,\mathrm{fm}^2$]	$-7(2)$	–

Summary: nuclear lattice simulations

- combining EFT and lattice simulations → access to (light) nuclei
- exciting results for the ^{12}C spectrum, first ab initio calculation of the Hoyle state
- Work in progress: spectrum of ^{16}O, volume dependence, reactions ...