Partial Wave Analysis of Nucleon-Nucleon Scattering below pion production threshold

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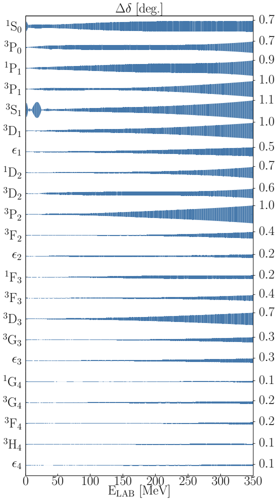
Motivation

- Study of the NN interaction for over 60 years
- More than 7800 experimental scattering data from 1950 to 2013
- Several partial wave analyses (PWA) and potentials since the 1950's
  - Hamada Johnston, Yale, Paris, Bonn, Nijmegen, Argonne, ...
- $\chi^2/\mathrm{d.o.f.} \sim 1$ possible by 1993

  [Stoks et al, Phys. Rev. C 48 (1993), 792]
- Chiral potentials appear in the mid 1990's

Motivation



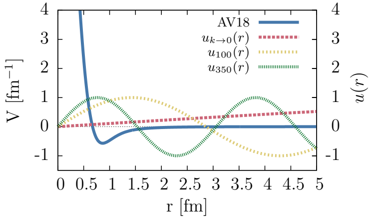
- No unique determination of the NN interaction
- Different phenomenological potentials
  - Fitted to experimental scattering data
  - High accuracy $\chi^2/\mathrm{d.o.f.} \sim 1$
  - Dispersion in Phaseshifts
  - OPE as the long range interaction
  - ~ 40 parameters for the short and intermediate range
  - Repulsive core for most of them
    - Short range correlations
- Nuclear structure calculations become complicated
- No statistical uncertainties estimates

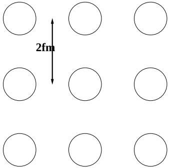
Motivation

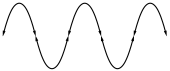
- Effective coarse graining
  - Oscillator Shell Model
  - Euclidean Lattice EFT
  - V_{lowk} interaction
- Characteristic distance $\sim 0.5 - 1.0$ fm
- Nyquist Theorem
  - Optimal sampling
  - Finite Bandwidth

$$\Delta r \Delta k \sim 1$$

  - de Broglie wavelength of the most energetic particle







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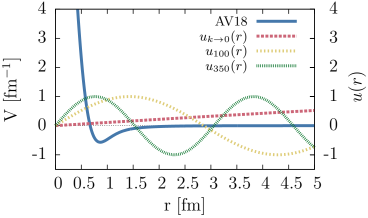
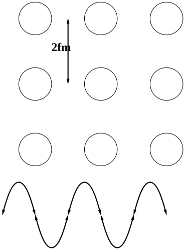




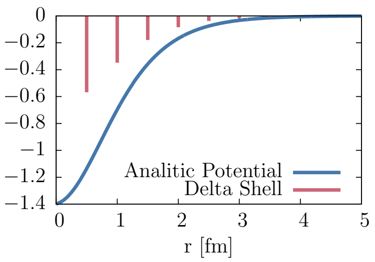
Delta Shell Potential

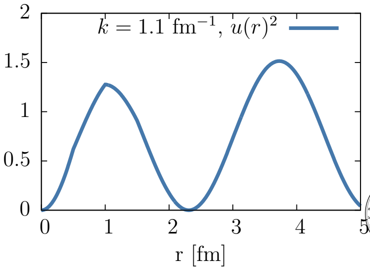
- A sum of delta functions

$$V(r) = \sum_i \frac{\lambda_i}{2\mu}\delta(r - r_i)$$

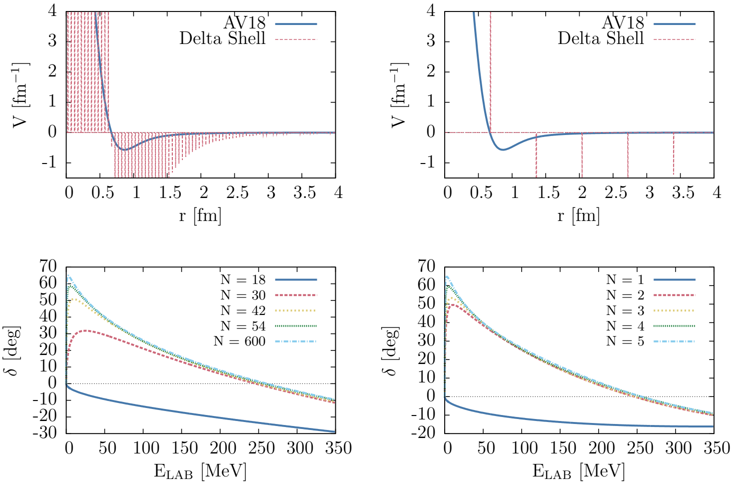
[Aviles, Phys.Rev. C6 (1972) 1467]

- Optimal and minimal sampling of the nuclear interaction
- Pion production threshold $\Delta k \sim 2$ fm^{-1}
- Optimal sampling, $\Delta r \sim 0.5$fm



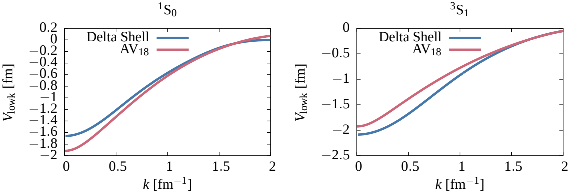


Coarse Graining the AV18 potential



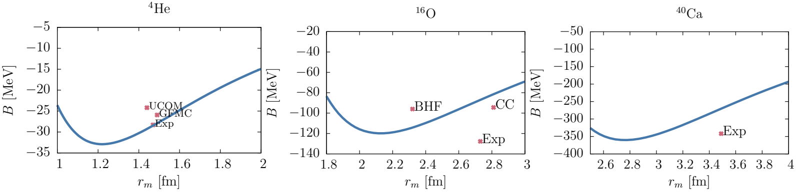
Delta Shell Potential

- Comparison with V_{lowk}



- Nuclear structure calculations

[Prog.Part.Nucl.Phys. 67 (2012) 359]



Delta Shell Potential

- 3 well defined regions
- Innermost region $r \leq 0.5$ fm
  - Short range interaction
  - No delta shell (No repulsive core)
- Intermediate region $0.5 \leq r \leq 3.0$ fm
  - Unknown interaction
  - λ_i parameters fitted to scattering data
- Outermost region $r \geq 3.0$ fm
  - Long range interaction
  - Described by OPE and EM effects
    - Coulomb interaction V_{C1} and relativistic correction V_{C2} (pp)
    - Vacuum polarization V_{VP} (pp)
    - Magnetic moment V_{MM} (pp and np)

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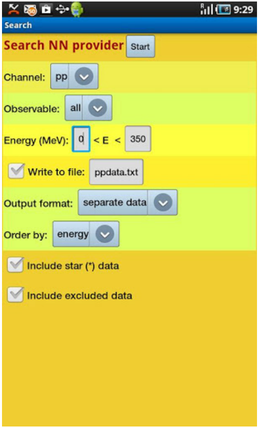
Fitting NN observables



- Database of NN scattering data obtained till 2013
  - http://nn-online.org/
  - http://gwdac.phys.gwu.edu/
  - NN provider for Android
    - Google Play Store

[J.E. Amaro, R. Navarro-Perez, and E. Ruiz-Arriola]

- 2868 pp data and 4991 np data
- 3σ criterion by Nijmegen to remove possible outliers

Fitting NN observables

- Delta shell potential in every partial wave

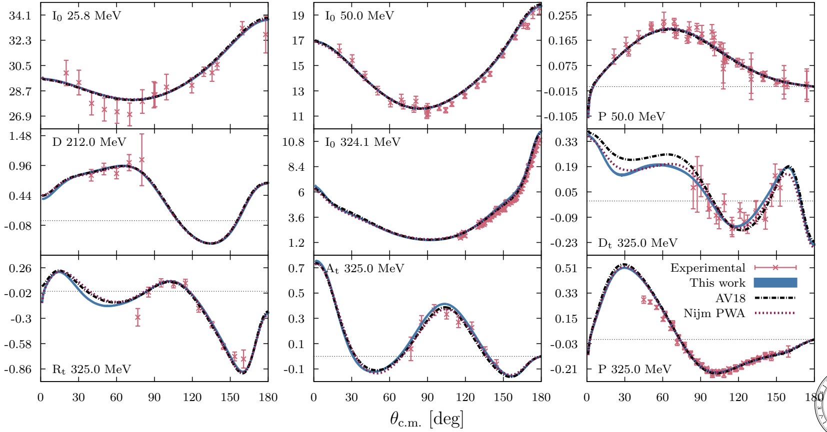
$$V_{l,l'}^{JS}(r) = \frac{1}{2\mu_{\alpha\beta}} \sum_{n=1}^{N} (\lambda_n)_{l,l'}^{JS} \delta(r - r_n) \qquad r \leq r_c = 3.0\text{fm}$$

- Strength coefficients λ_n as fit parameters
- Fixed and equidistant concentration radii $\Delta r = 0.6$ fm
- EM interaction is crucial for pp scattering amplitude

$$\begin{aligned} V_{C1}(r) &= \frac{\alpha'}{r} \,, \\ V_{C2}(r) &\approx -\frac{\alpha\alpha'}{M_p r^2} \,, \\ V_{VP}(r) &= \frac{2\alpha\alpha'}{3\pi r} \int_1^\infty dx\, e^{-2m_e r x} \left[1 + \frac{1}{2x^2}\right] \frac{(x^2-1)^{1/2}}{x^2} \,, \\ V_{MM}(r) &= -\frac{\alpha}{4M_p^2 r^3} \left[\mu_p^2 S_{ij} + 2(4\mu_p - 1)\mathbf{L}\cdot\mathbf{S}\right] \end{aligned}$$

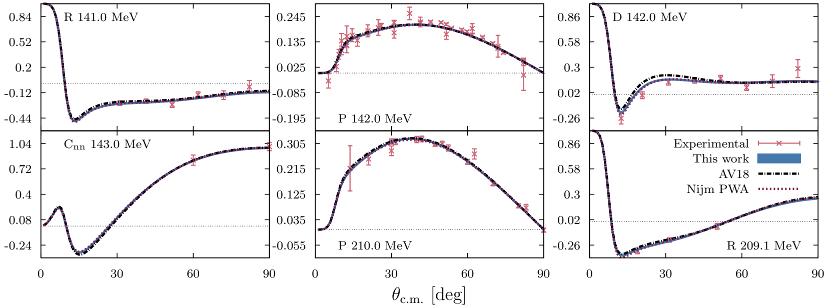
Scattering Observables

- Comparing with Potentials and Experimental data
- np data



Scattering Observables

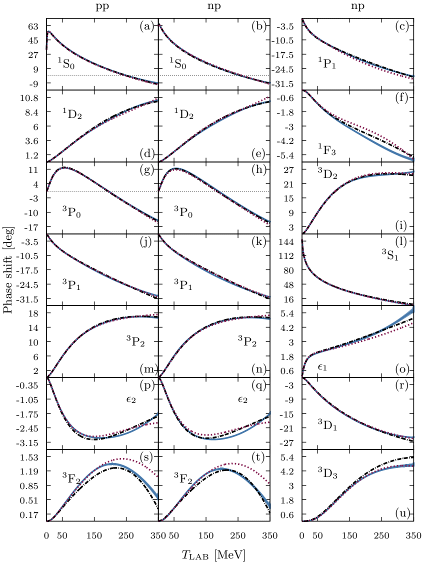
- Comparing with Potentials and Experimental data
- pp data



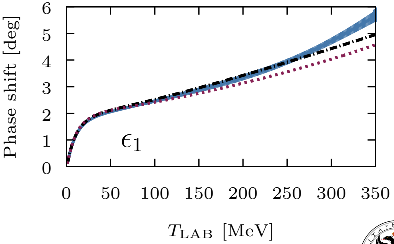
- $\chi^2/\mathrm{d.o.f.} = 1.06$ with $N = 2747|_{\rm pp} + 3691|_{\rm np}$

[arXiv:1304.0895]

Phase shifts



- Phase shifts for every partial
- Statistical uncertainty propagated directly from covariance matrix



Wolfenstein Parameters

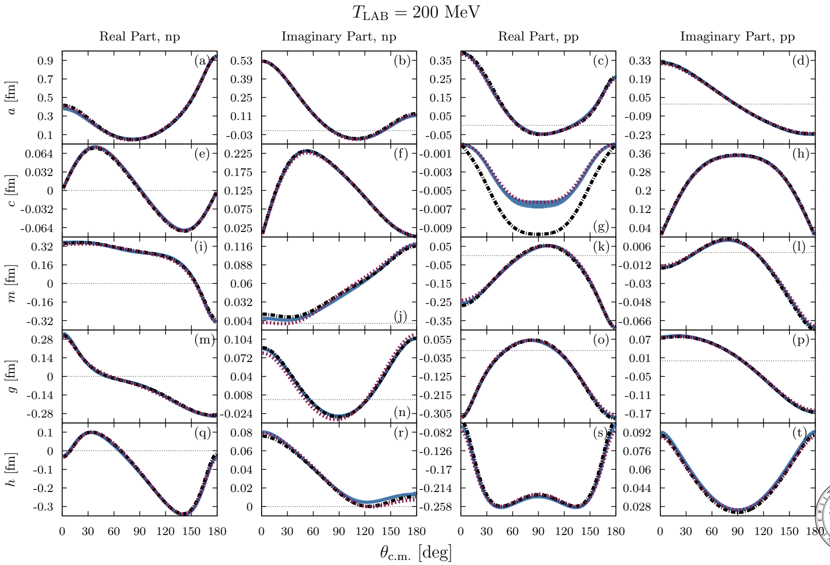
- A complete parametrization of the on-shell scattering amplitudes
- Five independent complex quantities
- Function of Energy and Angle

$$
\begin{aligned}
M(\mathbf{k}_f, \mathbf{k}_i) &= a + m(\sigma_1, \mathbf{n})(\sigma_2, \mathbf{n}) + (g - h)(\sigma_1, \mathbf{m})(\sigma_2, \mathbf{m}) \\
&\quad + (g + h)(\sigma_1, \mathbf{l})(\sigma_2, \mathbf{l}) + c(\sigma_1 + \sigma_2, \mathbf{n})
\end{aligned}
$$

- Scattering observables can be calculated from M

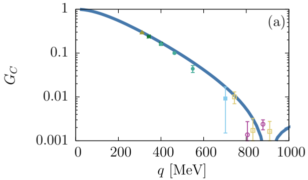
[Bystricky, J. et al, Jour. de Phys. 39.1 (1978) 1]

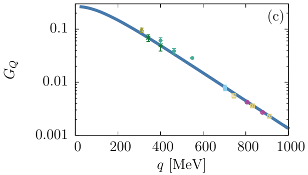
Wolfenstein Parameters

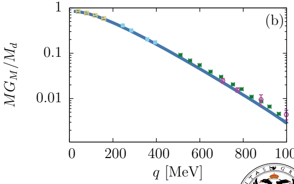


Deuteron Properties

	Delta Shell	Empirical	Nijm I	Nijm II	Reid93	AV18	CD-Bonn
E_d(MeV)	Input	2.224575(9)	Input	Input	Input	Input	Input
η	0.02493(8)	0.0256(5)	0.0253	0.0252	0.0251	0.0250	0.0256
$A_S(\mathrm{fm}^{1/2})$	0.8829(4)	0.8781(44)	0.8841	0.8845	0.8853	0.8850	0.8846
$r_m(\mathrm{fm})$	1.9645(9)	1.953(3)	1.9666	1.9675	1.9686	1.967	1.966
$Q_D(\mathrm{fm}^2)$	0.2679(9)	0.2859(3)	0.2719	0.2707	0.2703	0.270	0.270
P_D	5.62(5)	5.67(4)	5.664	5.635	5.699	5.76	4.85
$\langle r^{-1}\rangle(\mathrm{fm}^{-1})$	0.4540(5)			0.4502	0.4515		







Including Chiral Two Pion Exchange

- Inclusion of χTPE interactions at long and intermediate ranges
- pp PWA by the Nijmegen group

  [Rentmeester et al, Phys. Rev. Lett. 82 (1999), 4992]

  - Improvement in the χ^2 value compared to OPE only
  - Reduction of the number of parameters
  - Determination of chiral constants c_1, c_3, c_4
- Preliminary test using the δ-shell potential
  - OPE, TPE(l.o.) and TPE(s.o.)
  - Different cut radius, $r_c =$ 3.0, 2.4, 1.8fm

Comparing OPE and χTPE

- Fitting all NN data

r_c [fm]	1.8		2.4		3.0	
	N_p	χ^2/ν	N_p	χ^2/ν	N_p	χ^2/ν
OPE	31	1.80	39	1.56	46	1.54
TPE(l.o.)	31	1.72	38	1.56	46	1.52
TPE(s.o.)	30+3	1.60	38+3	1.56	46+3	1.52

- Fitting 3σ compatible NN data

	N_{Data}	N_p	χ^2/ν	N_{Data}	N_p	χ^2/ν	N_{Data}	N_p	χ^2/ν
OPE	5766	31	1.10	6363	39	1.09	6438	46	1.06
TPE(l.o.)	5841	31	1.10	6432	38	1.10	6423	46	1.06
TPE(s.o.)	6220	30+3	1.07	6439	38+3	1.10	6422	46+3	1.06

- OPE only at 3.0fm describes the data
- $1.8 \leq r \leq$ 3.0fm OPE + something else
- χTPE most of that something else

Summary

- Sampling of the NN interaction by a delta shell potential

$$1/\sqrt{m_\pi M} \lesssim \Delta r \lesssim 1/m_\pi$$

- 3 well defined regions
- Fit to NN scattering data
- Good description of scattering observables (over 6400 data)
- Statistical uncertainty propagation possible
- δ-shell representation allows straightforward calculations
  - Separable in momentum space
  - Finite nuclei Binding Energy
  - Phaseshifts
  - Scattering amplitudes
  - Deuteron properties and form factors
- Comparing OPE and χTPE
  - χTPE reduces number of parameters
  - Less 3σ compatible data