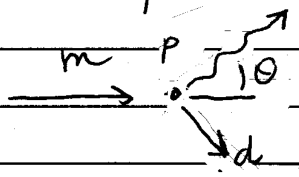
Esercitazione n-d capture

compute total unpolarized cross section in the LAB



$$\sigma = \frac{1}{4} \sum_{s_1 s_2} \sum_{\varepsilon \lambda} \sum_{\vec{k}\, \vec{p}_d} 2\pi \delta(E_i - E_f) \frac{\left| \langle \psi_d, \vec{q} \lambda | H_I | np \rangle \right|^2}{\text{flux}}$$

initial state in the CM $\qquad \text{flux} = \dfrac{p}{2\mu\Omega} = \dfrac{p}{m\Omega} = \dfrac{v}{\Omega}$

no interaction between n & p

$$\psi_{np} = \frac{e^{i\vec{p}\cdot\vec{r}} \chi_{s_1} \chi_{s_2} \xi_{\frac{1}{2}} \xi_{-\frac{1}{2}} - e^{-i\vec{p}\cdot\vec{r}} \chi_{s_2} \chi_{s_1} \xi_{-\frac{1}{2}} \xi_{\frac{1}{2}}}{\sqrt{2}\,\sqrt{\Omega}}$$

$$= \sum_{LM} 4\pi i^L Y^*_{LM}(\hat{p}) Y_{LM}(\hat{r}) j_L(pr)$$

$$\sum_{S S_z} \left(\tfrac{1}{2} s_1 \tfrac{1}{2} s_2 | S S_z\right) \chi_{S S_z} \sum_{T T_z} \left(\tfrac{1}{2} + \tfrac{1}{2}\, \tfrac{1}{2} - \tfrac{1}{2} | T 0\right) \xi_{T T_z}$$

$$\frac{1}{\sqrt{\Omega}} \left(\frac{1 - (-)^{L+S+1+T+1}}{\sqrt{2}} \right)$$

$$= \frac{\sqrt{2}}{\sqrt{\Omega}} \sum_{LM\, S S_z\, T T_z\, J J_z} 4\pi i^L Y^*_{LM}(\hat{p}) \left(\tfrac{1}{2} s_1 \tfrac{1}{2} s_2 | S S_z\right) \left(LM\, S S_z | J J_z\right)$$

$$\left(\tfrac{1}{2} + \tfrac{1}{2}\, \tfrac{1}{2} - \tfrac{1}{2} | T 0\right) \varepsilon_{LST} \left[Y_L \chi_S \right]_{J J_z} \xi_{T0}\, j_L(pr)$$

with the interaction

$$\psi_{np} = \frac{\sqrt{2}}{\sqrt{\Omega}} \sum_{LM\,SS_z T\;JJ_z} 4\pi\, i^L\, Y^*_{LM}(\hat{p})\, (\tfrac{1}{2} s_1 \tfrac{1}{2} s_2 | S S_z)(LM S S_z | J J_z)$$

$$(\tfrac{1}{2} + \tfrac{1}{2}\; \tfrac{1}{2} - \tfrac{1}{2} | T 0) \,.\quad \psi^{(T)}_{LSJJ_z}$$

$$\psi^{(T)}_{LSJJ_z} = \Omega_{LS}\, j_L + \sum_{L'S'} T_{LS,L'S'}\, \Omega_{L'S'} (-y_{L'} + i j_{L'})$$

thermal capture

$$E_n \cong .025\ \text{eV} = \tfrac{1}{2} m v^2 \qquad v = \sqrt{\frac{2E_n}{m}} = 7.338\ 10^{-6}$$

only S waves

$$\psi_{np} \cong \frac{\sqrt{2}}{\sqrt{\Omega}} \sum_{JJ_z} \sqrt{4\pi}\, (\tfrac{1}{2} s_1 \tfrac{1}{2} s_2 | J J_z)(\tfrac{1}{2} + \tfrac{1}{2}\; \tfrac{1}{2} - \tfrac{1}{2} | T 0)\, \psi_{0JJ_z}$$

$J = 0$ (singlet)

$J = 1$ (triplet)

we disregard the contribution of D waves in $J=1$

$$\psi^{(T)}_{LSJJ_z\,(r \gg b)} = \Omega_{0J} \left(\frac{\sin pr}{pr} + T \frac{e^{ipr}}{pr} \right) \qquad T = e^{i\delta_J} \sin \delta_J$$

$$\frac{T}{p} \to -a$$

$$p \to 0 \qquad = \Omega_{0J} \left(1 - \frac{a_J}{r} \right)$$

a_J = scattering length

$$= \Omega_{0J} \frac{u_J(r)}{r} \qquad u_J(r) \to r - a_J$$

Matrix element : one body contribution

$$H_I = -\frac{e}{\sqrt{2\omega\Omega}} \int d^3x \; \hat{\varepsilon}^*_{k\lambda} \, e^{-i\bar{k}\cdot\bar{x}} \cdot \vec{J}(\bar{x})$$

$$J_c(\bar{x}) = \sum_i \frac{e_i}{2im} \left[\delta(\bar{x}-\bar{r}_i) \vec{\nabla} + \bar{\nabla} \delta(\bar{x}-\bar{r}_i) \right]$$

$$e\bar{\mu}(\bar{x}) = \mu_N \sum_i \lambda_i \, \vec{\sigma}_i \, \delta(\bar{x}-\bar{r}_i)$$

$$e_i = \frac{1+\tau_z(i)}{2} \qquad \lambda_i = \frac{1+\tau_z(i)}{2}\mu_p + \frac{1-\tau_z(i)}{2}\mu_n$$

$$\mu_N = \frac{e}{2m} \qquad \mu_p = 2.79 \quad \mu_n = -1.91$$

$$H_I = -\frac{e}{\sqrt{2\omega\Omega}} \sum_i \left[\frac{e_i}{2im} \left(\vec{\nabla}_i - \overleftarrow{\nabla}_i \right) e^{-i\vec{k}\cdot\vec{r}_i} \hat{\varepsilon}_{k\lambda} + \frac{\mu_N}{e} \lambda_i \left(i\bar{k} \times \vec{\sigma}_i \right) e^{-i\bar{k}\cdot\bar{r}_i} \hat{\varepsilon}_{k\lambda} \right]$$

For simplicity we'll disregard the contribution from J_c

Matrix element - center of mass contribution

$$\int d^3r_1\, d^3r_2\; \psi_d^*(\bar r)\, \frac{e^{-i\bar P_d\cdot \bar R}}{\sqrt{\Omega}}\; \sigma\; \frac{e^{i\bar P_0\cdot \bar R}}{\sqrt{\Omega}}\; \psi_{LSJJ_z}(\bar r)$$

$$= \left\{\begin{array}{l} \bar r = \bar r_1 - \bar r_2 \\ \bar R = \frac{\bar r_1 + \bar r_2}{2} \end{array}\right. \quad \left\{\begin{array}{l} \bar r_1 = \bar R + \bar r/2 \\ \bar r_2 = \bar R - \bar r/2 \end{array}\right.$$

$$= \int d^3r\, d^3R\; \psi_d^*(\bar r)\, \frac{e^{-i\bar P_d\cdot \bar R}}{\sqrt{\Omega}} \left(\frac{-e}{\sqrt{2\omega\Omega}}\right) \sum_i \frac{\lambda_i}{2m}\, i(\bar k\times\sigma_i)\cdot\hat{\varepsilon}^*_{k\lambda}\; e^{-i\bar k(\bar R\pm\bar r/2)}\, \frac{e^{i\bar P_0\cdot\bar R}}{\sqrt{\Omega}}\, \psi_{LSJJ_z}(\bar r)$$

$$= \underbrace{\frac{-e}{\sqrt{2\omega\Omega}}}\; \delta_{\bar P_d+\bar k, P_0}\; \underbrace{\int d^3r\; \psi_d^*(\bar r) \sum_i \frac{\lambda_i}{2m}\, i(\bar k\times\bar\sigma_i)\cdot\hat{\varepsilon}^*_{k\lambda}\, e^{\mp i\bar k\frac{\bar r}{2}}\, \psi_{LSJJ_z}}_{\tilde H_{fi}}$$

$$\tilde H_{fi} = \frac{e}{\sqrt{2\omega\Omega}} \sum_{lm} (-i)^l \sqrt{2\pi}\, (-)^{J-J_z}\, (\lambda\, S\lambda\; J\, -J_z \mid lm)\; d^l_{m,-\lambda}(-\theta)\, \left(\lambda M_l^{LSJ} + E_l^{LSJ}\right)$$

to compute M & E we specialize our calculation in the system where $\hat{k} \parallel z$

$$\int d^3r\, \psi_f^*(\vec{r}) \sum_i \frac{\lambda_i}{2m} i(\vec{k}\times\vec{\sigma}_i)\, \hat{\varepsilon}^*_{k\lambda}\, e^{\mp i\vec{k}\vec{r}/2}\, \psi_{LSJJ_z} =$$

$$= \sum_{\ell \geq 1} (-i)^\ell \sqrt{2\pi}\, (1\, S_d\, J - J_z \,|\, \ell\, -\lambda)\, (\lambda M_\ell^{LSJ} + E_\ell^{LSJ})$$

selection rules

$$\begin{cases} J=0 & \ell=1 & M1 & \Pi_i \Pi_f (-)^\ell = +1 \ (E) \\ J=1 & \ell=1 & M1 & \Pi_i \Pi_f (-1)^{\ell+1} = +1 \ (M) \\ & \ell=2 & E2 & \end{cases}$$

$$\ell=1 \quad = -i\sqrt{2\pi}\, (1\, S_d\, 0\, 0 \,|\, \ell\, -\lambda)\, \lambda M_1^{000} = -i\sqrt{2\pi}\, \lambda M_1^{000}$$

$$\int d^3r\, \frac{u_0}{r} [Y_0 \chi_1]^+_{1S_d}\, \xi^+_{00} \sum_i \frac{\lambda_i}{2m} i(\vec{k}\times\vec{\sigma}_i)\, \hat{\varepsilon}^*_{k\lambda}\, e^{\mp i\vec{k}\vec{r}/2}$$

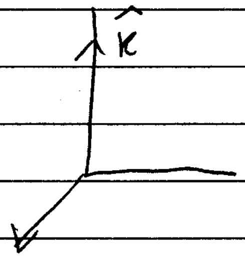
$$[Y_0 \chi_0]_{00}\, \xi_{10}\, \frac{u_0}{r}$$

$$\xi^+_{00}\, \lambda_1\, \xi_{10} = \frac{pn - np}{\sqrt{2}} \left(\frac{1+\tau_z(1)}{2}\mu_p + \frac{1-\tau_z(1)}{2}\mu_n \right) \frac{pn+np}{\sqrt{2}}$$

$$= \frac{pn-np}{\sqrt{2}} \left(\mu_p \frac{pn}{\sqrt{2}} + \mu_n \frac{np}{\sqrt{2}} \right)$$

$$= \frac{\mu_p - \mu_n}{2}$$

$$\xi_{00}^{+} \lambda_2 \xi_{10} = \frac{pn - np}{\sqrt{2}} \left(\mu_n \frac{pn}{\sqrt{2}} + \mu_p \frac{np}{\sqrt{2}} \right) = \frac{\mu_n - \mu_p}{2}$$



$$(\bar{k} \times \bar{\sigma}_1) \cdot \hat{\varepsilon} = (\hat{\varepsilon}^{*} \times \hat{k}) \cdot \bar{\sigma}$$

$$\hat{x} \times \hat{k} = -\bar{y}$$
$$\hat{y} \times \bar{k} = \bar{x}$$

$$(\hat{k} \times \bar{\sigma}_i) \cdot \hat{x} = \begin{vmatrix} x & y & z \\ 0 & 0 & 1 \\ \sigma_x & \sigma_y & \sigma_z \end{vmatrix} \cdot x = -\sigma_y$$

$\lambda = +1$ $\quad (\hat{k} \times \bar{\sigma}_i) \left[-\frac{(\hat{x} - i\hat{y})}{\sqrt{2}} \right] = -\frac{1}{\sqrt{2}} (-\sigma_{iy} - i\sigma_{ix}) =$

$$= \frac{\sigma_{iy} + i\sigma_{ix}}{\sqrt{2}}$$

$$= i \left(\frac{-i\sigma_{iy} + \sigma_{ix}}{\sqrt{2}} \right)$$

$$= i\sqrt{2}\, \sigma_{i-}$$

$\lambda = -1$ $\quad (\hat{\varepsilon}_{-1}^{*} \times \hat{z}) \cdot \bar{\sigma} = \left(\frac{\hat{x} + i\hat{y}}{\sqrt{2}} \times \hat{z} \right) \cdot \bar{\sigma}$

$$= \frac{1}{\sqrt{2}} (-\sigma_y + i\sigma_x)$$

$$= \frac{i}{\sqrt{2}} (i\sigma_y + \sigma_x)$$

$$= i\sqrt{2}\, \sigma_+$$

$J=0$

$\lambda=+1 \quad \chi^+_{1sd}\,(i\sqrt{2}\,\sigma_{1-})\,\frac{\uparrow\downarrow-\downarrow\uparrow}{\sqrt{2}} \qquad sd=-1 \quad \underline{\underline{OK}}$

$= \downarrow\downarrow \, i\sqrt{2} \, \frac{\downarrow\downarrow}{\sqrt{2}} = i \qquad \chi^+_{1sd}\, i\sqrt{2}\,\sigma_{2-}\,\frac{\uparrow\downarrow-\downarrow\uparrow}{\sqrt{2}} = -i$

$\lambda=-1 \quad \chi^+_{1sd}\, i\sqrt{2}\,\sigma_{1+}\,\frac{\uparrow\downarrow-\downarrow\uparrow}{\sqrt{2}} \qquad sd=+1$

$= \uparrow\uparrow \, i\sqrt{2} \, \frac{-\uparrow\uparrow}{\sqrt{2}} = -i$

$\chi^+_{1sd}\, i\sqrt{2}\,\sigma_{2+}\,\frac{\uparrow\downarrow-\downarrow\uparrow}{\sqrt{2}} = +i$

$(\lambda=+1) \quad H_{fi} = \int_0^\infty dr\, u_D u_0 \, \frac{1}{2m}\left[\frac{\mu_p-\mu_n}{2}\, ik \; i + \frac{\mu_n-\mu_p}{2}\, ik\,(-i)\right]$

$= \int_0^\infty dr\, u_D u_0 \, \frac{1}{2m}\, k(\mu_n-\mu_p)$

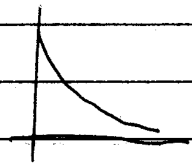
$(\lambda=-1) \quad H_{fi} = \int_0^\infty dr\, u_D u_0 \, \frac{1}{2m}\left[\frac{\mu_p-\mu_n}{2}\, ik\,(-i) + \frac{\mu_n-\mu_p}{2}\, ik\,(i)\right]$

$= \int_0^\infty dr\, u_D u_0 \, \frac{1}{2m}\, k(\mu_p-\mu_n)$

$-i\sqrt{2\pi}\, M_1 = -\frac{k(\mu_p-\mu_n)}{2m}\int_0^\infty dr\, u_D u_0 \qquad M_1 = -\frac{k\, i\,(\mu_p-\mu_n)}{2m\sqrt{2\pi}}\int_0^\infty dr\, u_D u_0$

Stima di M_1

$$u_D(r) = N e^{-\gamma r}$$



$$-\frac{1}{2\mu} u_D'' = -B_d u_D$$

$$\gamma = \sqrt{2\mu B_d}$$

$$\simeq 0{,}231 f^{-1}$$

$$B_d = \frac{2.225}{197.3} = 0.01128 f^{-1}$$

$$\mu = \frac{m}{2} = \frac{1}{2}\frac{mc^2}{\hbar c} = 2.377 f^{-1}$$

$$\int_0^\infty u_D^2 \, dr = 1 = N^2 \int_0^\infty e^{-2\gamma r} dr$$

$$= N^2 \left. \frac{e^{-2\gamma r}}{-2\gamma} \right|_0^\infty$$

$$= \frac{N^2}{2\gamma} \qquad N = \sqrt{2\gamma}$$

$$u_0 = r - a_0$$

$$\int_0^\infty dr \, u_d u_0 = \sqrt{2\gamma} \int_0^\infty dr \, e^{-\gamma r} (r - a_0)$$

$$= \sqrt{2\gamma} \, \frac{1 - a_0\gamma}{\gamma^2}$$

total cross section

The matrix element

$$\langle \Psi_d, \bar{q}\lambda | H_I | nd \rangle = \frac{\sqrt{2}}{\sqrt{\Omega}} \sum_{JJ_+} \sqrt{4\pi} \left(\tfrac{1}{2} S_1 \tfrac{1}{2} S_2 | J J_+\right) \left(\tfrac{1}{2} \tfrac{1}{2} \tfrac{1}{2} -\tfrac{1}{2} | 1 0\right)$$

$$\left(\frac{+e}{\sqrt{2\omega}\sqrt{\Omega}} \right) \delta_{\bar{p}_d + \bar{k}, 0}$$

$$\sum_{\ell m} (-i)^{\ell} \sqrt{2\pi} (-)^{J - J_+} (1 S_d J - J_+ | \ell m)$$

$$d^{\ell}_{m_1, -\lambda}(-\theta) \left(\lambda M_1^{0 J J} \right)$$

$$= -\frac{4\pi}{\Omega} \frac{e i}{\sqrt{2\omega}} \delta_{\bar{p}_d + \bar{k}, 0}$$

$$\sum_{J J_+ m} \left(\tfrac{1}{2} S_1 \tfrac{1}{2} S_2 | J J_+\right) \left(\tfrac{1}{2} \tfrac{1}{2} \tfrac{1}{2} - \tfrac{1}{2} | 1 0\right)$$

$$(-)^{J - J_+} (1 S_d J - J_+ | 1 m) \, d^{1}_{m_1, -\lambda}(-\theta)$$

$$\lambda M_1^{0 J J}$$

Properties of d

$$\int_{-1}^{1} d\cos\theta \; d^{j}_{m m'} \, d^{j'}_{\mu \mu'} = \frac{2}{(2j+1)} \delta_{m\mu} \delta_{m'\mu'} \delta_{j j'}$$

$$\sigma = \frac{1}{4}\sum_{s_1 s_2}\sum_{s_d \lambda}\sum_{\bar{k}\,\bar{P}_d} 2\pi\,\delta(E_i - E_f)\,\frac{(4\pi)^2}{\Omega^2}\,\frac{e^2}{2\omega}\,\delta_{\bar{P}_d+\bar{k},0}\,\frac{1}{\text{flux}}$$

$$\sum_{J J_z m} \left(\tfrac{1}{2} s_1 \tfrac{1}{2} s_2 | J J_z\right)\left(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2} -\tfrac{1}{2} | T 0\right)(-)^{J-J_z}$$

$$\left(1 s_d\ J -J_z | 1 m\right) d^1_{m,-\lambda}(-\theta)\ \lambda\ M_1^{0JJ}$$

$$\sum_{J' J_z' m'} \left(\tfrac{1}{2} s_1 \tfrac{1}{2} s_2 | J' J_z'\right)\left(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2} -\tfrac{1}{2} | T' 0\right)(-)^{J'-J_z'}$$

$$\left(1 s_d\ J' -J_z' | 1 m'\right) d^1_{m',-\lambda}(-\theta)\ \lambda \left(M_1^{0J'J'}\right)^*$$

$$\sum_{\bar{P}_d} \delta_{\bar{P}_d+\bar{k},0} \Rightarrow \bar{P}_d = -\vec{k} \qquad \sum_k \to \Omega \int \frac{d^3\vec{k}}{(2\pi)^3}$$

$$E_i = m_n + m_p + E_n \qquad \text{flux} = v/\Omega$$

$$\sum_{s_1 s_2} \left(\tfrac{1}{2} s_1 \tfrac{1}{2} s_2 | J J_z\right)\left(\tfrac{1}{2} s_1 \tfrac{1}{2} s_2 | J' J_z'\right) = \delta_{JJ'}\,\delta_{J_z J_z'}$$

therefore $L+S+T = \text{odd}$ $\quad L = L' = 0 \quad S = J' = J \Rightarrow T = T'$

$L'+S'+T' =$ "

$$\left(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}-\tfrac{1}{2} | T 0\right)^2 = 1/2$$

$$\sum_{s_d J_z} \left(1 s_d\ J -J_z | 1 m\right)\left(1 s_d\ J -J_z | 1 m'\right) = \delta_{mm'}$$

$$\int d\hat{k} \left(d^1_{m-\lambda}(-\theta)\right)^2 = \frac{2}{3}\cdot 2\pi = \frac{4\pi}{3}$$

$$\sigma = \frac{1}{4} \sum_{\lambda} \sum_{\hat{k}} 2\pi\, \delta(E_i - E_f) \frac{(4\pi)^2}{\Omega^2} \frac{e^2}{2\omega} \frac{\Omega}{v}$$

$$\sum_{Jm} \frac{1}{2} \left(d^1_{m,-\lambda}(-\theta)\right)^2 \left|M_1^{0JJ}\right|^2$$

$$\sum_{\lambda} \downarrow \quad \sum_{m} \downarrow$$

$$= \frac{1}{4} \quad \cancel{\Omega} \int_0^\infty \frac{k^2\, dk}{(2\pi)^3} \frac{(2\pi)(4\pi)^2}{\cancel{\Omega^2}} \frac{e^2}{2\omega} \frac{\cancel{\Omega}}{v} \frac{1}{2} \frac{4\pi}{3} \cdot 2 \cdot 3$$

$$\sum_{J} \left|M_1^{0JJ}\right|^2 \delta(E_i - E_f)$$

$$= 2\pi \frac{4\pi\alpha}{\omega v} \frac{k^2}{1 + \frac{k}{M_d}} \sum_{J} \left|M_1^{0JJ}\right|^2$$

$$= \frac{(4\pi)^2 \alpha}{2v} \frac{k}{1 + \frac{k}{M_d}} \sum_{J} \left|M_1^{0JJ}\right|^2$$

$v = 7,338\ 10^{-6}$

$\gamma = 0,231\ fm^{-1}$

$\mu_p - \mu_n = 4,70$

$$|M_1| = \frac{k}{2m}\frac{(\mu_p - \mu_n)}{\sqrt{2\pi}}\sqrt{2\gamma}\left(\frac{1 - a_0\gamma}{\gamma^2}\right) \simeq .183\ f^{+3/2}$$

$\sigma = 29,8\ fm^2 = 298\ mb$

sperimentale $\quad 334\ mb \pm .05\ mb$

332.6 ± 0.7