Electroweak responses of few-body systems at low energies.

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LECTURE 3

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Nuclear EM transition operators: $[\rho, \boldsymbol{J}]$

Standard Nuclear Physics Approach - SNPA

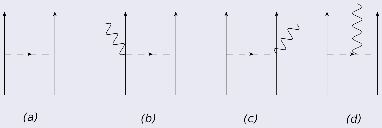
- One-body current

$$\boldsymbol{J}(\boldsymbol{x}) \approx \sum_j J_j^{(1)}(\boldsymbol{x}) = \sum_{j=1}^{A} \frac{1}{2M} e_j \left[\delta(\boldsymbol{x}-\boldsymbol{r}_j)\boldsymbol{p}_j + \boldsymbol{p}_j \delta(\boldsymbol{x}-\boldsymbol{r}_j)\right] + \boldsymbol{\nabla} \times \left[\sum_{j=1}^{A} \frac{1}{2M} \mu_j \boldsymbol{\sigma}_j \delta(\boldsymbol{x}-\boldsymbol{r}_j)\right]$$

- $\rightarrow$ large discrepancies with data
- Two-body currents: from $\pi-$, $\rho-$, $\omega-$, $\ldots$ exchanges
- Problem: is current conservation (CC) verified?

$$\boldsymbol{\nabla} \cdot \boldsymbol{J}(x) + i\left[H_0, \rho(\boldsymbol{x})\right] = 0 \qquad H = \sum_i \frac{\boldsymbol{p}_i^2}{2M} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

- v_{ij} and V_{ijk} depend on *isospin* and *momentum*: determined from fit of the NN dataset
- In principle $\boldsymbol{J} = \sum_i \boldsymbol{J}_i^{(1)} + \sum_{i<j} \boldsymbol{J}_{ij}^{(2)} + \sum_{i<j<k} \boldsymbol{J}_{ijk}^{(3)}$
- H_0 and $\boldsymbol{J}$ have to be derived consistently



- $\boldsymbol{J}_{ij}^{OPE,(2)}$ derived from (b-d) diagrams verifies CC with OPEP v_{ij}^{OPE}
- A simple prescription: if $v_{ij} = \sum_r c_r v_{ij}^{OPE}(m_r)$ then $\boldsymbol{J}_{ij} = \sum_r c_r \boldsymbol{J}_{ij}^{OPE,(2)}(m_r)$ verify CC
- [Buchmann, Leidemann, & Arenhövel, 1985], [Riska, 1985]
- [Marcucci *et al.*, PRC **72**, 014001 (2005)]

- One-body operators: non-relativistic reduction of $j_i^\mu \to O(1/M^2)$
- Two-body operators: families of $\pi-$, $\rho-$, $\omega-$exchanges
- Three-body operators: families of $\pi-$, $\rho-$, $\omega-$exchanges

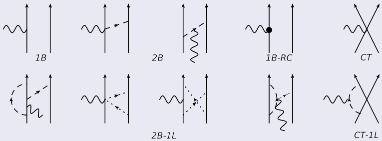
	$\mu(^3\mathrm{H})$	$\mu(^3\mathrm{He})$
1b	2.5745	-1.7634
Full	**2.9525**	**-2.1299**
Exp.	2.9790	-2.1276

AV18/UIX, $\Rightarrow$ Full=1b+2b+3b
[Marcucci *et al.*, PRC **72**, 014001 (2005)]

Nuclear EM operators from χEFT

Advantages

- NN potential and the EM current derived from the same $\mathcal{L}$
- Systematic inclusion of tterm using the power counting of χPT
- Most of the LECs entering the current are fixed by NN data



	LO	NLO	N^2LO	N^3LO
J	1B	2B	1B-RC	CT, 2B-1L

- [Koelling *et al.* (2009), (2011)]
- [Pastore *et al.*, 2009], [Piarulli *et al.*, 2012]
- In our current there are 4 undetermined LECs
- One fixed using Δ dominance
- The other three fitting $\mu(d)$, $\mu(^3\mathrm{H})$, and $\mu(^3\mathrm{He})$

$n - p$ capture

Deuteron wave function

- Spin state $\chi_{SS_z} = [s_1 s_2]_{SS_z}$, Isospin state $\xi_{TT_z} = [t_1 t_2]_{TT_z}$

$$\Phi^d_{1J_d}(\boldsymbol{r}) = \sum_{\ell=0,2} \frac{u_\ell(r)}{r} \left[Y_\ell(\hat{\boldsymbol{r}}) \chi_S \right]_{1,J_d} \xi_{00}$$

Scattering wave function; Lab frame: $\boldsymbol{p}$ along z

- j_L and y_L regular and irregular Bessel functions

$$\Psi_{m_1,m_2}(\boldsymbol{r}) = \frac{\sqrt{2}}{\sqrt{\Omega}} \sum_{LMSS_zJJ_z} 4\pi Y^*_{LM}(\hat{\boldsymbol{p}})(\frac{1}{2}, m_1, \frac{1}{2}, m_2 | S, S_z)$$
$$\times (L, M, S, S_z | J, J_z)(\frac{1}{2}, +\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} | T, 0) \Psi^{(+)}_{LSJJ_z}(\boldsymbol{r})$$

$$\Psi^{(+)}_{LSJJ_z}(\boldsymbol{r}) = j_l(pr)\Omega_{LSJJ_z} + \sum_{L'S'} \left(-\tilde{y}_L(pr) + i j_L(pr) \right) + \sum_{i=1}^{N} \underbrace{a_i f_i(r)}_{internal\ part}$$

$$\Omega_{LSJJ_z} = i^L \left[Y_L(\boldsymbol{r}) \chi_S \right]_{JJ_z} \xi_{TT_z} \qquad (-)^{L+S+T} = -1$$

Multipole Analysis (1)

RMEs

$$\langle \boldsymbol{q}\lambda; \Phi_{1J_d}|V|0; \Psi^{(+)}_{LSJJ_z}\rangle_L =$$

$$= \frac{e}{\sqrt{2\omega_q\Omega}} \sum_{\ell\geq 1,m} (-i)^\ell \sqrt{2\pi} d^\ell_{m,-\lambda}(-\theta)(-)^{1-J_d}(J,J_z,1,-J_d|\ell m)\left[E^{(LSJ)}_\ell + \lambda M^{(LSJ)}_\ell\right]$$

J^π	Wave	RMEs
0^+	$^1S_0$	$M1$
0^-	$^3P_0$	$E1$
1^+	$^3S_1 - {}^3D_1$	$M1, E2$
1^-	$^1P_1 - {}^3P_1$	$E1, M2$
2^+	$^1D_2 - {}^3D_2$	$M1, E2, M3$
2^-	$^3P_2 - {}^3F_2$	$E1, M2, E3$

$$\sigma_{np} = \frac{(4\pi)^2\alpha}{2v}\frac{q}{1+q/M_d}\sum_{LSJJ}\left(|E^{(LSJ)}_J|^2 + |M^{(LSJ)}_J|^2\right)$$

$$\sigma_{\gamma d} = \frac{2}{3}\left(\frac{p}{q}\right)^2\left(1+\frac{q}{M_d}\right)\sigma_{np}$$

Multipole analysis (2)

- At low energies
  - $L = 0$ wave dominates $\rightarrow M_1^{000}$ & M_1^{011}
  - $|M_1^{000}| \gg |M_1^{011}|$
- As E increases
  - $L > 0$ waves start to be important $\rightarrow E_1^{LSJ}$ RMEs
  - Giant dipole resonance

Results at thermal energies

J^π	Wave	RMEs	LO	NLO	N^{2}LO	N^{3}LO
0^+	$^1S_0$	$M1$	-0.185	-0.00445	0.00039	-0.00303
0^-	$^3P_0$	$E1$	–	–	–	–
1^+	$^3S_1 - {}^3D_1$	$M1$	-0.000023	–	–	–
1^-	$^1P_1 - {}^3P_1$	$E1$	–	–	–	–
2^+	$^1D_2 - {}^3D_2$	$M1$	–	–	–	–
2^-	$^3P_2 - {}^3F_2$	$E1$	–	–	–	–

Λ	LO	NLO	N^{2}LO	N^{3}LO
500 MeV	305.1	319.9	318.6	328.9
500 MeV*	303.9	–	–	–
600 MeV	302.6	316.9	315.8	326.9
Expt.			332.6(7)	

$n - p$ radiative capture cross section at thermal energies (mb)

*: disregarding $\boldsymbol{J}^C$ in the one-body current

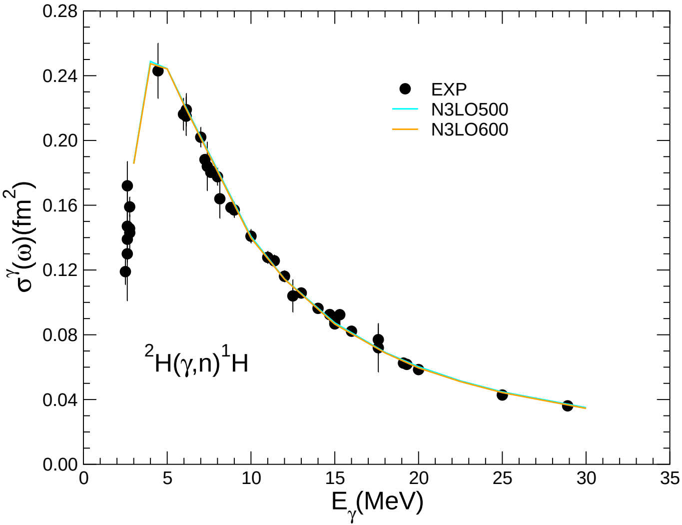
Results at $E = 1$ MeV

J^π	Wave	RMEs	LO+$\cdots$+N^{3}LO
0^+	$^1S_0$	$M1$	$-0.044307 + i0.021308$
0^-	$^3P_0$	$E1$	$-0.048302 - i0.000406$
1^+	$^3S_1$	$M1$	$0.000093 + i0.000098$
1^-	$^1P_1$	$E1$	$0.000095 - i0.000001$
1^-	$^3P_1$	$E1$	$-0.089569 + i0.000450$
2^-	$^3P_2$	$E1$	$-0.110992 - i0.000118$

Λ	LO	NLO	N^{2}LO	N^{3}LO
500 MeV	0.016305	0.017189	0.017148	0.017725
600 MeV	0.016285	0.017103	0.017083	0.018032

$n - p$ radiative capture cross section at thermal energies (nb)

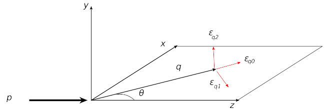
γd phodisintegration



Parity violation in $n - p$ radiative capture

Interest

- Study of the weak interaction between $u - d$ quarks ($\Delta S = 0$)
- Interplay between weak and strong interaction
- Goal: extraction of the PV $\pi - N$ coupling constant



NPDGAMMA experiment

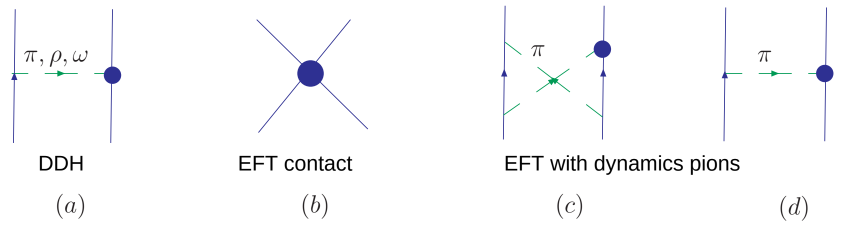
- $\vec{n} + p \rightarrow d + \gamma$
- Measurement of the A_z longitudinal asymmetry

$$A_z(\theta) = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

- Now there are PV component in the wave functions and in the PV terms in the current

PV interactions: models

meson-exchange & contact interaction



CP invariance assumed in all models

PV interaction (1): meson exchange models

Effective Lagrangian

$$\mathcal{L}_I^{PV} = -\frac{h_\pi^1}{\sqrt{2}}\bar{N}(\vec{\tau}\times\vec{\pi})_z N + \ldots$$

it contains 7 unknown parameters h_π^1, h_ρ^0, h_ρ^1, h_ρ^2, h_ω^0, h_ω^1, $h_\rho'^1$

DDH potential Desplanques *et al*, 1980

$$V_{DDH}^{PV}(1,2) = i\frac{h_\pi^1 g_{\pi NN}}{8\pi\sqrt{2}}(\vec{\tau}_1\times\vec{\tau}_2)_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\left[\frac{\boldsymbol{p}_1-\boldsymbol{p}_2}{2M},\frac{e^{-m_\pi r}}{r}\right] + \ldots$$

Parameters range

Param.	Best range	"DDH-best"	"DDH-adj" (*)
$10^7 \times h_\pi^1$	$0 \to 11.4$	4.56	4.56
$10^7 \times h_\rho^0$	$-30.78 \to 11.4$	-16.4	-11.4
$10^7 \times h_\rho^1$	$-0.38 \to 0$	-0.19	-2.77
$10^7 \times h_\rho^2$	$-11.02 \to -7.6$	-9.5	-13.7
$10^7 \times h_\omega^0$	$-10.26 \to 5.7$	-1.90	3.23
$10^7 \times h_\omega^1$	$-1.9 \to -0.76$	-1.14	1.94
$10^7 \times h_\omega'^1$	≈ 0	0	0

Experimental situation

Ramsey-Musolf & Page, 2006

Experiments in medium-heavy nuclei

Enhancement of PV effects

- circular polarization in the γ-decay of ^{18}F
- anapole moment of Cesium and other nuclei

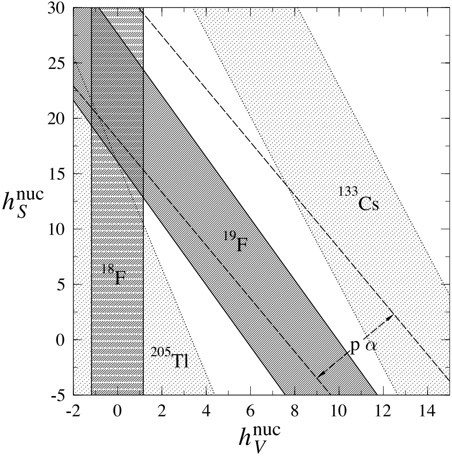
Theoretical analysis uncertain

Experiments in light nuclei

PV effects tiny - few-body dynamics under control

- longitudinal asymmetry in $\vec{p}p$ scattering
- NPDGAMMA experiments (LANSCE, ORNL)
- neutron spin rotation $\vec{n}p$, $\vec{n}d$, $\vec{n}\alpha$ (NIST, ORNL)
- $n + {}^3\mathrm{He} \rightarrow p + {}^3\mathrm{H}$ (ORNL)

Inconsistency



$$h_V^{nuc} = h_\pi^1 - 0.12h_\rho^1 - 0.18h_\omega^1$$
$$h_S^{nuc} = -h_\rho^0 - 0.7h_\omega^1$$

Taken from Ramsey-Musolf & Page, 2006

PV Lagrangian – EFT approach

Expression up to one four-gradient [Kaplan & Savage, 1992]

LEC'c $\sim G_F f_\pi^2 \approx 10^{-7}$

$u_\mu, X^a_{L,R}, \ldots$ quantities expressed in terms of the pion field

$$
\begin{aligned}
\mathcal{L}^{PV,-1}_{\Delta T=1} &= -\frac{h^1_\pi f_\pi}{2\sqrt{2}} \overline{N} X^3_- N \\
\mathcal{L}^{PV,0}_{\Delta T=0} &= -h^0_V \overline{N} u_\mu \gamma^\mu N \\
\mathcal{L}^{PV,0}_{\Delta T=1} &= +\frac{h^1_V}{2} \overline{N} \gamma^\mu N Tr(u_\mu X^3_+) - \frac{h^1_A}{2} \overline{N} \gamma^\mu \gamma^5 N Tr(u_\mu X^3_-) \\
\mathcal{L}^{PV,0}_{\Delta T=2} &= h^2_V I^{ab} \overline{N} (X^a_R u_\mu X^b_R + X^a_L u_\mu X^b_L) \gamma^\mu N \\
&\quad - \frac{h^2_A}{2} I^{ab} \overline{N} (X^a_R u_\mu X^b_R - X^a_L u_\mu X^b_L) \gamma^\mu \gamma^5 N
\end{aligned}
$$

+ contact terms of order Q (7 in [Zhu *et al.*] $\rightarrow$ 5 [Girlanda, 2008])

Multipole amalysis of A_z

NPDGAMMA experiment

- Ultracold neutrons from SNS at Oak Ridge
- Initial state: only S-waves
- Multipoles: only magnetic an electric dipoles
- $A_z(\theta) = a_z \cos\theta$

$$a_z = \frac{-\sqrt{2}\Re\left[M_1(^1S_0)^* E_1(^3S_1) + M_1(^3S_1)^* E_1(^1S_0)\right] + \Re\left[M_1(^3S_1)^* E_1(^3S_1)\right]}{|M_1(^1S_0)|^2 + |M_1(^3S_1)|^2}$$

- $E_1(^3S_1)$ and $E_1(^1S_0)$ deriving from PV interaction $\sim 10^{-7}$
- $a_z \times 10^8 \sim -0.11 h_\pi^1$ using DDH
- Calculation in progress for the EFT PV interaction & PV current

End of Lecture 3

Thank for your attention