Electroweak responses of few-body systems at low energies.

M. Viviani

INFN, Sezione di Pisa &
Department of Physics, University of Pisa
Pisa (Italy)

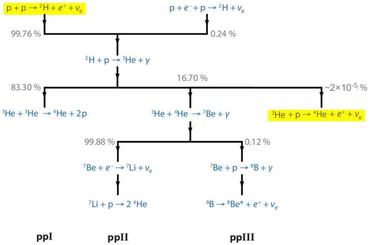
LECTURE 4

Raimondo Anni School, 6th Course, May 27-31, 2013

May 31, 2013

Outline

Reactions of astrophysical interest ($A \leq 4$)



Discussed in this talk:

- μ-capture on light nuclei: a stringent test for the theory
- $p + p \rightarrow {}^2\mathrm{H} + e^+ + \nu_e$
- $p + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + e^+ + \nu_e$

Historical perspective

- *pp* fusion:
  - first estimate: Bethe & Critchfield, 1938
  - "Standard Nuclear Physics Approach" (SNPA): $\rightarrow$ Schiavilla *et al.*, 1998
- "hep" reaction:
  - "SNPA": Marcucci *et al.*, 2001 (four nucleon dynamics)

New interest

Re-compute the cross sections (astrophysical factors) using χEFT

- more contact with QCD
- systematic and controlled expansion of nuclear potential/transition operators

Re-compute the cross sections (astrophysical factors) using χEFT

Weak current $\mathcal{J}_\mu = \mathcal{V}_\mu - \mathcal{A}_\mu$

- Weak transition operators: [Park, Rho & Kubodera (1995)]
- "Vector" part: from CVC it is derived from the EM operators
  - EM current $j_\mu^{EM} = \bar{\psi}\gamma_\mu(1+\tau_z)/2\psi$
  - Weak current $j_\mu^{weak} = \bar{\psi}(\gamma_\mu - g_A\gamma_\mu\gamma^5)\tau_+\psi$
- CVC hypothesis: $\mathcal{V}_\mu$ obtained from the isovector part of j_μ^{EM} via the substitution $\tau_z \rightarrow \tau_x + i\tau_y$
- Verified in the standard model

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$$

Wave functions for $A > 2$

NN potentials

- "Old models": Argonne V18, CD-Bonn, Nijmengen ($\chi^2 \approx 1$)
- Fit of 3N data using non-locality in P-waves (INOY [Doleschall, 2008])
- Effective field theory (EFT)
  - J-N3LO – [Epelbaum and Coll, 1998-2006]
  - N3LO – [Entem & Machleidt, 2003]

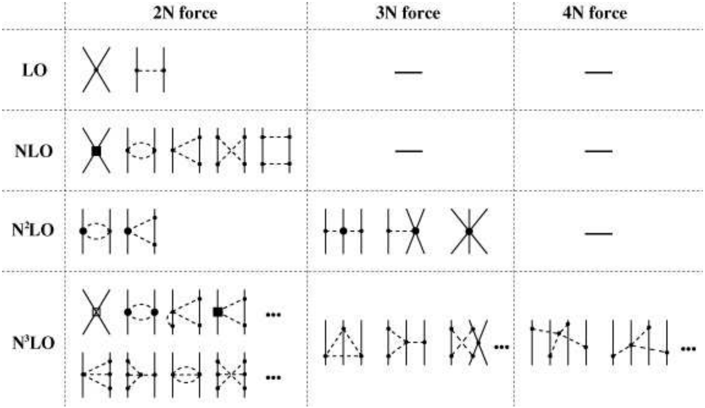
3N potentials

- "Old models": Tucson-Melbourne [Coon *et al*, 1979, Friar *et al*, 1999]; Brazil [Robilotta & Coelho, 1986]; Urbana [Pudliner *et al*, 1995]
- Effective field theory
  - at N2LO [Epelbaum *et al*, 2002], [Navratil, 2007]
- Illinois [Pieper *et al*, 2001]
- Under progress: N3LO, N4LO

Accurate nuclear wave functions

- Methods for $A \geq 3$: Faddeev-Yakubovsky Equations, GFMC, Variational methods (Gaussians, NCSM, HH)
  - HH method [Kievsky, MV, *et al.*,J. Phys. G **35**, 063101 (2008)]
  - EIHH method [Barnea *et al.*, PRC **61**, 054001 (2000)], [Bacca *et al.*, `arXiv:1210.7255`]

NN potential from χEFT



- NN potential: N^3LO "N3LO" model – [Entem & Machleidt, 2003]
- 3N potential: N^2LO "N2LO" model – [Navratil, 2007], [Marcucci *et al.*, 2012]

The HH method

HH functions

- hyperradius $\rho^2 = \frac{2}{A} \sum_{i<j} r_{ij}^2$
- hyperangles $\Omega = \{\frac{\mathbf{x}_1}{\rho}, \ldots, \frac{\mathbf{x}_{A-1}}{\rho}\}$ ($\mathbf{x}_i$ Jacobi vectors)
- $T = T\rho + T_\Omega$
- The HH functions $\mathcal{Y}_{[K]}(\Omega)$ are the eigenstates of T_Ω

The HH method

HH functions

- hyperradius $\rho^2 = \frac{2}{A}\sum_{i<j} r_{ij}^2$
- hyperangles $\Omega = \{\frac{\mathbf{x}_1}{\rho}, \dots, \frac{\mathbf{x}_{A-1}}{\rho}\}$ ($\mathbf{x}_i$ Jacobi vectors)
- $T = T\rho + T_\Omega$
- The HH functions $\mathcal{Y}_{[K]}(\Omega)$ are the eigenstates of T_Ω

$$
\begin{aligned}
|\Psi\rangle &= \sum_\mu a_\mu |\mu\rangle \\
\langle \mathbf{r}_1, \dots, \mathbf{r}_A|\mu\rangle &= L_n^{(3A-4)}(\gamma\rho)e^{-\gamma\rho/2}\mathcal{Y}_{[K]}(\Omega)
\end{aligned}
$$

The HH method

HH functions

- hyperradius $\rho^2 = \frac{2}{A}\sum_{i<j} r_{ij}^2$
- hyperangles $\Omega = \{\frac{\mathbf{x}_1}{\rho}, \ldots, \frac{\mathbf{x}_{A-1}}{\rho}\}$ ($\mathbf{x}_i$ Jacobi vectors)
- $T = T\rho + T_\Omega$
- The HH functions $\mathcal{Y}_{[K]}(\Omega)$ are the eigenstates of T_Ω

$$
\begin{aligned}
|\Psi\rangle &= \sum_\mu a_\mu |\mu\rangle \\
\langle \mathbf{r}_1, \ldots, \mathbf{r}_A|\mu\rangle &= L_n^{(3A-4)}(\gamma\rho)e^{-\gamma\rho/2}\mathcal{Y}_{[K]}(\Omega)
\end{aligned}
$$

Advantages - bound state

Simplified calculation of the matrix elements of

- local/non-local NN & 3N potentials
- given in coordinate/momentum space

Scattering calculation (1)

Example: $A - B$ elastic scattering for a given J^π

$$\Omega^F_{LS}(A,B) = \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A\phi_B]_S \right]_{JJ_z} \frac{F_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}}$$

$$\Omega^G_{LS}(A,B) = \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A\phi_B]_S \right]_{JJ_z} \frac{G_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}} (1 - e^{-\gamma r_{AB}})^{2L+1}$$

$$\Omega^\pm_{LS}(A,B) = \Omega^G_{LS}(A,B) \pm i\Omega^F_{LS}(A,B)$$

$$|\Psi_{LS}\rangle = \sum_\alpha c_{LS,\alpha}\Phi_\alpha + |\Omega^F_{LS}(p, {}^3\mathrm{He})\rangle + \sum_{L'S'} T_{LS,L'S'}|\Omega^+_{L'S'}(p, {}^3\mathrm{He})\rangle$$

Scattering calculation (1)

Example: $A - B$ elastic scattering for a given J^π

$$\Omega^F_{LS}(A,B) = \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A\phi_B]_S \right]_{JJ_z} \frac{F_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}}$$

$$\Omega^G_{LS}(A,B) = \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A\phi_B]_S \right]_{JJ_z} \frac{G_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}} (1 - e^{-\gamma r_{AB}})^{2L+1}$$

$$\Omega^\pm_{LS}(A,B) = \Omega^G_{LS}(A,B) \pm i\Omega^F_{LS}(A,B)$$

$$|\Psi_{LS}\rangle = \sum_\alpha c_{LS,\alpha}\Phi_\alpha + |\Omega^F_{LS}(p, {}^3\mathrm{He})\rangle + \sum_{L'S'} T_{LS,L'S'} |\Omega^+_{L'S'}(p, {}^3\mathrm{He})\rangle$$

- Sum over LS such that $\vec{L} + \vec{S} = \vec{J}$ and $(-)^L = \pi$
- $T_{LS,L'S'}$ = T-matrix elements
- $c_{LS,n}$ and $T_{LS,L'S'}$ determined using the Kohn variational principle (KVP)

Scattering state calculation (2)

Kohn Variational Principle

$$\mathcal{F}(c_{LS,\alpha}, T_{LS,L'S'}) = T_{LS,L'S'} - \langle \mathcal{T}\Psi_{L'S'}|H-E|\Psi_{LS}\rangle$$

- $\mathcal{T}\Psi_{L'S'}$ = "time reversed" wave function
- Problem: evaluation of the matrix elements $A^X_{\alpha,LS} = \langle \mathcal{T}\Phi_\alpha|H-E|\Omega^X_{LS}\rangle$ and $B^{XX'}_{LS,L'S'} = \langle \mathcal{T}\Omega^X_{LS}|H-E|\Omega^{X'}_{L'S'}\rangle$ $(X = F, G)$
- Ω^X_{LS} are decomposed in partial waves

$$\begin{pmatrix} H_{1,1}-E & \cdots & H_{1,\mathcal{N}} & A^G_{1,LS} \\ \cdots & \cdots & \cdots & \cdots \\ H_{\mathcal{N},1} & \cdots & H_{\mathcal{N},\mathcal{N}}-E & A^G_{\mathcal{N},LS} \\ A^G_{1,LS} & \cdots & A^G_{\mathcal{N},LS} & B^{GG}_{LS,LS} \end{pmatrix} \begin{pmatrix} c_{LS,1} \\ \cdots \\ c_{LS,\mathcal{N}} \\ T_{LS,LS} \end{pmatrix} = \begin{pmatrix} -A^X_{LS,1} \\ \cdots \\ -A^X_{LS,\mathcal{N}} \\ 1 - B^{GF}_{LS,LS} - B^{FG}_{LS,LS} \end{pmatrix}$$

A high-precision variational approach to three- and four-nucleon bound and zero-energy scattering states, A. Kievsky, S. Rosati, M. Viviani, L.E. Marcucci, and L. Girlanda J. Phys. G, **35**, 063101 (2008)

Some results for $A = 2$–4

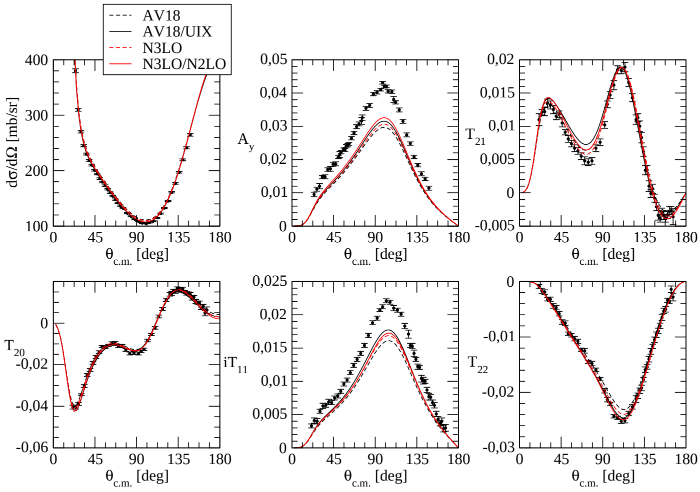
$A=2$	AV18	N3LO	Exp.
B_d (MeV)	2.22457	2.22456	2.224574(9)
a_{nn} (fm)	–18.487	–18.900	–18.9(4)
${}^1a_{np}$ (fm)	–23.732	–23.732	–23.740(20)
${}^3a_{np}$ (fm)	5.412	5.417	5.419(7)
$A=3$	AV18/UIX	N3LO/N2LO	Exp.
$B_{^3\mathrm{H}}$ (MeV)	8.479	8.474	8.482
$B_{^3\mathrm{He}}$ (MeV)	7.750	7.733	7.718
${}^2a_{nd}$ (fm)	0.590	0.675	0.645(10)
${}^4a_{nd}$ (fm)	6.343	6.342	6.35(2)
$A=4$	AV18/UIX	N3LO/N2LO	Exp.
$B_{^4\mathrm{He}}$ (MeV)	28.45	28.36	28.30
${}^0a_{n^3\mathrm{He}}$ (fm)	7.81	7.61	7.57(3)
${}^1a_{n^3\mathrm{He}}$ (fm)	3.39	3.37	3.36(1)

Accuracy of the calculation tested in several benchmarks

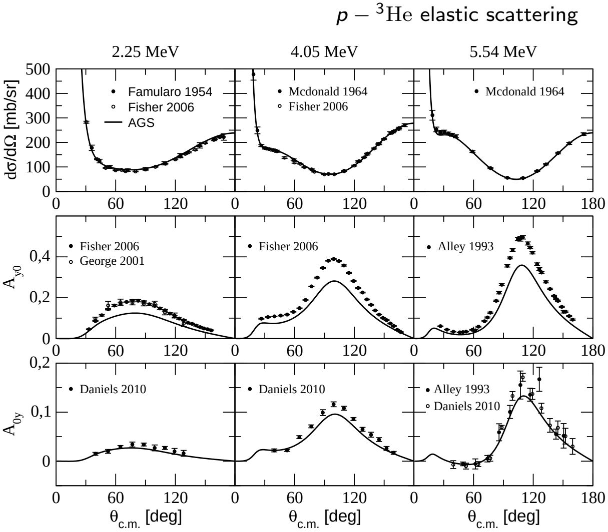
Bound states: [Kamada *et al.*, 2001] – Scattering states [MV *et al.*, 2011]

N-d elastic scattering

p-d scattering at $E_p = 2.5$ MeV



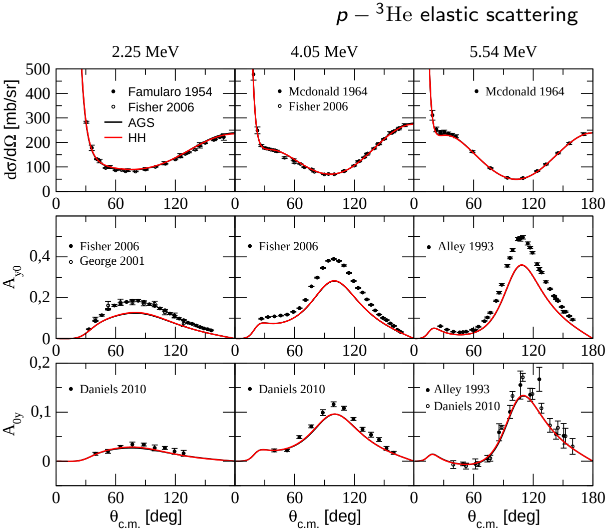
Benchmark test of 4N scattering calculations [PRC **84**, 054010 (2011)]



N3LO potential

AGS= Deltuva & Fonseca
FY= Lazauskas & Carbonell

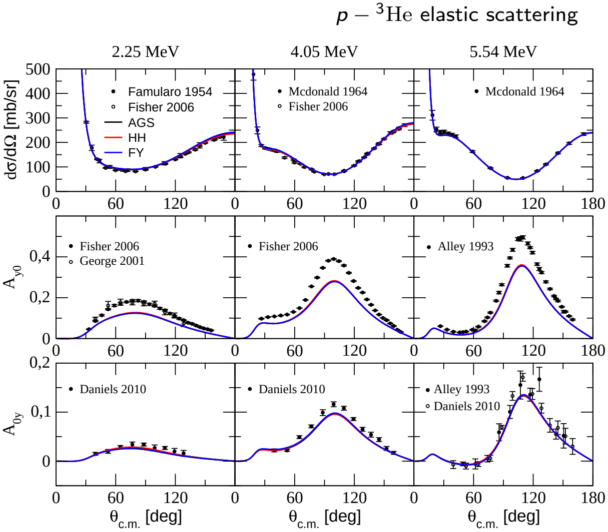
Benchmark test of 4N scattering calculations [PRC **84**, 054010 (2011)]



N3LO potential

AGS= Deltuva & Fonseca
FY= Lazauskas & Carbonell

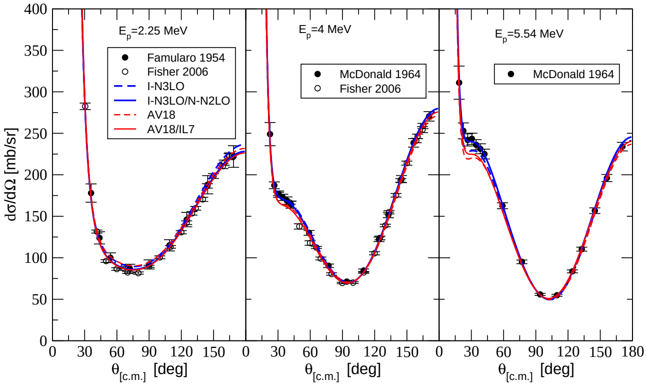
Benchmark test of 4N scattering calculations [PRC **84**, 054010 (2011)]



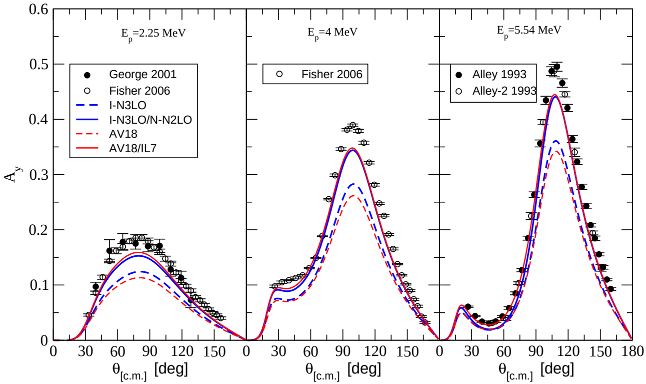
N3LO potential

AGS= Deltuva & Fonseca
FY= Lazauskas & Carbonell

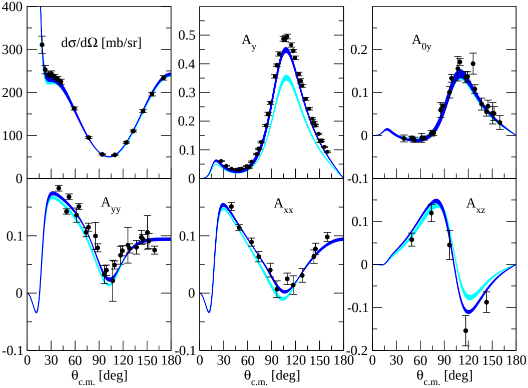
$p - {}^3\mathrm{He}$ differential cross section



$p - {}^3\mathrm{He}\ A_y$



$p - {}^3\mathrm{He}$ Observables



$E_p = 5.54$ MeV

cyan band: only NN potentials   blue band: inclusion of 3N potentials

Nuclear weak transition operators

- Nuclear weak transition operators: $[\rho^{(A,V)}, \mathbf{j}^{(A,V)}]$
  - Standard Nuclear Physics Approach - SNPA [Schiavilla *et al.*, PRC **58**, 1263 (1998); Marcucci *et al.*, PRC **63**, 015801 (2000)]
  - Chiral Effective Field Theory Approach - χEFT [Park, Min, & Rho, Phys. Rep. **233**, 341 (1993); Park *et al.*, PRC **67**, 055206 (2003)]

SNPA transition operators

- One-body operators: NRR of $j_i^\mu \rightarrow O(1/m^2)$
- Two-body operators: $\pi-$, $\rho-$, $\omega-$, ... exchanges + Δ d.o.f. excitations
- $\rho^{(V)}$ and $\mathbf{j}^{(V)}$: CVC $\rightarrow$ EM operators (constructed to verify current conservation)

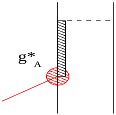
	$\mu(^3\mathrm{H})$	$\mu(^3\mathrm{He})$
1b	2.5745	-1.7634
Full	**2.9525**	**-2.1299**
Exp.	2.9790	-2.1276

AV18/UIX, $\Rightarrow$ Full=1b+2b+3b
[Marcucci *et al.*, PRC **72**, 014001 (2005)]

SNPA transition operators (2)

- Two-body $\rho^{(A)}$: PCAC + low-energy theorem $\rightarrow$ π-exchange and short-range terms
- Two-body $\mathbf{j}^{(A)}$: π- and ρ-exchange, $\pi\rho$ mechanism, and $\mathbf{j}^{(A)}(\Delta)$

Largest contribution to $\mathbf{j}^{(A)}(\Delta)$ from



g_A^* fit to observable: GT_{exp} of tritium β-decay

$$t_{1/2}(^3\mathrm{H}) = \frac{K/G_V^2}{f_V|\langle F\rangle|^2 + f_A g_A^2 |\langle GT\rangle|^2} \frac{1}{1+\delta_R}$$

- $\langle F\rangle \equiv \langle ^3\mathrm{He}|| \sum_i \tau_i^+ ||^3\mathrm{H}\rangle$, $\langle GT\rangle \equiv \langle ^3\mathrm{He}|| \sum_i \tau_i^+ \boldsymbol{\sigma}_i ||^3\mathrm{H}\rangle$
- δ_R radiative corrections ($\sim 2\%$)
- $\rightarrow \langle GT\rangle_{\mathrm{expt}} = 1.657 \pm 0.005$
- Only one-body contribution $\langle GT\rangle \sim 1.60$: fix g_A^* to reproduce $\langle GT\rangle_{\mathrm{expt}}$

Nuclear transition operators: χEFT at N^{3}LO

- One-body operators $\equiv$ SNPA
- Two-body operators: from [Park, Min, & Rho, Phys. Rep. **233**, 341 (1993)], [Song *et al.*, PRC **79**, 064002 (2009)]
  - Two-body $\rho^{(A)}$: soft π-exchange dominant
  - Two-body $\rho^{(V)} = 0$ at N^{3}LO
  - Two-body $\mathbf{j}^{(V)}$: CVC $\rightarrow$EM current

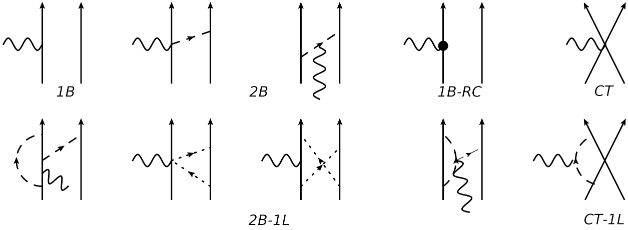
    here $1\pi + 2\pi$ + CT $\rightarrow$ two LECs (g_{4S} & g_{4V}) $\Rightarrow$ from $\mu(^3\mathrm{H} - {}^3\mathrm{He})$
  - Two-body $\mathbf{j}^{(A)}$: 1π + CT $\rightarrow$ one LEC (d_R) $\Rightarrow$ from GT_{exp} of ^{3}H β-decay

- "hybrid" (χEFT*) $\Rightarrow$ AV18/UIX $\Rightarrow$ current and potentials "uncorrelated"
- "Consistent" χEFT: the same LEC's enter the nuclear potential and transition operators
- $\Rightarrow$ N3LO/N2LO interaction
  - Warning: in the current derived by Park *et al.* several LEC's are determined indipendently
  - Fully consistent calculations in progress [Epelbaum *et al.*], [Barone *et al.*] $\rightarrow$ a step forward: d_R LEC

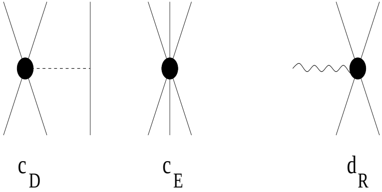
χ charge & current: chiral counting

Contributions from each type of current at $\mathbf{q} = \mathbf{p}_e + \mathbf{p}_\nu = 0$.

J^μ	LO	NLO	N^{2}LO	N^{3}LO	N^{4}LO
$\mathbf{j}^A$	1B	–	1B-RC	2B	1B-RC, 2B-1L and 3B
ρ^A	–	1B	2B	1B-RC	1B-RC, 2B-1L
$\mathbf{j}^V$	–	1B	2B	1B-RC	1B-RC, 2B-1L
ρ^V	1B	–	–	2B	1B-RC, 2B-1L and 3B



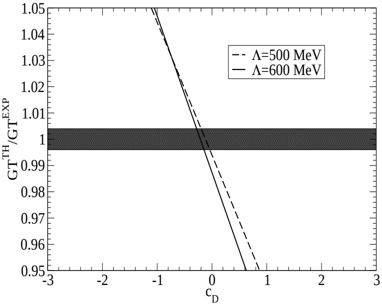
Example: the LEC d_R

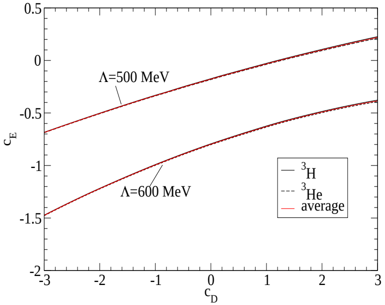


$$d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

Gardestig and Phillips, PRL **96**, 232301 (2006)
Gazit *et al.*, PRL **103**, 102502 (2009)

fit c_D (d_R) to GT_{exp} and c_E to $B(A=3)$ (using the N3LO/N2LO model)
$\Rightarrow$ $\{c_D; c_E\}_{\rm MAX}$ and $\{c_D; c_E\}_{\rm MIN}$





Remaining LEC's: g_{4S} and g_{4V} in the vector current $\Rightarrow$ fit to the $A = 3$ magnetic moments

	$\{c_D; c_E\}$	g_{4S}	g_{4V}
Λ=500 MeV	$\{-0.20; -0.208\}$	0.207 ± 0.007	0.765 ± 0.004
	$\{-0.04; -0.184\}$	0.200 ± 0.007	0.771 ± 0.004
Λ=600 MeV	$\{-0.32; -0.857\}$	0.146 ± 0.008	0.585 ± 0.004
	$\{-0.19; -0.833\}$	0.145 ± 0.008	0.590 ± 0.004

Radiative corrections[1] ARE included

[1] Czarnecki *et al.*, PRL **99**, 032003 (2007)

Multipoles for the weak trnasitions

$\mathcal{J}^{(h),\mu} = \{\rho^V - \rho^A, \mathbf{J}^V - \mathbf{J}^A\}$

- charge, longitudinal, electric and magnetic operators

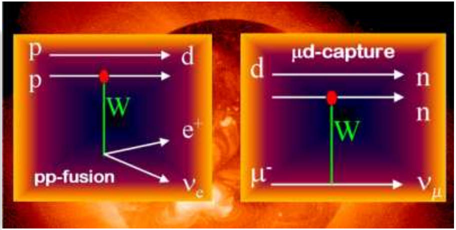
$$
\begin{aligned}
T^C_{JM} &= \int d^3x\, \rho^{(h)}(\mathbf{x}) \left[j_J(qx) Y_{J,0}(\hat{\mathbf{x}}) \right] \\
T^L_{JM} &= \frac{i}{q} \int d^3x\, \mathbf{J}^{(h)}(\mathbf{x}) \cdot \boldsymbol{\nabla} \left[j_J(qx) Y_{J,0}(\hat{\mathbf{x}}) \right] \\
T^E_{J,M} &= \frac{1}{q} \int d^3x\, \mathbf{J}^{(h)}(\mathbf{x}) \cdot \boldsymbol{\nabla} \times j_J(qx) \mathbf{Y}^{\lambda}_{J,J,1}(\hat{\mathbf{x}}) \\
T^M_{J,M} &= \frac{1}{q} \int d^3x\, \mathbf{J}^{(h)}(\mathbf{x}) \cdot j_J(qx) \mathbf{Y}^{\lambda}_{J,J,1}(\hat{\mathbf{x}})
\end{aligned}
$$

Parity selection rules $\Pi_i \Pi_f =$			
C(V)	L(V)	E(V)	M(V)
$(-)^J$	$(-)^J$	$(-)^J$	$(-)^{J+1}$
C(A)	L(A)	E(A)	M(A)
$(-)^{J+1}$	$(-)^{J+1}$	$(-)^{J+1}$	$(-)^J$

Muon capture

Muon catpures:

- $\mu^- + d \to n + n + \nu_\mu$
- $\mu^- + {}^3\mathrm{He} \to {}^3\mathrm{H} + \nu_\mu$ (70%)
- $\mu^- + {}^3\mathrm{He} \to n + d + \nu_\mu$ (20%)
- $\mu^- + {}^3\mathrm{He} \to n + n + p + \nu_\mu$ (10%)



- $\Gamma(\mu - d) = 300 - 500\ \mathrm{s}^{-1}$ →[MuSun Experiment (PSI)]
- $\Gamma(\mu - {}^3\mathrm{He}) = 1496(4)\ \mathrm{s}^{-1}$ [Ackerbauer *et al.*, (1998)]

Stringent test of the nuclear wave functions/transition operators

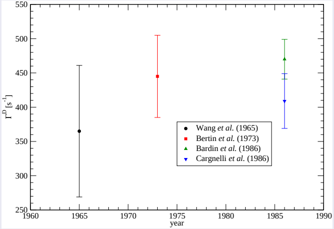
Extraction of the pseudoscalar form factor of the nucleon factor

$$j^\mu = \overline{u_p}\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N} - G_A(q^2)\gamma^\mu\gamma^5 - G_{PS}(q^2)\frac{q^\mu\gamma^5}{2M_N}\right] u_n$$

Muon capture: Experimental situation

$\mu^- + d \to n + n + \nu_\mu$

- Two hyperfine states: $f = 1/2$ and $3/2$
- Dominant capture from $f = 1/2$ $\to \Gamma^D$



- New measurement in progress: MuSun

$\mu^- + {}^3\mathrm{He} \to {}^3\mathrm{H} + \nu_\mu$

- Hyperfine states: $(f, f_z) = (1, \{\pm 1, 0\})$ and $(0, 0)$

$$\frac{d\Gamma}{d(\cos\theta)} = \frac{1}{2}\Gamma_0 \Big[1 + A_v P_v \cos\theta + A_t P_t \Big(\frac{3\cos^2\theta - 1}{2}\Big) + A_\Delta P_\Delta\Big]$$

$$P_v = P_{1,1} - P_{1,-1}$$

$$P_t = P_{1,1} + P_{1,-1} - 2P_{1,0}$$

$$P_\Delta = 1 - 4P_{0,0}$$

- Γ_0= total capture rate, $A_{v,t,\Delta}$= angular correlation parameters
- Ackerbauer *et al.*, (1998):

$$\boxed{\Gamma_0 = 1496(4)\ \mathrm{s}^{-1}}$$

- Souder *et al.*, (1998): $A_v = 0.63 \pm 0.09\ (\mathrm{stat.})^{+0.11}_{-0.14}\ (\mathrm{syst.})$

Multipole analysis of the $\mu^- + {}^3\mathrm{He}$ capture

$$
\begin{aligned}
T_{f,f_z} &= \langle {}^3\mathrm{H}, s_3'; \nu_\mu, s_\nu' | V_W | (\mu^-, {}^3\mathrm{He}), ff_z \rangle \\
&= \frac{G_V}{\sqrt{2}} \psi_{1s}(0) \sum_{s_\mu s_3} (\frac{1}{2}, s_\mu, \frac{1}{2}, s_3 | f, f_z) l^\sigma(s_\nu', s_\mu) \langle {}^3\mathrm{H}, s_3' | \int d^3x\, \mathcal{J}_\sigma^{(h)} e^{-i\mathbf{q}\cdot\mathbf{x}} | {}^3\mathrm{He}, s_3 \rangle
\end{aligned}
$$

$\frac{1}{2}^+ \to \frac{1}{2}^+$ transition

Contributing RMEs			
C	L	E	M
$C_0(V), C_1(A)$	$L_0(V), L_1(A)$	$E_1(A)$	$M_1(V)$

- $\psi_{1s}(0)$ atomic wave function $= \mathcal{R}(2\alpha\mu)^3/\pi$
- Factor $\mathcal{R} = 0.98$: finite extent of the nuclear charge distribution

Observables

$$
\begin{aligned}
\frac{d\Gamma}{d(\cos\theta)} &= \frac{1}{2}\Gamma_0\left[1 + A_v P_v \cos\theta + A_t P_t\left(\frac{3\cos^2\theta - 1}{2}\right) + A_\Delta P_\Delta\right] \\
P_v &= P_{1,1} - P_{1,-1} \\
P_t &= P_{1,1} + P_{1,-1} - 2P_{1,0} \\
P_\Delta &= 1 - 4P_{0,0}
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_0 &= |C_0(V) - L_0(V)|^2 + |C_1(A) - L_1(A)|^2 + |M_1(V) - E_1(A)|^2 \\
A_v &= 1 + \frac{1}{\Gamma_0}\left\{2\Im\left[(C_0(V) - L_0(V))(C_1(A) - L_1(A))^*\right] - |M_1(V) - E_1(A)|^2\right\} \\
\ldots &\quad \ldots
\end{aligned}
$$

Results: $\Gamma_0(\mu^- + {}^3\mathrm{He})$

SNPA(AV18/UIX)	Γ_0
g_A=1.2654(42)	1486(8)
g_A=1.2695(29)	1486(5)
χEFT(N3LO/N2LO)	**Γ_0**
IA – $\Lambda = 500$ MeV	1362
IA – $\Lambda = 600$ MeV	1360
FULL – $\Lambda = 500$ MeV	1488(9)
FULL – $\Lambda = 600$ MeV	1499(9)

[Marcucci *et al.*, PRL **108**, 052502 (2012)]

Γ_0 =**1494(13) s^{-1}**

vs. Γ_0**(exp)=1496(4) s^{-1}**

- [Gazit, PLB **666**, 472 (2008)] χEFT* – AV18/UIX $\rightarrow$ 1499(16) s^{-1}
- If G_{PS} is left free $\Rightarrow$ G_{PS} =8.2 $\pm$0.7 vs. $G_{PS}^{\chi PT}$ =7.99$\pm$0.20
- MUCAP experiment ($\mu^- - p$ capture) $G_{PS}^{\chi expt}$ =8.06$\pm$0.48$\pm$0.28

  [arXiv:1210.6545]

Results: $\Gamma^D(\mu^- + d)$

SNPA(AV18)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
g_A=1.2654(42)	246.6(7)	20.1	46.7	71.6	4.5	0.9	390.4(7)
g_A=1.2695(29)	246.8(5)	20.1	46.8	71.8	4.5	0.9	390.9(7)
χEFT (N3LO)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
IA $-\ \Lambda = 500$ MeV	238.8	21.1	44.0	72.4	4.4	0.9	381.7
IA $-\ \Lambda = 600$ MeV	238.7	20.9	43.8	72.0	4.4	0.9	380.8
FULL $-\ \Lambda = 500$ MeV	254(1)	20.5	46.8	72.1	4.4	0.9	399(1)
FULL $-\ \Lambda = 600$ MeV	255(1)	20.3	46.6	71.6	4.4	0.9	399(1)

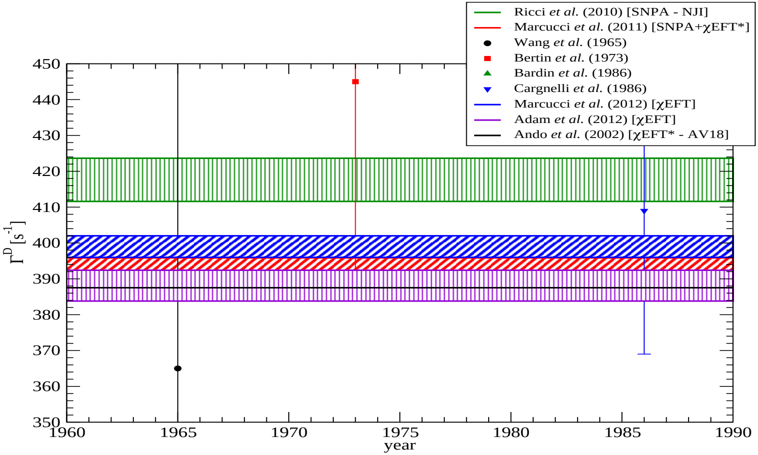
Γ^D =**399(3) s^{-1}** (conservative assumption)

Theoretical "error": from the uncertainties in c_D, c_E, g_{4S}, g_{4V}

Calculation performed assuming $G_{PS} = G_{PS}^{\chi\mathrm{PT}} = 7.99 \pm 0.20$

[Marcucci *et al.*, PRC **83**, 014002 (2011), PRL **108**, 052502 (2012)]

Comparison with data and previous calculations



$p + p \to d + e^+ + \nu_e$ astrophysical factor

$$\sigma(E) = \frac{1}{(2\pi)^3} \frac{G_V^2}{v} m_e^5 f(E) \sum_M |\langle d, M | \mathbf{A}_- | pp \rangle|^2 \qquad S_{11}(E) = S_{11}(0) + S'_{11}(0)E + \frac{1}{2} S''_{11}(0)E^2 + \ldots$$

Goal: $< 1\%$ accuracy

- Dominant contribution from the $^1S_0$ wave
- *P*-wave contribution: $\sim 1\%$
- Two-body contribution: $\sim 1\%$

pp wave function

- EM interaction: $V_{C1} + V_{C2} + V_{DF} + V_{VP} + \cdots$
- $V_{VP} \sim \exp(-2m_e r)$: sizeable effect at low energies
- Necessity to solve the Schroedinger equation up to $1,000$ fm
- 1% effect

SNPA: [Schiavilla *et al.*, 1998], χEFT*: [Park *et al.*, 2003]

$S_{11}(0) = 3.94(1 \pm 0.0015 \pm 0.0010)$ (only $^1S_0$ wave)

errors from uncertainties in g_A, fit of the tritium β-decay, etc; fully χEFT calc. needed

See also the review paper: [E. G. Adelberger *et al.*, Rev. Mod. Phys. **83**, 195 (2011) [arXiv:1004.2318]]

Results with the 'Less hybrid" χEFT

N3LO potential – PRELIMINARY

d_R, g_{4S}, g_{4V} fixed using the N3LO/N2LO wave functions - only V_{C1} EM int.

calculation performed between $0 < E < 10$ keV

	$S_{11}(0)$ [$\times 10^{-25}$ MeV b]		$S'_{11}(0)/S_{11}(0)$ [MeV^{-1}]	
	$^1S_0$	$S+P$	$^1S_0$	$S+P$
IA(500)	3.96	3.98	11.16	11.68
IA(600)	3.94	3.96	11.17	11.68
FULL(500)	4.025(5)	4.052(5)	11.17	11.68
FULL(600)	4.007(5)	4.033(5)	11.17	11.68

Summary:

$^1S_0$

$$S_{11}(0) = 4.00 \div 4.03 \times 10^{-25}\,\mathrm{MeV\ b}$$

$$\frac{S'_{11}(0)}{S_{11}(0)} = 11.17\,\mathrm{MeV}^{-1}$$

All waves

$$S_{11}(0) = 4.03 \div 4.06 \times 10^{-25}\,\mathrm{MeV\ b}\ \ (1\%)$$

$$\frac{S'_{11}(0)}{S_{11}(0)} = 11.68\,\mathrm{MeV}^{-1}\ \ (4\%)$$

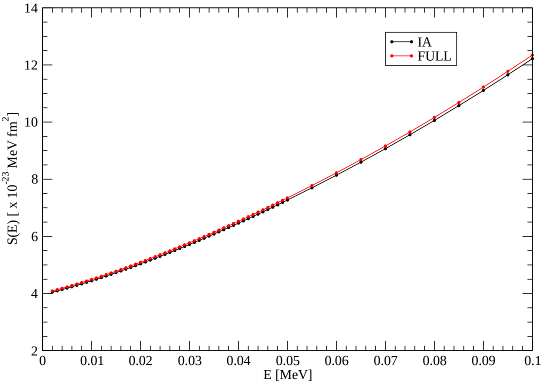
Pionless EFT at N^{2}LO [Wei *et al.*, 2012]

$S_{11}(0) = (3.99 \pm 0.14)10^{-25}$ MeV b, $S'_{11}(0)/S_{11}(0) = (11.3 \pm 0.1)$ MeV^{-1}

New interest: $S''_{11}(0)$

$S(E)$ calculated in the range $0 - 100$ keV

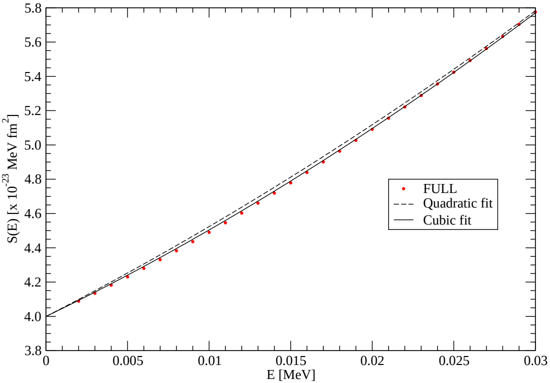
Effect of two-body currents



Calculation with AV18 - only the $^1S_0$ wave

$S(E)$ calculated in the range $0 - 100$ keV

Test of the quadratic approximation



Calculation with AV18 - only the $^1S_0$ wave

Small effects

Quadratic vs. cubic fit

$$S(E) = S(0) + a_0 E + a_1 E^2$$
$$S(E) = S(0) + a_0 E + a_1 E^2 + a_2 E^3$$

	$S(0) \times 10^{23}$ [MeV fm^2]	$S'(0)/S(0)$ [MeV^{-1}]	$S''(0)/S(0)$ [MeV^{-2}]
quadratic fit	4.00	12.23	175.0
cubic fit	4.00	11.47	233.3
Wei *et al.*	3.99 ± 0.14	11.3 ± 0.1	170 ± 2

Effect of the EM interactions $V_{C2} + V_{DF} + V_{VP} + \cdots$

Calculation performed so far only for the AV18 interaction

	$S(0) \times 10^{23}$ [MeV fm^2]	$S'(0)/S(0)$ [MeV^{-1}]	$S''(0)/S(0)$ [MeV^{-2}]
AV18+V_{C1}	4.03	11.59	226.5
AV18+V_{EM}	4.00	11.47	233.3

"hep" reaction

$p + {}^3\text{He} \to {}^4\text{He} + e^+ + \nu_e$

- Source of the most energetic neutrinos from the sun
- Calculation of the reaction rate rather difficult ($A = 4$ bound and scattering states)
- Gamow peak $E \approx 10$ keV
- Astrophysical factor $S(E) = E\sigma(E)\exp(4\pi\alpha/v)$
- relative incoming momentum $\mathbf{p}$ along z

$$\Psi^{p^3\text{He}}_{s_1,s_3} = \sqrt{4\pi} \sum_{LSJJ_z} (\frac{1}{2}, s_1, \frac{1}{2}, s_3 | SJ_z)(L, 0, S, J_z | J, J_z)\Psi_{LSJJ_z}$$

	Contributing RMEs			
Wave ${}^{2S+1}L_J$	C	L	E	M
${}^1S_0$	$C_0(V)$	$L_0(V)$	–	–
${}^3S_1$	$C_1(A)$	$L_1(A)$	$E_1(A)$	$M_1(V)$
${}^3P_0$	$C_0(A)$	$L_0(A)$	–	–
${}^1P_1$	$C_1(V)$	$L_1(V)$	$E_1(V)$	$M_1(A)$
${}^3P_1$	$C_1(V)$	$L_1(V)$	$E_1(V)$	$M_1(A)$
${}^3P_2$	$C_2(A)$	$L_2(A)$	$E_2(A)$	$M_2(V)$

- In $p^3\mathrm{He}$ S-wave scattering is suppressed (Pauli repulsion)
- Interaction in P-waves is attractive
- 4 resonances in $^3P_2$, $^3P_1$, $^1P_1$, $^3P_0$ waves
- → non-negligible contribution from P-waves
- (due to Coulomb repulsion also S-wave capture is suppressed)
- Calculation performed only in SNPA [Marcucci *et al.*, PRC **63** 015801 (2000)

	"hep" $S(E=0)$ [10^-20 keV b]	
Wave $^{2S+1}L_J$	AV18/UIX	AV18
$^1S_0$	0.02	0.01
$^3S_1$	6.38	7.69
$^3P_0$	0.82	0.89
$^1P_1$	1.00	1.14
$^3P_1$	0.30	0.52
$^3P_2$	0.97	1.78
TOT	9.64	12.1

Suppression of $^1S_0$ contribution

Suppression of $^1S_0$ contribution

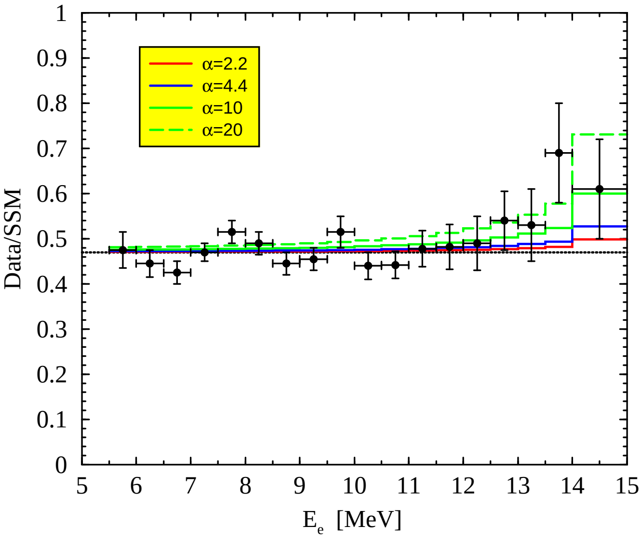
- LWA approximation for T^C_{00} and T^L_{00}

$$T^C_{00} \approx \frac{1}{\sqrt{4\pi}} \sum_{j=1}^{A} \tau_{j+} = \frac{1}{\sqrt{4\pi}} T_+ \qquad T^C_{00} \approx -\frac{1}{\sqrt{4\pi}\, q} [H, T_+]$$

- $^4\mathrm{He}$ has total isospin $= 0$, $p - {}^3\mathrm{He}$ has total isospin $= 1$
- $\rightarrow \langle T = 0 | T_+ | T = 1 \rangle = 0$

- Final result: $S(E)$ at $E = 10$ keV
- $S(E) = (10.16 \pm 0.6) \times 10^{-20}$ keV b
- Around a factor 4.5 larger than assumed in Solar Standar Model (SSN) calculations [Bahcall and Krastev, Phys. Lett. B **436**, 243 (1998)]

SuperKamiokande enhancement



Collaborators

I would like to thank

- L. Girlanda - *INFN & Salento University, Lecce (Italy)*
- R. Schiavilla - *Jefferson Lab. & ODU, Norfolk (VA, USA)*
- M. Piarulli & A. Barone (PhD students) - *ODU, Norfolk (VA, USA)*
- A. Kievsky & L. E. Marcucci - *INFN & Pisa University, Pisa (Italy)*
- F. Spadoni (graduate student) & D. Cartisano (undergraduate student) *Pisa University, Pisa (Italy)*

Thank you for your attention!