The Lorentz Integral Transform Method

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Introduction

Consider an observable R(E) and an integral transform $\Phi(\sigma)$:

$$\Phi(\sigma) = \int dE\ K(\sigma,E)\ R(E)$$

with some kernel $K(\sigma,E)$

Often it is easier to calculate $\Phi(\sigma)$ than R(E). Then the observable R(E) can be obtained via inversion of the integral transform.

In order to make the inversion sufficiently stable the kernel $K(\sigma,E)$ should resemble a kind of energy filter (Lorentzians, Gaussians, ...); best choice would be a δ-function.

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For the LIT we consider Lorentzians: $K(\sigma,E) = [(E-\sigma_R)^2 + \sigma_I^2]^{-1}$

Introduction

Reactions of particle systems induced by external probes (photons, electrons, neutrinos) can be divided in inclusive and exclusive processes.

Inclusive reaction: final state of particle system after reaction is **not** observed

Corresponding cross sections have the form

$$\frac{d^2\sigma}{d\Omega d\omega} = \left.\frac{d^2\sigma}{d\Omega d\omega}\right|_{zero} \sum_{i=1}^{N} f_i(\text{kinematics})\, R_i(\omega,q)$$ **Inclusive**

with N inclusive response functions R_i containing information on the dynamics of the particle system

Electron scattering: $\left.\frac{d^2\sigma}{d\Omega d\omega}\right|_{zero} = \left.\frac{d^2\sigma}{d\Omega d\omega}\right|_{Mott}$

Photo absorption: $\frac{d^2\sigma}{d\Omega d\omega} \longrightarrow \frac{d\sigma}{d\omega}$

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Exclusive reaction: final state of particle system after reaction **is identified**

For example, final state consists of a knocked out proton and a residual nucleus, energy and angle of proton have to be measured:

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$$\frac{d^3\sigma}{d\Omega d\omega d\Omega_p} = \left.\frac{d^3\sigma}{d\Omega d\omega d\Omega_p}\right|_{zero} \sum_{i=1}^{M} f_i(\text{kinematics})\, g(\phi_p)\, r_i(\omega, q, \theta_p)$$

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$$\int r_i(\omega,q,\theta_p)\, d\Omega_p = R_i(\omega,q)\ ,\quad i = 1, \ldots, N$$

$$\int r_i(\omega,q,\theta_p)\, d\Omega_p = 0\ ,\quad i = N+1, \ldots, M$$

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Example: unpolarized (e,e'):

$r_1 = r_L$, $r_2 = r_T$

$r_3 = r_{LT}$, $r_4 = r_{TT}$

LIT method: Theory

Inclusive response functions have the following form

$$R(\omega) = \sum_n\!\!\!\!\!\!\int |\langle n|\Theta|0\rangle|^2\, \delta(\omega - E_n + E_0)$$

where we have set for q=const: $R(\omega,q) \rightarrow R(\omega)$

$|0\rangle, |n\rangle$ and E_0, E_n are eigen states and corresponding eigen energies of Hamiltonian H and Θ is transition operator inducing the reaction

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Exclusive response functions have more complicated forms. They are sums of products of T-matrix elements

$$T^{\alpha,\beta}_{n0}(\omega) = \langle n_\alpha \,|\, \Theta \,|\, 0_\beta \rangle$$

For a calculation of response functions one needs initial and final state wave functions of the particle system. With increasing particle number such calculations become more and more difficult

	bound-state calculation	continuum state calculation
A=2	easy	easy
A=3	not easy	difficult
A=4	difficult	very difficult
A>4	today possible up to relatively large A (GFMC, NCSM, CC)	today: only below three-body breakup threshold

In last decade much progress in bound-state calculations applying different methods

AB INITIO BOUND STATE CALCULATIONS

BE of ^{4}He (exp. 28.296 MeV)

TABLES

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	-128.33(10)	-25.94(5)	1.485(3)
CRCGV	102.30	-128.20	-25.90	1.482
SVM	102.35	-128.27	-25.92	1.486
HH	102.44	-128.34	-25.90(1)	1.483
GFMC	102.3(1.0)	-128.25(1.0)	-25.93(2)	1.490(5)
NCSM	103.35	-129.45	-25.80(20)	1.485
EIHH	100.8(9)	-126.7(9)	-25.944(10)	1.486

from H.Kamada et al. (18 auhors 7 groups) PRC 64 (2001) 044001

Motivation of LIT method

Aim: calculation of reactions involving A-body systems in the continuum

calculation of A-body continuum state tremendously more difficult than A-body bound state calculation

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Aim: calculation of reactions involving A-body systems in the continuum

calculation of A-body continuum state tremendously more difficult than A-body bound state calculation

Question: Is it possible to calculate continuum observables without explicit knowledge of the corresponding continuum wave function **?**

YES, via the LIT method!

Continuum state problem $\xrightarrow{\text{LIT}}$ bound-state-like problem

LIT - Theory for Inclusive Reactions

Cross section described by response functions $R(\omega)$

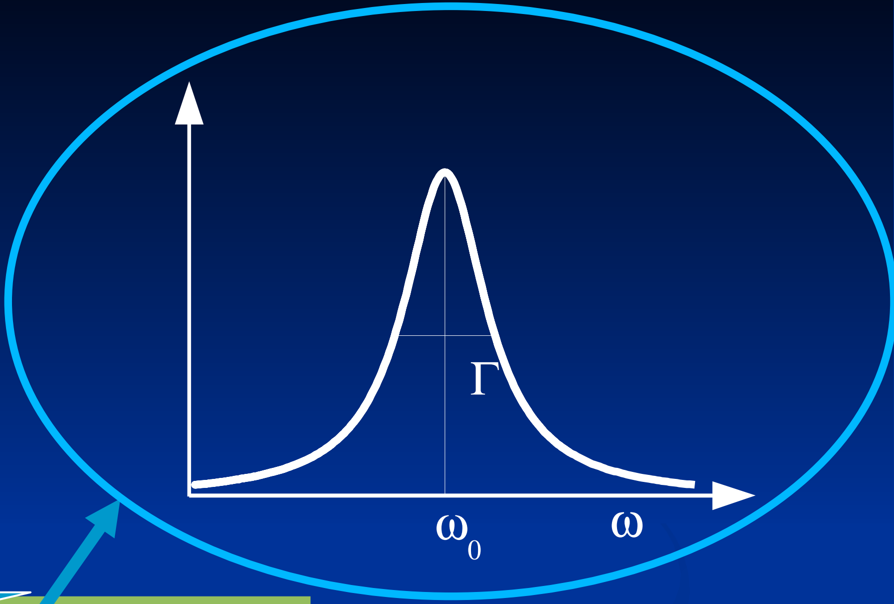
$$R(\omega) = \sum\!\!\!\!\!\!\!\int_n \left|\langle n|\Theta|0\rangle\right|^2 \delta(\omega - E_n + E_0)$$

steps:

1. Solve for many ω_0 and fixed Γ

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = \Theta\,|0\rangle$$

2. Calculate



for given ω_0 and Γ

$$\left\langle \tilde{\Psi} | \tilde{\Psi} \right\rangle = \int R(\omega) L(\omega, \omega_0, \Gamma) d\omega$$

for a Theorem based on closure

3. Invert transform

$$\int_{E^-_{th}}^{\infty} d\omega \frac{R(\omega)}{(\omega - \omega_0)^2 + \Gamma^2}$$

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$$= \int_{E^-_{th}}^{\infty} d\omega \frac{\int dn \;\; <0|\Theta^\dagger|n><n|\Theta|0> \;\; \delta(\omega - E_n - E_0)}{(\omega - \omega_0 - i\Gamma)\,(\omega - \omega_0 + i\Gamma)}$$

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$$= \langle 0|\Theta^\dagger\,(H-E_0-\omega_0-i\Gamma)^{-1}\ (H-E_0-\omega_0+i\Gamma)^{-1}\,\Theta\,|0\rangle = \langle\tilde{\psi}|\tilde{\psi}\rangle$$

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with $\quad (H - E_0 - \omega_0 + i\Gamma)\,|\tilde{\psi}\rangle = \Theta\,|0\rangle$

LIT - Theory for Exclusive Reactions

General form of final state wave function for a given channel

$$|\Psi(E)\rangle = |\Phi(E)\rangle + (E - H + i\eta)^{-1} V |\Phi(E)\rangle$$

$|\Phi(E)\rangle$ is "channel function" (with proper antisymmetrization),

in general fragment bound states times their free relative motion,
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trivial part: T_{Born}

non trivial part: T_{FSI}

Spectral representation for non trivial part

$$\langle \Phi(E) \,|\, V\, (E - H + i\eta)^{-1}\, \Theta\, |0\rangle \;=\; \Sigma_n\, (E - E_n)\, F_{fi}(E,E_n)$$

$$+ \int_{E\_th}^{\infty} (E - E' + i\eta)^{-1} F_{fi}(E,E')\, dE'$$

$$F_{fi}(E,E') = \Sigma\!\!\!\!\int d\gamma\, \langle \Phi(E) \,|\, V | \Psi_\gamma \rangle \langle \Psi_\gamma |\, \Theta\, |0\rangle\, \delta(E-E')$$

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$$(H - \sigma_R + i\sigma_I)\, \tilde{\Psi}_1 = \Theta |0\rangle \;, \quad (H - \sigma_R + i\sigma_I)\, \tilde{\Psi}_2 = V|\Phi(E)\rangle$$

LIT: $\langle \tilde{\Psi}_1 | \tilde{\Psi}_2 \rangle$

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3) Calculate T_{FSI}

$$T_{FSI}(E) = -\, i\pi\, F_{fi}(E,E) + P\int_{E\_th}^{\infty} (E - E')^{-1} F_{fi}(E,E')\, dE'$$

Consider the following exclusive reaction:

$$^4He + \gamma \longrightarrow n + {}^3He$$

For a conventional calculation one needs to know the four-body continuum wave function

Very difficult to go above three-body break-up threshold:

$$^4He + \gamma \longrightarrow n + p + d$$

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$T(E) = T_{BORN}(E) + T_{FSI}(E)$ with $T_{BORN}(E) = \langle PW(E) | \Theta | \Psi(^{4}He) \rangle$

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With $F_{fi}(E,E')$ from inversion of the LIT

LIT - Inversion

Standard LIT inversion method

Take the following ansatz for the response function $R(\omega)$ (or $F_{fi}(E,E')$)

$$R(\omega') = \sum_{m=1}^{M_{max}} c_m \, \chi_m(\omega',\alpha_i)$$

with $\omega'=\omega-\omega_{th}$, given set of functions χ_m , and unknown coefficients c_m

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and expand in set $\tilde{\chi}_m$: $L(\sigma_R,\sigma_I) = \sum_{m=1}^{M_{max}} c_m \, \tilde{\chi}_m(\omega',\alpha_i)$

Determine c_m via best fit

Increase M_{max} up to the point that stable result is obtained for $R(\omega)$. Even further increase of M_{max} might lead to oscillations in $R(\omega)$

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As basis set χ_m we normally use

$$\chi_m(\omega',\alpha_i) = (\omega')^{\alpha} \exp(-\alpha_2 \omega'/m) \quad \text{with} \quad m = 1, 2, \ldots, M_{max}$$

main point of the LIT :

Schrödinger-like equation with a source

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = S$$

The $\tilde{\Psi}$ solution is unique and has ***bound state like*** asymptotic behavior

→ one can apply ***bound state methods***

LIT - Example

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deuteron photodisintegration in unretarded dipole approximation

unretarded dipole approximation $\Rightarrow \Theta = \sum_{i=1}^{A} z_i \frac{1+\tau_{i,z}}{2}$,

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$\stackrel{\Theta}{\Longrightarrow}$ $\sigma_\gamma(\omega) = 4\pi^2 \alpha\, \omega\, R(\omega)$ with $R(\omega) = \sum\!\!\!\!\!\!\int_f |\langle f|\,\Theta|0\rangle|^2\, \delta(\omega - E_f - E_0)$

with $|0\rangle$ and E_0 bound-state wave function and energy

$|f\rangle$ and E_f final-state wave function and energy

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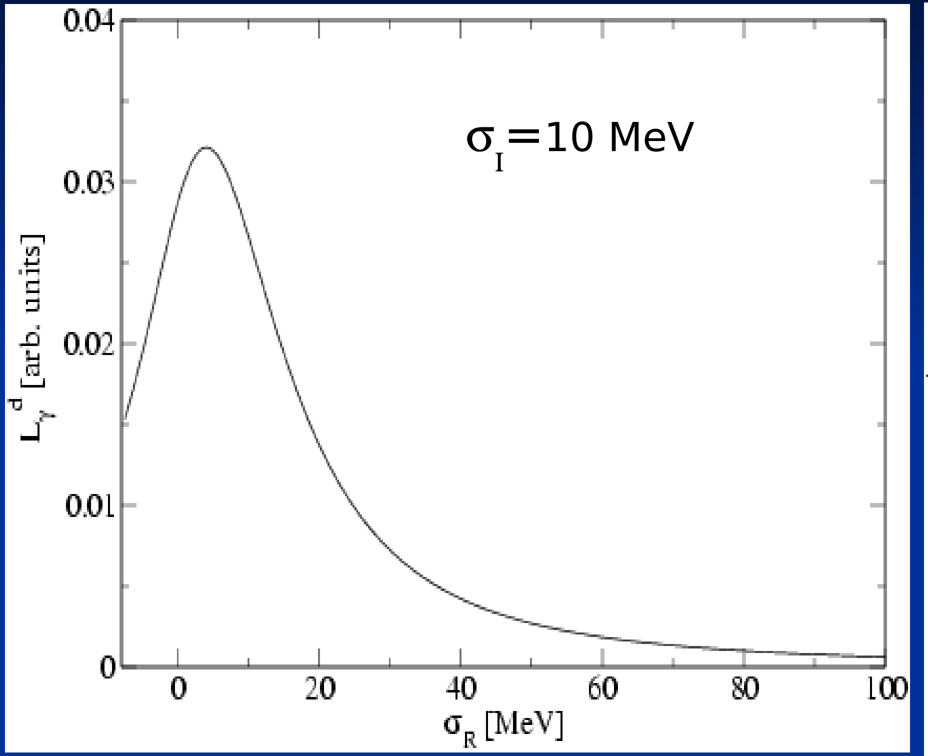
with $|0\rangle$ and E_0 bound-state wave function and energy

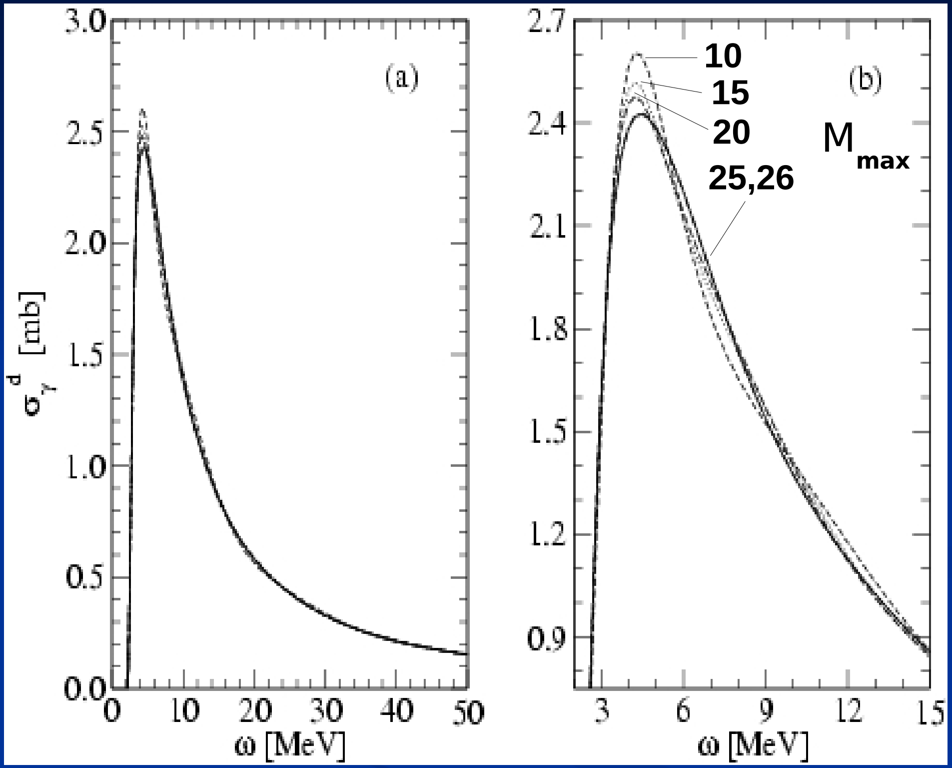
$|f\rangle$ and E_f final-state wave function and energy

In unretarded dipole approximation $|f\rangle$ contains only $^3P_0$, $^3P_1$, $^3P_2 - {}^3F_2$ NN states

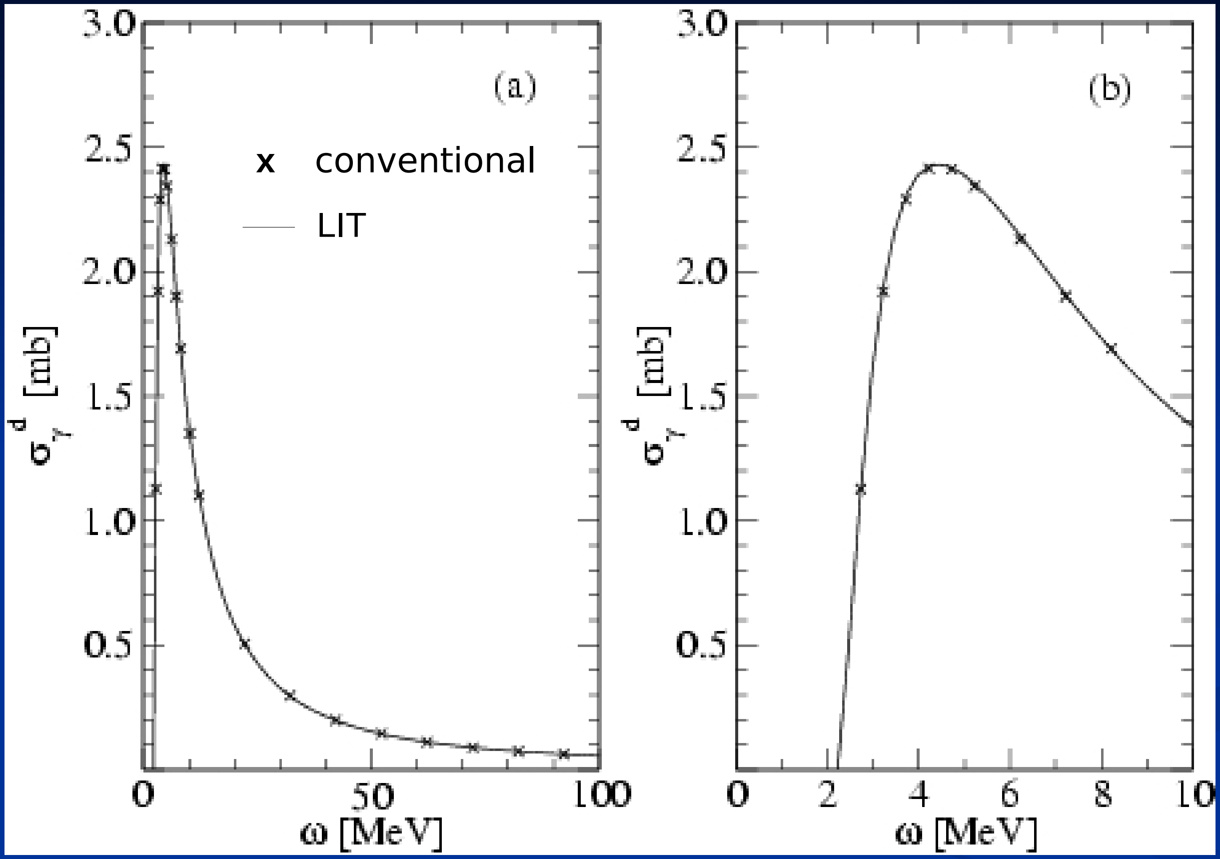
NN interaction: Argonne V14 potential



LIT



$\sigma_\gamma(\omega)$ from inversion with various M_{max}



$\sigma_\gamma(\omega)$ from inversion with various M_{max} = 25

and result from conventional calculation with explicit np continuum wave functions

LIT - Applications A>2

Total photoabsorption cross section
in unretarded dipole approximation

LIT - Applications A>2

Our method for calculating bound-state and bound-state-like equations:

Hyperspherical **H**armonics Expansions (HH): **CHH** and **EIHH**

CHH: Additional two-body correlation functions are introduced
EIHH: Effective Interaction is constructed via Lee-Suzuki transformation

EIHH: N. Barnea, WL, G. Orlandini, PRC 61, 054001 (2000), NPA 693, 565 (2001), PRC 67, 054003 (2003), PRC 81, 064001 (2010)

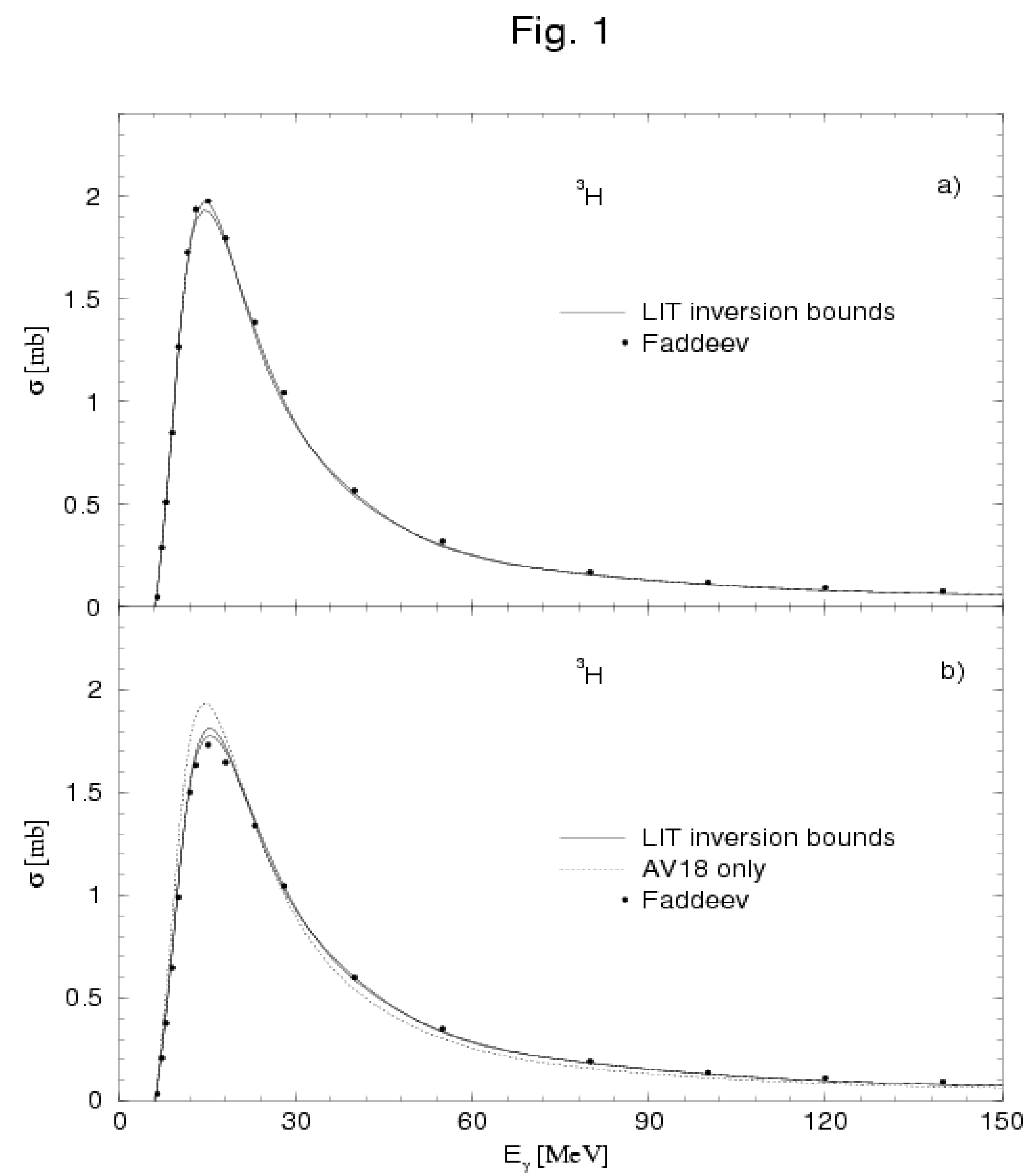
Total photoabsorption cross section of three-nucleon systems

First calculation with realistic NN and 3N forces was made with the LIT method: V.D. Efros, WL, G. Orlandini, E.L. Tomusiak, PLB 484, 223 (2000)

Later a benchmark calculation with the Faddeev technique was made (Golak et al., Nucl. Phys. A 707, 365 (2002))

^{3}H Total photoabsorption cross section in unret. dipole appr. (AV18 +UIX force)

LIT versus Faddeev calculation of Golak et al. NPA 707, 365 (2002)

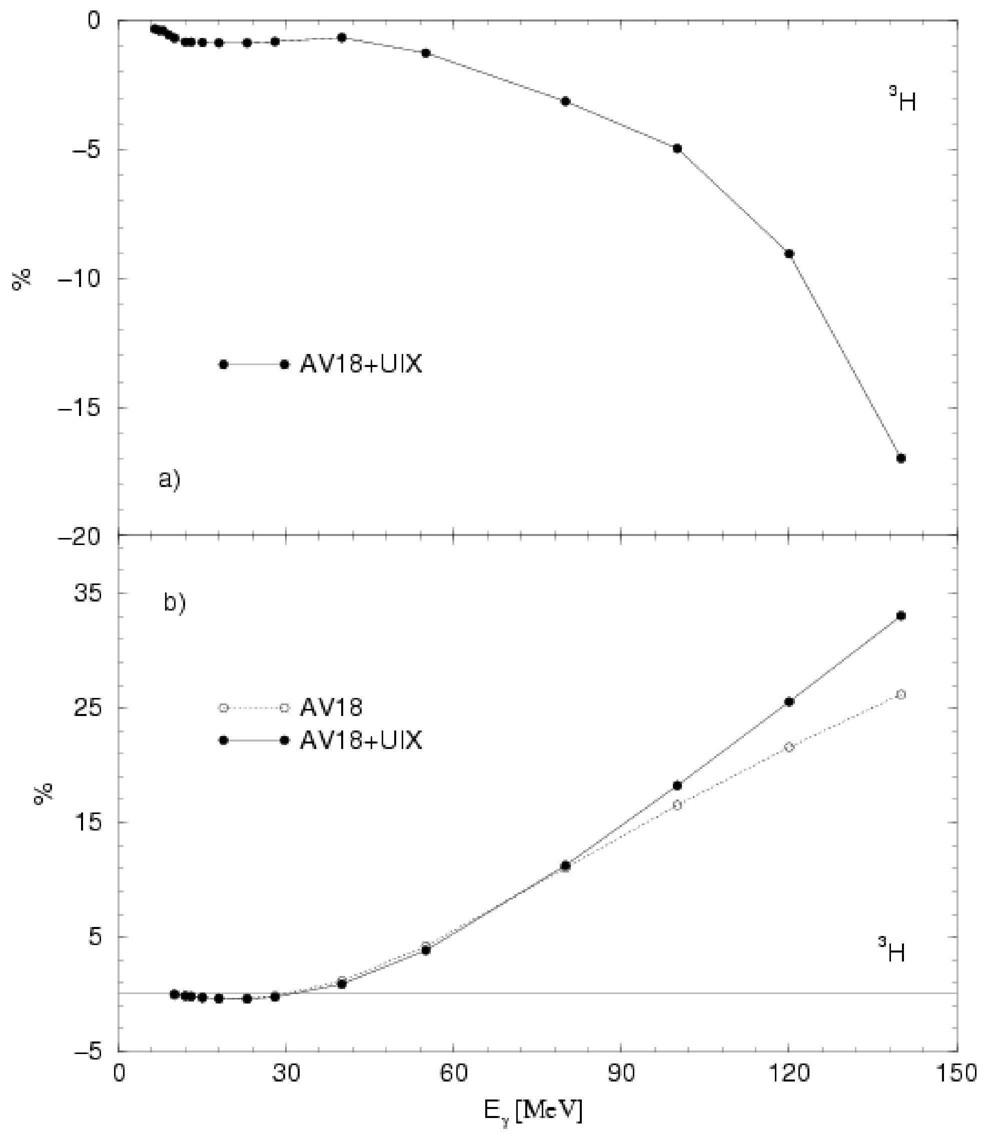


$^{3}H(\gamma)$

Effect of Retardation

Combined Effects of Retardation and further Eλ and Mλ multipoles

Fig. 2



^{4}He total photoabsorption cross section

- LIT method
- Nuclear potential: central S-wave NN potentials
- Calculation in unretarded dipole approximation

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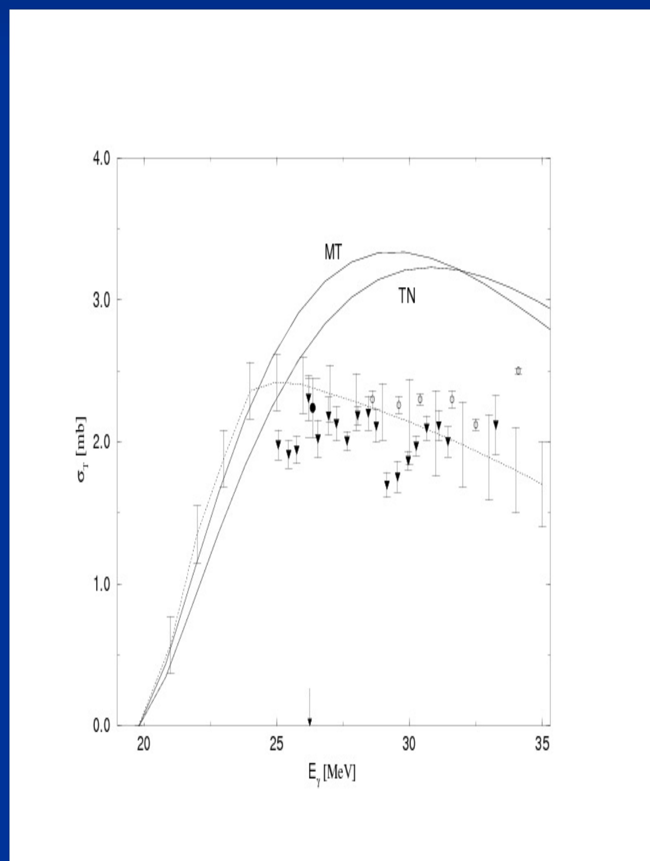

experimental data:
Berman *et al*. (1980)
Feldman *et al*. (1990)
+ others

V. Efros, WL, G. Orlandini,
PRL 78, 4015 (1997)

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- LIT method
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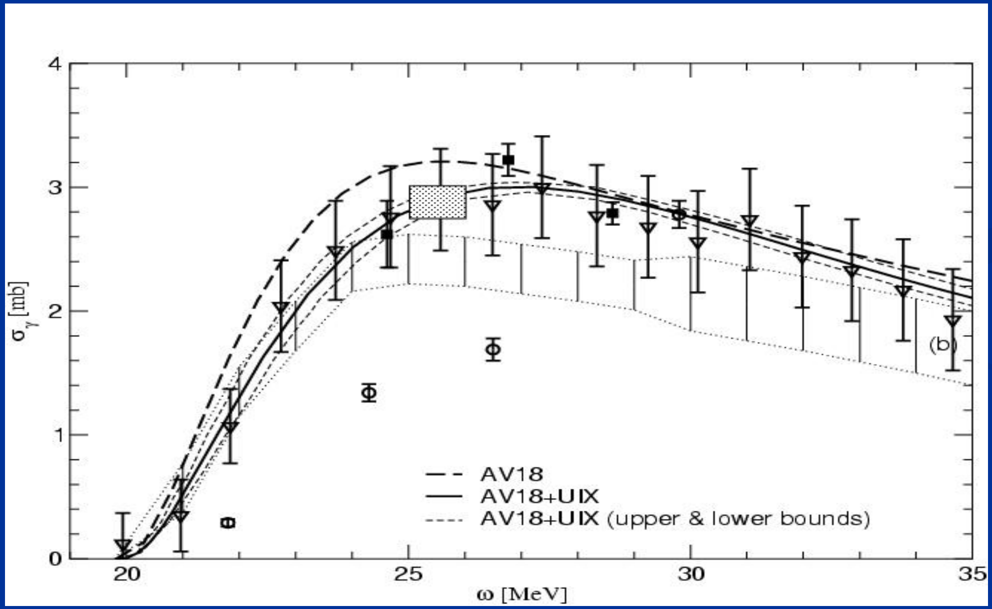
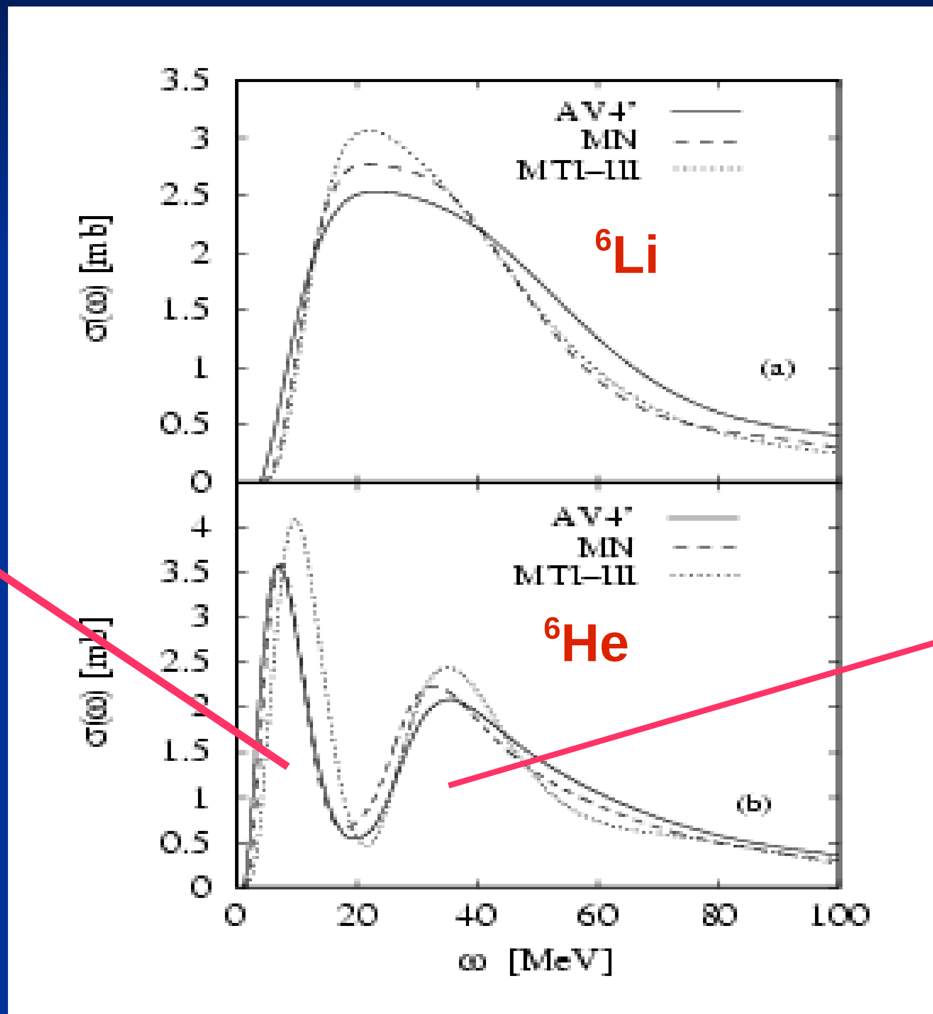


experimental data:
Berman *et al*. (1980)
Feldman *et al*. (1990)
Wells *et al*. (1992)
Nilsson *et al*. (2005)
Shima *et al*. (2005)
Nakayama *et al*. (2007)

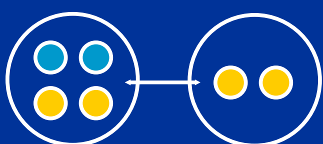
D. Gazit, S. Bacca, N. Barnea, WL, G. Orlandini, PRL 96, 112301 (2006)

6-Body total photodisintegration

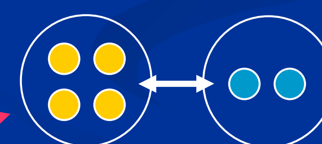
Appearance of collective motion



EIHH



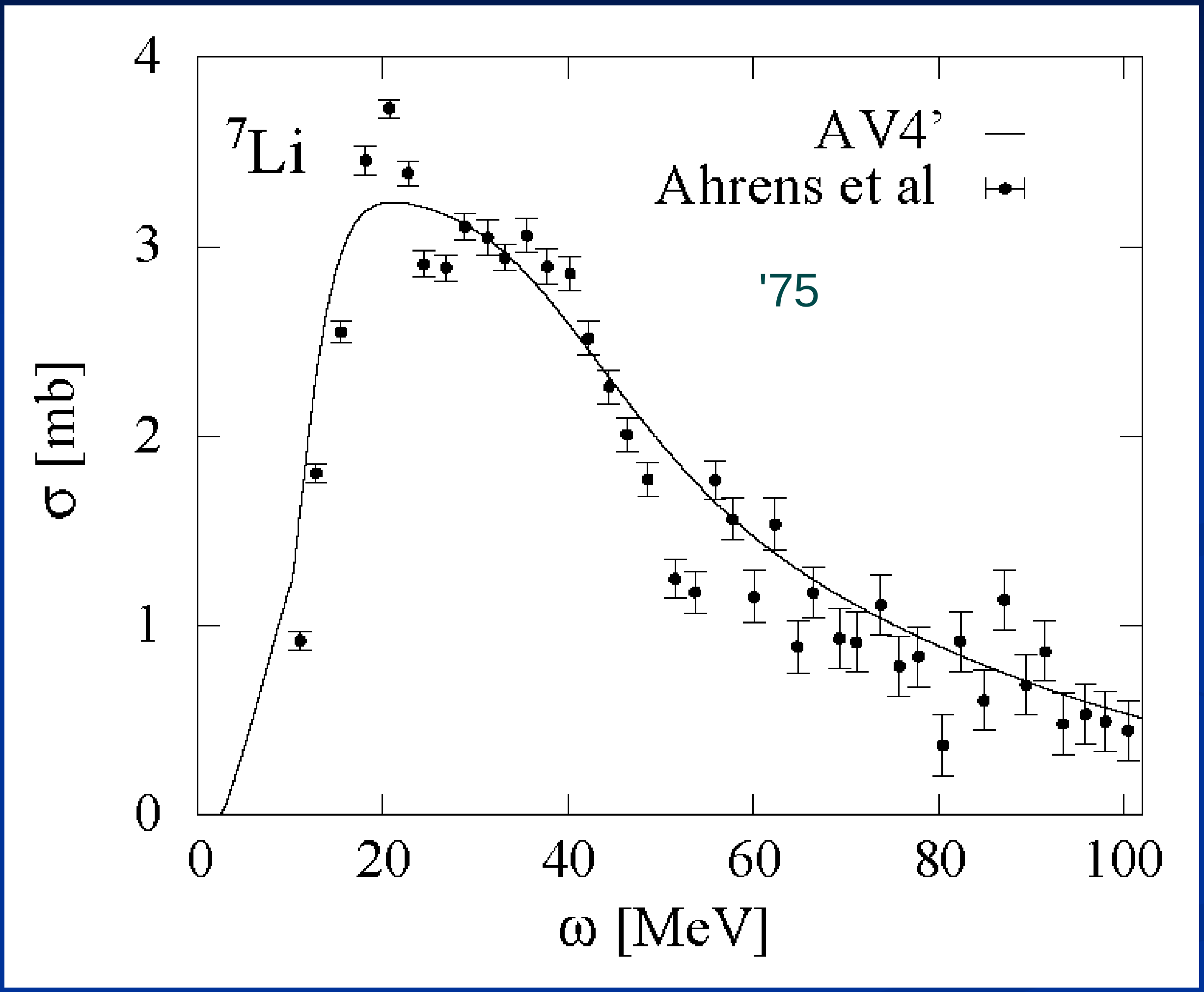
soft mode



classical GT mode

S. Bacca, M. Marchisio, N. Barnea, WL, G. Orlandini PRL89, 052502 (2002)

7-Body total photodisintegration



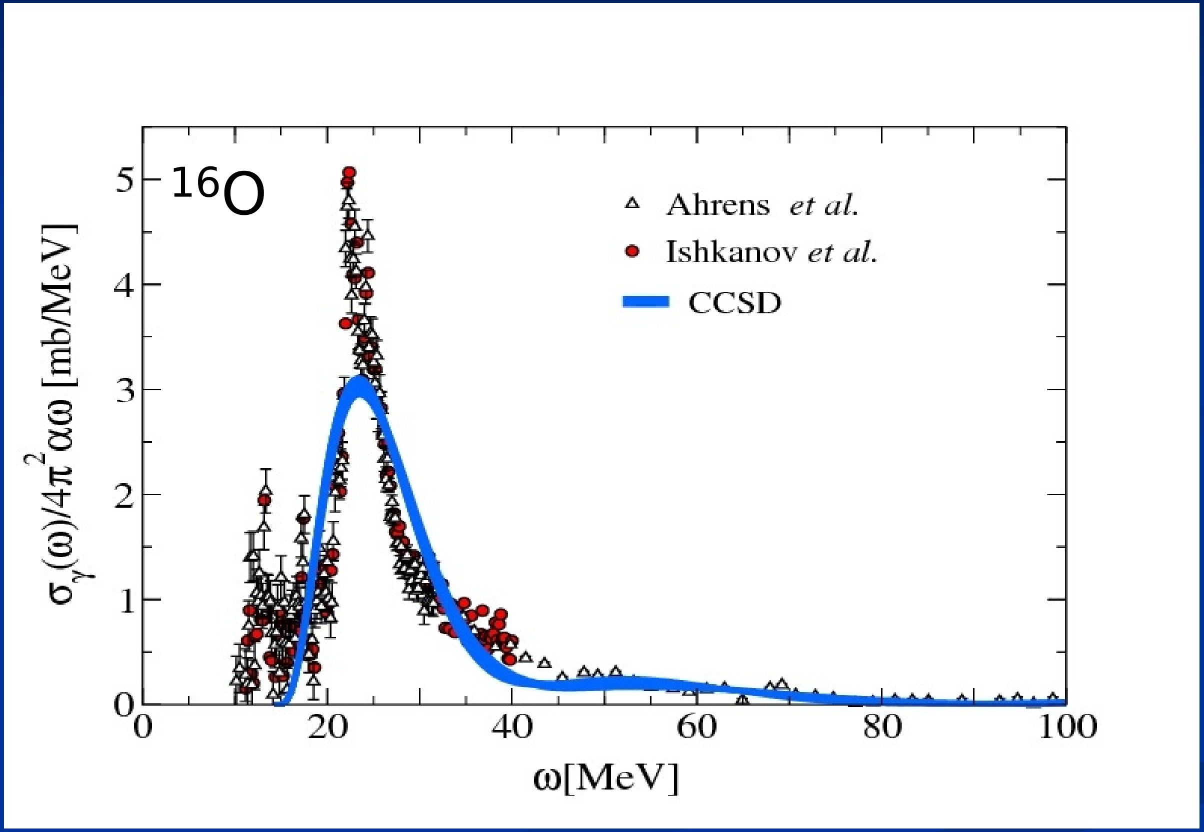
S.Bacca et al.
PLB 603(2004) 159

EIHH

16-Body total photodisintegration

Coupled Cluster
Idaho-N3LO

S. Bacca, N. Barnea,
G. Hagen, G. Orlandini,
Th. Papenbrock,
arXiv:1303.7446



Exclusive Reactions

$^4He(\gamma,p)^3H$ and $^4He(\gamma,n)^3He$ (S. Quaglioni et al., PRC 69, 044002 (2004))

$^4He(e,e'p)^3H$ (S. Quaglioni et al., PRC 72, 064002 (2005))

$^4He(e,e'd)^2H$ (D. Andreasi et al., EPJA 27, 47 (2006))

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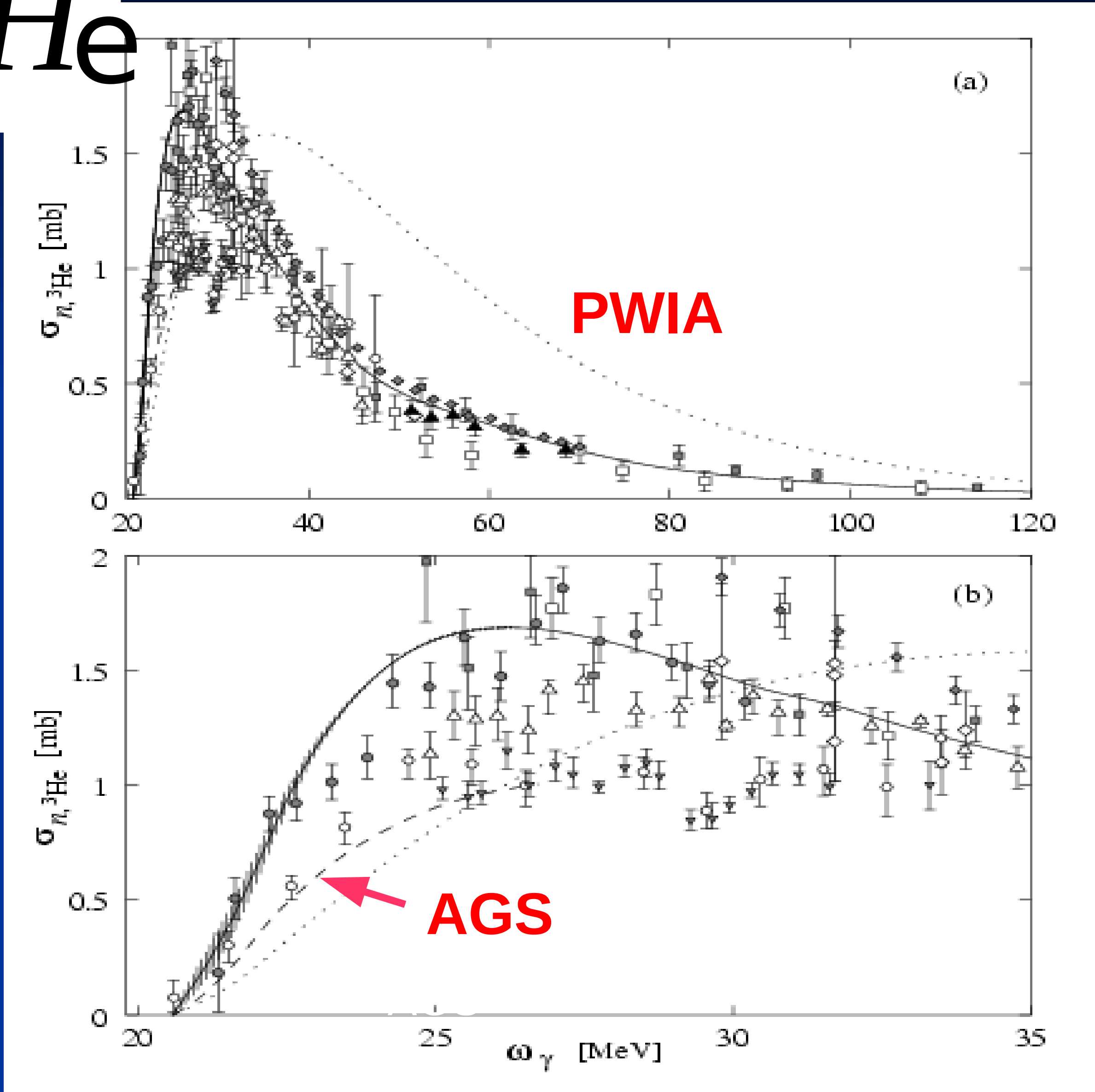
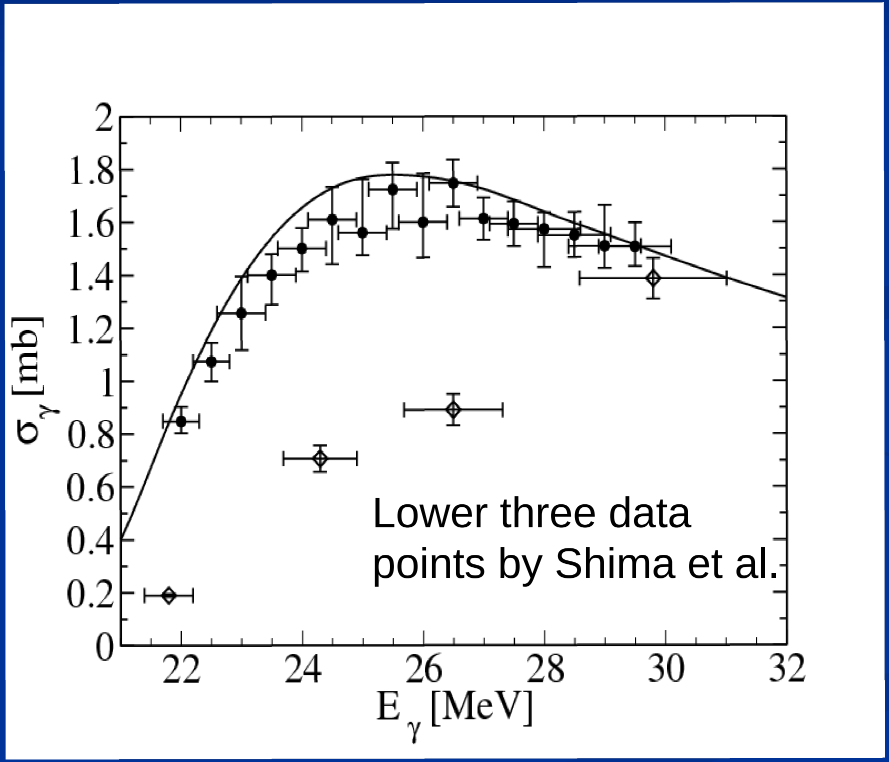
$^{4}He(\gamma,\mathrm{n})^{3}He$

LIT calculation with
MTI/III potential by
Quaglioni et al.,
PRC 69, 044002 (2004)



New results from Hiγs for $^{4}He(\gamma,p)^{3}H$

R. Raut et al., PRL 108, 042502 (2012)



LIT calculation with MTI/III potential by Quaglioni et al., PRC 69, 044002 (2004)

LIT method and resonances

LIT method and resonances

The LIT: a method with a controlled resolution

LIT method and resonances

The LIT: a method with a controlled resolution

Case study for deuteron photodisintegration

NN potential with fictitious resonance in $^3P_1$ partial wave

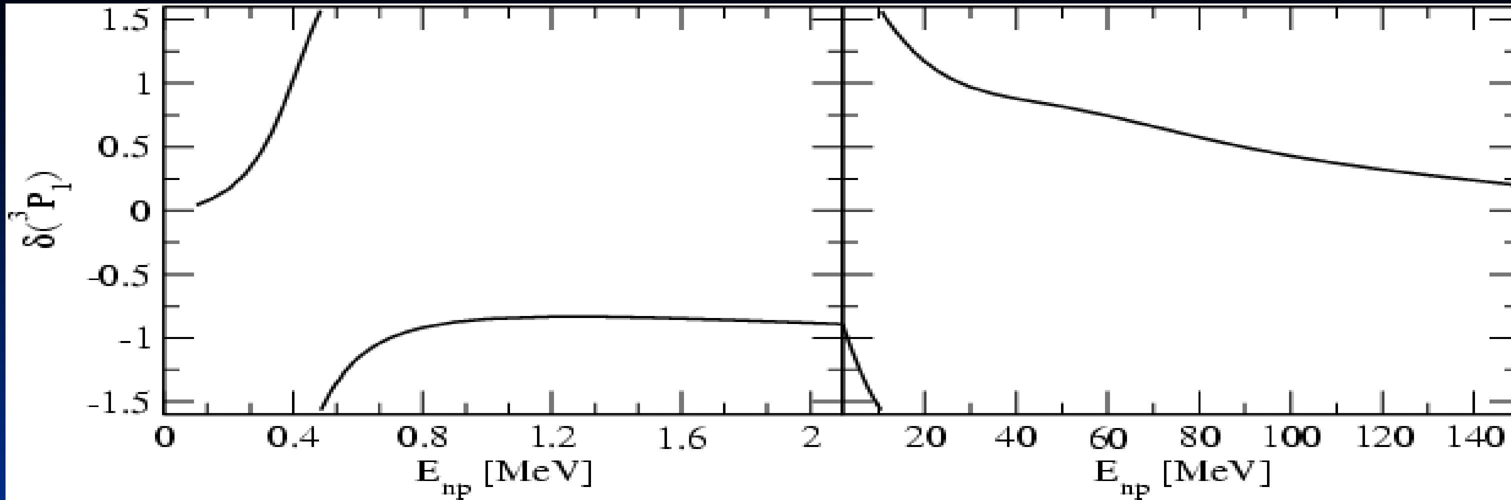
$$V(^3P_1) \longrightarrow V(^3P_1) + V_{add}$$

With $$V_{add} = -\frac{57.6\ \text{MeV}}{r}\,(1-\exp(-2r^2))(1+\exp(\frac{r-5}{0.2}))^{-1}$$

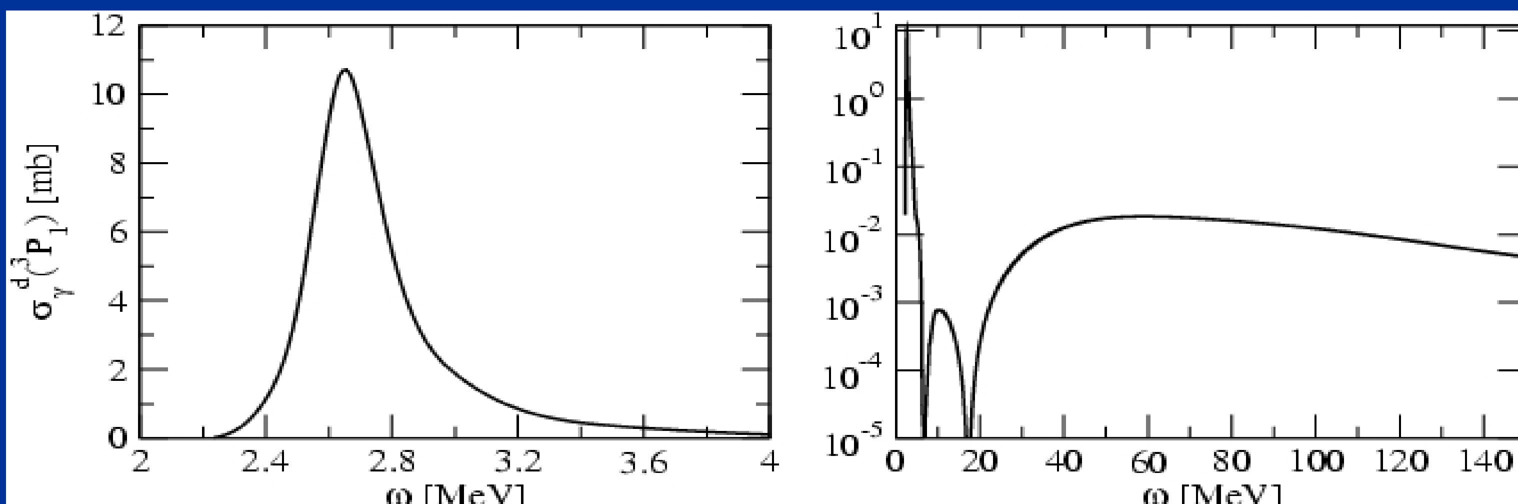
and relative coordinate r in units of fm

Why such a potential?

To understand this better let us have a look on corresponding phaseshift $^3P_1$ and deuteron photoabsorption cross section in $^3P_1$ partial wave



Phase shifts shows two resonances one at E_{np} = 0.48, 10.5 MeV



$\sigma_\gamma(^3P_1)$ shows two corresponding resonances: low-energy resonance very pronounced with small width Γ=270 KeV, the other one is much weaker and has a larger width

What has to be done in the LIT calculation to resolve the pronounced low-energy resonance?

$\tilde{\Psi}$ is localized state of finite norm, but what is the radial extension of the state. Cross section structures with small width require smaller σ_I $\Rightarrow$ $\tilde{\Psi}$ is longer ranged

In our LIT calculation for the deuteron photodisintegration we are able to check it for the modified $^3P_1$ interaction

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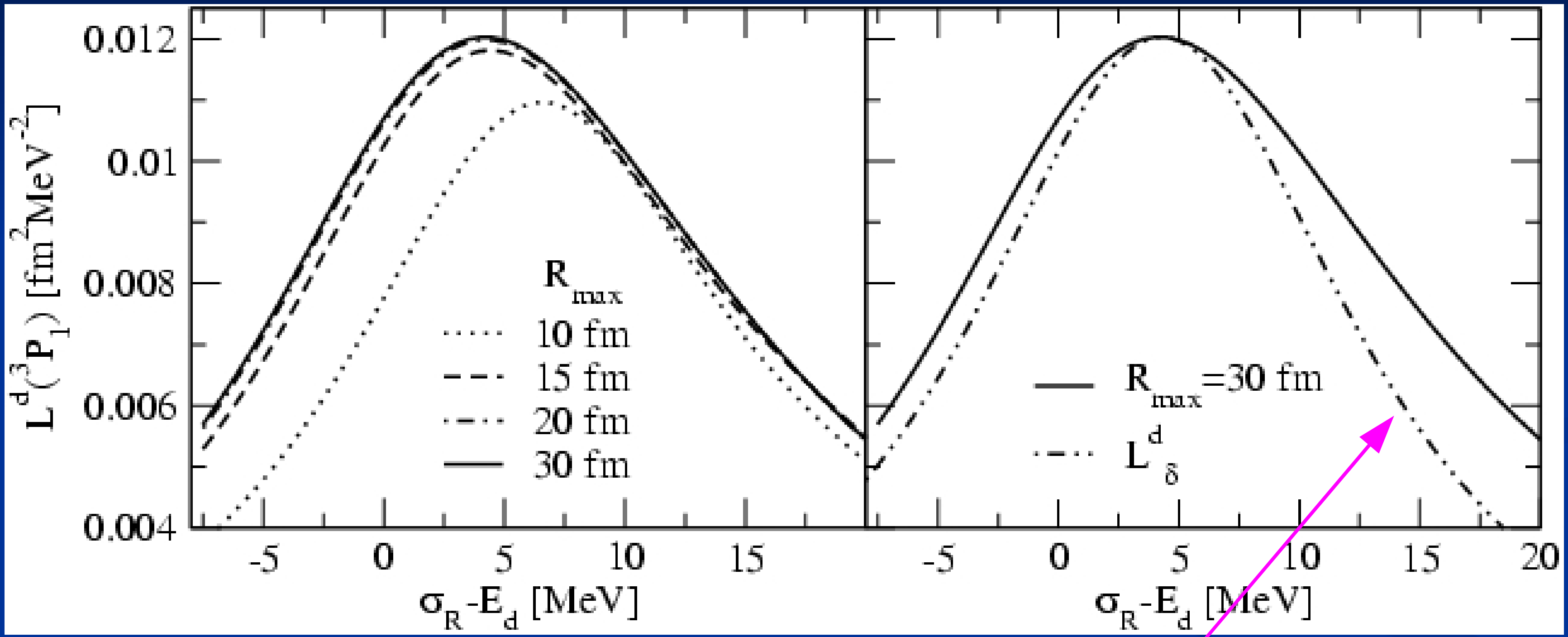
Let us first check better the case for the true deuteron photodisintegration using the following procedure. At a distance $r=R_{max}$ we take as boundary condition a very strong fall-off for the solution $\tilde{\Psi}$ and evaluate the norm

$$<\tilde{\Psi}|\tilde{\Psi}> = \int_0^{R_{max}} dr\, |\tilde{\psi}(r,\sigma_R,\sigma_I)|^2$$

LIT for deuteron total photoabsorption cross section considering only transitions to $^3P_1$ channel with unchanged interaction (no resonance)

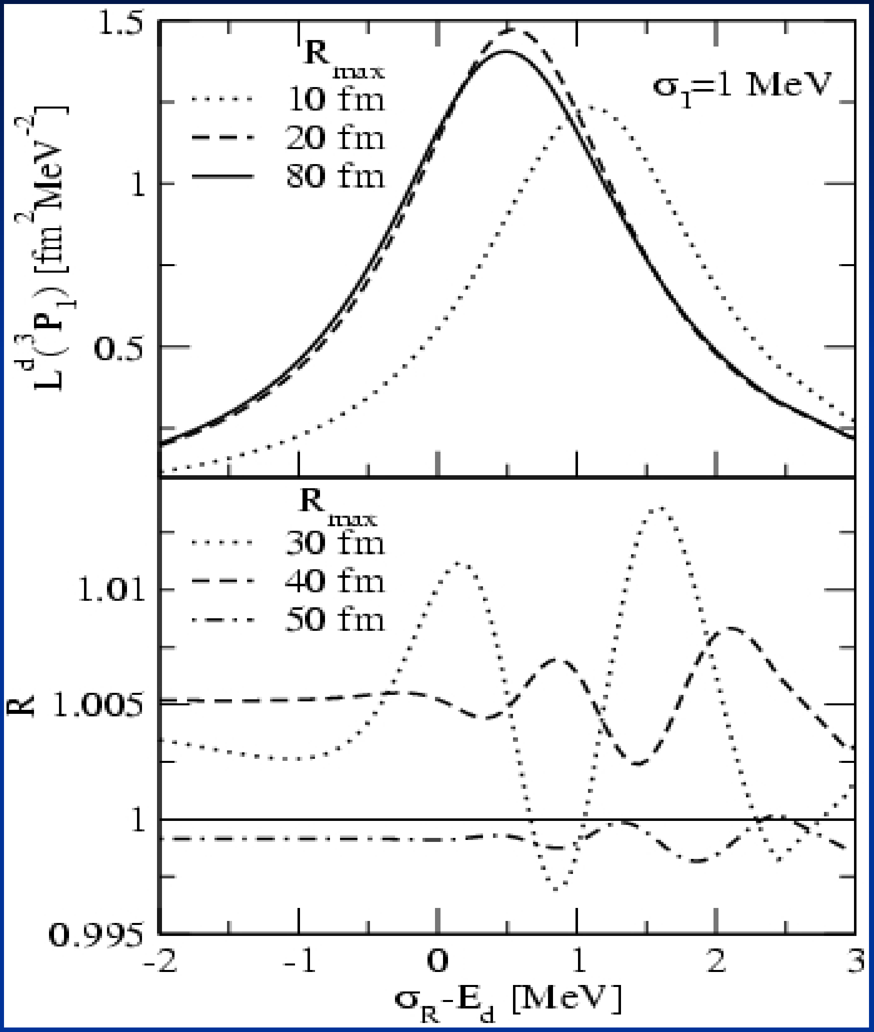


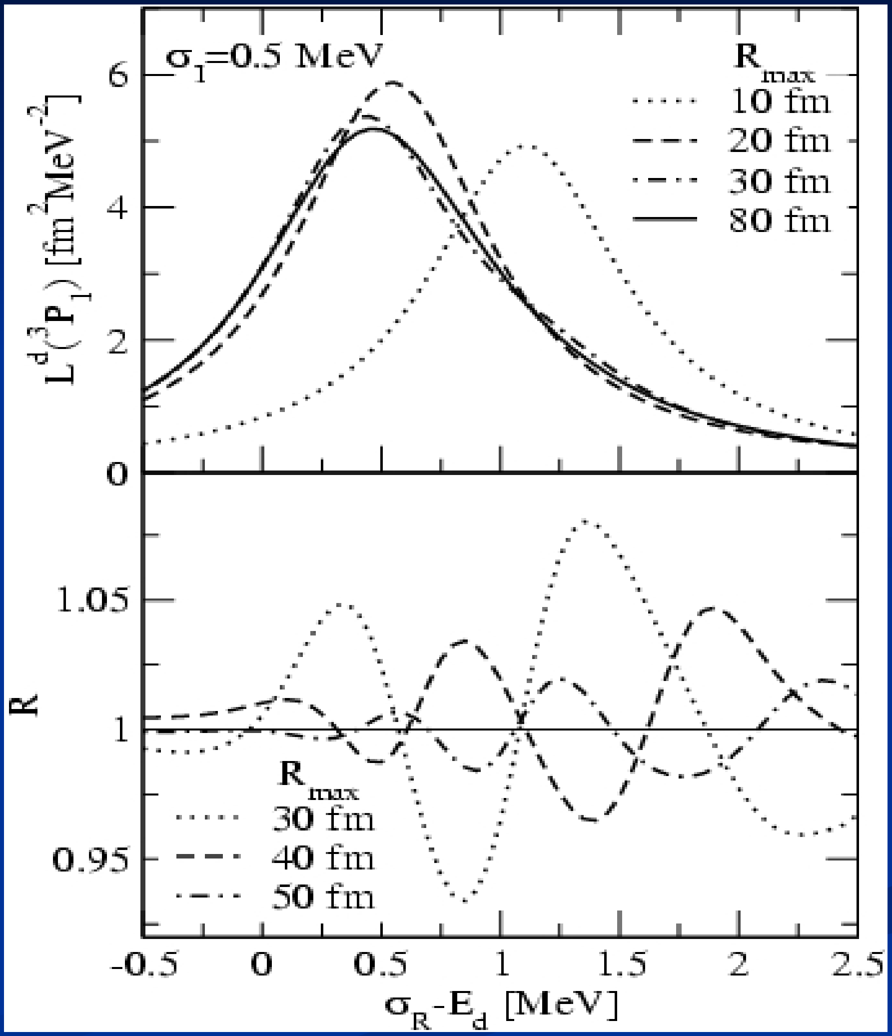
single Lorentzian with σ_I=10 MeV

Results with modified $^3P_1$ potential

First LIT in the region of the low-energy resonance

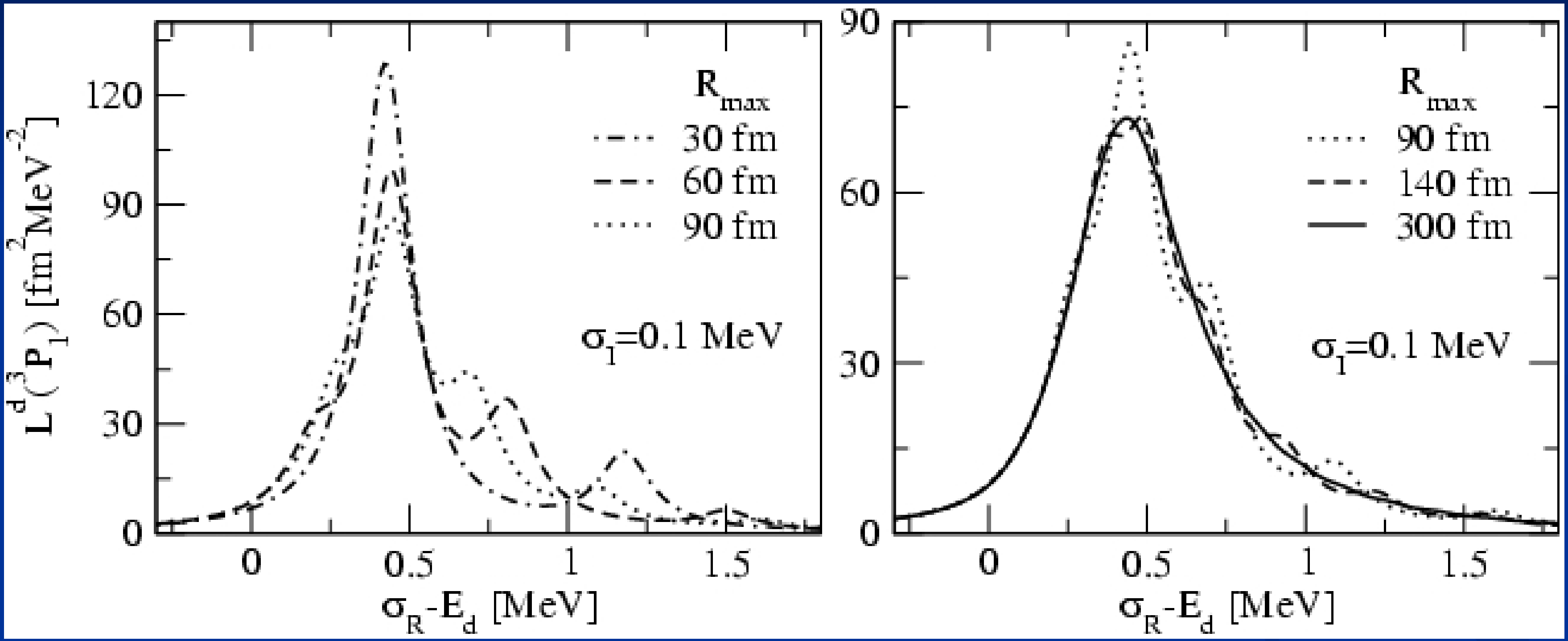
LIT for deuteron total photoabsorption cross section considering only transitions to $^3P_1$ channel with modified interaction





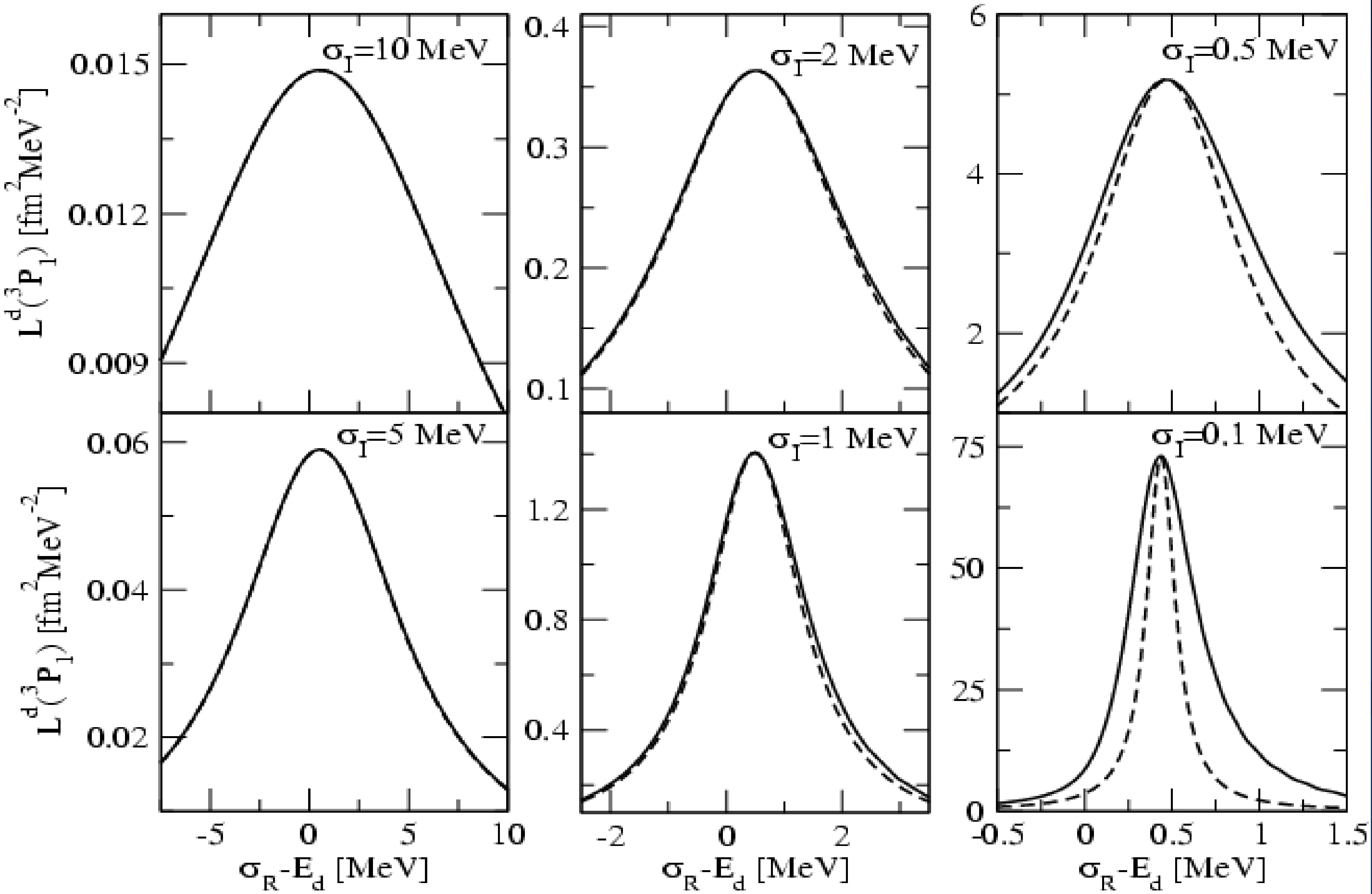
σ_I=1 MeV

σ_I=0.5 MeV



σ_I=0.1 MeV

LITs in the resonance region with various σ_I (full curves);
comparison with single Lorentzians of corresponding σ_I (dashed curves)

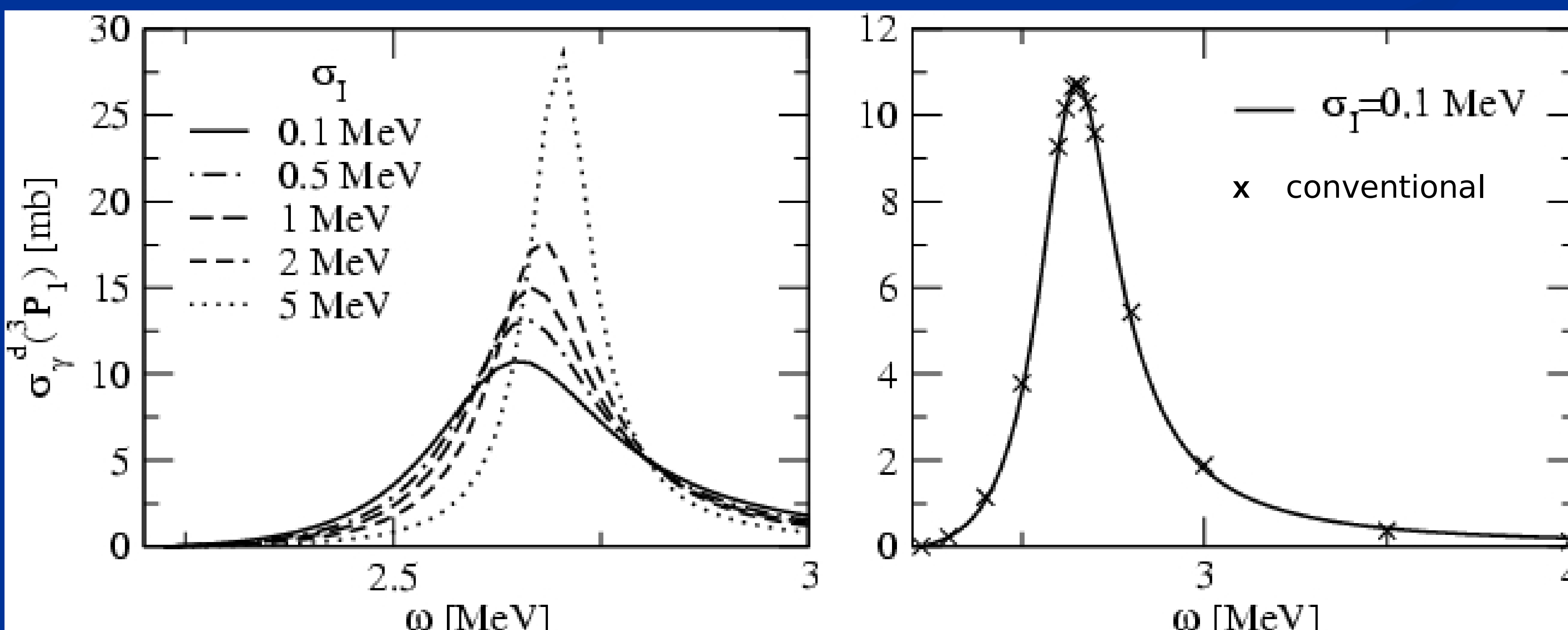


Incomplete Inversion

Instead of using set χ_m defined previously we take M_{max}=1 and take

$$\chi_1^{res} = \frac{1}{(E_{np} - E_{res})^2 + (\Gamma/2)^2}\left(\frac{1}{1+\exp(-1)} - \frac{1}{1+\exp((E_{np} - \alpha_3)/\alpha_3)}\right)$$

E_{res} , Γ, and α_3 are fit parameters

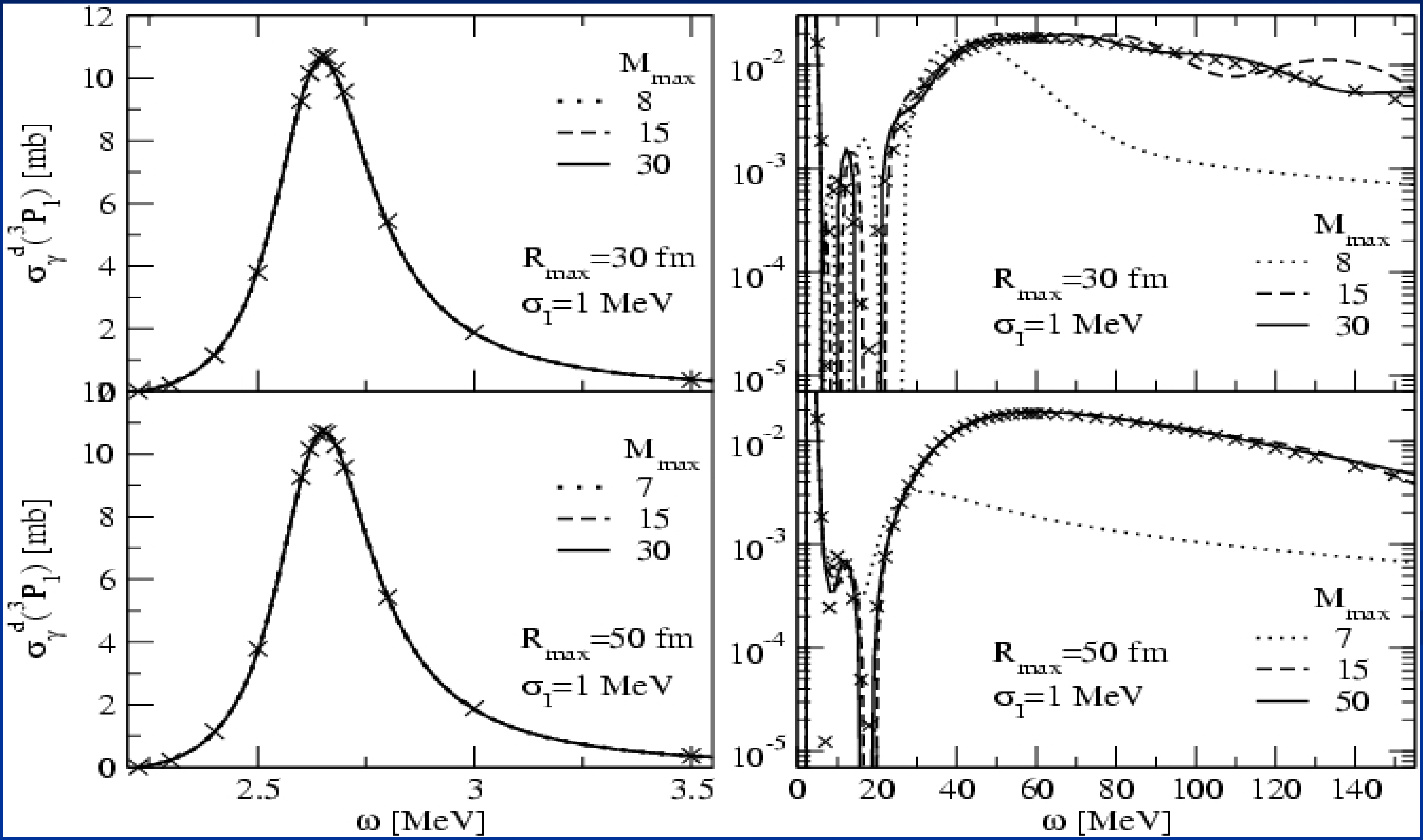


Results with modified $^3P_1$ potential

Now to the LIT results beyond low-energy resonance

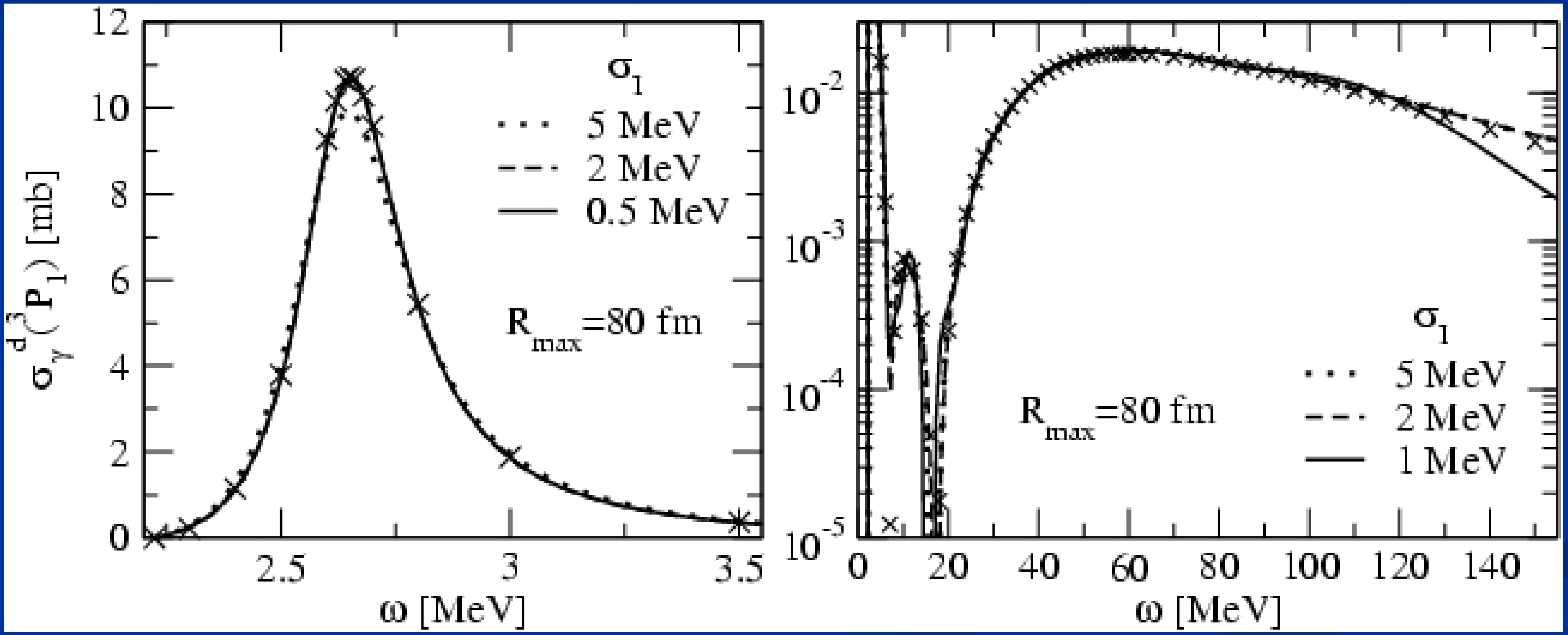
Complete inversion with set χ_m defined previously using in addition as new first basis function χ_1^{res}

σ_I = 1 MeV, R_{max} = 30 and 50 fm, various M_{max}



Complete inversion with set χ_m defined previously using in addition as new first basis function χ_1^{res}

various σ_I , R_{max} = 80 fm, M_{max} =30



Up to now direct numerical solutions of Schrödinger equation for bound state and LIT equation for $\widetilde{\Psi}$

For A > 2 it is more convenient to use expansions in complete sets using expansions in HH or HO functions

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Reformulation of the LIT

$$LIT(\sigma_R,\sigma_I) = -\frac{1}{\sigma_I}\,\mathrm{Im}\left\{\langle\Psi_0|\Theta^\dagger\,(\sigma_R + E_0 - H + i\,\sigma_I)^{-1}\,\Theta\,|\Psi_0\rangle\right\}$$

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$$R(E=\sigma_R) = -\frac{1}{\pi}\,\mathrm{Im}\left\{\lim_{\sigma_I\to 0}\langle\Psi_0|\Theta^\dagger\,(\sigma_R + E_0 - H + i\,\sigma_I)^{-1}\,\Theta\,|\Psi_0\rangle\right\}$$

New example:

deuteron photodisintegration with the
LIT method using expansion techniques

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deuteron photodisintegration with the LIT method using expansion techniques

First we use the JISP-6 NN potential which is defined on an HO basis:
$\langle n'| V | n\rangle$ up n=n'=4 (n=0,1,2,...; HO quantum number, Ω = 40 MeV)

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Also deuteron wave function and $\tilde{\Psi}$ are expanded on HO basis
Note: radial parts contain Laguerre polynomials up to order N times Gaussians

Alternatively exponential fall-off exp(-r/b) instead of Gaussians

JISP-6 potential: deuteron binding energy E_d

Slow convergence for E_d

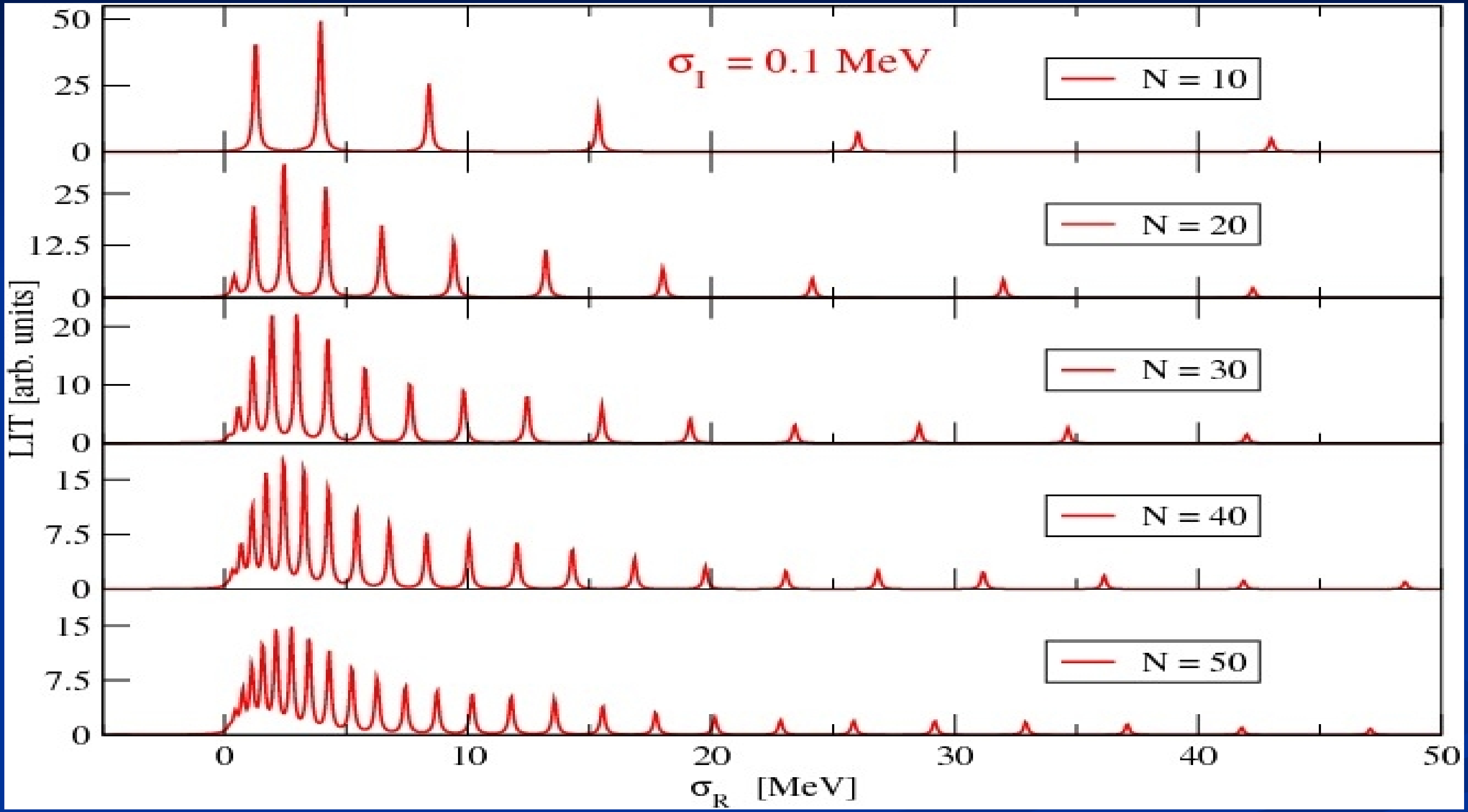
N_{max} in expansion of deuteron wave function	E_d [MeV]
10	2.057
20	2.195
50	2.2236
100	2.224555
150	2.224574

Deuteron photodisintegration with the JISP-6 NN potential

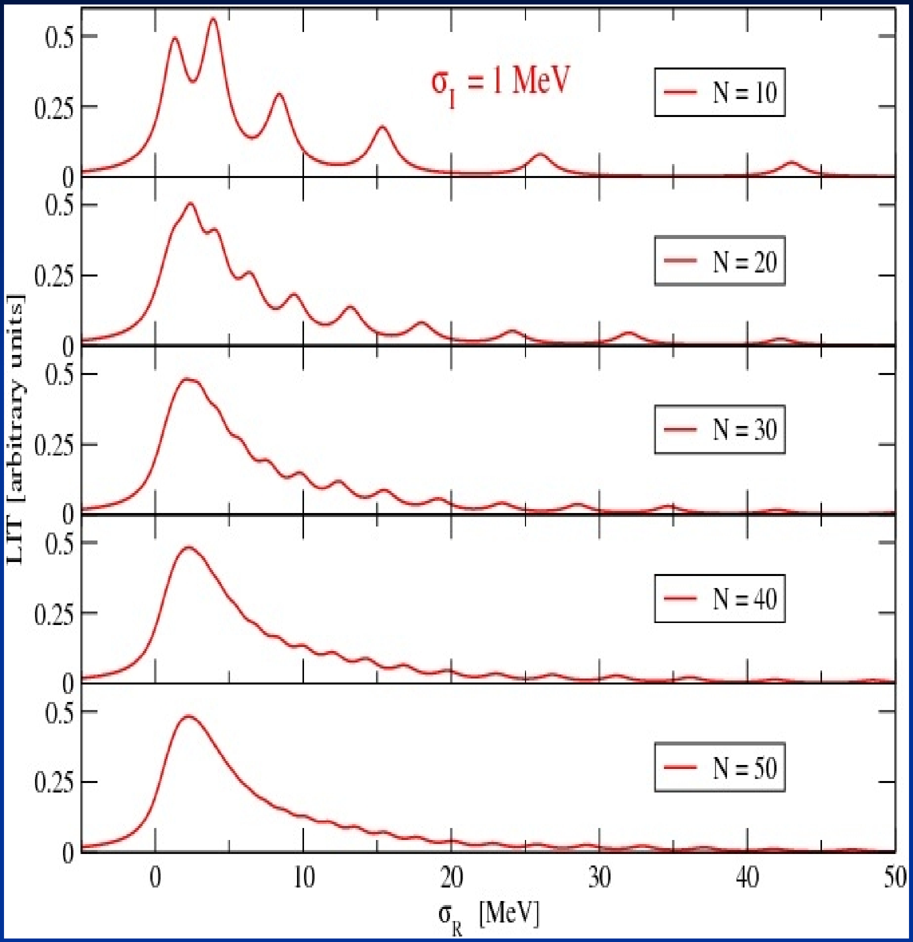
Deuteron photodisintegration with the JISP-6 NN potential

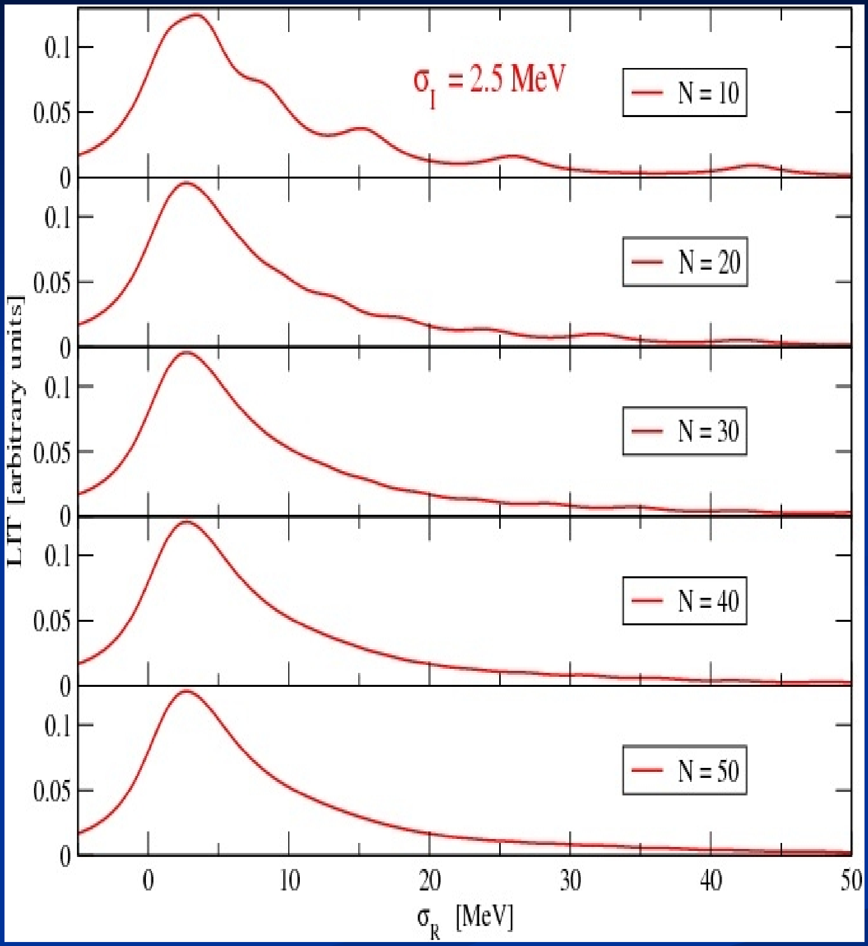
First, only considerations of transitions to the $^3P_1$ np final state

This leads to the following LITs with Laguerre polynomials up to order N with exponential fall-off (b=0.5 fm):

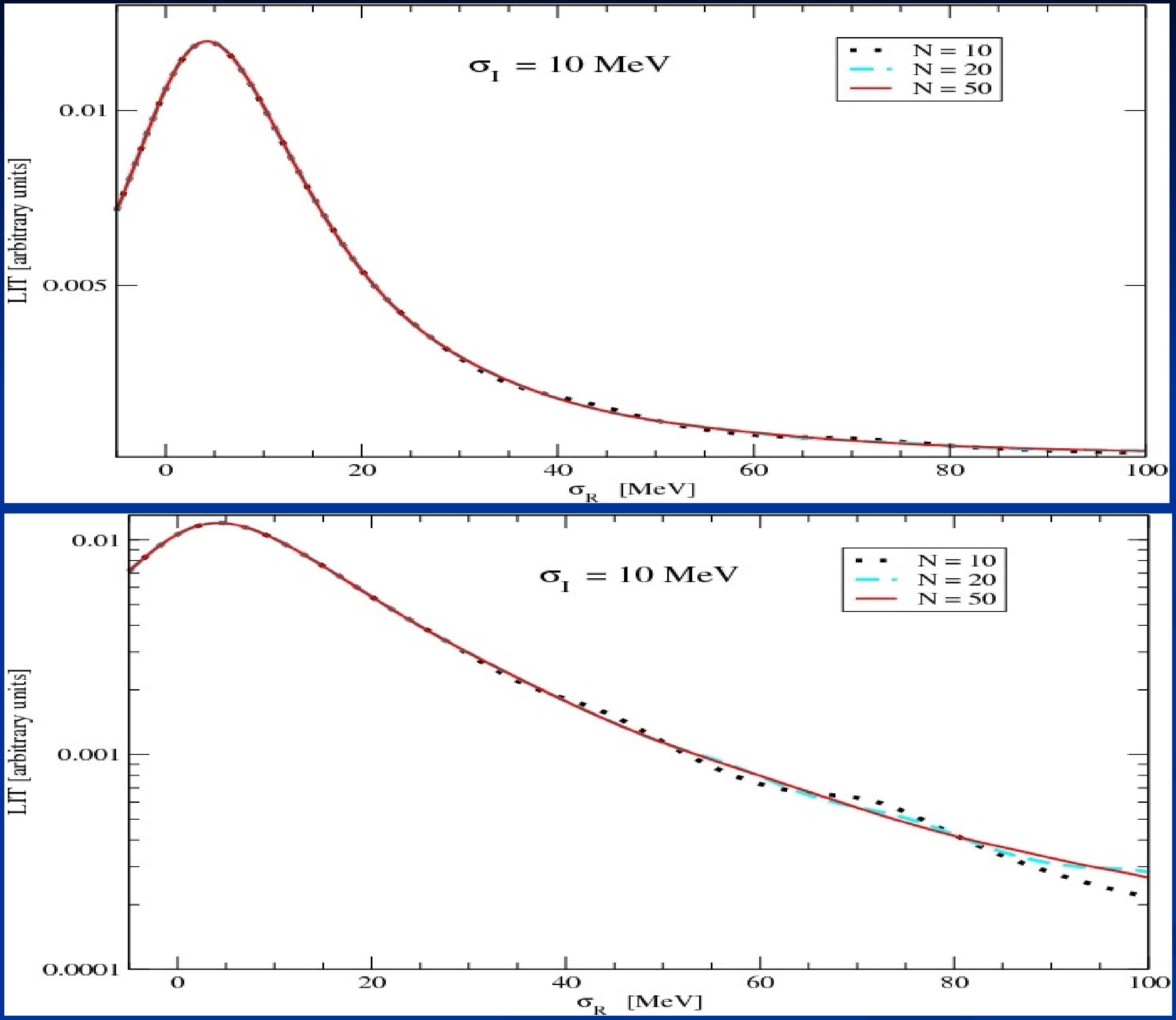


Laguerre polynomials up to order N (exponential fall-off)





Laguerre polynomials up to order N (exponential fall-off)

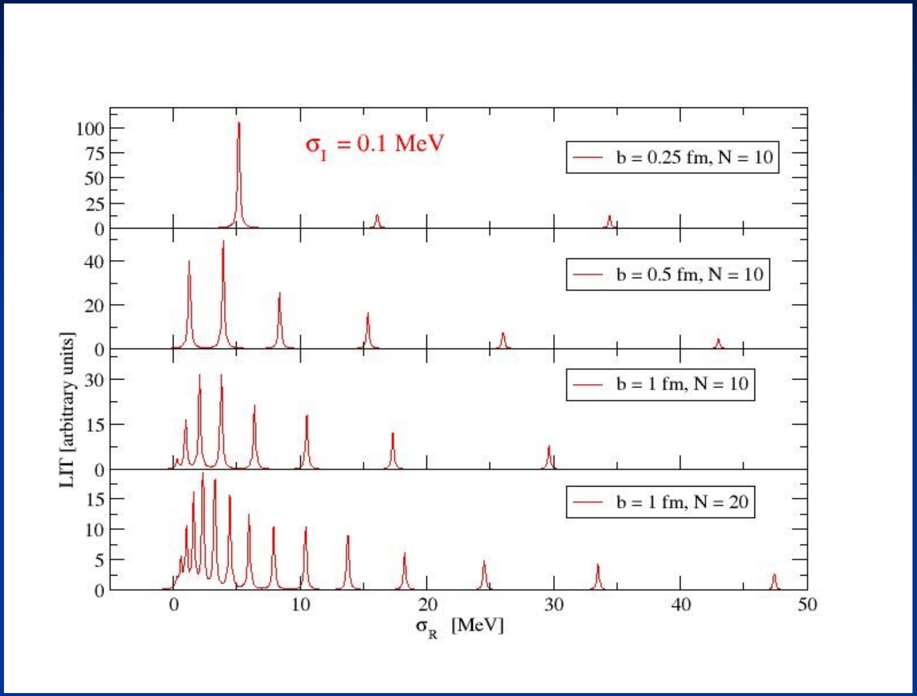


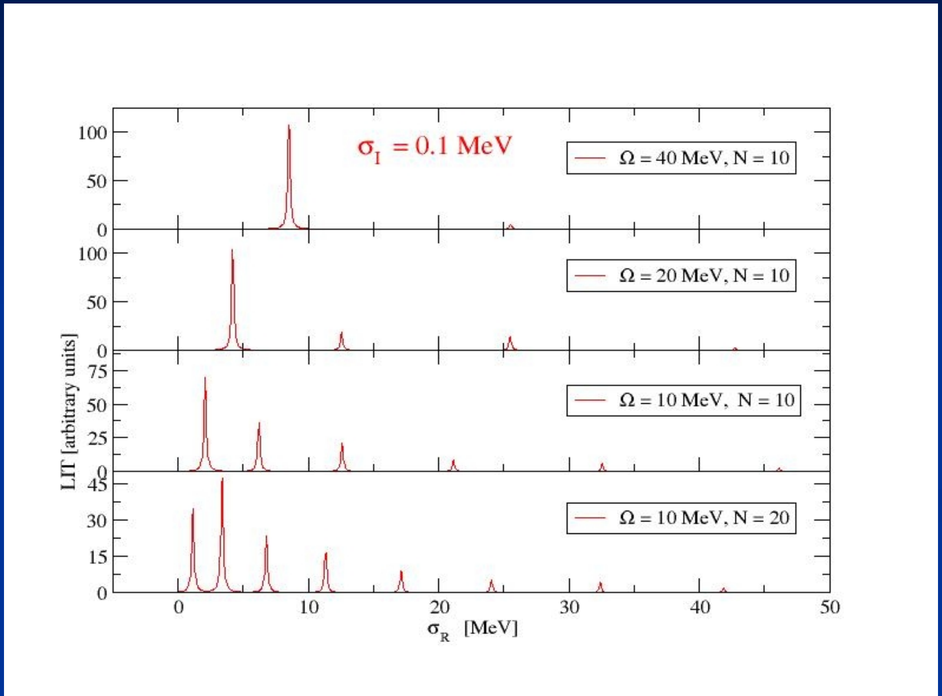
LIT approach is a method with a controlled resolution!

Next: Effect of changing fall-off parameter b

In addition: consideration of Gaussians instead of an exponential fall-off exp(-r/b) exponential fall-off exp(-r/b)

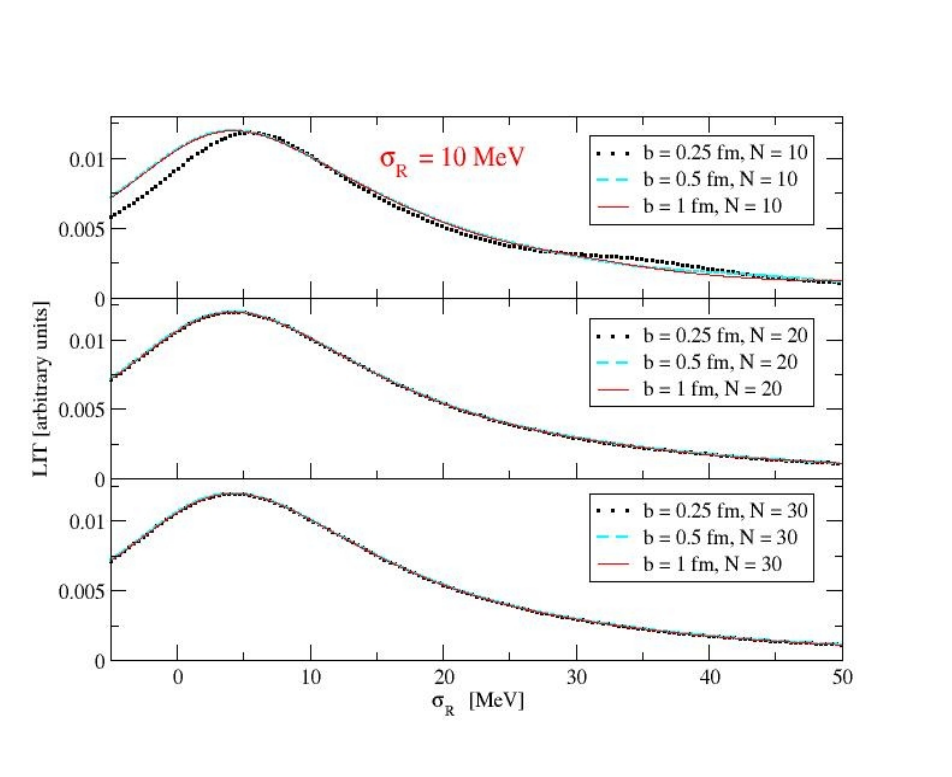
Gaussians

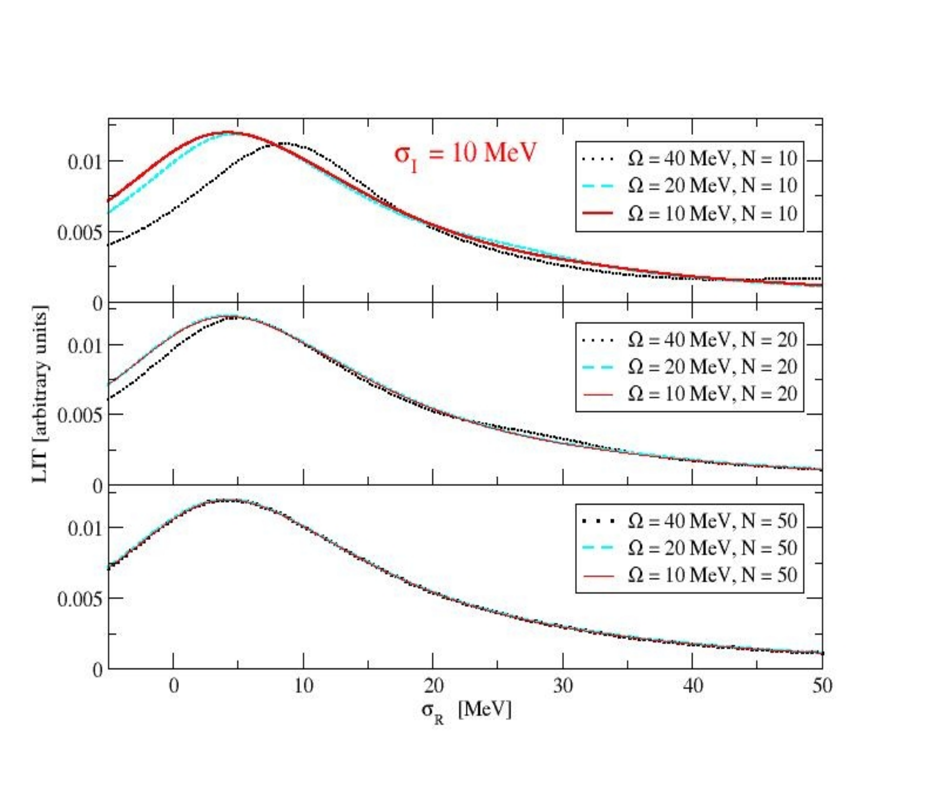




exponential fall-off exp(-r/b)

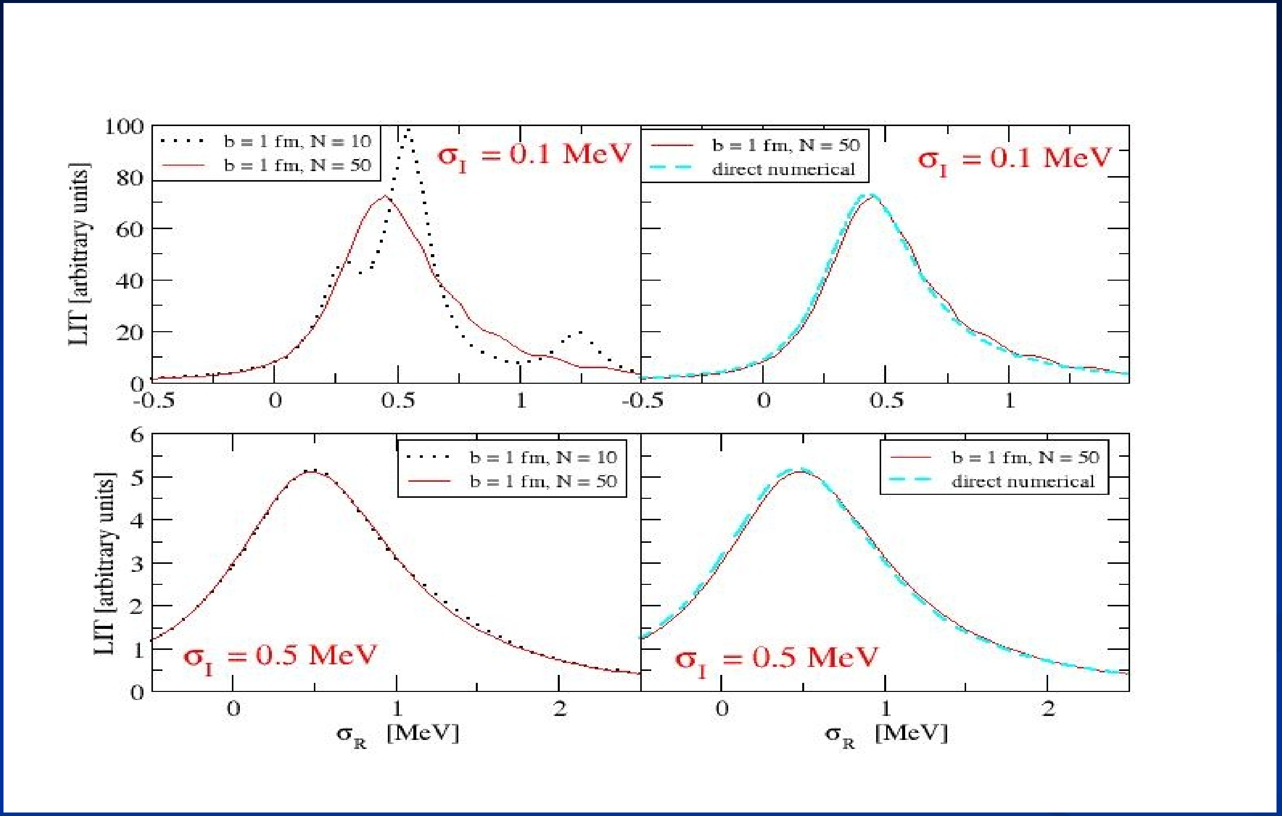
Gaussians





Now we consider the modified interaction for $^3P_1$ with resonance

Comparison of LITs from direct numerical solution and those from expansions with exponential fall-off exp(-r/b)



LIT [arbitrary units]
b = 1 fm, N = 10
b = 1 fm, N = 50
σ_I = 0.1 MeV
direct numerical
σ_I = 0.5 MeV
σ_R [MeV]

Lanczos technique

Lanczos technique is used, e.g., for diagonalization of Hamiltonian matrix (dimension: M) in a bound-state calculation.

Very efficient: total diagonalization is avoided instead only $N \ll M$ Lanczos steps are needed.

They lead to N energy eigenvalues ε_ν , which are very good approximations of the lower energy eigenvalues of H, especially for $\nu \ll N$.

Lanczos technique is also applicable to solve LIT equation.

Lanczos response

Since the Lorentzian function is a representation of the δ-function one could think of calculating R(ω) as the limit of $L(\omega,\sigma_R,\sigma_I)$ for $\sigma_I \to 0$.

The extrapolation would give

$$R(\omega) = \sum_\nu^N r_\nu \, \delta(\omega - \varepsilon_\nu^N)$$

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Lanczos response: δ-function is replaced by Lorentzian with small σ_I

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Deuteron photodisintegration:

Consider all three transitions $^3P_0$, $^3P_1$, $^3P_2 - {}^3F_2$

now expansion of radial LIT part in HO functions

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Consider all three transitions $^3P_0$, $^3P_1$, $^3P_2 - {}^3F_2$

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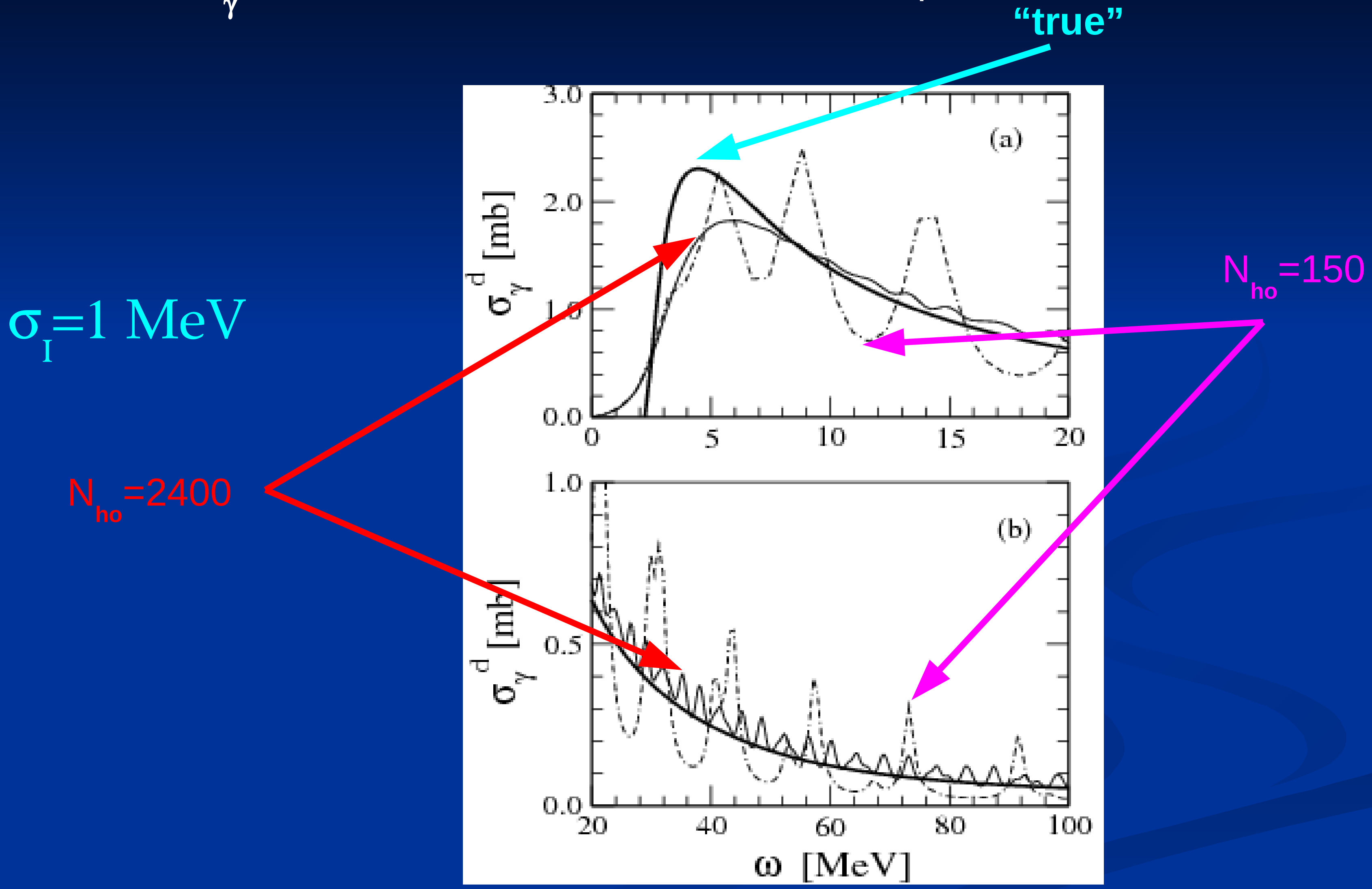
NN potential: JISP6

$\sigma_\gamma(\omega)$ from inversion and Lanczos response



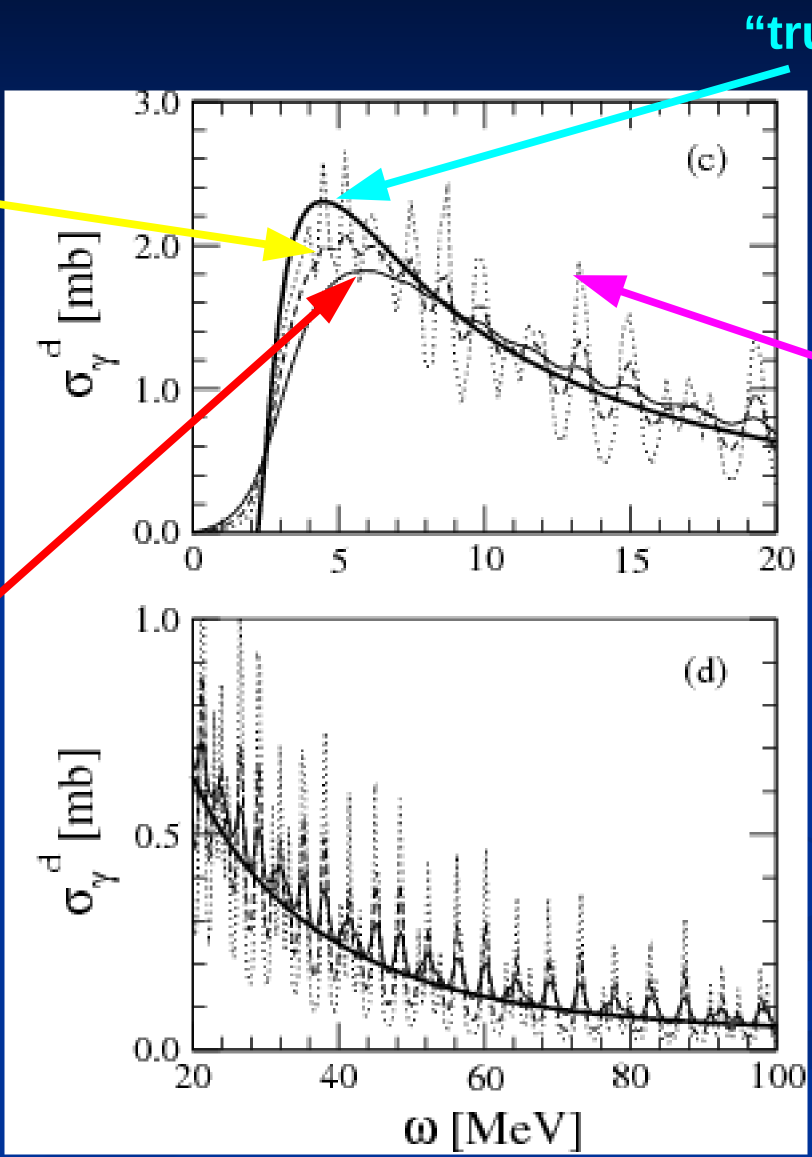
$\sigma_\gamma(\omega)$ from inversion and Lanczos response

$\Gamma = 0.5$ MeV

HO basis:

fixed $N_{HO}=2400$

$\Gamma=1$ MeV



$\Gamma=0.25$ MeV

Conclusion

Strength for a given discrete state of energy E is not the actual strength for this energy, but can only be interpreted correctly within an integral transform approach.

The correct distribution of strength is obtained via the inversion of the integral transform.

LIT application for inclusive electron scattering

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- 0^+ resonance of ^4He

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- Longitudinal response function $R_L(\omega,q)$ for A = 3 and 4

LIT application for inclusive electron scattering

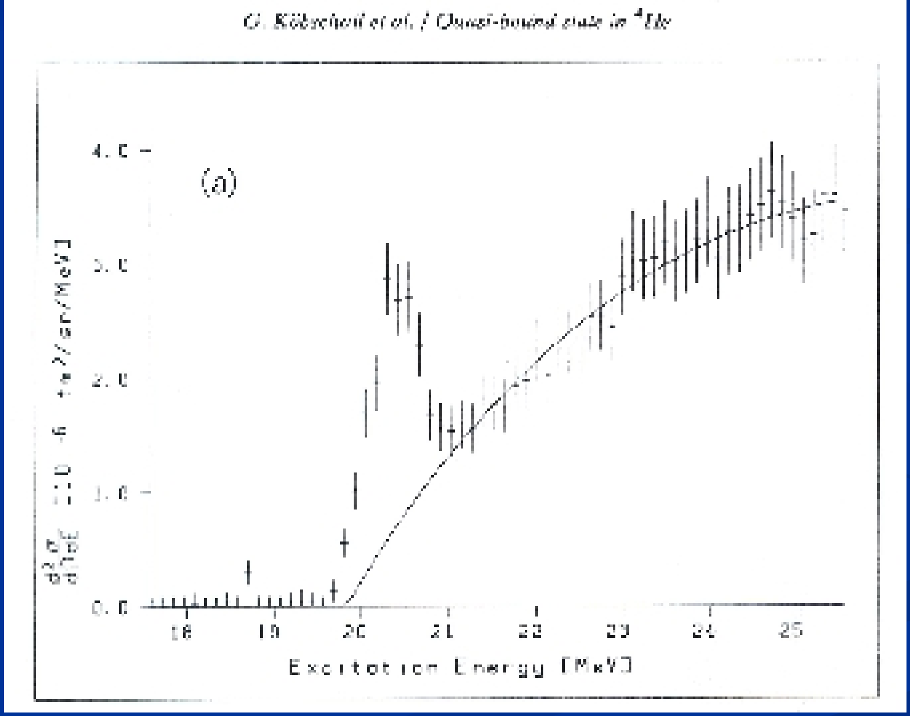
- 0^+ resonance of ^{4}He
- Longitudinal response function $R_L(\omega,q)$ for A = 3 and 4
- Transverse response function $R_T(\omega,q)$ for A = 3
    - Δ degrees of freedom
    - Quasi-elastic response at higher q (q=500-700 MeV/c)

0^+ resonance in longitudinal response function R_L in $^4He(e,e')$

S. Bacca, N. Barnea, WL, G. Orlandini, PRL 110, 042503 (2013)

0^+ Resonance in the ^{4}He compound system

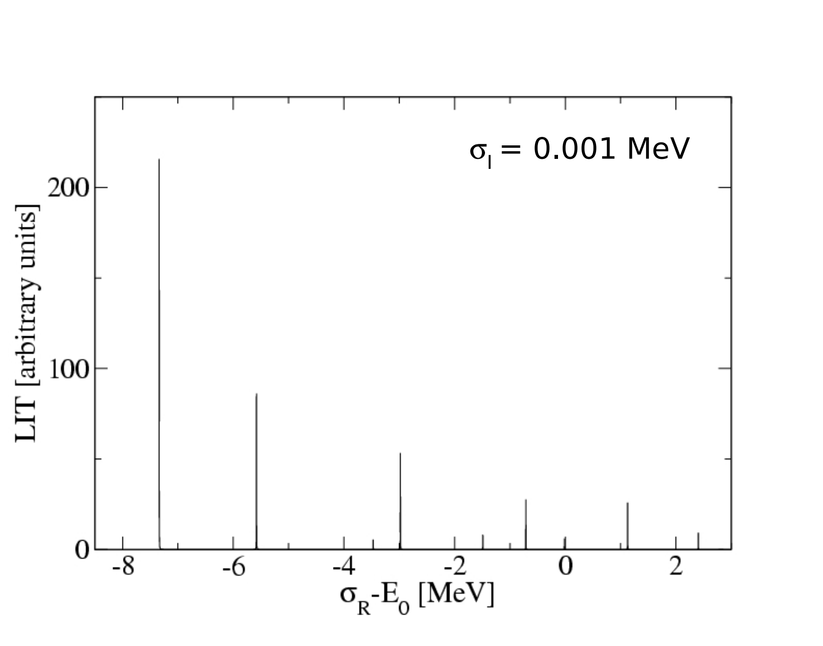
Resonance at E_R = -8.2 MeV, i.e. above the ^{3}H-p threshold. Strong evidence in electron scattering off ^{4}He



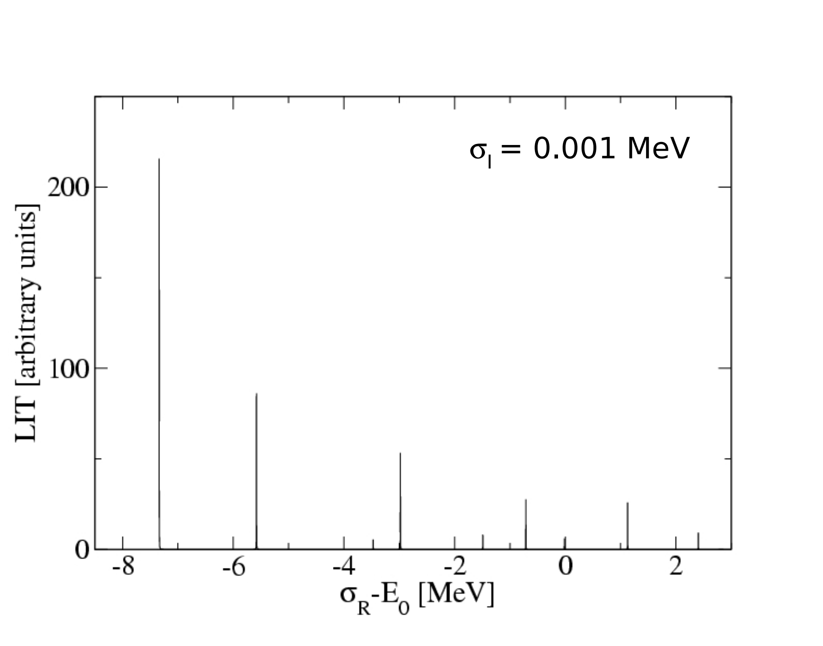
Γ = 270±70 keV

G. Köbschall et al., NPA 405, 648 (1983)

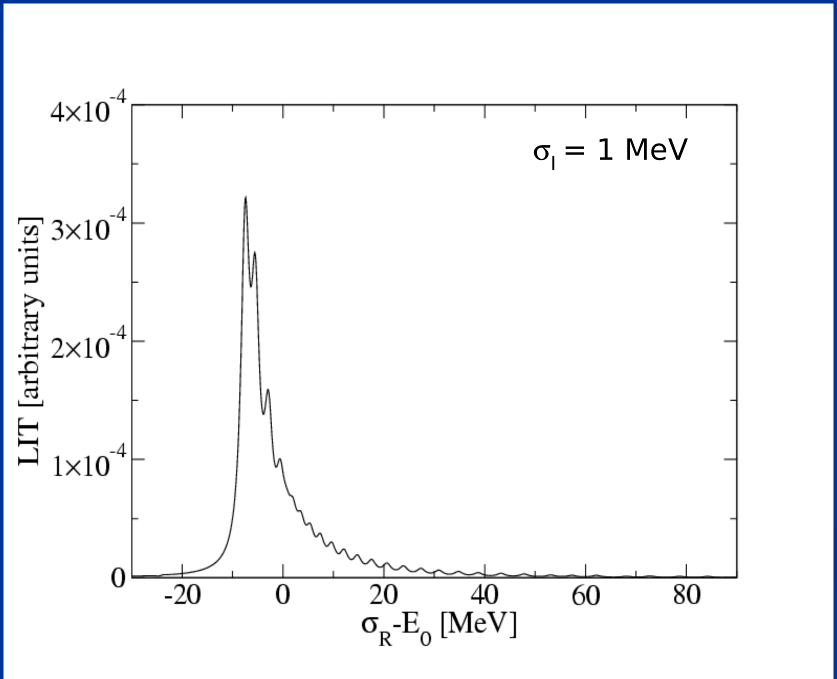
Results of our LIT calculation



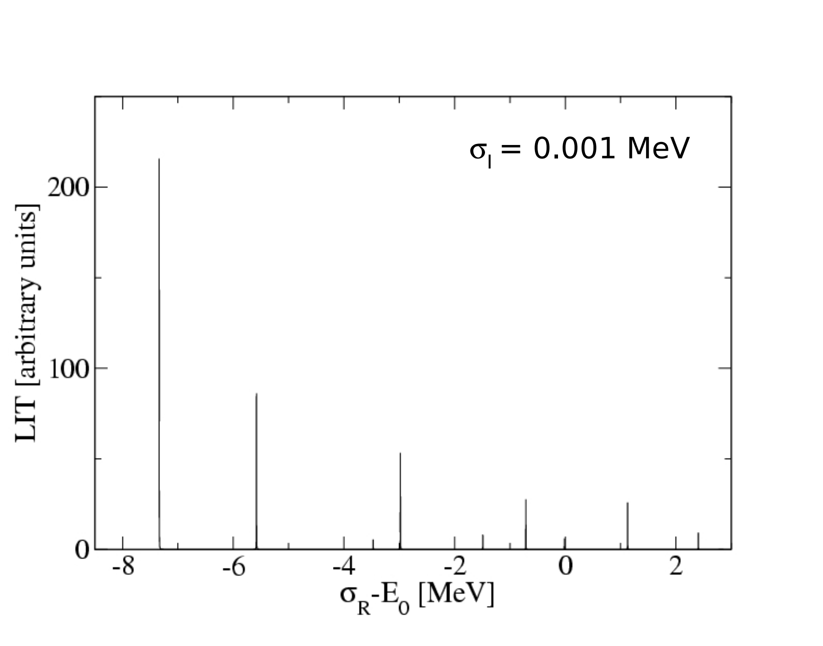
σ_I = 0.001 MeV
LIT [arbitrary units]
200
100
0
-8
-6
-4
-2
0
2
σ_R-E_0 [MeV]



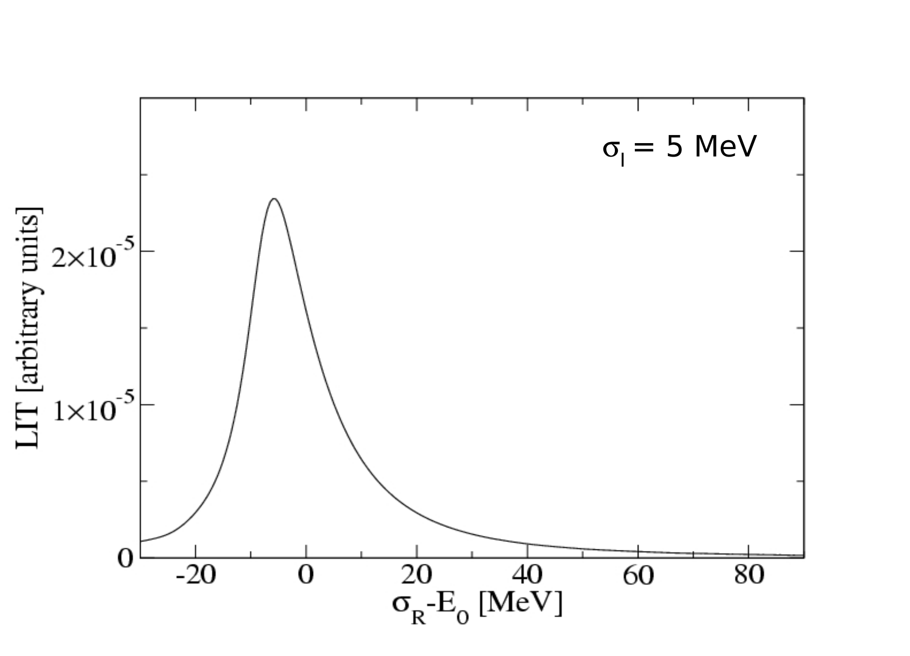
σ$_I$ = 0.001 MeV
LIT [arbitrary units]
0
100
200
-8
-6
-4
-2
0
2
σ$_R$-E$_0$ [MeV]



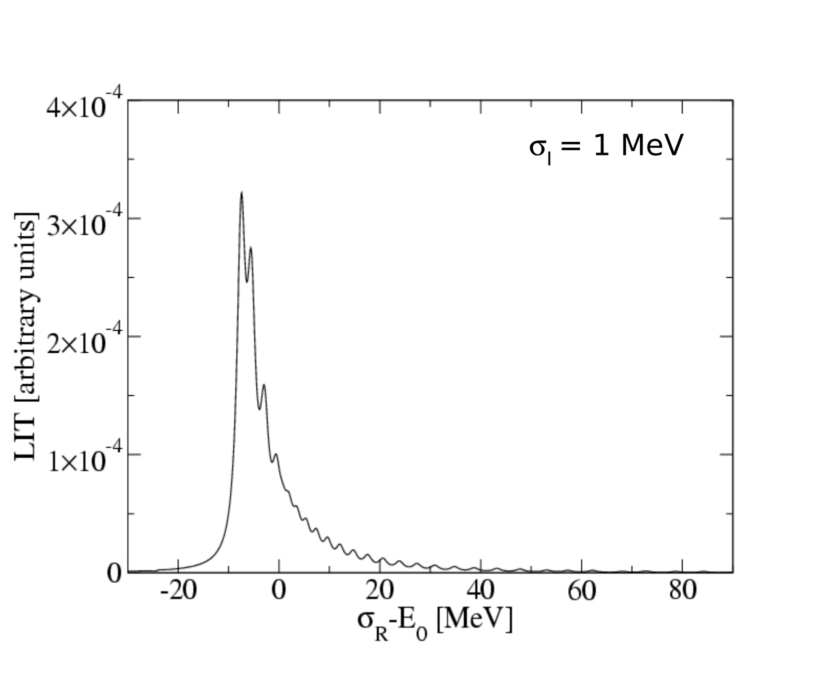
σ$_I$ = 1 MeV
LIT [arbitrary units]
0
1×10^{-4}
2×10^{-4}
3×10^{-4}
4×10^{-4}
-20
0
20
40
60
80
σ$_R$-E$_0$ [MeV]



σI = 0.001 MeV
LIT [arbitrary units]
200
100
0
-8
-6
-4
-2
0
2
σR-E0 [MeV]



σI = 5 MeV
LIT [arbitrary units]
2×10-5
1×10-5
0
-20
0
20
40
60
80
σR-E0 [MeV]



σI = 1 MeV
LIT [arbitrary units]
4×10-4
3×10-4
2×10-4
1×10-4
0
-20
0
20
40
60
80
σR-E0 [MeV]

The present precision of the calculation does not allow to resolve the shape of the resonance, therefore the width cannot be determined.

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However, the strength of the resonance can be determined!

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Of course not by taking the strength to the discretized state, but by rearranging the inversion in a suitable way:

The present precision of the calculation does not allow to resolve the shape of the resonance, therefore the width cannot be determined.

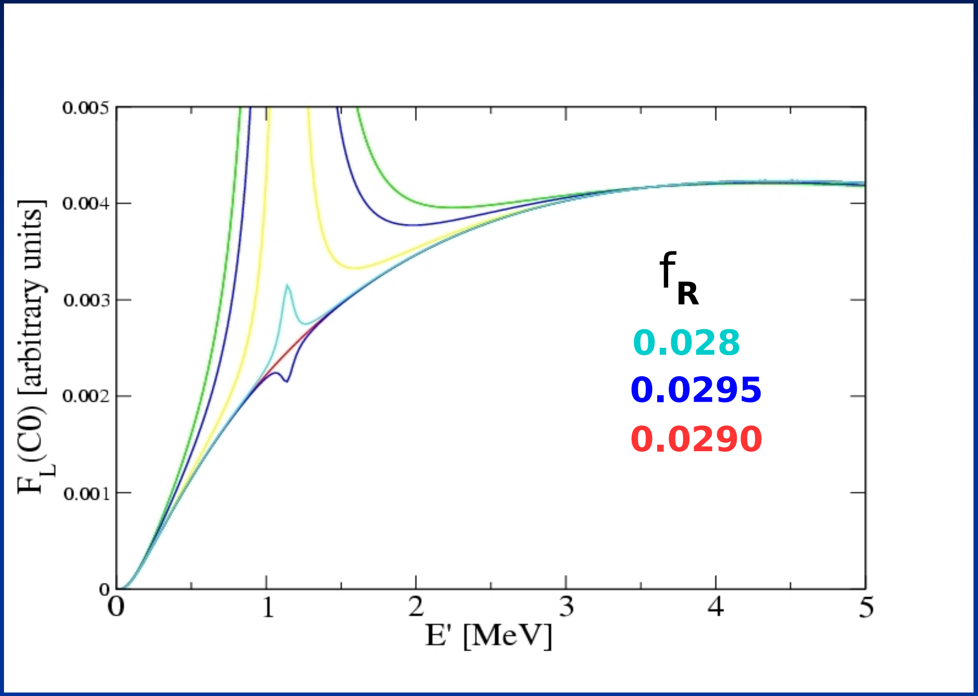
However, the strength of the resonance can be determined!

Of course not by taking the strength to the discretized state, but by rearranging the inversion in a suitable way:

Reduce strength to the state up to the point that the inversion does not show any resonant structure at the resonance energy E_R:

$$\mathrm{LIT}(\sigma_R, \sigma_I) \rightarrow \mathrm{LIT}(\sigma_R, \sigma_I) - f_R \,/\, [(E_R - \sigma_R)^2 + \sigma_I^2] \equiv \mathrm{LIT}(\sigma_R, \sigma_I, f_R)$$

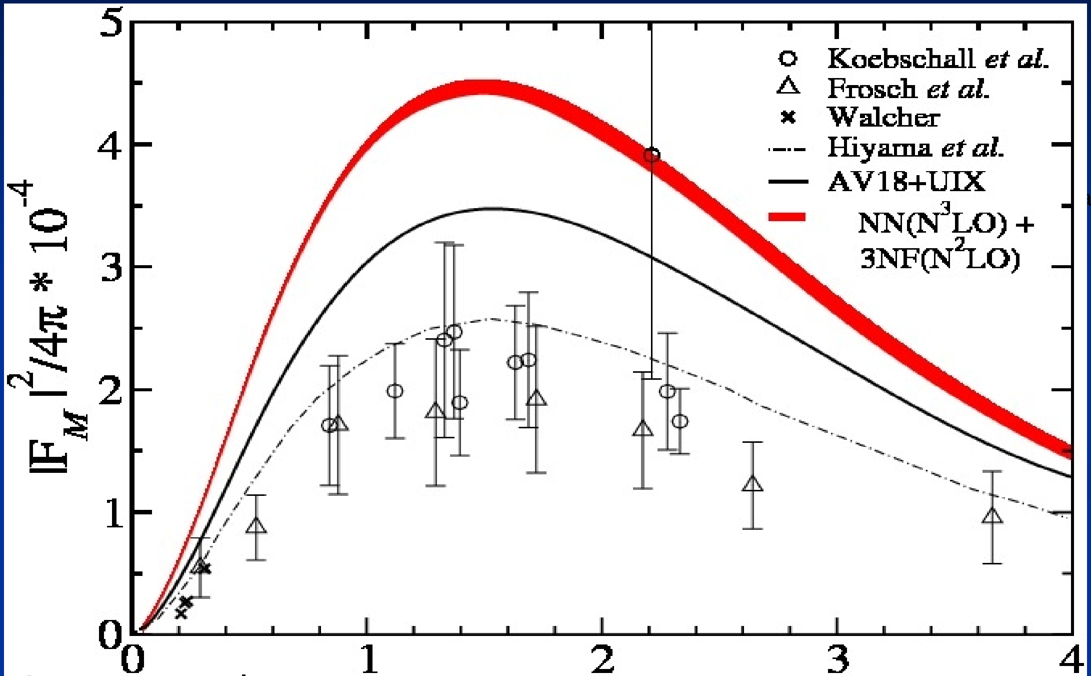
with resonance strength f_R



Inversion results with different f_R values

AV18+UIX, q=300 MeV/c

Comparison to experimental results



S. Bacca et al.
PRL 110, 042503 (2013)

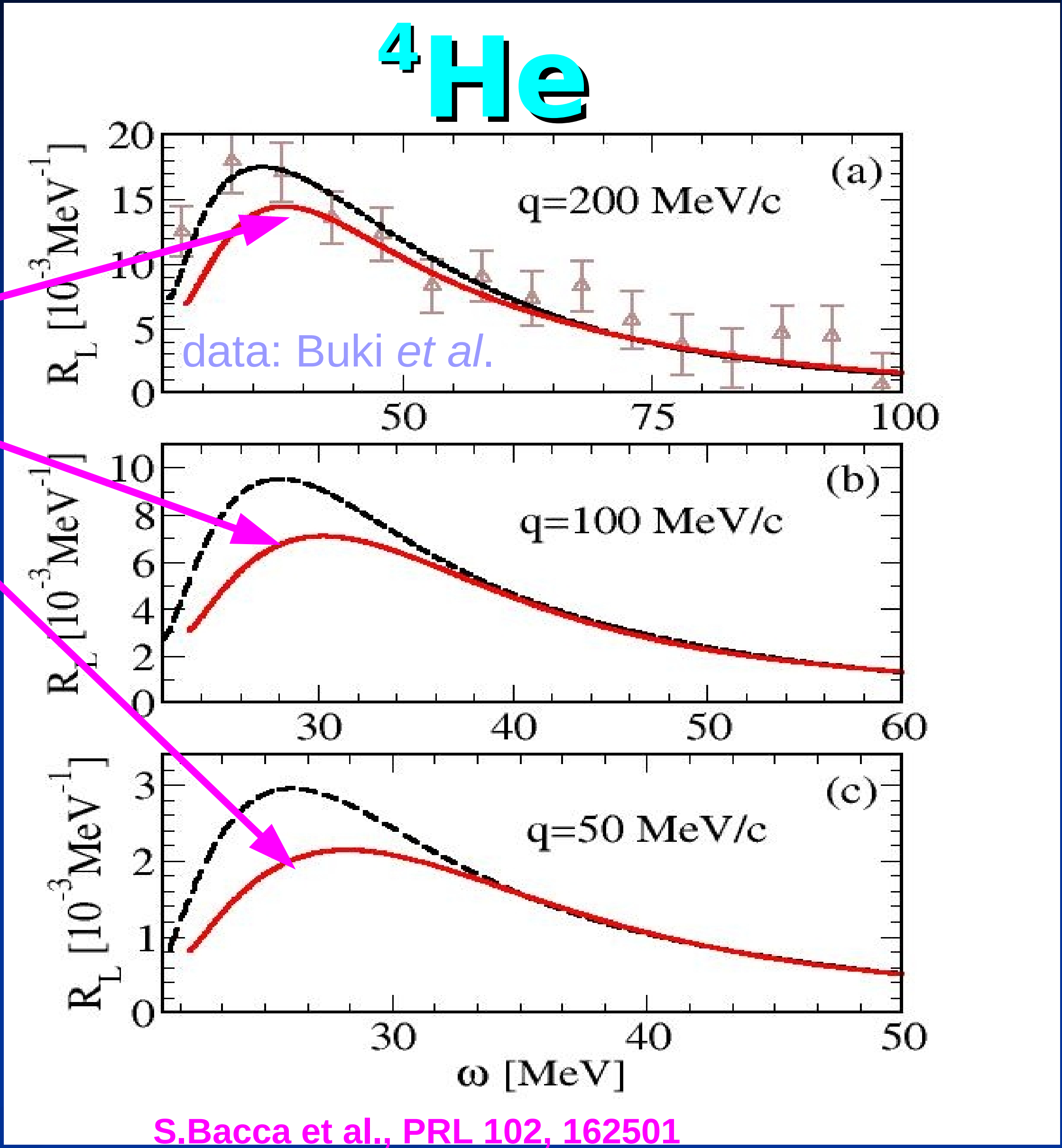
LIT/EIHH Calculation for AV18+UIX and Idaho-N3LO+N2LO

Dotted: AV8' + central 3NF (Hiyama et al.)

(e,e') Longitudinal Response

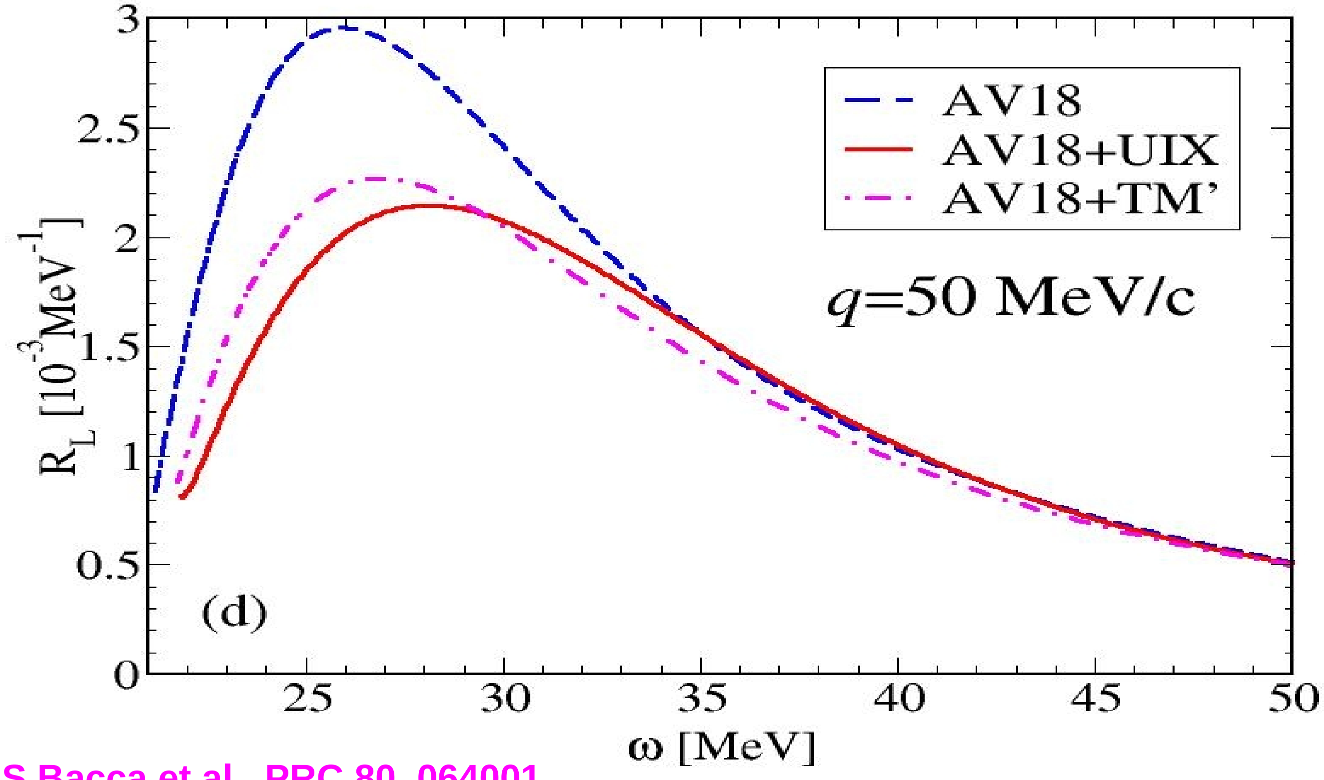
SURPRISE:
LARGE EFFECT OF 3-BODY FORCE AT LOW q

Calculation via EIHH with force model: AV18 + UIX



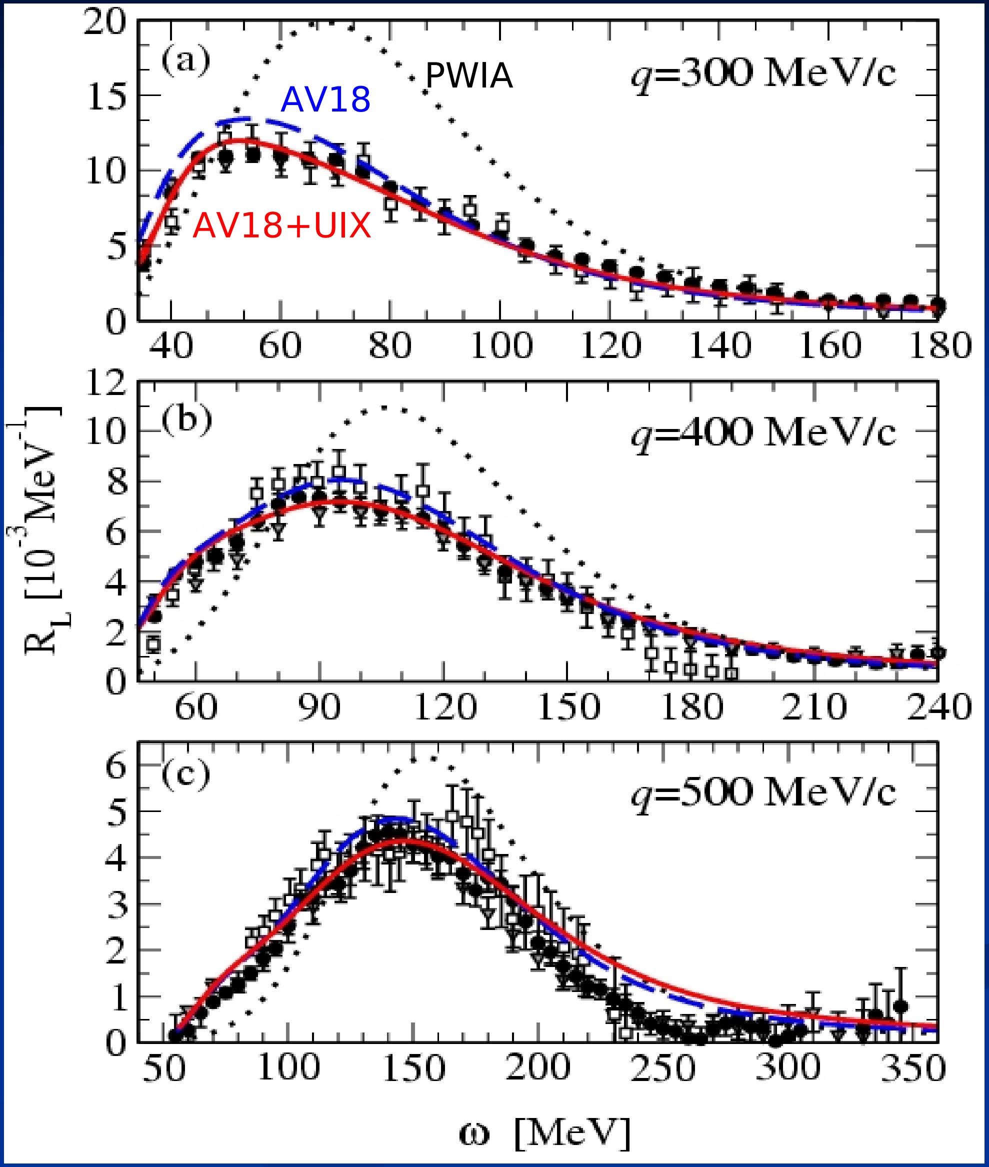
S.Bacca et al., PRL 102, 162501

Dependence on different 3-nucleon forces



S.Bacca et al., PRC 80, 064001

^{4}He (e,e') Longitudinal Response



SMALL EFFECT OF 3-BODY FORCE AT HIGH q

Exp.: Saclay
Bates
world data (J. Carlson et *al*.)

3-Body inclusive electrodisintegration
Role of 3-Nucleon force

LONGITUDINAL RESPONSE

"low" q

------ **AV18**

____ **AV18 + UIX**

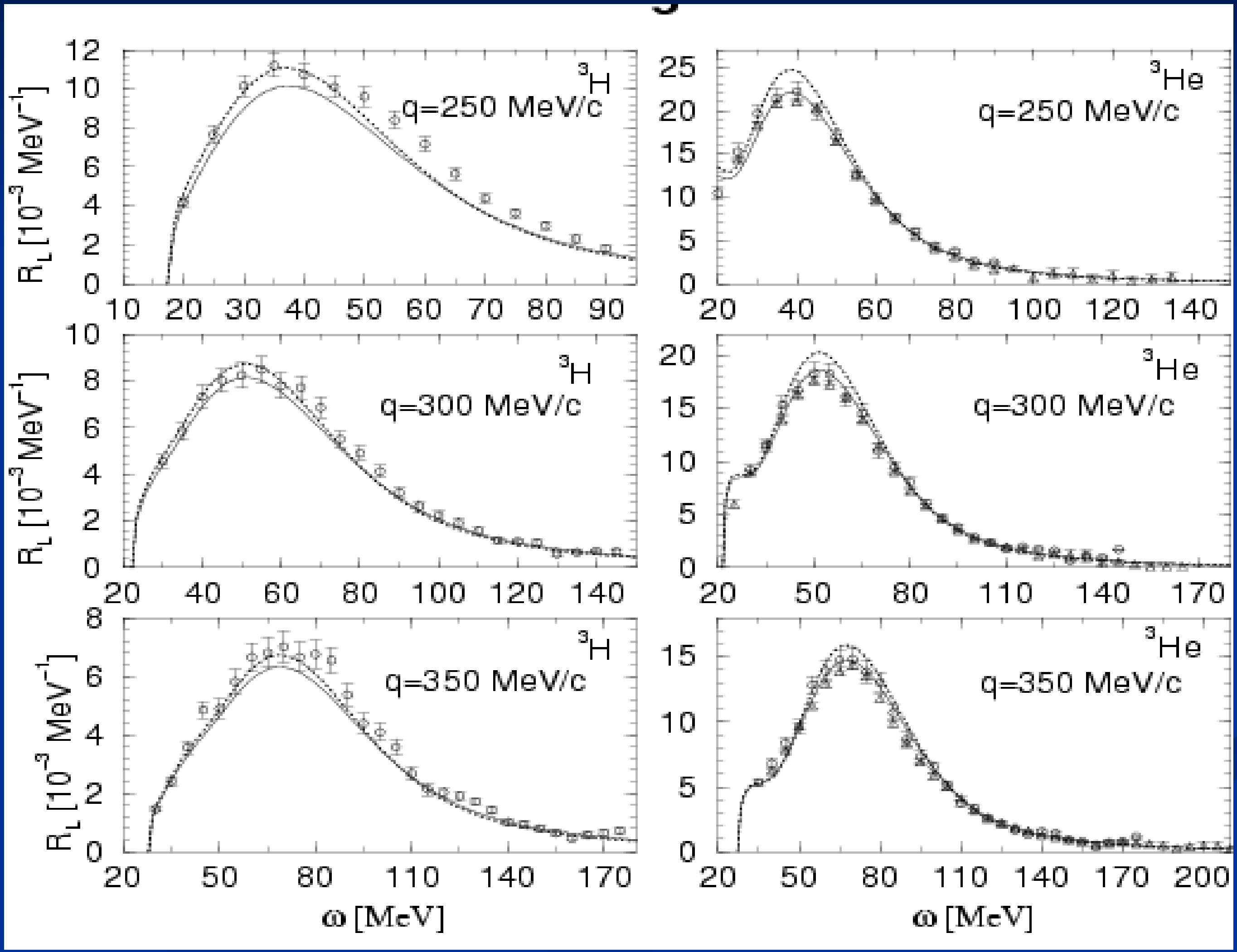
CHH

V. Efros, W.L., G. Orlandini
E. Tomusiak
PRC69, 044001 (2004)

Exp:

Dow

Marchand



Transverse response function $R_T(\omega,q)$

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Subnuclear degrees of freedom can become important

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Subnuclear degrees of freedom can become important

- Meson exchange currents (MEC)

MEC with LIT method: S. Della Monaca, V.D. Efros, A. Khugaev, WL, G. Orlandini, E.L. Tomusiak, L. Yuan, PRC 77, 044007 (2008)

Transverse response function $R_T(\omega,q)$

Subnuclear degrees of freedom can become important

- Meson exchange currents (MEC)

MEC with LIT method: S. Della Monaca, V.D. Efros, A. Khugaev, WL, G. Orlandini, E.L. Tomusiak, L. Yuan, PRC 77, 044007 (2008)

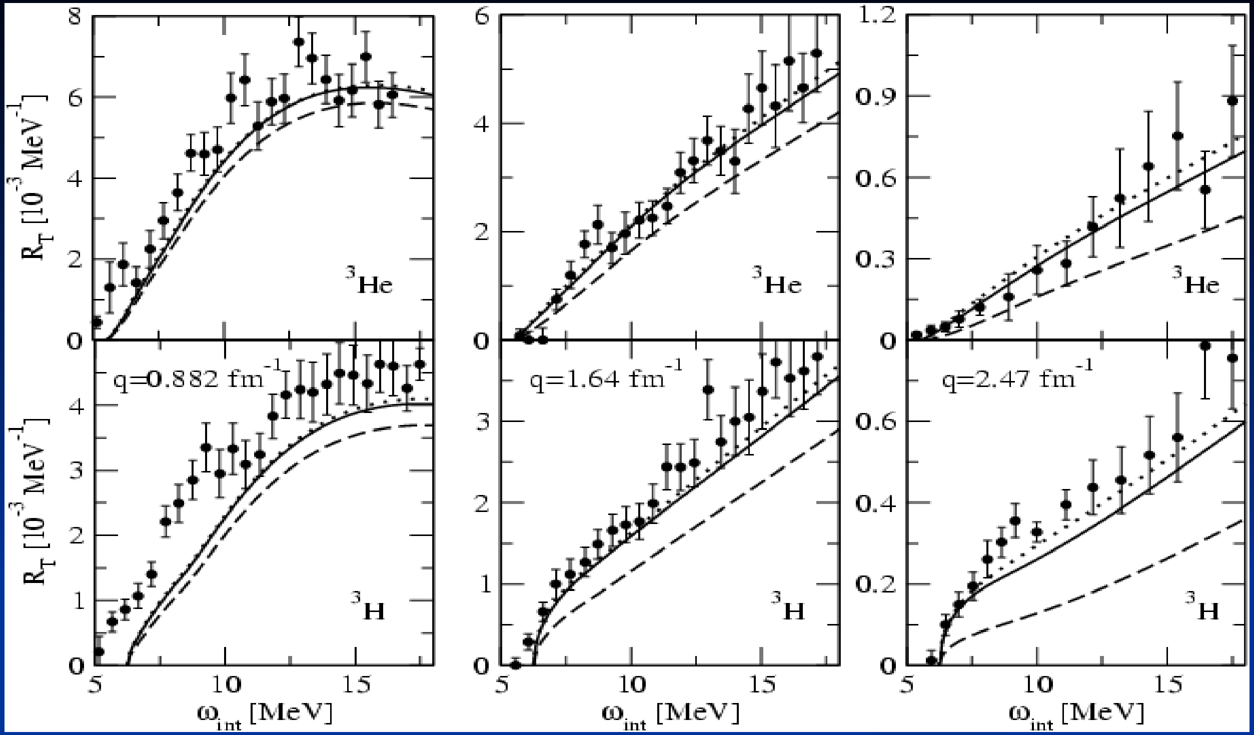
- Δ isobar currents (Δ-IC)

Δ-IC with LIT method: L. Yuan, V.D. Efros, WL, E.L. Tomusiak, PRC 81, 064001 (2010)

NR: dashed

NR+MEC: dotted

Rel.+MEC: full



q = 174 MeV/c q = 324 MeV/c q = 487 MeV/c

R_T close to break-up threshold

(V.D. Efros, WL, G. Orlandini, E.L. Tomusiak, Few-Body Syst. 47, 157 (2010))

Δ degrees of freedom

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta$$

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta$$ coupled channel calculation

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$ solve eqs. simultaneously

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta$$ Impulse approximation

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$ Solve formally for Ψ_Δ

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

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$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta$$

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$$= H_\Delta$$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ $(V_{N\Delta})$ transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

$$\Psi_\Delta = -(H_\Delta - E)^{-1}\,V_{N\Delta,NN}\,\Psi_N$$

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta \quad (*)$$

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$

$$= H_\Delta$$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ $(V_{N\Delta})$ transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

$$\Psi_\Delta = -(H_\Delta - E)^{-1}\,V_{N\Delta,NN}\,\Psi_N$$ Insert formal solution in (*)

$$(T_N + V_{NN} - V_{NN,N\Delta}(H_\Delta - E)^{-1}\,V_{N\Delta,NN} - E)\,\Psi_N = 0$$

$$\cong V_{NN}^{realistic}$$

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta \quad (*)$$

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

$$\Psi_\Delta = -(H_\Delta - E)^{-1}\,V_{N\Delta,NN}\,\Psi_N \quad \textbf{(IA)}$$

$$(T_N + V_{NN} - V_{NN,N\Delta}(H_\Delta - E)^{-1}\,V_{N\Delta,NN} - E)\,\Psi_N = 0 \quad \textbf{(**)}$$

$\cong V_{NN}^{realistic}$

Step 1: solve (**) with realistic V_{NN} +3NF

Step 2: solve Ψ_Δ in IA

LIT equation with Δ degrees of freedom

$$\tilde{\Psi} = \tilde{\Psi}_N + \tilde{\Psi}_\Delta$$

$$(T_N + V_{NN} - \sigma)\, \tilde{\Psi}_N = - V_{NN,N\Delta}\, \tilde{\Psi}_\Delta + O_{NN}\, \Psi_{0,N} + O_{N\Delta}\, \Psi_{0,\Delta}$$

$$(\delta m + T_\Delta + V_{N\Delta} - \sigma)\, \tilde{\Psi}_\Delta = - V_{N\Delta,NN}\, \tilde{\Psi}_N + O_{\Delta N}\, \Psi_{0,N} + O_{\Delta\Delta}\, \Psi_{0,\Delta}$$

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

LIT equation with Δ degrees of freedom

$$\widetilde{\Psi} = \widetilde{\Psi}_N + \widetilde{\Psi}_\Delta$$

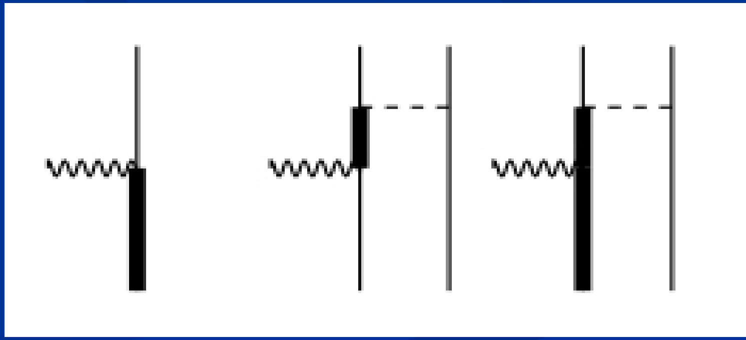
$$(T_N + V_{NN} - \sigma)\, \widetilde{\Psi}_N = -\, V_{NN,N\Delta}\, \widetilde{\Psi}_\Delta + O_{NN}\, \Psi_{0,N} + O_{N\Delta}\, \Psi_{0,\Delta}$$

$$(\delta m + T_\Delta + V_{N\Delta} - \sigma)\, \widetilde{\Psi}_\Delta = -\, V_{N\Delta,NN}\, \widetilde{\Psi}_N + O_{\Delta N}\, \Psi_{0,N} + O_{\Delta\Delta}\, \Psi_{0,\Delta}$$

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

We take into account electromagnetic operators with the Δ (Δ-IC) represented by the following graphs



LIT equation with Δ degrees of freedom

$$\widetilde{\Psi} = \widetilde{\Psi}_N + \widetilde{\Psi}_\Delta$$

$$(T_N + V_{NN} - \sigma)\, \widetilde{\Psi}_N = -V_{NN,N\Delta}\, \widetilde{\Psi}_\Delta + O_{NN}\, \Psi_{0,N} + O_{N\Delta}\, \Psi_{0,\Delta}$$

$$(\delta m + T_\Delta + V_{N\Delta} - \sigma)\, \widetilde{\Psi}_\Delta = -V_{N\Delta,NN}\, \widetilde{\Psi}_N + O_{\Delta N}\, \Psi_{0,N} + O_{\Delta\Delta}\, \Psi_{0,\Delta}$$

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

$$(T_N + V^{realistic} - \sigma)\, \widetilde{\Psi}_N = -V_{NN,N\Delta}(H_\Delta - \sigma)^{-1}(O_{\Delta N}\, \Psi_{0,N} + O_{\Delta\Delta}\, \Psi_{0,\Delta})$$

$$+ O_{NN}\, \Psi_{0,N} + O_{N\Delta}\, \Psi_{0,\Delta}$$

^{3}He (e,e') Response Functions in the Quasielastic Region

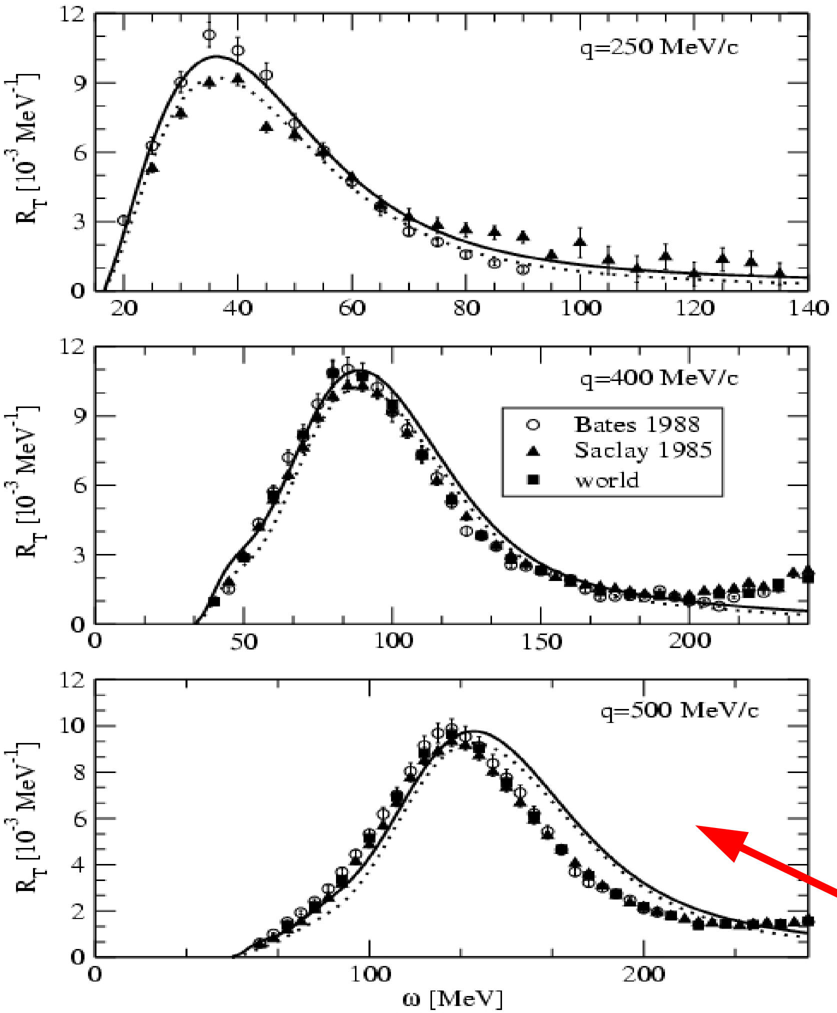
The quasielastic region is dominated by the one-body parts of ρ and **J**, but relativistic contributions become increasingly important with growing momentum transfer q

Our aim: non-rel. calculation + rel. corrections

with realistic nuclear forces

Motivation

$R_T(\omega,q)$ at various q



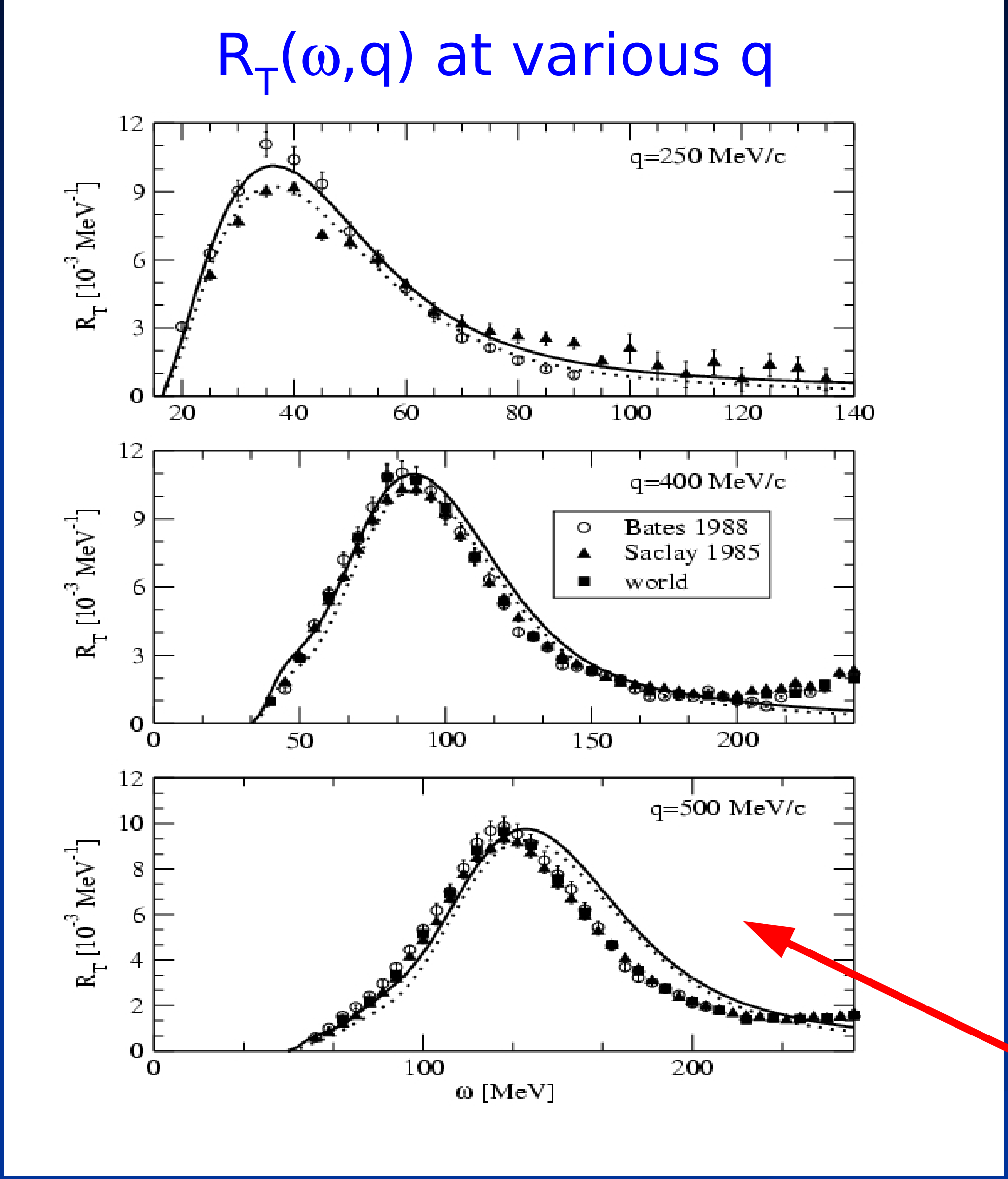
Potential: BonnRA +TM'

one-body current: dashed
one+two-body current: full

(S. Della Monaca et al., PRC 77, 044007 (2008))

Bad agreement between theory and experiment because of non considered relativistic effects

Motivation



Potential: BonnRA +TM'

one-body current: dashed
one+two-body current: full

Quasi-elastic kinematics (q=500 MeV/c),
Kinetic energy of outgoing nucleon:

non-rel. : $T = q^2/2m = 133$ MeV
rel.: $T = (m^2+q^2)^{1/2} - m = 125$ MeV

Bad agreement between theory and experiment because of non considered relativistic effects

We already considered this problem for R_L and studied R_L in various reference frames:

Laboratory: $P_T = 0$

Breit: $P_T = -q/2$

Anti-Lab: $P_T = -q$

Active Nucleon Breit: $P_T = -Aq/2$

non-rel.: $\omega_{frame} + (P_T)^2/2Am = E_{internal} + (P_T+q)^2/2Am$

$R_L(\omega,q)$ at higher q

Frame dependence

calculation in various frames:

Laboratory: $P_T = 0$

Breit: $P_T = -q/2$

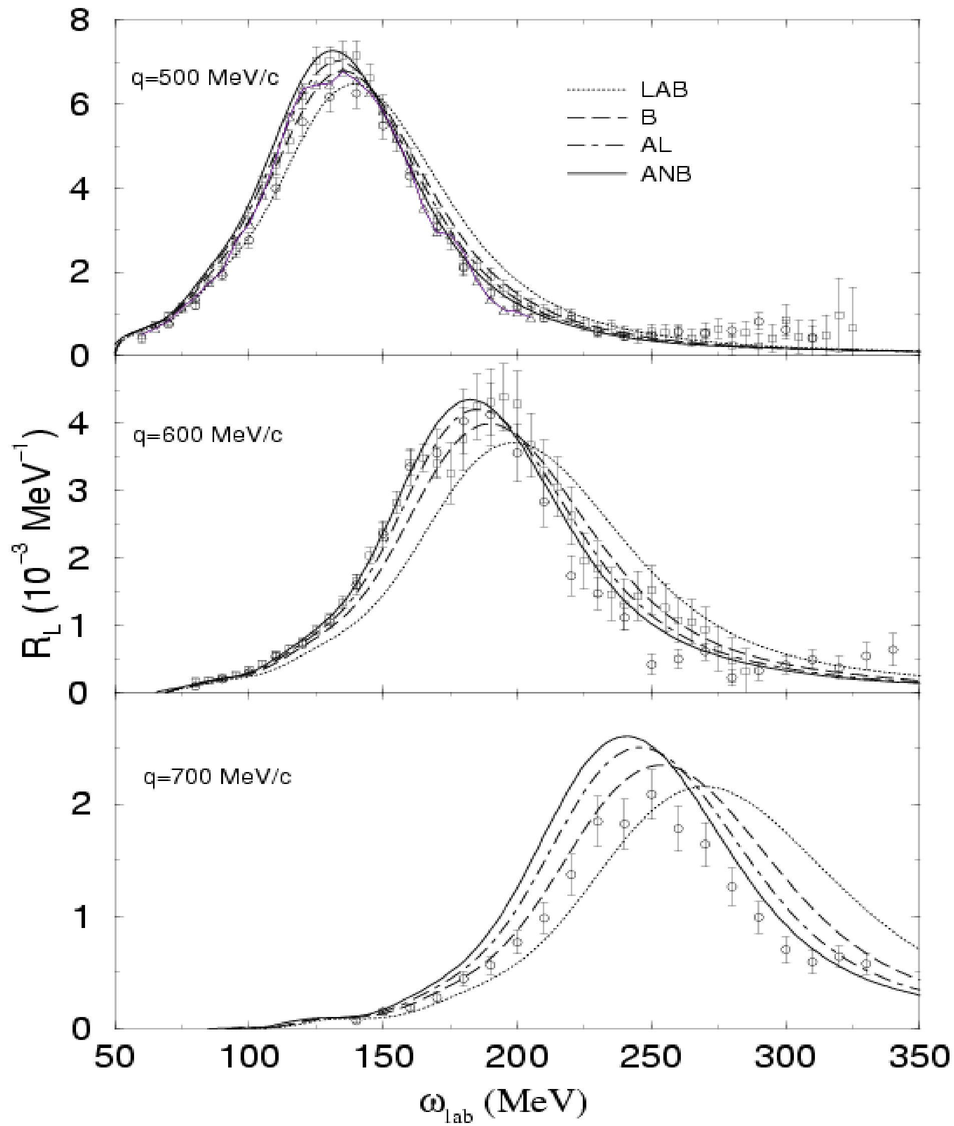
Anti-Lab: $P_T = -q$

Active Nucleon Breit: $P_T = -Aq/2$

Potential: AV18+UIX

Result in LAB frame

$$R_L(\omega,q) = \frac{q^2}{(q_{fr})^2}\frac{E_T^{fr}}{M_T} R_L^{fr}(\omega^{fr},q^{fr})$$



Exp: Marchand 1985, Dow 1988, Carlson 2002

V. Efros, W.L., G. Orlandini, E. Tomusiak
PRC 72 (2005) 011002(R)

How to get more frame independent results?

Assume quasi-elastic kinematics:

whole energy and momentum transfer taken by the knocked out nucleon (residual two-body system is in its lowest energy state)

$\Longrightarrow$ Effective two-body problem
Treat kinematics relativistically correct

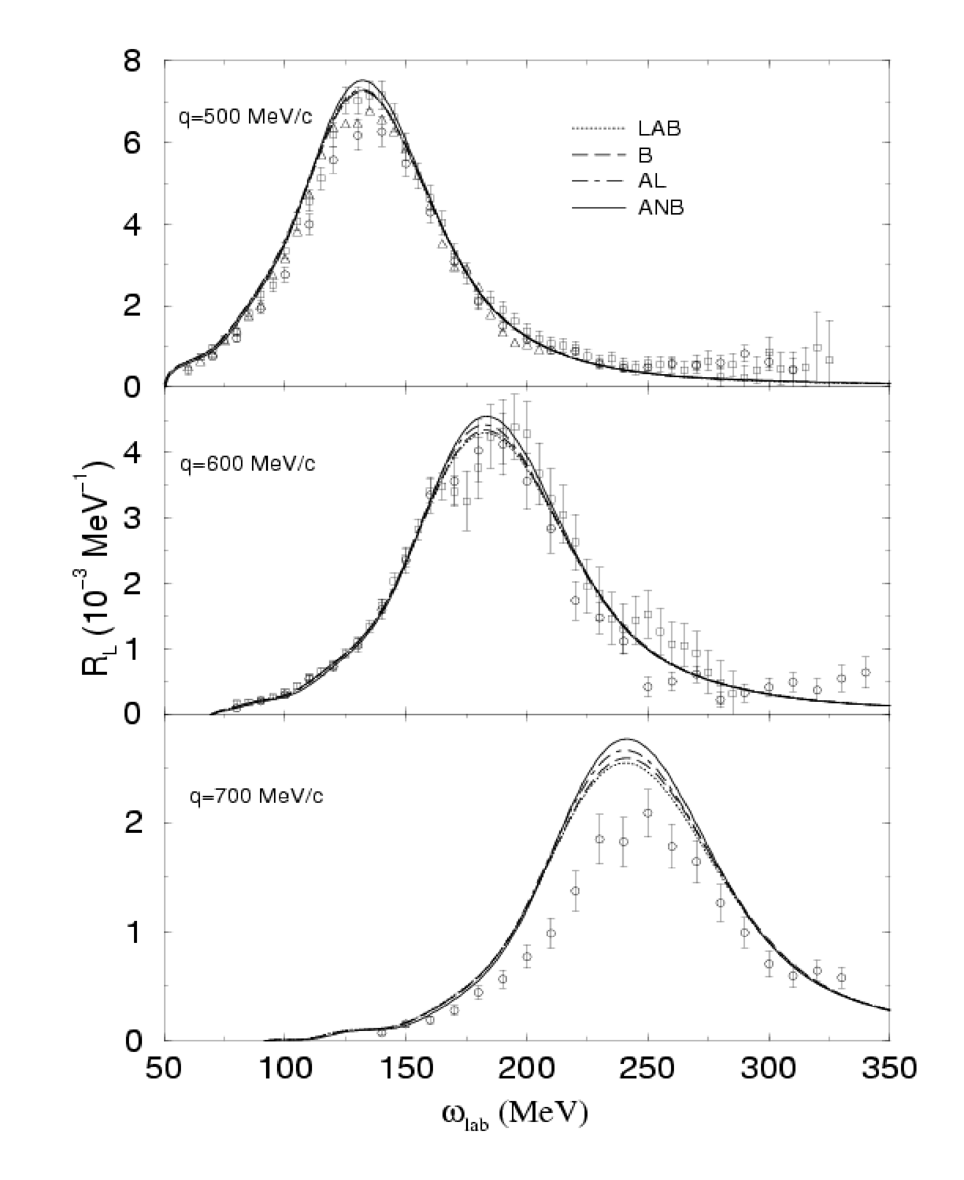
Take the correct relativistic relative momentum k_{rel} and calculate the corresponding non-relativistic relative energy

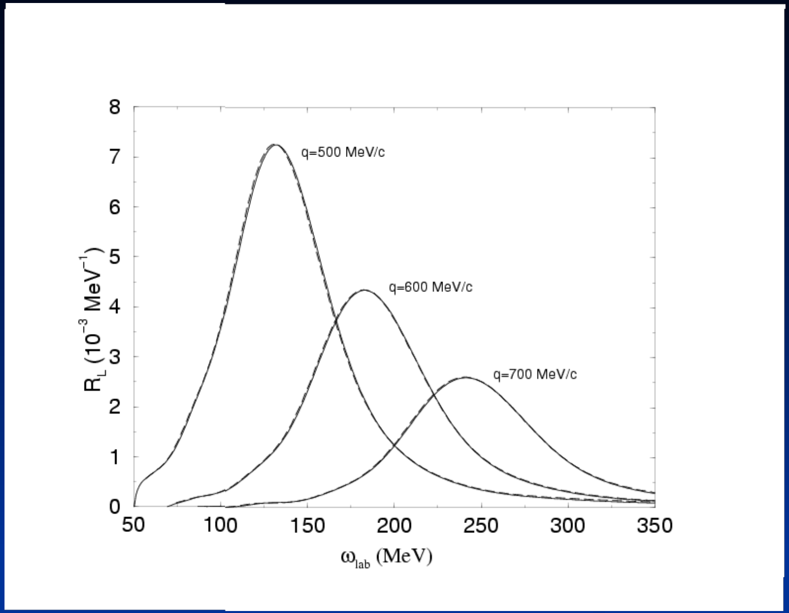
$$E_{nr} = (k_{rel})^2/2\mu$$

with reduced mass μ of nucleon and residual system

use E_{nr} as internal excitation energy in your calculation

$R_L(\omega,q)$ at higher q





R_L calculated in ANB frame with (dashed) and without (full) assumption of a two-body break-up

Quasielastic region: assume two-body break-up and use the correct relativistic relative momentum

Transverse response function $R_T(q,\omega)$ of ^{3}He in the quasi-elastic region

Nuclear current operator includes besides the usual non-relativistic one-body currents also meson exchange currents and Δ-isobar currents as well as relativistic corrections for the one-body current

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Calculation in active nucleon Breit (ANB) frame ($P_T = -Aq/2$) and subsequent transformation to laboratory system

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Calculation of bound state wave function and solution of LIT equation with the help of expansions in correlated hyperspherical harmonics

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Calculation of bound state wave function and solution of LIT equation with the help of expansions in correlated hyperspherical harmonics

Nuclear force model: Argonne v18 NN potential and Urbana 3NF

Further calculation details

The current operator $\mathbf{J}$

$$\mathbf{J} = \mathbf{J}^{(1)} + \mathbf{J}^{(2)}$$

$$\mathbf{J}^{(1)} = \mathbf{J}^{(1)}(q,\omega,P_T) = \mathbf{J}_{spin} + \mathbf{J}_{p} + \mathbf{J}_{q} + (\omega/M)\,\mathbf{J}_{\omega}$$

for instance spin current

$$\mathbf{J}_{spin} = \exp(i\mathbf{q}\cdot\mathbf{r})\; i\;\boldsymbol{\sigma}\times\mathbf{q}/2M\,[G_M(1\text{-}q^2/8M^2) - G_E\,\kappa^2 q^2/8M^2]$$

with $\kappa=1+2P_T/Aq$

Further calculation details

The current operator **J**

$\mathbf{J} = \mathbf{J}^{(1)} + \mathbf{J}^{(2)}$

$\mathbf{J}^{(1)} = \mathbf{J}^{(1)}(q,\omega,P_T) = \mathbf{J}_{spin} + \mathbf{J}_p + \mathbf{J}_q + (\omega/M)\,\mathbf{J}_\omega$

for instance spin current

$\mathbf{J}_{spin} = \exp(i\mathbf{q}\cdot\mathbf{r})\; i\,\boldsymbol{\sigma}\times\mathbf{q}/2M\;[G_M(1-q^2/8M^2) - G_E\,\kappa^2 q^2/8M^2]$

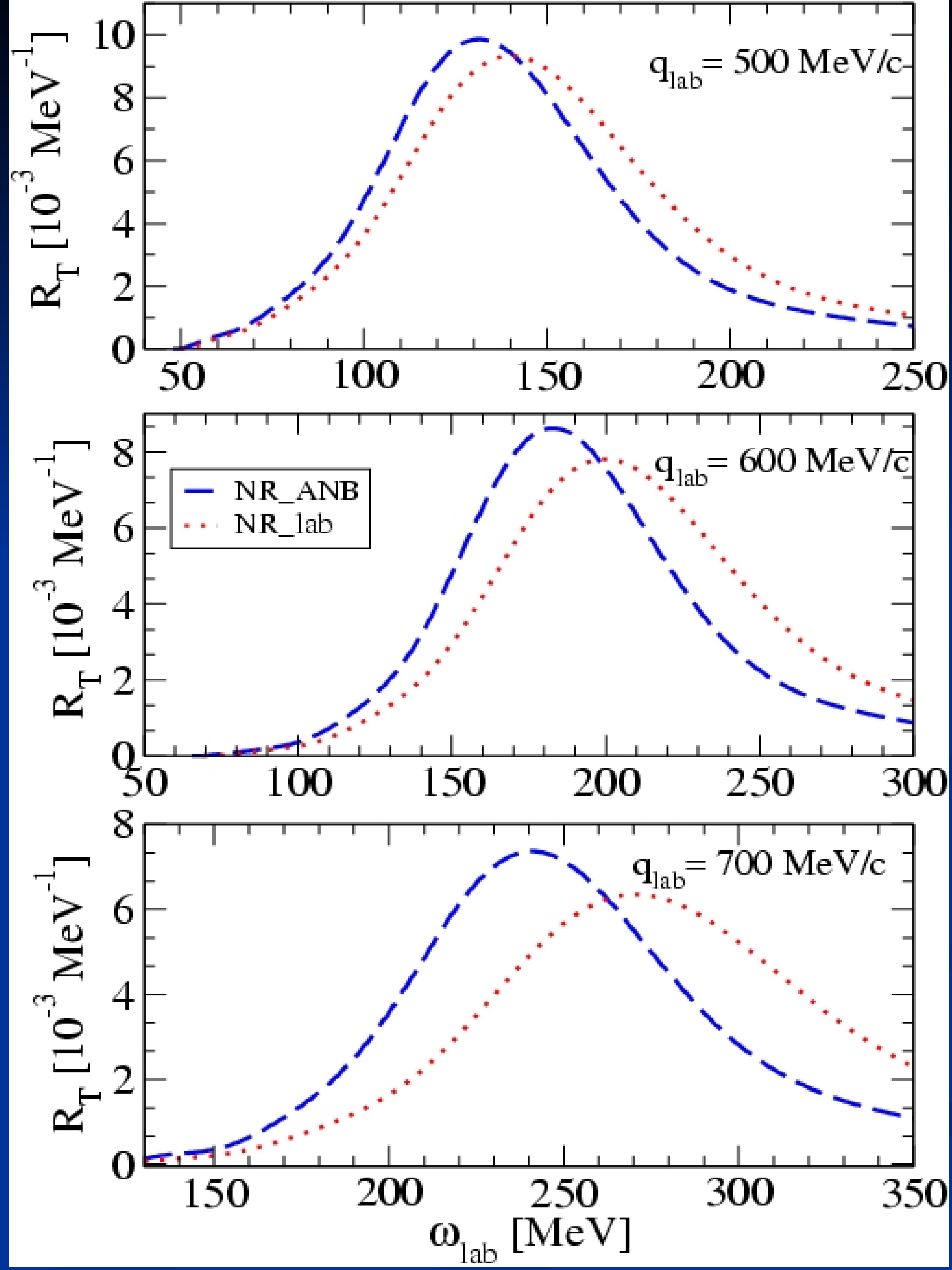
with $\kappa = 1 + 2P_T/Aq$

Transformation from ANB frame to LAB frame

$R_T^{LAB}(\omega^{LAB}, q^{LAB}) = R_T^{ANB}(\omega^{ANB}, q^{ANB})\;\; E_T^{ANB}/M_T$

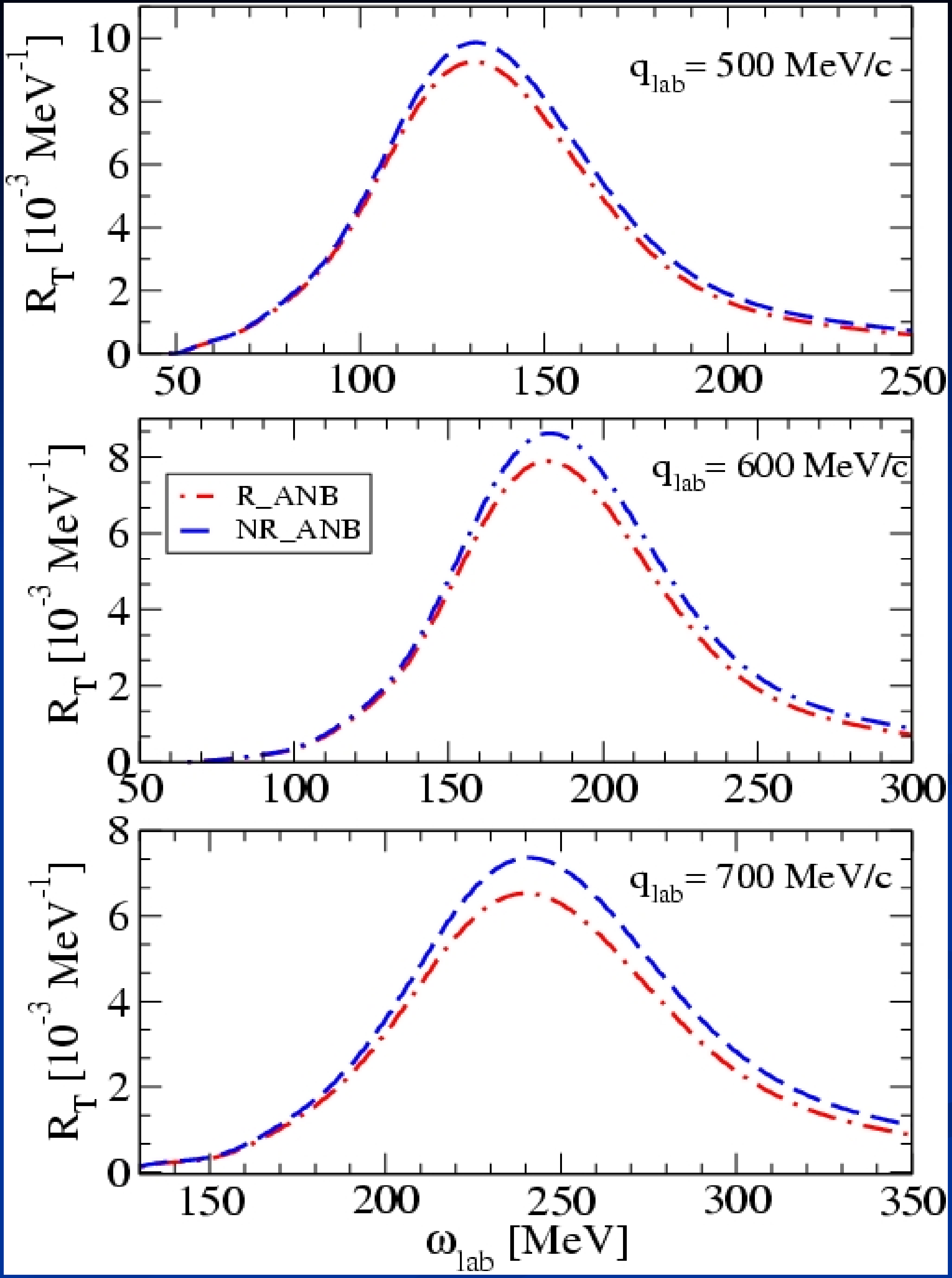
Results

- Comparison of ANB and LAB calculation: strong shift of peak to lower energies! (8.7, 16.7, 29.3 MeV at q=500, 600, 700 MeV/c)



Results

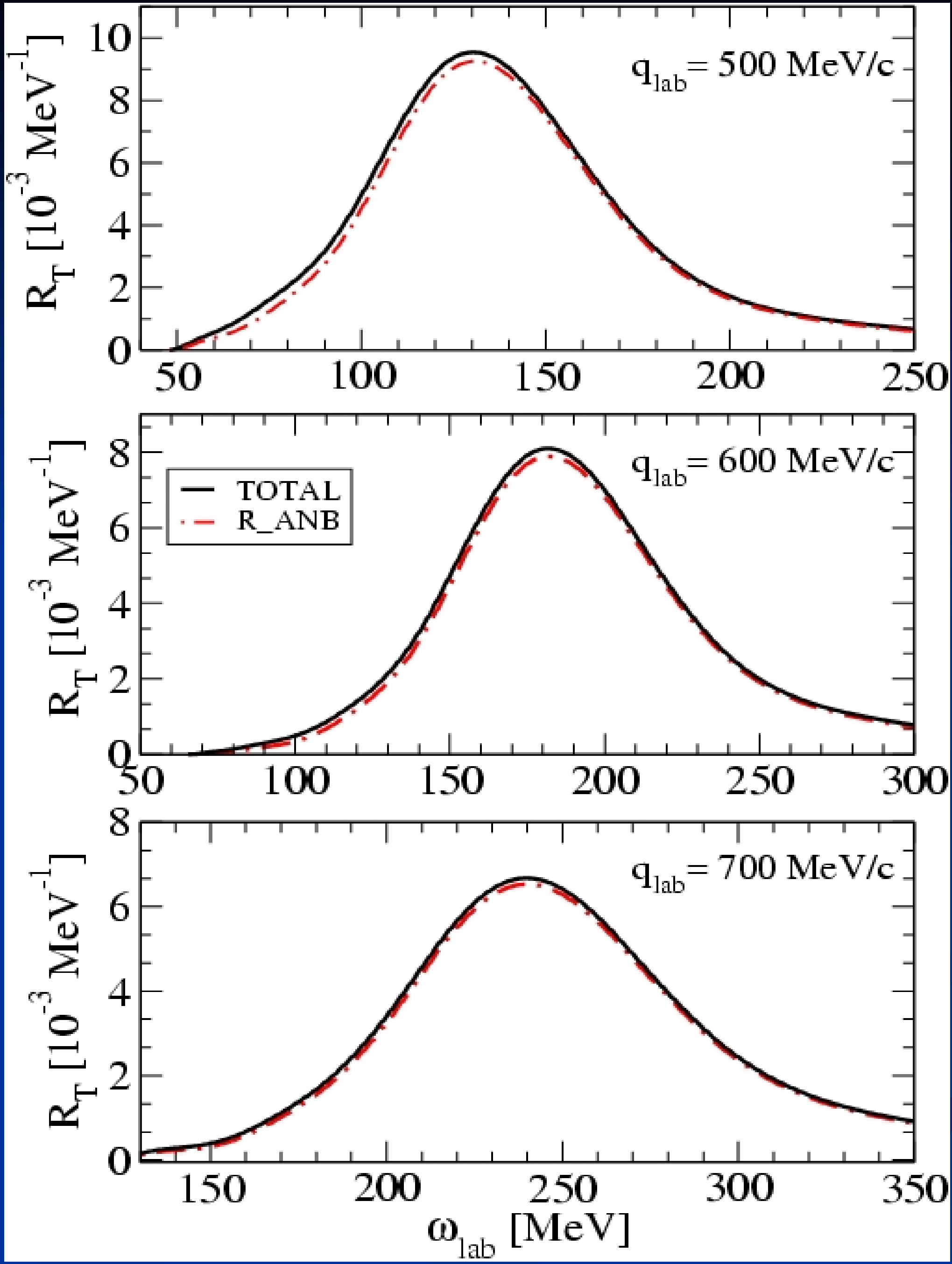
- Rel. contribution: reduction of peak height (6.2%, 8.5%, 11.3 % at q=500, 600, 700 MeV/c)

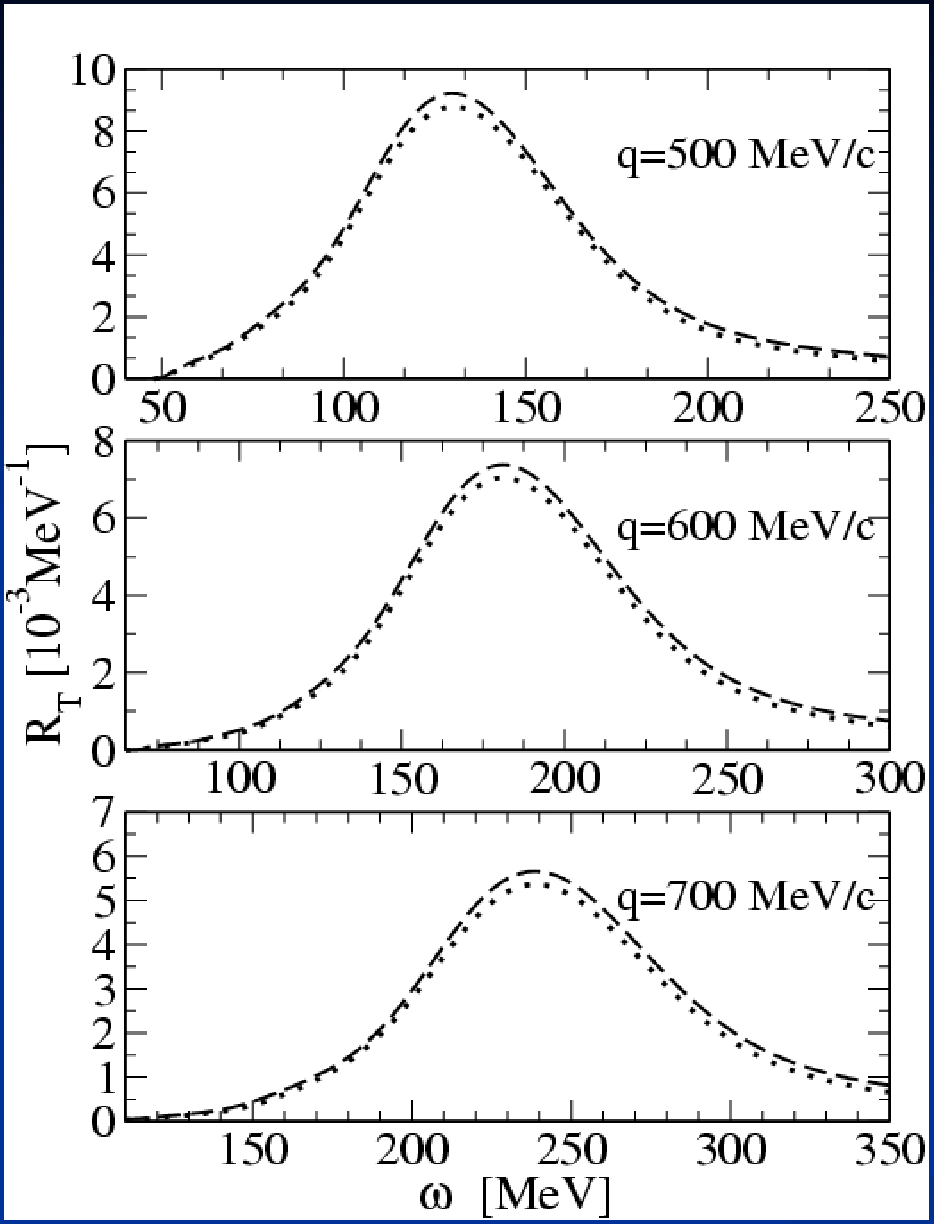


Results

- MEC:

small increase of peak height (3.2%, 2.7%, 2.2% at q=500, 600, 700 MeV/c)



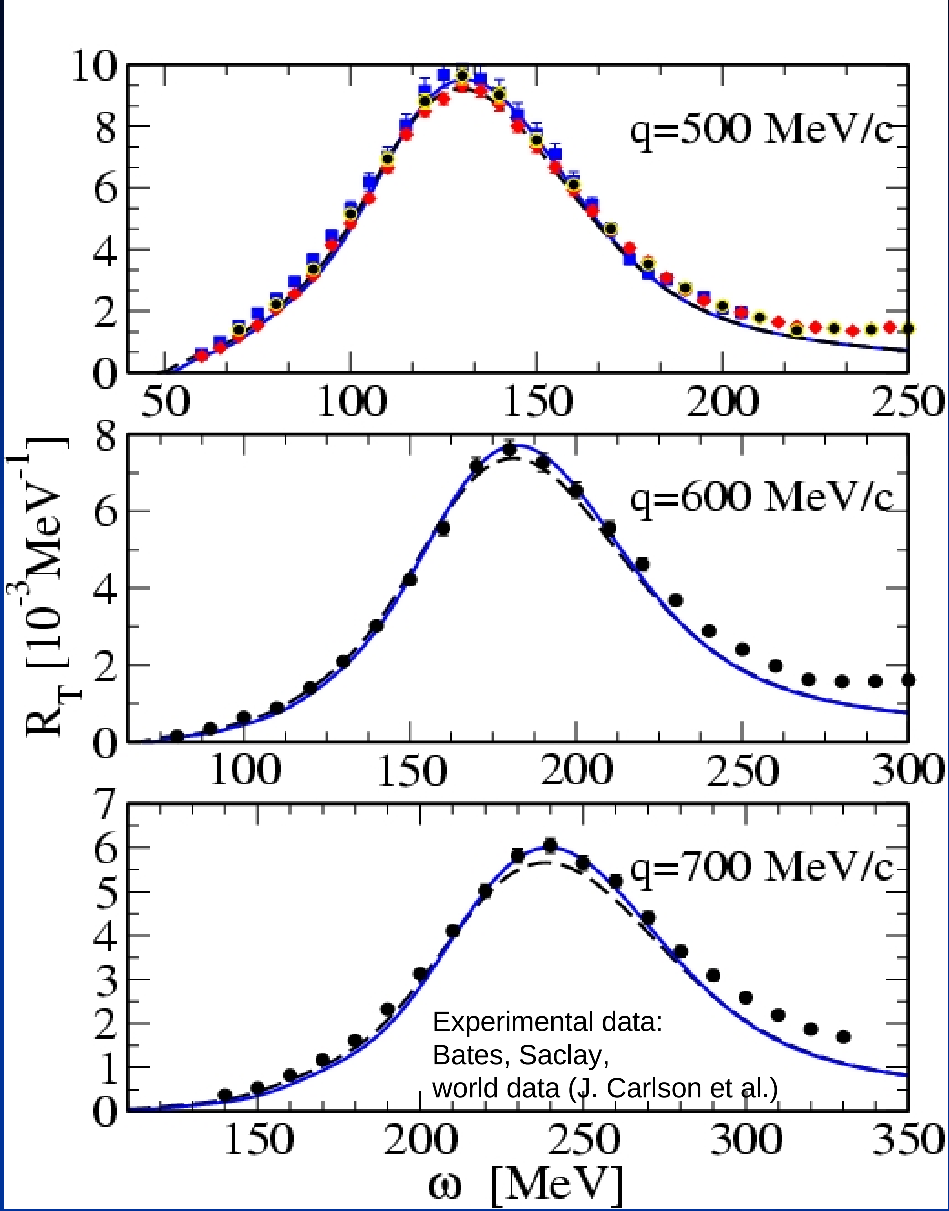


Δ-IC contribution

Dotted: without Δ

Dashed with Δ

Effect of two-fragment model



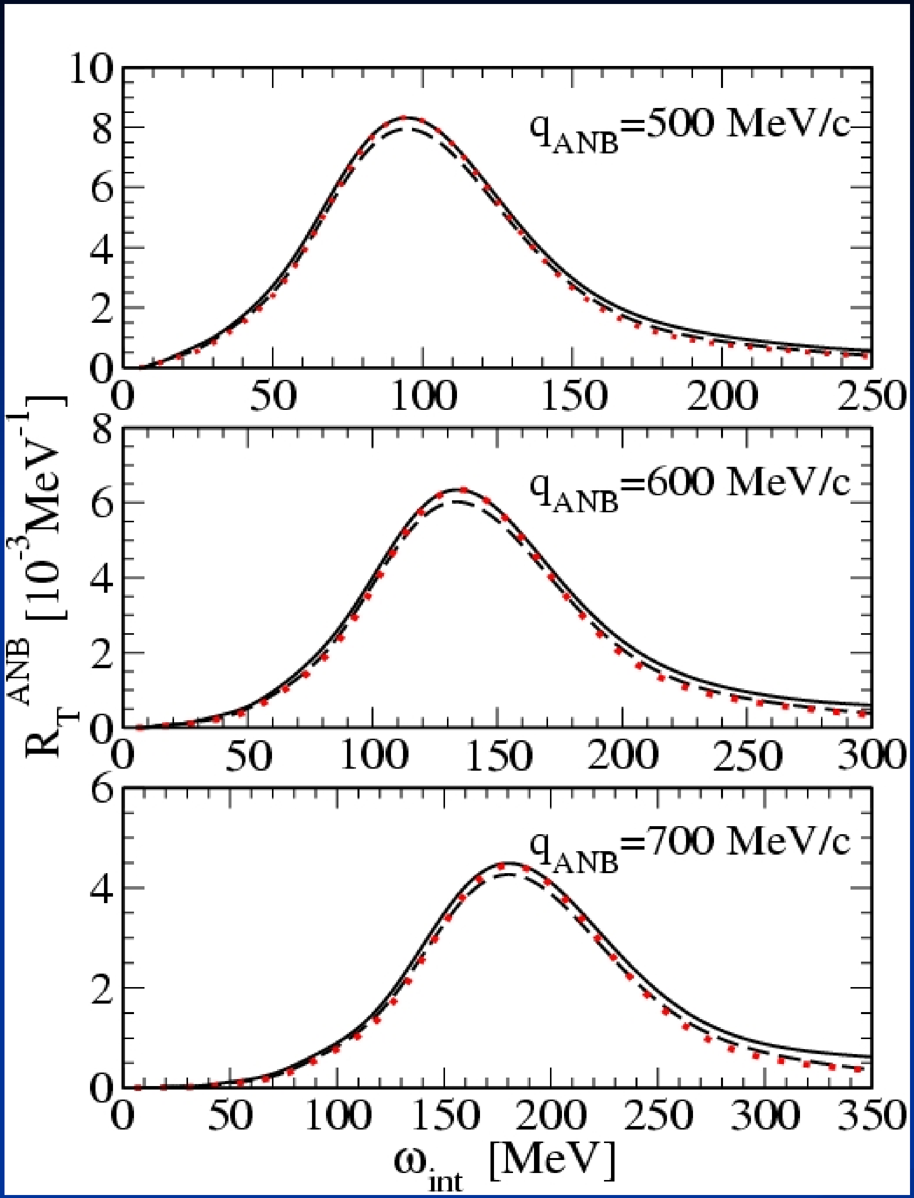
Dashed: with Δ (as before)

Solid: same but with two-fragment model

Deltuva et al. (PRC70, 034004,2004):

Calculation of R_T of ^{3}He with CDBonn and CDBonn+Δ:

no Δ effects in peak region!



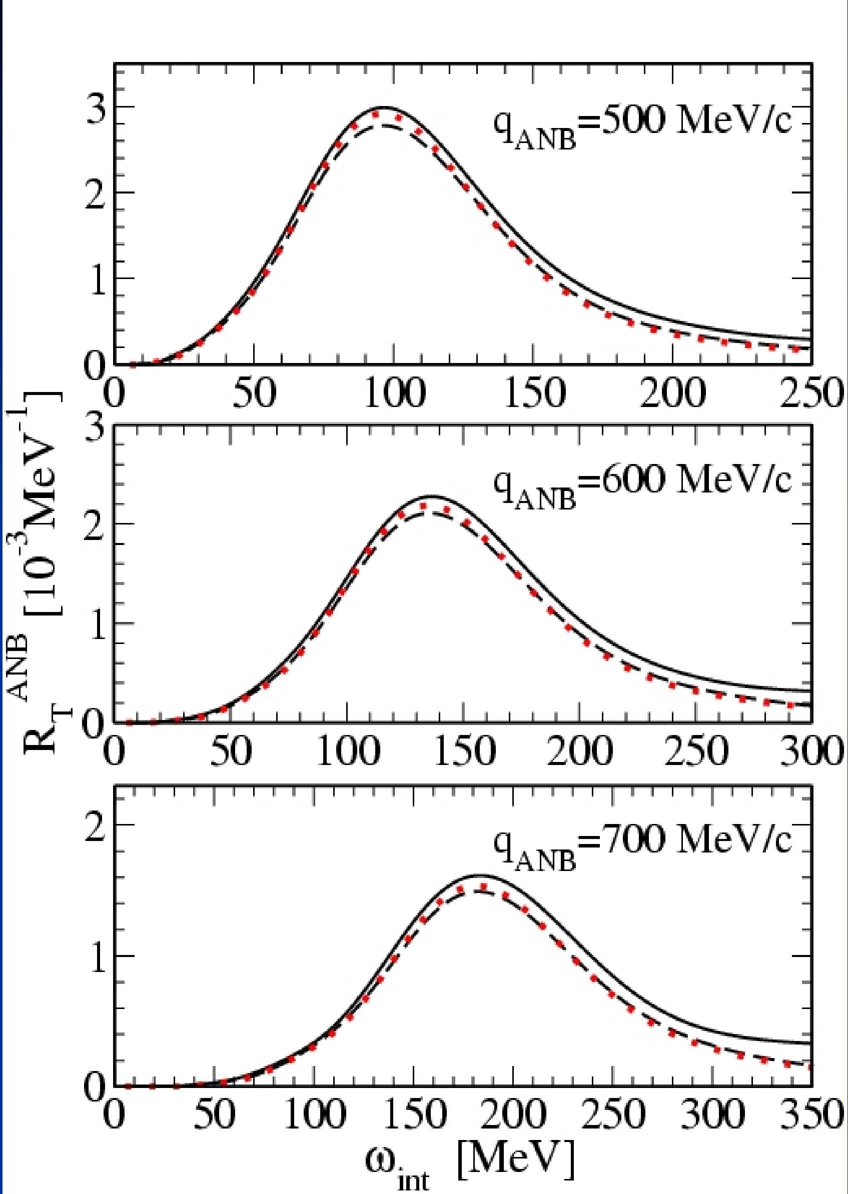
Partial compensation of Δ-IC and 3NF

Dotted: no Δ and no 3NF

Dashed: no Δ but with 3NF

Solid: with Δ and with 3NF

No Δ effect in peak region
In a CC calculation!



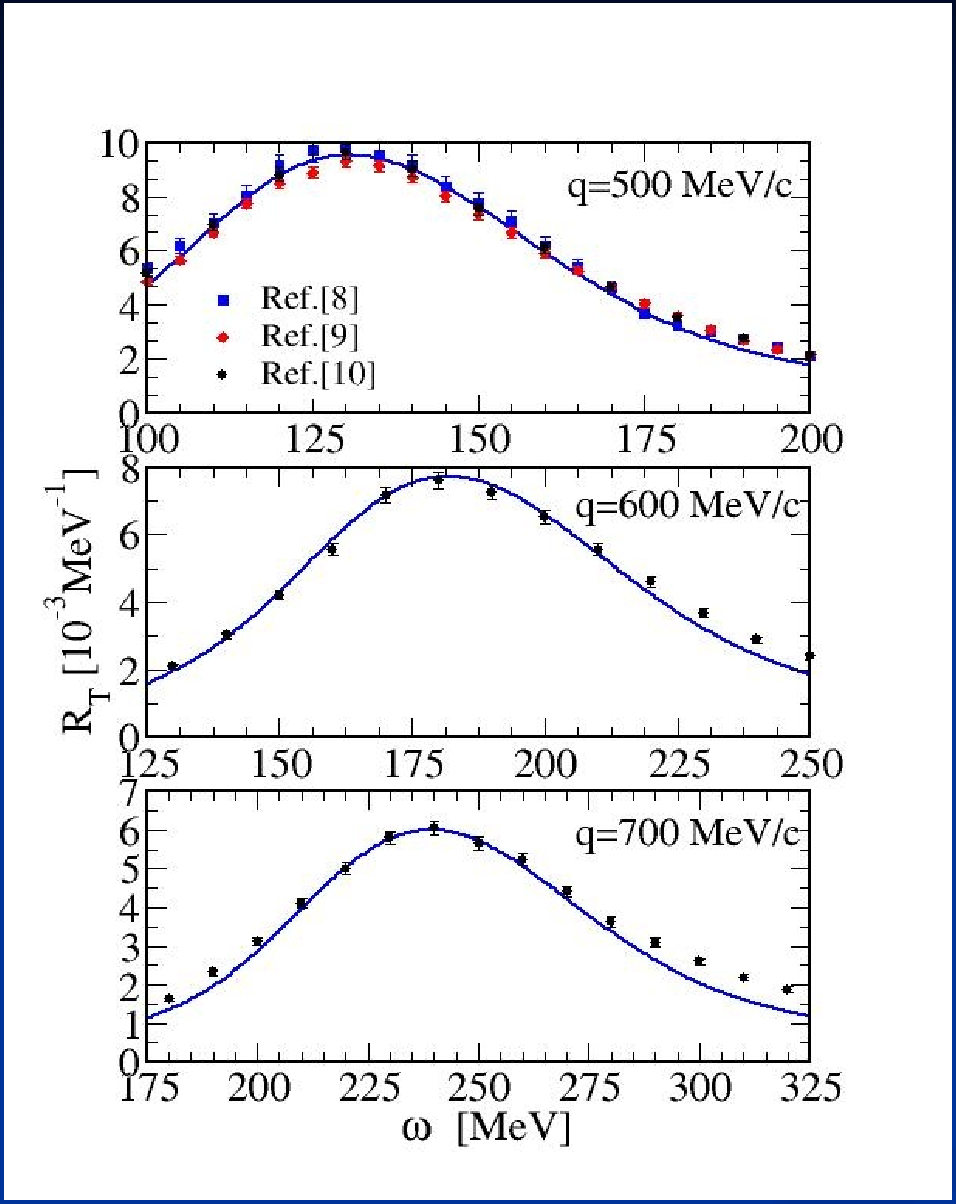
Only Isospin channel T=3/2

Dotted: no Δ and no 3NF

Dashed: no Δ but with 3NF

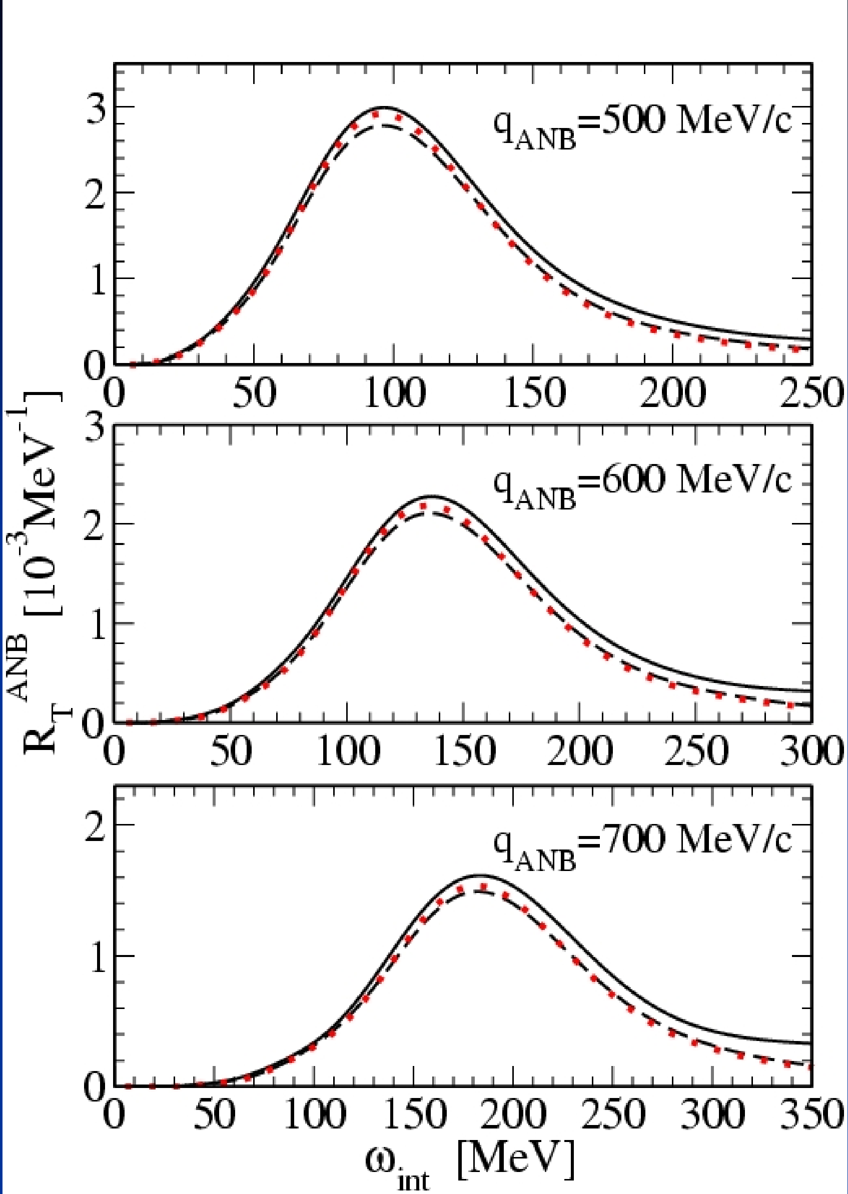
Solid: with Δ and with 3NF

Δ-IC contribution larger than 3NF effect in peak region!



L. Yuan et al., PLB 706, 90 (2011)

Experimental data:
Bates, Saclay,
world data (J. Carlson et al.)



Only Isospin channel T=3/2

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Strong Δ-IC effect also beyond peak

⇒ for this kinematics Δ-IC are important in 3-body breakup reactions

Conclusions

- the LIT metod opens up the possibility to carry out ab-initio calculations of reactions into the A-body continuum for A > 2
- only **bound states** techniques are needed
- the LIT is a method with controlled resolution

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- the LIT is a method with controlled resolution

We have discussed quite a few applications, there are still more (Compton scattering, pion production, weak nuclear responses)

Summary

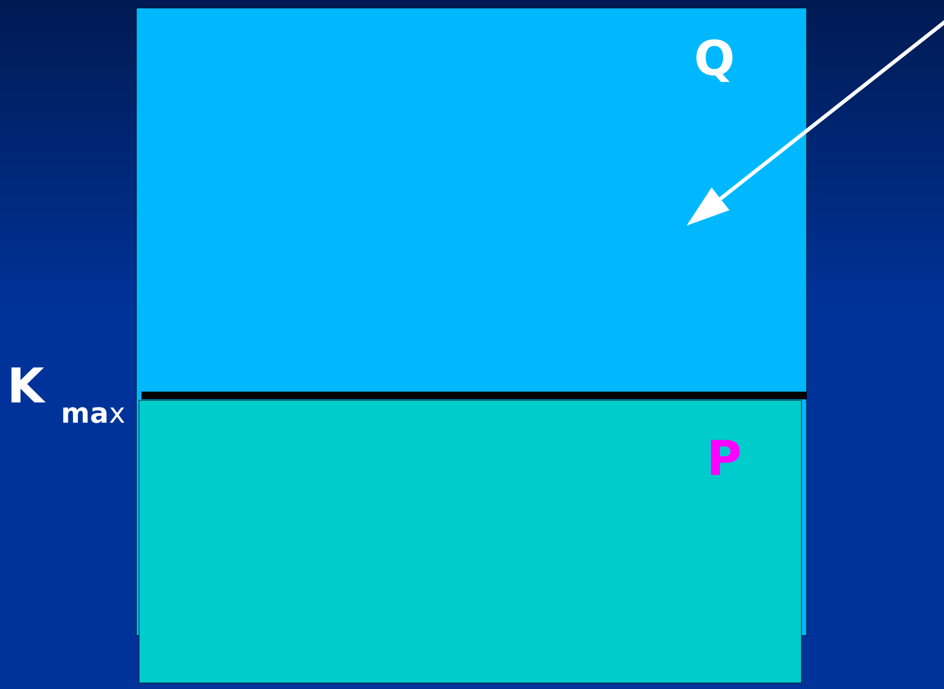
HOW TO SPEED UP THE CONVERGENCE?

SOLUTION

Here comes the idea of **EFFECTIVE INTERACTION**

same idea as for No Core Shell Model.
there the many particle basis is HO
here the many particle basis is HH

What is the main idea of an **effective interaction?**



whole Hilbert space

P and Q are projection operators

$$P + Q = 1$$

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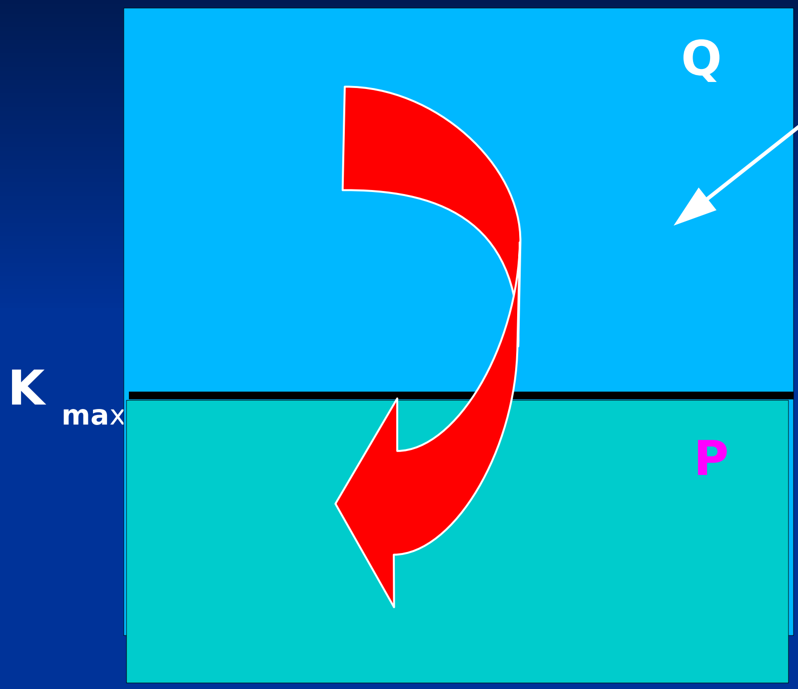

whole Hilbert space

P and Q are projection operators

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Find a transformation **V** $\xrightarrow{T}$ **V**$_{eff}$ such that

$$\langle \Psi | P H_{eff} P | \Psi \rangle = \langle \Psi | H | \Psi \rangle$$

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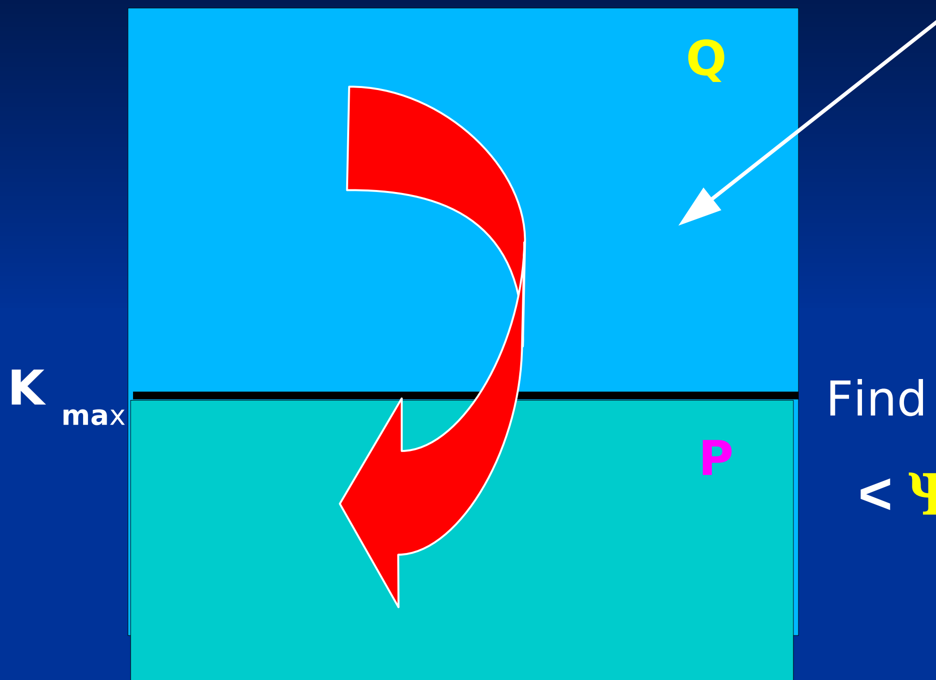

whole Hilbert space

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formally this transformation exists (Bloch-Horowitz, **Lee-Suzuki**), however,

1) V_{eff} becomes an **A-body** operator
2) T is written in function of **Q**

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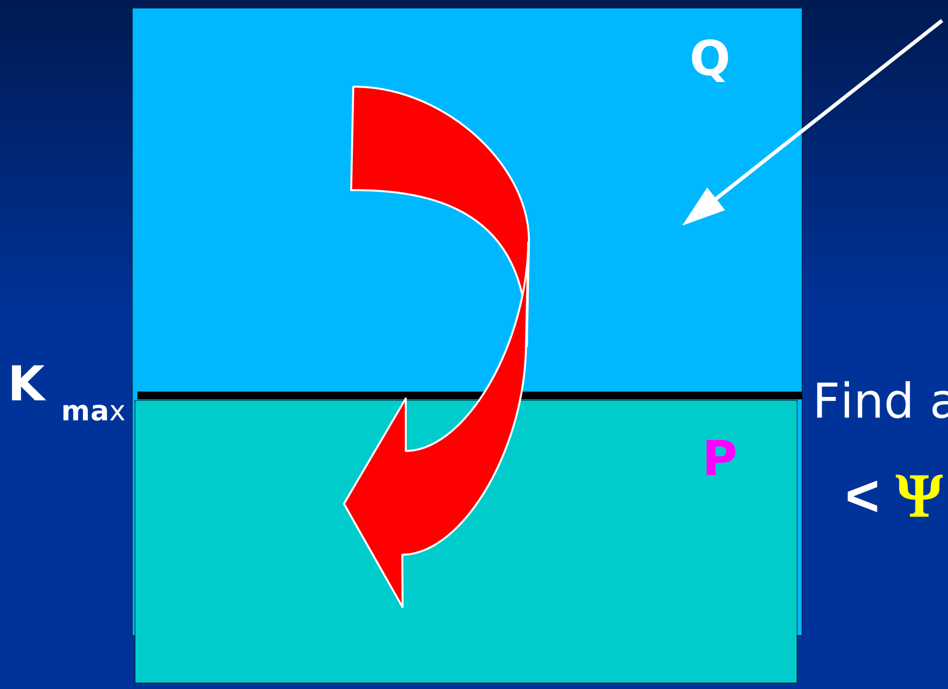

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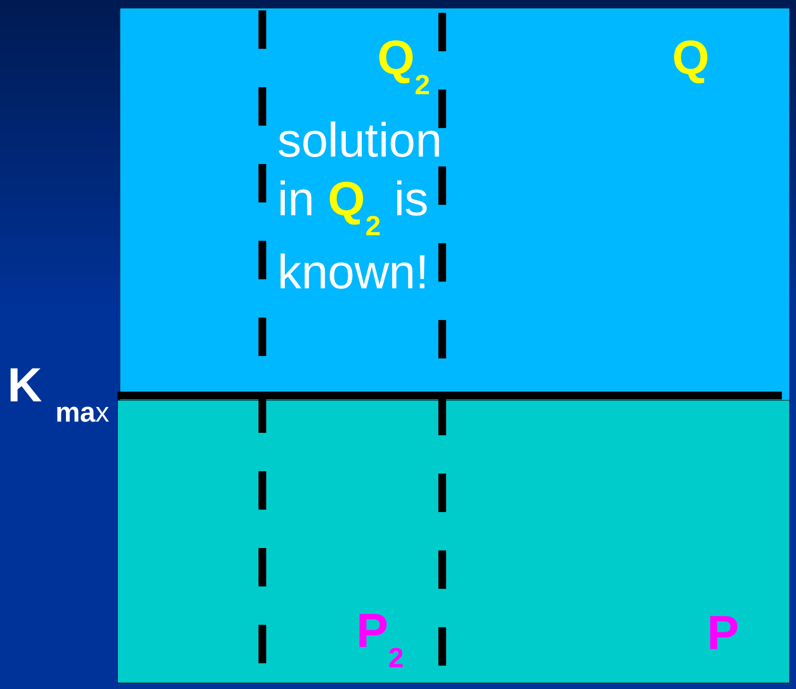
formally this transformation exists (Bloch-Horowitz, **Lee-Suzuki**), however,

1) V_{eff} becomes an **A-body** operator $V_{eff}^{[A]}$

2) T is written in function of **Q**

Useless for practical purposes, the same as solving the full problem

PRACTICALLY:



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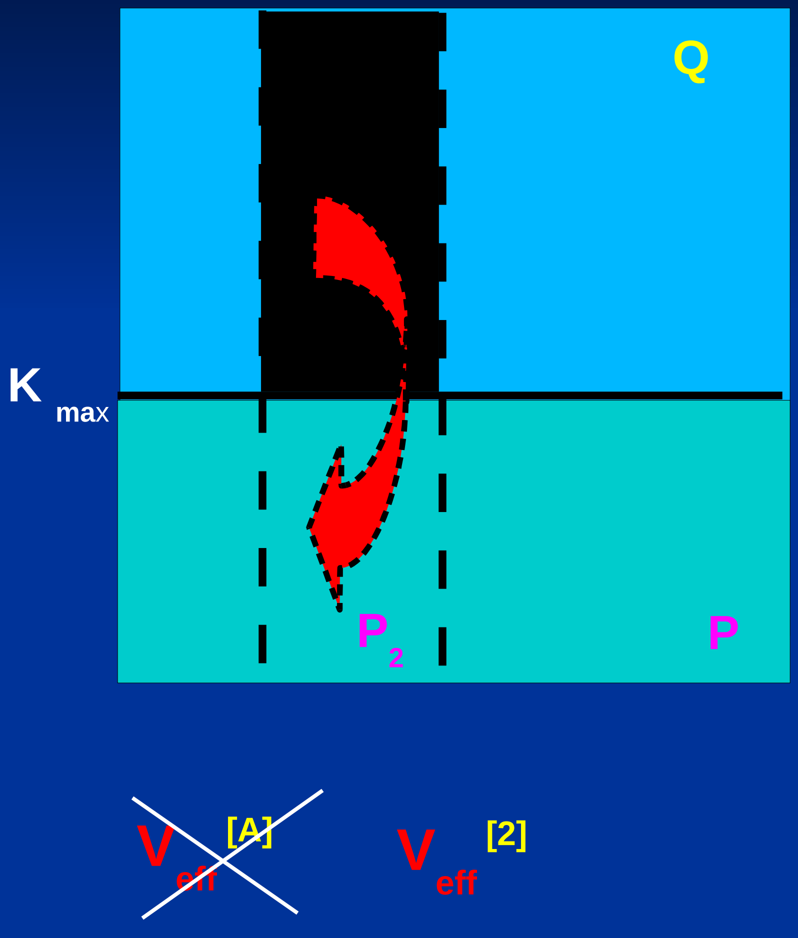

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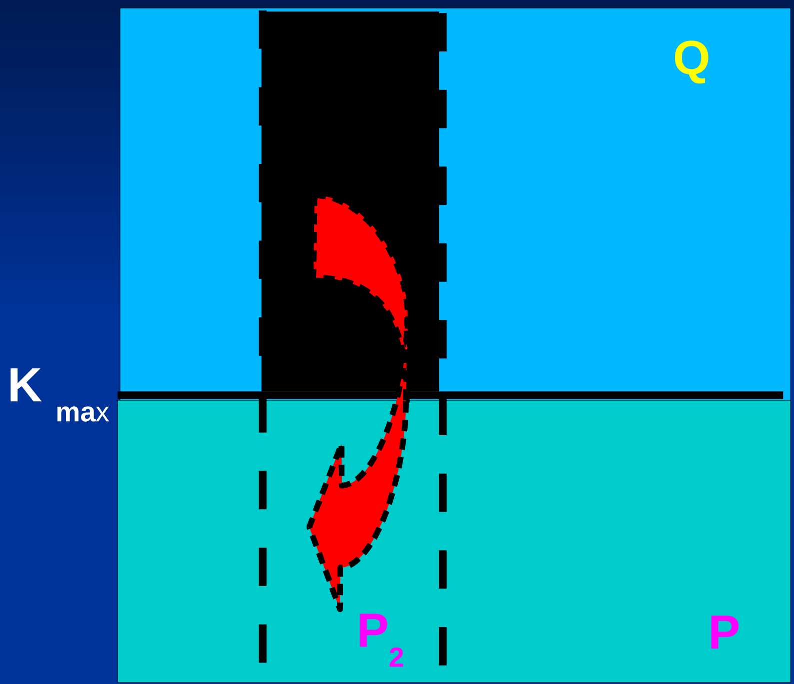


PRICE: I have to increase **P** (i.e. $\mathbf{K}_{\mathbf{max}}$) up to convergence

GAIN: what is missing is **less** than before -------> **faster convergence!**

~~$V_{eff}^{[A]}$~~ $V_{eff}^{[2]}$

Where, in the full H, is the two-body H_2 which I have to solve ?

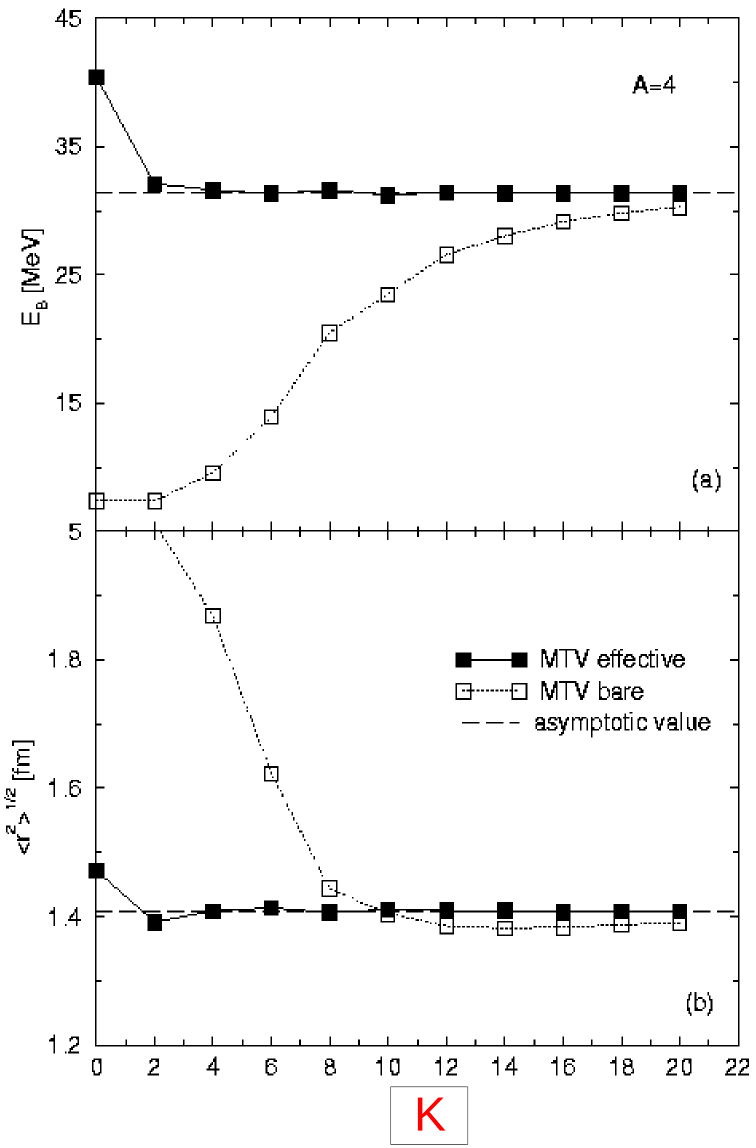
$$H_{NCSM} = \Sigma_k^{A-1} h_k^{ho} + (V_{12} - V_{12}^{HO}) + (V_{13} - V_{13}^{HO}) + \ldots..$$

$$= h^{ho}(\xi_1) + h^{ho}(\xi_2) + \ldots + V(\vec{\xi}_1) - V^{HO}(\xi_1^2) + \ldots$$

$$H_{EIHH} = T + V_{12} + V_{13} + \ldots..$$

$$= 1/\mu\,(\Delta_\rho - K^2/\rho^2) + V(\xi_1) + V(\xi_1, \xi_2, \ldots. \xi_{A-1})$$

convergence:



^{4}He with MTV NN Potential