Representation Theory in Venice

ABSTRACTS

Monday 16 September, 9:30 – 10:20.
George Lusztig: Lie theory over semifields.
In this talk we show how several basic objects in Lie theory usually defined over a field can be also defined over a semifield such as the set of strictly positive real numbers.

Monday 16 September, 11:00 – 11:50.
Mario Salvetti: The $k(\pi, 1)$ conjecture for affine Artin groups.
In a recent paper with Giovanni Paolini (`arXiv: 1907.11795`) we proved the $K(\pi, 1)$ conjecture for the family of affine Artin groups. We give a sketch of the main ideas and tools used in the proof.

Monday 16 September, 12:00 – 12:50.
Valerio Toledano Laredo: From wonderful models of hyperplane complements to Coxeter categories.
In recent work with Andrea Appel, we introduced the notion of a Coxeter category associated to a Coxeter group W. Such a category gives rise to representations of the corresponding braid group B_W, and is to the latter what a braided tensor category is to Artin's braid groups $\{B_n\}$.

Just as braided tensor categories are closely related to $\overline{\mathcal{M}}_{0,n}$, the definition of a Coxeter category takes its cue from, and is very much inspired by, the geometry of the wonderful model of the reflection arrangement of W constructed by De Concini-Procesi

Time permitting, I will outline how to construct such a structure on integrable, category O representations of a symmetrisable Kac-Moody algebra $\mathfrak{g}$, in a way that incorporates the monodromy of the KZ and Casimir connections of $\mathfrak{g}$. The rigidity of this structure implies in particular that the monodromy of the latter connection is described by Lusztig's quantum Weyl group operators for the quantum group $U_h(\mathfrak{g})$.

Monday 16 September, 14:30 – 15:20.
Hanspeter Kraft: Perpetuants – A lost treasure.
This talk is about the following beautiful result conjectured by MacMahon in 1884 and proved by Emil Stroh in 1890.

Theorem. *The dimension of the space of perpetuants of degree $k > 2$ and weight g is the coefficient of x^g in*

$$\frac{x^{2^{k-1}-1}}{(1-x^2)(1-x^3)\cdots(1-x^k)}.$$

For $k = 1$ there is just one perpetuant, of weight 0, and for $k = 2$ the number is given by the coefficient of x^g in $x^2/(1-x^2)$.

We will explain the notion since it has some mathematical interest, and also Stroh's proof which is quite remarkable and in a way very modern. With our method we shall in fact exhibit a *basis of perpetuants*, the main new result.

Let us shortly explain what a perpetuant is. In the classical invariant theory of binary forms $R_n := \Bbbk[x, y]_n$, the U-invariants (sometimes called semi-invariants) $S(n) = \mathcal{O}(R_n)^U$ play a central role. So far, the indecomposable U-invariants, i.e. the generators of the algebra of U-invariants $S(n)$ are only known for small n, and there seems to be no hope to find the Hilbert-series nor the dimensions of the spaces of indecomposables U-invariants in general.

However, there is a natural embedding $S(n) \subset S(n+1)$, i.e. every U-invariant of binary forms of degree n is also a U-invariant of binary forms of degree $n+1$. (This was efficiently used by the classics, in particular in their computations.) It is not difficult to see that an indecomposable U-invariant of R_n might become decomposable as a U-invariant of R_m for some $m > n$. This leads to the definition of a perpetuant.

Definition. A *perpetuant* is an indecomposable element of $S(n)$ which remains indecomposable in all $S(m)$, $m \geq n$.

Setting $I_n \subset S(n)$ to be the homogeneous maximal ideal, then a perpetuant gives an element of I_n/I_n^2 which *lives forever*, that is it remains nonzero in all I_m/I_m^2 for $m \geq n$. In this sense it is *perpetuant.*

Although there is no hope to describe I_n/I_n^2 *in general, we have the beautiful formula above for the perpetuants* $P_n \subseteq I_n/I_n^2$.

(joint work with Claudio Procesi)

Monday 16 September, 15:30 – 16:20.

Michel Brion: Lines on Schubert varieties and minimal rational curves on their desingularizations.

Lines in flag varieties have been extensively investigated. In particular, for a semi-simple group G and a maximal parabolic subgroup P, it is known that the lines in G/P passing through the base point form a smooth projective variety on which P acts with one or two orbits.

By contrast, very little seems to be known about lines in Schubert varieties. The talk will discuss these and the related notion of minimal rational curves on natural desingularizations of Schubert varieties, based on joint work in progress with S. Senthamarai Kannan.

Monday 16 September, 17:00 – 17:50.

Enrico Arbarello: Quiver representations and singularities of irreducible symplectic varieties.

We will give an overview on some older and some more recent results on the connections between Nakajima quiver varieties and the local structure of a class of irreducible symplectic varieties (among which Bridgeland moduli spaces), describing resolution of singularities in terms of wall crossing.

Tuesday 17 September, 9:30 – 10:20.

Martina Lanini: Moment graphs and localisation of Wakimoto flags.

Wakimoto modules are representations of affine Kac-Moody algebras. They were introduced in the 80s by Wakimoto (in the $\mathfrak{sl}_2$ case) and by Feigin and Frenkel in the general case. The geometric realisation of Feigin and Frenkel allows one to think of Wakimoto modules as a "semi-infinite" analogue of Verma modules. Motivated by this, we propose to adapt Fiebig's moment graph approach to the study of modules in a non-critical block which admit a Verma flag to the semi-infinite setting. This is joint work in progress with Tomoyuki Arakawa.

Tuesday 17 September, 11:00 – 11:50.

Pavel Etingof: Symmetric tensor categories in positive characteristic.

I will talk about my joint work with Dave Benson which constructs new symmetric tensor categories in characteristic 2 arising from modular representation theory of elementary abelian 2-groups, and about its conjectural generalization to characteristic $p > 2$. I will also discuss my work with Gelaki and Coulembier which shows that integral symmetric tensor categories in characteristic $p > 2$ whose simple objects form a fusion category are super-Tannakian (i.e., representation categories of a supergroup scheme), and discuss what happens in characteristic 2.

Tuesday 17 September, 12:00 – 12:50.

Victor Kac: Cohomology of algebraic structures: from Lie algebras all the way to vertex algebras.

I will explain an operadic approach to cohomology of algebraic structures. This approach provides a unified point of view on cohomology theory of a number of algebraic structures playing an important role in integrable systems and quantum field theory, incuding Lie conformal algebras, Poisson vertex algebras, and vertex algebras. Some of concrete computations will be explained as well.

This theory is being developed in a series of papers with Bakalov, De Sole, Heluani, and Vignioli.

Tuesday 17 September, 15:30 – 16:20.

Andrea Maffei: Abelian varieties as automorphism groups of projective varieties.

I describe a result obtained together with Davide Lombardo in which we characterize the abelian varieties that are the group of automorphism of a smooth variety.

Tuesday 17 September, 17:00 – 17:50.

UMBERTO ZANNIER: Abelian varieties not isogenous to any Jacobian.

For dimensional reasons it follows from well-known results that for any integer $g > 3$ there exist complex abelian varieties of dimension g not isogenous to any Jacobian. KATZ and OORT raised the question whether the same holds if we restrict to objects defined over the field of algebraic numbers.

CHAI and OORT in 2012 gave an affirmative answer, relying however on the ANDRE'-OORT conjecture, unproven at that time; shortly afterwards TSIMERMAN gave an unconditional affirmative answer.

Both arguments produced examples of CM type, so the question remained open to provide existence of such abelian varieties with 'generic' properties.

In the talk I will briefly illustrate recent joint work with MASSER, where we use a different method to achieve this goal. For instance we prove that there exist abelian varieties defined over the algebraic numbers, with trivial endomorphism ring, and not isogenous to any Jacobian.

Wednesday 18 September, 9:30 – 10:20.

DAVID HERNANDEZ: Stable maps, category O and categorified exchange relations.

We define and construct stable maps on tensor products of representations in the Borel category O of an arbitrary quantum affine algebra. The construction is based on the study of the action of the Drinfeld-Cartan subalgebra. In A,D,E-cases, these stable maps generalize Maulik-Okounkov stable maps on finite-dimensional standard modules obtained from K-theoretic stable maps on quiver varieties. As an application, we obtain new R-matrices in the category O and categorified exchange relations of corresponding cluster algebra structures.

Wednesday 18 September, 11:00 – 11:50.

PETER LITTELMANN: Valuations and Standard Monomial Theory.

The theory of standard monomials is closely related to semitoric degenerations. The theory of Newton-Okounkov bodies provides new tools to approach this problem. This is a report on joint work in progress with R. CHIRIVÌ and X. FANG. Starting with an embedded projective variety admitting a nice family of subvarieties, we will present some results and many questions.

Wednesday 18 September, 12:00 – 12:50.

JERZY WEYMAN: Finite Free Resolutions and Root Systems.

In this talk I discuss the recent advances regarding generic rings for resolutions of length three. To a format of three ranks (r_1, r_2, r_3) of the maps in the resolution we associate a triple $(p, q, r) = (r_1+1, r_2-1, r_3+1)$. One can construct a specific generic ring $\hat{R}_{gen}$ for resolutions of that format which deforms to the ring with the multiplicity free action of Lie algebra $\underline{gl}(F_0) \times \underline{gl}(F_2) \times \underline{g}(T_{p,q,r})$. The ring $\hat{R}_{gen}$ is Noetherian only for triples (p, q, r) such that $T_{p,q,r}$ is a Dynkin graph.

Moreover, the pattern involving graph $T_{p,q,r}$ allows to construct the families of perfect ideals with resolutions of all Dynkin formats in a uniform way. They come from certain Schubert varieties in homogeneous spaces corresponding to $T_{p,q,r}$. These resolutions are expected to play important role in classifying the perfect ideals of codimension three.

Wednesday 18 September, 15:00 – 15:50.

MICHÈLE VERGNE: Splines, Representation theory and Geometry.

Let T be a torus with lattice of characters Λ. Given a complex vector space M with T action, the multiplicity of t^λ in the space $\mathrm{Sym}(M)$ is a locally quasi-polynomial function on Λ, called the vector partition function. I will recall some of our past work with DE CONCINI and PROCESI on the relation of the vector partition functions and the geometry of the moment map. Then I will discuss asymptotic analysis (work in common with P-E PARADAN and Y. LOIZIDES) on a space of locally quasi polynomial functions. This analysis allows us to prove a Riemann-Roch infinitesimal formula for the quantization of a K-Hamiltonian manifold M with proper moment map which is functorial with respect to branching rules.

Wednesday 18 September, 16:00 – 16:50.

CLAUDIO PROCESI: Cayley Hamilton algebras.

I will discuss old and recent results on algebras with trace or with norm and their representations.