Deeply Virtual Neutrino Scattering at Leading Twist
(Electroweak Nonforward Parton Distributions)

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Work in collaboration with Marco Guazzi (Lecce)
Hadronic Interactions mediated by weak currents

Generalized Bjorken region

NONFORWARD PARTON DISTRIBUTIONS (weak) (nonforward RG Evolution)

Distribution Amplitudes (hadronic Wave Function) ERBL

The link is COMPTON SCATTERING (WEAK)

DGLAP, “structure functions”
Leading twist amplitudes for exclusive neutrino interactions in the deeply virtual limit. 


Deeply virtual neutrino scattering (DVNS), with M. Guzzi and P. Amore

JHEP 0502:038,2005

Generalized Bjorken region: more than 1 scaling variable:

1) Bjorken x

2) asymmetry parameter between the initial and final momenta of the nucleon
Charged Current

Cross section for $\nu N \rightarrow \mu X$

- ANL
- BEBC
- BNL
- CCFR
- Total
- QE
- DIS
- SPP

$\sigma/E \left(10^{-38} \text{cm}^2/\text{GeV}\right)$

Neutrino energy (GeV)

- Quasi elastic
- Single pion production
- DIS

TOTAL
In neutrino factories the range of the interaction between the weak currents and the nucleons reaches the intermediate region of QCD “the Few GeV’s region”

This kinematical window, pretty large indeed (from $2-3 \text{ GEV}^2$ up to $20 \text{ GeV}^2$ or so) can be described by perturbative methods using **FACTORIZATION THEOREMS**.

Factorization means that

1) **the theory is light-cone dominated** and in a given process
2) We can “separate” the non perturbative part of the interaction, due to confinement, from the “valence” part which is described by a standard perturbative expansion.

We can predict the intermediate energy behaviour of weak form factors and describe elastic processes with high accuracy
Factorization at intermediate energy is associated with a class of Renormalization Group Equations

EFREMOV-RADYUSHKIN-BRODSKY-LEPAGE (ERBL)
RG Evolution of hadronic wave functions

Complementary to the usual DGLAP Evolution in DIS

Both evolutions can be unified in a new class of evolution equations

Nonforward RG Evolution

The nonforward evolution summarizes both limits (DGLAP/ERBL)
$$P_{1,2} = \bar{P} \pm \frac{\Delta}{2} \quad q_{1,2} = \bar{q} \pm \frac{\Delta}{2}$$

with $-\Delta = P_2 - P_1$ being the momentum transfer. Clearly

$$\bar{P} \cdot \Delta = 0, \quad t = \Delta^2 \quad \bar{P}^2 = M^2 - \frac{t}{4}$$

$$T_{\mu\nu}(q_1^2, \nu) = i \int d^4 z e^{iq \cdot z} \langle \bar{P} - \frac{\Delta}{2} | T(J_\nu^\mu(-z/2) J_{\gamma}^\nu(z/2)) | \bar{P} + \frac{\Delta}{2} \rangle.$$
Kinematics of the process $\nu(l)N(P_1) \rightarrow \nu(l')N(P_2)\gamma(q_2)$
No Bethe-Heitler (large) background for neutral currents

Recoiling nucleon
Virtual Compton Amplitude

Bethe-Heitler
(pr) = 0, \quad p^2 = m^2 - \frac{t}{4}, \quad t = r^2.

p_{1,2} = p \pm \frac{r}{2}, \quad q_{1,2} = q \mp \frac{r}{2}.

T_{\mu\nu} = i \int d^4z e^{i(qz)} (p - r/2|T J_\mu(-z/2)J_\nu(z/2)|p + r/2).

(q_1 = q_2 = q, p_1 = p_2 = p, r = 0)

1) DIS Limit

\[ q_1^2 \to \infty, (p_1q_1) \to \infty, \text{ with } x_B \equiv -q_1^2/[2(p_1q_1)] \]

2) DVCS/DVNS Limit

\[ q_2^2 = 0, \text{ in the limit } q_1^2 \to \infty, (p_1q_1) \to \infty, \text{ again with } x_B \equiv -q_1^2/[2(p_1q_1)] \text{ fixed.} \]

(p_1 - p_2 \equiv r \neq 0), \quad t \text{ fixed.}

\[ (rq_1) \to \infty \text{ proportional to } q_1^2. \]

where \( t \equiv r^2 < 0 \) does not grow with \( q_1^2 \).
\[ \xi = \frac{-q^2}{2(pq)}, \quad \eta = \frac{(rq)}{2(pq)}, \]

\[ q_1^2 = \left(1 + \frac{\eta}{\xi}\right)q^2 + \frac{t}{4}, \quad x_B = \frac{-q_1^2}{2(p_1 q_1)} = \frac{\xi + \eta}{1 + \eta}. \]

\[ q_2^2 = \left(1 - \frac{\eta}{\xi}\right)q^2 + \frac{t}{4}, \]

**DIS:** \( \eta = 0, \quad x_B = \xi, \)

**DVCS:** \( \eta = \xi, \quad x_B = \frac{2\xi}{1 + \xi}. \)

**DVNS kinematics remains invariant w.r.t. DVCS**
Nonforward (Radyushkin) vs Off-forward (Ji)

\[ \gamma^*(q_1) \rightarrow \gamma(q_2) \]
\[ N(p_1) \rightarrow N(p_2) \]

\[ X = \frac{x + \xi}{1 + \xi} \]
\[ \zeta = \frac{2\xi}{1 + \xi}. \]

Nonforward pdf’s: 0 < X < 1

Off forward: -1 < x < 1

\[ p = \frac{p_1 + p_2}{2}. \]
\[ \int \frac{d\lambda}{(2\pi)} e^{i\lambda z} \langle P' | \bar{\psi} \left( -\frac{\lambda n}{2} \right) \gamma^\mu \psi \left( \frac{\lambda n}{2} \right) | P \rangle = \]

\[ H(z, \xi, \Delta^2) \overline{U}(P') \gamma^\mu U(P) + E(z, \xi, \Delta^2) \overline{U}(P') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} U(P) + ..... \]

\[ \int \frac{d\lambda}{(2\pi)} e^{i\lambda z} \langle P' | \bar{\psi} \left( -\frac{\lambda n}{2} \right) \gamma^\mu \gamma^5 \psi \left( \frac{\lambda n}{2} \right) | P \rangle = \]

\[ \tilde{H}(z, \xi, \Delta^2) \overline{U}(P') \gamma^\mu \gamma^5 U(P) + \tilde{E}(z, \xi, \Delta^2) \overline{U}(P') \frac{\gamma^5 \Delta^\mu}{2M} U(P) + ..... \]

\[ H^i(z, \xi, \Delta^2, Q^2) = F_1^i(\Delta^2) q^i(z, \xi, Q^2) \]

\[ \tilde{H}^i(z, \xi, \Delta^2, Q^2) = G_1^i(\Delta^2) \Delta q^i(z, \xi, Q^2) \]

\[ E^i(z, \xi, \Delta^2, Q^2) = F_2^i(\Delta^2) r^i(z, \xi, Q^2) \]
\[ q(z, \xi, Q^2) = \int_{-1}^{1} dx' \int_{-1+|x'|}^{1-|x'|} dy' \delta(x' + \xi y' - z) f(y', x', Q^2) \]

\[ f(y, x) = \pi(y, x) f(x), \]

\[ \pi(x, y) = \frac{\Gamma(2b + 2)}{2^{2b+1} \Gamma^2(b + 1)} \frac{[(1 - |x|)^2 - y^2]^b}{(1 - |x|)^{2b+1}} \]
\[ M_{fi} = J_\mu^Z(q_1) D(q_1) \epsilon^{\nu*}(q_1 - \Delta) \]
\times \left\{ \frac{i}{2} \tilde{g} g_u U_v \int_{-1}^{1} dz \left( \tilde{n}^\mu n^\nu + \tilde{n}^\nu n^\mu - g^{\mu\nu} \right) \right. \\
\left. \alpha(z) \left[ H^u(z, \xi, \Delta^2) \bar{U}(P_2) \gamma^5 U(P_1) + E^u(z, \xi, \Delta^2) \bar{U}(P_2) \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} U(P_1) \right] + \right. \\
\beta(z) i\epsilon^{\mu\nu\alpha\beta} \tilde{n}_\alpha n_\beta \left[ \tilde{H}^u(z, \xi, \Delta^2) \bar{U}(P_2) \gamma^5 U(P_1) + \tilde{E}^u(z, \xi, \Delta^2) \bar{U}(P_2) \gamma^5 (\Delta \cdot n) U(P_1) \right] + \\
\left. \frac{i}{2} \tilde{g} g_d D_v \int_{-1}^{1} dz \left\{ u \to d \right\} - \right. \\
\left. \frac{i}{2} \tilde{g} g_u \int_{-1}^{1} dz \left( -\tilde{n}^\mu n^\nu - \tilde{n}^\nu n^\mu + g^{\mu\nu} \right) \right. \\
\left. \alpha(z) \left[ \tilde{H}^u(z, \xi, \Delta^2) \bar{U}(P_2) \gamma^5 U(P_1) + \tilde{E}^u(z, \xi, \Delta^2) \bar{U}(P_2) \frac{i\gamma^5 \Delta \cdot n}{2M} U(P_1) \right] + \right. \\
\left. \beta(z) i\epsilon^{\mu\nu\alpha\beta} \tilde{n}_\alpha n_\beta \left[ H^u(z, \xi, \Delta^2) \bar{U}(P_2) \gamma^5 U(P_1) + E^u(z, \xi, \Delta^2) \bar{U}(P_2) \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} (P_1) \right] - \right. \\
\left. \frac{i}{2} \tilde{g} g_d \int_{-1}^{1} dz \left\{ u \to d \right\} \right\}. \]
\[ |\mathcal{M}|^2 = P.V. \int_{-1}^{1} dz \int_{-1}^{1} dz' \left( K_1(z, z') \alpha(z) \alpha^*(z') + K_2(z, z') \beta(z) \beta^*(z') \right) \]
\[ + \pi^2 \left( K_1(\xi, \xi) - K_1(\xi, -\xi) - K_1((-\xi, \xi) + K_1(-\xi, -\xi)) \right) \]
\[ + \pi^2 \left( K_2(\xi, \xi) + K_2(\xi, -\xi) + K_2((-\xi, \xi) + K_2(-\xi, -\xi)) \right) \]

\[ A_1(z, z'x, t, Q^2) = \tilde{g}^4 Q^2 \left[ 4 q_d^2 [\tilde{E}'_d (4 \tilde{H}_d M^2 + \tilde{E}_d t) x^2 \right. \]
\[ + 4 \tilde{H}'_d M^2 (4 \tilde{H}_d (x - 1) + \tilde{E}_d x^2) \]
\[ + 4 g_d g_u [(4 \tilde{E}'_u \tilde{H}_d M^2 + 4 \tilde{E}'_d \tilde{H}_u M^2 + \tilde{E}_u \tilde{E}_d t + \tilde{E}'_d \tilde{E}_u t) x^2 \right. \]
\[ + 4 \tilde{H}'_d M^2 (4 \tilde{H}_d (x - 1) + \tilde{E}_d x^2) + 4 \tilde{H}'_u M^2 (4 \tilde{H}_u (x - 1) + \tilde{E}_u x^2) \]
\[ + D_v U_v g_d g_u [4 E'_d E_d t + 4 E'_d E_d t - 4 E'_d E_d t x - 4 E'_d E_u t x + 4 E'_d E_d M^2 x^2 \right. \]
\[ + 4 E'_d E_u M^2 x^2 + 4 E'_u H_d M^2 x^2 + 4 E'_d H_u M^2 x^2 + E'_d E_d t x^2 + E'_d E_u t x^2 \]
\[ + 4 H'_d M^2 (4 H_d (x - 1) + E_d x^2) + 4 H'_d M^2 (4 H_u (x - 1) + E_u x^2) \]
A. Cafarella, M. Guzzi, C.C.
Inelastic Scattering

\[ W^2 \equiv (P + q)^2 = M^2 + 2(P \cdot q) - Q^2. \]

\[ W^{\mu\nu} = \frac{1}{2\pi M} T^{\mu\nu} \]

\[ \frac{d^2 \sigma_{(eN)}}{d\Omega d\omega'} = \frac{\alpha^2 \omega'}{Q^4 \omega} L^{(e)}_{\mu\nu} W^{\mu\nu}. \]

\[ W^{\mu\nu} = \frac{1}{4\pi M} \int d^4 z \frac{1}{2} \sum_s \langle N(P, S) | J_{EM}^{\mu}(z) J_{EM}^{\nu}(0) | N(P, S) \rangle. \]

\[ T^{\mu\nu} = i \int d^4 z \ e^{i(q \cdot z)} \langle N(P, S) | T \{ J_{EM}^{\mu}(z) J_{EM}^{\nu}(0) \} | N(P, S) \rangle. \]
Deep Inelastic limit

\[ \begin{align*}
F_1 \left( x_B, Q^2 \right) & \equiv MW_1 \left( \nu, Q^2 \right), \\
F_2 \left( x_B, Q^2 \right) & \equiv \nu W_2 \left( \nu, Q^2 \right).
\end{align*} \]

\[ \frac{d^2 \sigma_{(eN)}}{dx_B dy} = \left( \frac{2\pi M \omega y}{\omega'} \right) \frac{d^2 \sigma_{(eN)}}{d\Omega d\omega'} \]

\[ = \frac{4\pi \alpha^2}{Q^2 y} \left[ y^2 F_1 \left( x_B, Q^2 \right) + \left( \frac{1-y}{x_B} - \frac{M^2 y}{s-M^2} \right) F_2 \left( x_B, Q^2 \right) \right] \]

\[ e^-(k) \quad \rightarrow \quad e^-(k') \]

\[ \gamma^*(q) \]

\[ q_a(p=xP) \quad \rightarrow \quad q_a(p'=xP+q) \]

\[ N(P) \]

emission of a parton from a light-cone dominated process
\[ q_\mu W^{\mu\nu} = 0 \quad \text{and} \quad W^{\mu\nu} q_\nu = 0, \]

\[ \partial_\mu J^\mu_{EM} = 0, \]

\[ W^{\mu\nu} = W_1(\nu, Q^2) \left[ -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right] + \frac{W_2(\nu, Q^2)}{M^2} \left[ p^\mu - q^\mu (P \cdot q) \right] \left[ p^\nu - q^\nu (P \cdot q) \right] \]

\[ x_B = \frac{Q^2}{W^2 - M^2 + Q^2} = \frac{1}{1 + (W^2 - M^2) / Q^2} \]

\[ s \equiv (k + P)^2 \]

\[ Q^2 = x_B y (s - M^2) \]

\[ y \equiv \frac{(P \cdot q)}{(P \cdot k)}. \]
Total cross section

\[ \sum_X \left| \frac{N(P) \rightarrow X}{N(P) \rightarrow N(P)} \right|^2 = 2 \]

Forward parton distributions

Partons are emitted and re-absorbed on the light-cone with momentum \( xP \)
\begin{align*}
  f_1(x) &= \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P | \bar{\psi}(0) \gamma_\lambda \psi(\lambda n) | P \rangle \\
  g_1(x) &= \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS_\parallel | \bar{\psi}(0) \gamma_\lambda \gamma_5 \psi(\lambda n) | PS_\parallel \rangle \\
  h_1(x) &= \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS_\perp | \bar{\psi}(0) [S_\perp, \gamma_\lambda] \gamma_5 \psi(\lambda n) | PS_\perp \rangle
\end{align*}
Neutral current

\[ j_{Z}^{\mu} \equiv \overline{\nu}(l')\gamma^{\mu} \left( -1 + 4\sin^{2}\theta_{W} + \gamma_{5} \right) u(l) \]

\[
T_{\mu\nu}(q_{1}^{2}, \nu) = i \int d^{4}z e^{i\mathbf{q} \cdot \mathbf{z}} \langle P_{1} | T(\mathcal{J}_{Z}^{\mu}(\xi) \mathcal{J}_{Z}^{\nu}(0)) | P_{1} \rangle
\]

weak hadronic tensor
neutral current
unitarity
The analysis at higher twists is far more involved and one isolates 14 structures if spin and mass effects are included. Use: Lorenz covariance

T- invariance

neglect CP violating effects from CKM matrix

If we impose Ward Identities (current conservation) we reduce them to 8.

Ward identities are broken in a spontaneously broken theory, so this is equivalent to set to zero the quark masses.
Summary: Weak Unitarity for neutral currents

\[
\hat{P}_1^\mu = P_1^\mu - q_1^\mu P_1 \cdot q_1 / q_1^2.
\]

\[
\eta^{\gamma|2}(Q^2) = 1,
\]

\[
\eta^{\gamma Z|}(Q^2) = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \frac{Q^2}{Q^2 + M_Z^2},
\]

\[
\eta^{Z|2}(Q^2) = (\eta^{\gamma Z|})^2(Q^2).
\]

\[
W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_{1\mu} q_{1\nu}}{q_1^2}\right) W_1(\nu, Q^2) + \frac{\hat{P}_1^\mu \hat{P}_1^\nu}{P_1^2} \frac{W_2(\nu, Q^2)}{M^2} - i\epsilon_{\mu\nu\lambda\sigma} q_1^\lambda P_1^\sigma \frac{W_3(\nu, Q^2)}{2M^2}
\]

\[
L^i_{\mu\nu} = \sum_{\nu'} \left[ \bar{u}(k', \lambda') \gamma_\mu (g_V^{i1} + g_A^{i1} \gamma_5) u(k, \lambda) \right]^* \bar{u}(k', \lambda') \gamma_\nu (g_V^{i2} + g_A^{i2} \gamma_5) u(k, \lambda).
\]

Generic em/weak cross section

\[
\frac{d^3\sigma}{dx dy d\theta} = \frac{y\alpha^2}{Q^4} \sum_i \eta_i(Q^2) L^i_{\mu\nu} W^i_{\mu\nu},
\]

\[
MW_1(Q^2, \nu) = F_1(x, Q^2)
\]

\[
\nu W_2(Q^2, \nu) = F_2(x, Q^2)
\]

\[
\nu W_3(Q^2, \nu) = F_3(x, Q^2),
\]

\[
g_V^\gamma = 1, \quad g_V^Z = -\frac{1}{2} + 2\sin^2\theta_W, \quad g_A^\gamma = 0, \quad g_A^Z = \frac{1}{2}.
\]
\[ q^0(x, Q^2) = \left[ \frac{u_v(x, Q^2) + d_v(x, Q^2)}{2} + \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} \right] (L_u^2 + L_d^2) + \left[ \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} \right] (R_u^2 + R_d^2) \]

\[ \bar{q}^0(x, Q^2) = \left[ \frac{u_v(x, Q^2) + d_v(x, Q^2)}{2} + \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} \right] (R_u^2 + R_d^2) + \left[ \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} \right] (L_u^2 + L_d^2) \]

Parton distributions

\[ L_u = 1 - \frac{4}{3} \sin^2 \theta_W, \quad L_d = -1 + \frac{2}{3} \sin^2 \theta_W \]

\[ R_u = -\frac{4}{3} \sin^2 \theta_W, \quad R_d = \frac{2}{3} \sin^2 \theta_W \]

\[ x \approx 0.1 \]
Quark-antiquark distributions using $H(x)$

$$q(x) - \bar{q}(x) = H_q(x) + H_q(-x) \equiv H_q^V(x)$$

$$\sum_q [q(x) + \bar{q}(x)] = \sum_q [H_q(x) - H_q(-x)] \equiv H^S(x).$$

In a similar way we may introduce $H_g(x) \equiv x g(x)$ where $g(x)$ is the familiar gluon distribution.

$$H_g(x) = \frac{1}{P^+} \int \frac{dy^-}{2\pi} e^{-ixP^+y^-}$$

$$\times \langle P | F^{+\nu}(0, y^-/2, 0) F^{+\nu}_\nu(0, -y^-/2, 0) | P \rangle,$$

where $F^{\mu\nu}$ is the gluon field strength tensor

$$H_q(x) = \frac{1}{2P^+} \int \frac{d^2k_T}{2\pi} \sum_\lambda \left[ \langle P | b^{+\lambda}(xP^+, k_T) b^{-\lambda}(xP^+, k_T) | P \rangle \theta(x) \right.$$}

$$\left. - \langle P | d^{+\lambda}_{\lambda}(-xP^+, k_T) d^{-\lambda}_{\lambda}(-xP^+, k_T) | P \rangle \theta(-x) \right]$$
In order to introduce off-diagonal distributions it is most convenient to first recall the definition of the conventional (diagonal) parton distributions in terms of light-cone coordinates \( x^\pm = (x^0 \pm x^3)/\sqrt{2}, x^1, x^2 \) and in the light-cone gauge \( (A^+ = 0) \):

\[
H_q(x) = \frac{1}{2} \int \frac{dy^-}{2\pi} e^{-ix^-p^+y^-} \langle P | \bar{\psi}_q(0, y^-/2, 0) \gamma^+ \psi_q(0, -y^-/2, 0) | P \rangle.
\]

Note that the matrix element is diagonal in the four momentum \( P \) of the proton.

(a) \( x > 0 \): \( q(x) = H_q(x) \)

(b) \( x < 0 \): \( \bar{q}(-x) = -H_q(x) \)

\[
H_q(x) = \begin{cases} 
q(x) & \text{for } x > 0 \\
-\bar{q}(-x) & \text{for } x < 0.
\end{cases}
\]
Hadronic Interactions mediated by weak currents

**ENERGY**

**INCLUSIVE PROCESSES**

DGLAP, “structure functions”

**EXCLUSIVE PROCESSES**

NONFORWARD PARTON DISTRIBUTIONS
(weak)
(nonforwar RG Evolution)

Distribution Amplitudes
(hadronic Wave Function)
ERBL

The link is COMPTON SCATTERING (WEAK)
\[
\frac{1}{Q} \\
\pi(p_1) \quad \pi(p_2) \\
\quad \\
(1-x)p_1 \quad (1-y)p_2 \\
\]

(a)

(b)

\[
(p_2 + p_1) \mu F_\pi(Q^2) = \langle \pi(p_2)|J_\mu(0)|\pi(p_1)\rangle
\]

\[
p_1^+ = Q/\sqrt{2}, \quad p_1^- = 0,
\]

\[
p_2^+ = Q/\sqrt{2}, \quad p_2^- = 0
\]

\[
(p_2 - p_1)^2 = -2p_1^+ p_2^- = -Q^2.
\]

\[
F_\pi(Q) \sim (1/Q^2)
\]

\[
F_\pi(Q^2) = \int_0^1 dx \, dy \, \phi_\pi(y, \mu^2) \, T(y, x, Q^2, \mu^2) \, \phi_\pi(x, \mu^2)
\]
At small separation $b$, the hadronic Wave function reproduces the collinear one

$$T_H = 16\pi C_F \alpha_s(\mu^2) \left[ \frac{2}{3} \frac{1}{xyQ^2} + \frac{1}{3} \frac{1}{(1-x)(1-y)Q^2} \right]$$

![Diagram](image)

(a) (b)

Inclusion of transverse momentum

$$F_\pi(Q^2) = \int_0^1 dx \, dy \left( \int \frac{d^2b_1}{(2\pi)^2} \frac{d^2b_2}{(2\pi)^2} \mathcal{P}(y, b_2, p_2, \mu) \times T(y, x, p_1, b, \mu) \mathcal{P}(x, b_1, p_1, \mu) \right)$$

$$\mathcal{P}(x, \mu = 1/\mu, p_i, \mu) \sim \phi(x, \mu^2),$$

At small separation $b$, the hadronic Wave function reproduces the collinear one
The inclusion of transverse momentum allows to lower the validity of the Factorization picture.

\[ \mu \frac{d}{d\mu} F_\pi(Q^2) = 0. \]

\[ 0 = \int_0^1 dx dy \left[ \frac{d\phi_\pi(y)}{d\mu} T\phi_\pi(x) + \phi_\pi(y) \frac{dT}{d\mu} \phi_\pi(x) + \phi_\pi(y) T \frac{d\phi_\pi(x)}{d\mu} \right]. \]

\[ \mu \frac{d\phi(y, \mu^2)}{d\mu} = \int_0^1 dz V(y, z, \alpha_s(\mu^2)) \phi_\pi(z, \mu^2) \]

\[ \phi_\pi(x, \mu^2) = x(1-x) \sum_{n \geq 0} a_n C_n^{3/2} (2x - 1) \left( \ln \frac{\mu^2}{\Lambda^2} \right)^{-\gamma_n/2\beta_2} \]
Inclusion of transverse momentum

\[ Q \frac{\partial}{\partial Q} \mathcal{P}(x, b, p, \mu) = [K(b\mu) + G(x, Q/\mu)] \mathcal{P}(x, b, p, \mu) \]

\[ \mathcal{P}(x, b; p, \mu) = e^{-S(x, b; Q, \mu)} \left( \phi_{\pi}(x, 1/b^2) + \mathcal{O}(\alpha_s^2(1/b)) \right) \]

Sudakov suppression

Asymptotic Solution

\[ \phi_{\pi}(x, \mu^2) \to \sqrt{3} f_{\pi} x(1 - x) \]

But….where is “asymptotia”?

\[ \phi_{CZ}(x, \mu_0^2) = 5\sqrt{3} f_{\pi} x(1 - x)(1 - 2x)^2 \]

Nothing prevents us from applying these consideration to weak processes
FORM FACTORS

\[ |P\uparrow\rangle_{1/2} = \frac{1}{12} \int \frac{[dx][d^2k]}{\sqrt{x_1x_2x_3}} \psi_1(\kappa_1, \kappa_2, \kappa_3) \]

\[ \times \varepsilon^{abc} u_{a\uparrow}^\dagger(\kappa_1) \left\{ u_{b\downarrow}^\dagger(\kappa_2) d_{c\uparrow}^\dagger(\kappa_3) - d_{b\downarrow}^\dagger(\kappa_2) u_{c\uparrow}^\dagger(\kappa_3) \right\} |0\rangle \]

Transverse momentum dependence Sudakov suppression (Li-Sterman)

Fock vacuum
DISTRIBUTION AMPLETTUES

\[ \langle 0 | \bar{\psi}_d (-z/2) \gamma^\mu \gamma_5 \psi_u (z/2) | \pi^+ (P) \rangle_{z=0} = i P^\mu f_{\pi} \int_{-1}^{1} d\alpha \ e^{i\alpha (P \cdot z)/2} \varphi_{\pi^+} (\alpha) \]

The fractions of the pion momentum carried by the quarks are \((1 \pm \alpha)/2\).
The Feynman mechanism we may be unable to resolve the partonic structure of the nucleon, Overlap of wave functions
However: CS has a life of its own

Feynman mechanism of overlapping wave functions

Intrinsically SOFT, not factorizable.
Use interpolating currents (Dispersive description)
Nonforward (Radyushkin) vs Off-forward (Ji)

\[ \gamma^*(q_1) \gamma(q_2) \]

\[ N(p_1) \rightarrow N(p_2) \]

Longitudinal/transverse momentum exchange

\[ X = \frac{x + \xi}{1 + \xi} \quad \zeta = \frac{2\xi}{1 + \xi} \]

Nonforward pdf’s: \( 0 < X < 1 \)

Off forward: \( -1 < x < 1 \)

\[ p = \frac{p_1 + p_2}{2} \]
$F^f_\zeta(X,t)$ is the probability amplitude that the initial fast-moving hadron, having longitudinal momentum $P^+$, emits a parton of flavor $f$ carrying the momentum $XP^+$ while the final hadron, having longitudinal momentum $(1 - \zeta)P^+$, absorbs a parton of flavor $f$ carrying the momentum $(X - \zeta)P^+$. 
$H_q(x, \xi, t) = \frac{1}{2} \int \frac{dy^-}{2\pi} e^{-ix\bar{P}^+ y^-} \langle P' | \bar{\psi}_q(0, y^- / 2, 0) \rangle \times \gamma^+ \psi_q(0, -y^- / 2, 0) | P \rangle$.

The variable $t$ is the usual $t$-channel invariant, $t = \Delta^2$,

$\Delta \equiv (P - P') = \xi (P + P')$

the distribution $H_q(x, \xi, t)$ now contains two extra scalar variables, in addition to the Bjorken $x$ variable. 

Averaged momentum: $p = (p_1 + p_2) / 2$. 
\[ H_q(x, \xi) = \frac{1}{2P^+} \int \frac{d^2 k_T}{2 \sqrt{|x^2 - \xi^2|} (2\pi)^3} \sum \lambda \left[ \langle P' | b_\lambda^\dagger ((x-\xi) P^+, k_T - \Delta_T) b_\lambda ((x + \xi) P^+, k_T) | P \rangle \theta(x \geq \xi) \right. \\
- \left. \langle P' | d_\lambda^\dagger ((-x-\xi) P^+, k_T - \Delta_T) d_\lambda ((-x + \xi) P^+, k_T) | P \rangle \theta(x \leq -\xi) \right]. \]

\[ + \langle P' | d_\lambda ((-x + \xi) P^+, -k_T - \Delta_T) b_{-\lambda} ((x + \xi) P^+, k_T) | P \rangle \theta(-\xi < x < \xi) \]

(a) $x \geq \xi$: DGLAP-type region for the quark distribution
(b) $-\xi < x < \xi$: ERBL-type probability amplitude
(c) $x < -\xi$: DGLAP-type region for the antiquark distribution
\[
\frac{d\sigma}{dxdQ^2d|\Delta^2|d\phi_r} = \frac{y}{Q^2} \frac{d\sigma}{dx dy d|\Delta^2|d\phi_r} = \frac{xy^2}{8\pi Q^4} \left(1 + \frac{4M^2x^2}{Q^2}\right)^{-\frac{1}{2}} |M_{fi}|^2.
\]

\[
\sigma_{(x=0.2)} [\text{nb}/\text{GeV}^4]
\]

\[
|\Delta^2| = 0.1 \text{ GeV}^2 \quad \text{---} \quad |\Delta^2| = 0.2 \text{ GeV}^2 \quad \text{---} \quad |\Delta^2| = 0.5 \text{ GeV}^2
\]

\[
ME = 10 \text{ GeV}^2
\]
Charged Currents

\[
T_{\text{BH}}^{W^+} = -|e| \frac{g}{2\sqrt{2}} \frac{g}{\sqrt{2}} \bar{u}(l') \left[ \gamma^\mu \frac{(l - \Delta)}{(l - \Delta)^2 + i\epsilon} \gamma^\nu (1 - \gamma^5) \right] \\
\times u(l) \frac{D^\nu\delta(q_1)}{\Delta^2 - M_W^2 + i\epsilon} \epsilon^*_\mu(q_2) U(P_2) \\
\times \left[ [F_1^u(\Delta^2) - F_1^d(\Delta^2)]\gamma^\delta + [F_2^u(\Delta^2) - F_2^d(\Delta^2)]i \frac{\sigma^{\delta\alpha}\Delta^\alpha}{2M} \right] U(P_1),
\]

\[
T_{\mu\nu} = i \int d^4x e^{iqx} \langle P_2 | T[J_\nu^\gamma(x/2)J_\mu^{W^z},Z_0(-x/2)] | P_1 \rangle,
\]

\[
T_{\mu\nu}^{W^+} = i \int d^4x \frac{e^{iqx} \chi^\alpha U_{ud}}{2\pi^2(\chi^2 - i\epsilon)} \langle P_2 | [iS_{\mu\alpha\nu\beta}(\tilde{O}_\nu^{\beta} + O_{\nu\beta}^{5\beta}) + \epsilon_{\mu\alpha\nu\beta}(O_\nu^{\beta} + \tilde{O}_\nu^{5\beta})] | P_1 \rangle,
\]

(17)
Expressed in terms of nfpd's

\[
\begin{align*}
\bar{O}_a^\beta (x/2, -x/2) &= \bar{\psi}_a(x/2) \gamma^\beta \psi_a(-x/2) + \bar{\psi}_a(-x/2) \gamma^\beta \psi_a(x/2), \\
\bar{O}_a^{5\beta} (x/2, -x/2) &= \bar{\psi}_a(x/2) \gamma^5 \gamma^\beta \psi_a(-x/2) - \bar{\psi}_a(-x/2) \gamma^5 \gamma^\beta \psi_a(x/2), \\
O_a^\beta (x/2, -x/2) &= \bar{\psi}_a(x/2) \gamma^\beta \psi_a(-x/2) - \bar{\psi}_a(-x/2) \gamma^\beta \psi_a(x/2), \\
O_a^{5\beta} (x/2, -x/2) &= \bar{\psi}_a(x/2) \gamma^5 \gamma^\beta \psi_a(-x/2) + \bar{\psi}_a(-x/2) \gamma^5 \gamma^\beta \psi_a(x/2), \\
\bar{O}_{ud}^\beta (x/2, -x/2) &= g_u \bar{\psi}_u(x/2) \gamma^\beta \psi_d(-x/2) + g_d \bar{\psi}_u(-x/2) \gamma^\beta \psi_d(x/2), \\
\bar{O}_{ud}^{5\beta} (x/2, -x/2) &= g_u \bar{\psi}_u(x/2) \gamma^5 \gamma^\beta \psi_d(-x/2) - g_d \bar{\psi}_u(-x/2) \gamma^5 \gamma^\beta \psi_d(x/2), \\
O_{ud}^\beta (x/2, -x/2) &= g_u \bar{\psi}_u(x/2) \gamma^\beta \psi_d(-x/2) - g_d \bar{\psi}_u(-x/2) \gamma^\beta \psi_d(x/2), \\
O_{ud}^{5\beta} (x/2, -x/2) &= g_u \bar{\psi}_u(x/2) \gamma^5 \gamma^\beta \psi_d(-x/2) + g_d \bar{\psi}_u(-x/2) \gamma^5 \gamma^\beta \psi_d(x/2).
\end{align*}
\]

\[
\langle P_2 \parallel \bar{\psi}_a(-kx) \gamma^5 \gamma^\mu \psi_a(kx) \parallel P_1 \rangle^{\text{twist-2}} = \int Dz e^{-i(k \cdot P_z) F^{5a}(\nu)}(z_1, z_2, P_i \cdot P_j x^2, P_i \cdot P_j) \\
\times \bar{U}(P_2) [\gamma^5 \gamma^\mu - ikP_\mu \gamma^5 \gamma^\beta] U(P_1) \\
+ \int Dz e^{-i(k \cdot P_z) G^{5a}(\nu)}(z_1, z_2, P_i \cdot P_j x^2, P_i \cdot P_j) \\
\times \bar{U}(P_2) \gamma^5 \left[ \frac{i \sigma^{\mu \alpha} \Delta_\alpha}{M} - ikP_\mu \frac{(i \sigma^{\alpha \beta} x_\alpha \Delta_\beta)}{M} \right] U(P_1).
\]
CONCLUSIONS

Plenty of new applications of pQCD at intermediate energy

1) Perturbative analysis of weak form factors
2) Study of coherence effects
3) Will be able to explore hadronic/weak interactions in a new territory