Learning Many-Body Physics with Adelchi

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We present an overview of the contributions that Adelchi Fabrocini made to the field of many-body physics during the last thirty years. He left us while he was still in full activity, and his work, which is an enduring resource for all of us, will continue to motivate and guide future research in many-body physics.

I. INTRODUCTION

It is already five months since Adelchi left us, and it becomes evident that we will not get used to his absence. We often talk as if he is still present, or we think of common future projects. More than once, during these months, we have found ourselves trying to make impossible phone calls. Both from the personal and scientific points of view, we feel an emptiness hard to fill. In this short article, we want to remember his contributions to science, and to give a feeling for his attitude with respect to physics.

For Adelchi, physics, and science in general, was not just a job, but a way of life, a way of understanding the world. Rigor in analysis, imagination and creativity in the invention of new formalisms, curiosity and the urge to ask “why” and “how”, were all characteristics of his scientific personality, and they were projected in his daily life. We are of similar ages, and from the very beginning we have made our careers together, sharing and enjoying common scientific interests. In fact, the many-body problem as it arises in several fields has occupied us for about thirty years.

Adelchi often recalled to us his first participation in a many-body conference, the seminal one that took place in 1978 in Trieste. The main discussions at this conference were driven by the discrepancies between the results of the Brueckner-Hartree-Fock theory and those of the variational calculations for nuclear matter. At that time, many-body problems were centered around nuclear physics, although early Monte Carlo calculations for Bose systems were also presented at the conference. But what stood out in Adelchi’s memory was the special seminar that P. A. M. Dirac gave about the time dependence of universal constants.

The Trieste conference convinced Adelchi that Many-Body Physics was a good subject to study, and he dedicated his efforts to it. The guidance of Sergio Rosati and Stefano Fantoni was crucial in helping him make the initial steps in this direction. Adelchi’s motivations are aptly summarized at the beginning of one of the review articles that Adelchi wrote together with Stefano Fantoni [1]: “Nearly all of physics is many-body physics at the most microscopic level of understanding, appropriate to the energy scale of the particular branch of physics under consideration. Thus, the subject of quantum many-body theory (QMBT) can fairly be said to virtually strengthen from beneath all of modern physics. The fundamentally many-particle nature of nuclei, atoms, molecules, solids, and fluids are all manifestly apparent, but even the single nucleon problem is itself becoming a multiparticle problem at the deepest level of understanding”.

QMBT was applied by Adelchi to describe many kinds of quantum many-particle systems, from cold atoms to neutron stars. In spite of the different scales of length and energy of these systems, the salient physical phenomena can often be described by using similar strategies. The microscopic description of these systems is not only an important intellectual goal, it is also the appropriate framework for exhibiting correspondences and relationships that unify our understanding of many-body phenomena.

Microscopic description of a many-body system first requires an appropriate identification of its constituents. The proper choice depends on the experimental findings that are to be explained and hence depends on the energy scale used to probe the system. Determination of the interaction between the constituents is the next step. A knowledge of the masses of the constituents and of their interactions is sufficient information to define the Hamiltonian of the system. At this point, the goal is to solve the corresponding Schrödinger equation, evaluate the binding and excitation energies of the system, and describe its dynamics.

The many-body systems we have studied with Adelchi are characterized by strong interactions between their
constituent particles, producing large correlations which strongly modify the independent-particle picture. These strong interactions give rise to serious difficulties in the application of traditional perturbative approaches. A way out of this difficulty is to construct and implement effective interactions that are weak enough to allow rapid convergence of the traditional perturbation expansion. An alternative approach is provided by Correlated Basis Function (CBF) theory, which Adelchi did so much to improve and develop in the last thirty years.

The core idea of CBF theory is to incorporate the correlations from the very beginning into a trial wave function $\Psi_T$ of the form

$$\Psi_T(1,\ldots,N) = F(1,\ldots,N)\Phi(1,\ldots,N) \ .$$

In this expression, the factor $\Phi(1,\ldots,N)$ describes the system in the Independent Particle Model (IPM) picture, i.e., in absence of interaction between the constituents, and, evidently, devoid of correlations other than those due to quantum statistics. The factor $F(1,\ldots,N)$ represents a correlation operator which takes into account in a direct way (though not exhaustively) the correlations induced by the interactions.

The quantum statistics of the system are taken into account by the IPM (wave) function $\Phi(1,\ldots,N)$. In the case of a homogeneous bosonic liquid, $\Phi(1,\ldots,N)$ is a constant, i.e. all the particles are assumed to occupy the zero-momentum state. In the case of an infinite Fermi system, $\Phi$ is a Slater determinant of plane waves, with all the momenta occupied up to the Fermi level. For finite systems, $\Phi(1,\ldots,N)$ is built from single-particle wave functions generated by a mean-field potential.

The variational principle

$$\frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} = E_T \geq E_0 \quad (2)$$

gives the first estimate $E_T$ of the true ground-state energy $E_0$.

Actual calculation of the expectation value (2) is rather involved, and several methods, for example Monte Carlo techniques, have been devised to evaluate it. During the second half of the seventies, remarkable advances were made with methods based on integral equations, most notably hypernetted-chain resummation and its generalization to Fermi systems (Fermi Hypernetted-Chain, FHNC).

The CBF theory provides a way to systematically improve on a trial wave function of type (1) by doing perturbation theory with correlated basis functions. Since many correlations are already contained in the variational wave function (1), one expects that a satisfactory description of the system can be obtained at low perturbative order in CBF theory. Obviously, perturbation theory formulated in a correlated basis is more complicated than the standard perturbation theory developed for weak interactions. Adelchi has made pivotal contributions to this technically difficult subject, also defining paths to be followed in the future. In the next sections, we briefly describe the impact of these contributions on different subfields of quantum many-body physics.

In ending this introduction, we would first like to emphasize Adelchi’s important role in writing several review articles which, still at this time, are the best places to learn about the latest developments in CBF theory. We also recall, with gratitude, the great amount of energy he devoted to organizing and coordinating activities that served to catalyze the Italian theoretical nuclear physics community. These efforts included organization of the Cortona meetings, which have been especially beneficial to the younger generations of nuclear physicists in Italy.

Finally, we want to say that we have been very fortunate to know and to be with Adelchi, to enjoy his friendship, to share projects with him, and to enjoy his sweet and clever irony which always helped us to look at problems, in both life and physics, from new and truer perspectives.

II. QUANTUM LIQUIDS AND ULTRACOLD TRAPPED ATOMS

In his first article [2], published in Il Nuovo Cimento in March 1980, Adelchi studied an infinite-hard-sphere Fermi system using Jastrow variational wave functions. The FHNC equations were used to obtain the corresponding energy and momentum distribution. Special attention was also devoted to evaluation of the different types of elementary diagrams that appear in the diagrammatic representation of the cluster expansion of the two-body distribution function. The elementary diagrams cannot be calculated in a closed form, and their evaluation is a recurrent problem in all integral-equation resummation methods [3]. The existence of low-density expansions and of Brueckner-Hartree-Fock calculations for this system opened the possibility of a critical comparison between the results obtained with different many-body methods. Recently, there has been renewed interest in this system in connection with the microscopic description of both fermionic and bosonic cold atomic gasses.

Immediately after his work on hard-sphere fermions, Adelchi began to study more realistic systems, notably $^3$He-$^4$He mixtures. This was a time of intense activity in ab initio, microscopic treatment of both $^4$He and $^3$He quantum liquids,
which are prototypes of strongly interacting Bose and Fermi systems, respectively. One motivation for focusing on these systems is that the interaction between the basic constituents, i.e. the helium atoms, is rather simple, depending as it does only on their spatial separation. The interaction features a repulsive hard core at short distances and a relatively weak attraction at larger distances. The interaction, electromagnetic in origin, is the same for both isotopes. The simplicity of the He-He interaction, in concert with the high excitation energy of the helium atom and hence its “almost elementary” character, make the helium liquids excellent laboratories to test the accuracy of quantum many-body methods.

These systems – i.e., bulk $^4\text{He}$ and $^3\text{He}$ – remain liquid at zero temperature. In the case of isotopic mixtures of helium, there is incomplete phase separation, the maximum solubility of $^3\text{He}$ in liquid $^4\text{He}$ being about 6.5% at zero pressure. The coexistence of Fermi and Bose statistics, along with the small concentration of the fermionic component in solution, make $^3\text{He}–^4\text{He}$ mixtures a very interesting class of systems. To study the ground state of such a mixture, in 1982 we derived [4] a system of seven coupled, nonlinear HNC/FHNC integral equations. Soon afterward, we studied the momentum distributions in the mixture and the dependence of the condensate fraction of $^4\text{He}$, i.e., the fraction of $^4\text{He}$ atoms in the zero-momentum state, on the concentration of $^4\text{He}$ [5]. We predicted a small increase of the condensate fraction upon increase of the $^3\text{He}$ concentration, based on the fact that the total density of the mixture decreases, due to the larger mobility of $^3\text{He}$ atoms. The discrepancies of our predictions from the results of subsequent deep-inelastic neutron-scattering experiments required us to revise our calculational approach. Upon introducing much more sophisticated wave functions, including both two- and three-body correlations, we ultimately obtained results [6] that are in agreement with recent experimental data.

The limit of zero $^3\text{He}$ concentration defines the impurity problem. The experimental chemical potential of the $^3\text{He}$ impurity at the saturation density of liquid $^4\text{He}$ is $-2.785$ K, which is to be compared with the binding energy per particle of liquid $^4\text{He}$, $-2.5$ K. Another important property is the excitation spectrum of the impurity, characterized by its effective mass $m^*/m_3$ of 2.3 at zero momentum. In 1986 we considered all these observables at the variational level using backflow correlations, obtaining an effective mass of $m^*/m_3 = 1.7$ – clearly far away from the experimental value. This result forced us to carry out an exhaustive analysis within correlated-basis perturbation theory. We improved the wave function by including backflow correlations not only around the impurity, but also around the $^4\text{He}$ atoms in the medium. The additional correlations are described in terms of a basis in which the momentum of the excitation is shared between the Feynman phonons that can be excited in the medium. Finally, by including all the perturbative diagrams involving up to two independent phonons, we arrived at an effective mass of $m^*/m_3 = 2.2$. This analysis was extended to the full range of momentum in order to study the momentum dependence of the effective mass [8]. This dependence is important for an understanding of the two branches of the spectrum of elementary excitations of $^3\text{He}–^4\text{He}$ mixtures, namely the phonon and roton excitations of the $^4\text{He}$ medium, which are affected very little by the presence of $^3\text{He}$ atoms, and the $^3\text{He}$ quasiparticle excitations characterized by the effective mass studied in the impurity problem. We were then naturally led to investigate the response of $^3\text{He}–^4\text{He}$ mixtures [9] in the range of low momentum transfer were the two branches of the response appear well separated in energy.

At the same time, we have analyzed the variational contents of the Average Correlation Approximation, i.e., the approximation in which the same correlations are assumed for all pair of particles. This enabled us to use the impurity as a probe in liquid $^4\text{He}$ and to obtain a lower bound on the kinetic energy per particle of liquid $^4\text{He}$,

$$t_4(\rho) \geq \left( \frac{m_4}{m_3} - 1 \right)^{-1} \left( \mu_3^{exp}(\rho) - \mu_4^{exp}(\rho) \right).$$

The lower bound (3) was useful in constraining the value of the kinetic energy of liquid $^4\text{He}$ extracted from deep-inelastic neutron scattering data [10]. Elucidation of the analogies between the deep-inelastic regime for liquid $^3\text{He}$ and that of inelastic electron scattering off nuclei was also an important piece of work [11, 12]. In addition, the $^4\text{He}$ impurity problem in $^3\text{He}$ liquid received our attention. In this case, the microscopic calculation of the binding energy and effective mass of the $^4\text{He}$ impurity, based on an extended Jastrow-Slater wave function including two- and three-body correlations along with backflow correlations between the $^4\text{He}$ atom and the particles in the medium, produced the result $m^*/m_4 = 1.21$ at the $^3\text{He}$ saturation density, in very good agreement with the experimental value [13].

In 1995 Bose-Einstein condensation (BEC) was realized experimentally in a system of dilute, ultracold, magnetically trapped alkali atoms. In this case all of the atoms reside in the condensate, in contrast to the case of liquid $^4\text{He}$, for which the bosons are strongly interacting and the condensate fraction near absolute zero is only about 8%. We have dedicated some efforts to the microscopic description of the dilute trapped-atom systems.

Specifically, we have studied the ground state of Bose hard spheres confined by a harmonic trap, in order to cast light on the effects of the interatomic correlations and the accuracy of the mean-field Gross-Pitaevskii commonly used to describe the trapped-atom systems. This study led us to propose a modified Gross-Pitaevskii equation [14–16], based on a local density approximation. This is still a mean-field description, with all the atoms residing in the condensate, but it incorporates additional terms of the low-density expansion of the energy of a homogeneous hard-sphere system.
The existence of Feshbach resonances has made it possible to tune the interatomic potential experimentally, so it has become necessary to give serious consideration to correlation effects in the field of cold atoms.

It is fitting that in one of his last publications [17], Adelchi put his stamp, with some irony, on an old problem that is yet of great current importance, both within nuclear physics and in the new field of fermionic cold atoms. We refer to the pairing problem in infinite nucleonic matter, which can reasonably be approached with a trial ground state that incorporates state-dependent Jastrow-type two-body correlations as well as BCS pairing correlations. At present, the results from this approach, as followed by Adelchi and his collaborators, do not show very significant differences from those obtained in the standard BCS approximation. Certainly, more efforts will be devoted to this line of research, which continues to be full of both promise and surprise.

### III. CORRELATED-BASIS-FUNCTION THEORY OF NUCLEAR MATTER

Nuclear matter can be thought of as a giant nucleus, with given numbers of protons and neutrons interacting through nuclear forces only. Calculation of the binding energy of such a system, whose equilibrium value can be inferred from nuclear systematics, is greatly simplified by translational invariance. A quantitative understanding of the properties of nuclear matter, besides being a necessary intermediate step toward the description of real nuclei, is needed to develop realistic models of neutron-star matter.

When Adelchi was beginning his scientific career at the end of the 1970s, the study of nuclear matter was regarded as the hottest topic in microscopic many-body theory. The accuracy of calculations carried out using the widely accepted formalisms of G-matrix perturbation theory and the hole-line expansion was being questioned by the results of new variational approaches based on correlated wave functions and cluster-expansion techniques. In the spring of 1977 a number of outstanding physicists, including the Nobel Laureate Hans Bethe, had gathered at the University of Illinois in Urbana–Champaign to attend a Workshop on Nuclear and Dense Matter, aimed at assessing the status of the field and tracking down the sources of the striking disagreements between the results of different many-body approaches. As mentioned in Section I, the Conference on Recent Progress in Many-body Theories, held the following year at ICTP in Trieste, was also largely devoted to nuclear matter.

Adelchi played an important role in the development of CBF theory for nuclear matter and its quantitative implementation. Collaborating with Bob Wiringa in the 1980s, he carried out a detailed study of the equation of state of charge-neutral nucleon matter in weak equilibrium. The results set the standard of the field for over a decade and have been employed in a number of calculations of neutron-star properties.

At about the same time, the large body of electron-scattering data flowing from the new facilities operating in both Europe and the U.S. provided crucial information on nuclear dynamics, exposing the limitations of the nuclear shell model and confirming the importance of nucleon-nucleon correlations. Extrapolation of the available data to the limit of an infinite target allowed one to extract empirical information on the linear response and Green’s function of nuclear matter at equilibrium density.

The CBF formalism is ideally suited to carry out theoretical studies of correlation effects on electron-scattering observables. Adelchi was quick to realize this potential and engaged in a number of projects aimed at producing quantitative predictions to be compared with the data.

Calculations at moderate momentum transfers (|q| < 0.5 GeV) can be performed using nonrelativistic wave functions to describe both the initial and final nuclear states, and expanding the nuclear current operator in powers of |q|/m, m being the nucleon mass. Within this approach, Adelchi and Stefano Fantoni carried out a CBF calculation of the response of nuclear matter to longitudinally polarized photons [19], which is measured in inclusive electron-nucleus scattering. Their results clearly demonstrated that correlation effects dominate the nuclear cross section in the regions of both low and high electron energy loss.

At higher values of |q|, corresponding to beam energies larger than ~1 GeV, the description of the final states in terms of nonrelativistic nucleons is no longer possible. Calculations of the nuclear cross section in this regime require a set of simplifying assumptions that enable treatment of the relativistic motion of final-state particles carrying momenta ~q, as well as inelastic processes leading to the appearance of hadrons other than protons and neutrons.

The impulse-approximation scheme is based on the assumption that, since the spatial resolution of a probe delivering momentum q is ~1/|q|, at large enough |q| the process of e-scattering off a nuclear target reduces to the incoherent sum of elementary processes involving individual nucleons. As a consequence, the nuclear cross section can be written in terms of the spectral function, i.e., the Green’s function yielding the energy and momentum distribution of the target nucleons.

Adelchi made relevant contributions both to the analysis of the limits of the impulse approximation [12] and to the determination of nuclear-matter Green’s functions [20, 21] within CBF. The results of these calculations are still routinely used in the analysis of data from electron-scattering experiments. Further studies of the analytic structure of the nuclear-matter Green’s function also led to a generalization of Migdal’s theorem to momenta different from...
the Fermi momentum, thus providing a clearcut identification of correlation effects in the spectroscopic strengths measured in high-resolution proton knock-out experiments [22].

While being very fond of the beauty of the mathematical formalism of many-body theory, Adelchi was also keenly interested in phenomenology and managed to interact with experimentalists in a remarkably productive fashion. Starting in the early 1990s, he was part of a collaboration including Ingo Sick that developed a practical methodology for describing final-state interactions in electron-nucleus scattering within CBF [23, 25]. During this period, Adelchi coauthored a paper that was very peculiar for a theorist: a letter on the interpretation of the ratios of inclusive nuclear cross sections that contained no equations [26].

Perhaps the most amazing aspect of Adelchi’s personality was the contrast between his capacity to carry out truly outstanding research of lasting impact, and his ironic and almost self-mocking attitude. He certainly believed that, in the words of the Latin poet Horace, “humor does not prevent one from speaking the truth” (“ridentem dicere verum, quid vetat?” (Horace, Sat I.1.24)).

IV. CORRELATED BASIS FUNCTIONS FOR FINITE NUCLEI

At the beginning of the 1990s, we started a major project aimed at applying the FHNC/SOC computational scheme to finite nuclear systems. The idea of the project was triggered by Stefano Fantoni who, together with Sergio Rosati, had in the late 1970s proposed a formal extension of FHNC theory to finite Fermi systems. The original FHNC equations must be reformulated to account for the effects of the nonuniform density of atomic nuclei; this is done by the introduction of so-called vertex corrections. The modified FHNC equations are known as Renormalized Fermi Hypernetted Chain (RFHNC) equations.

The results of the first numerical application of the RFHNC equations to finite nuclear systems were presented in Ref. 27. In that article, model nuclei were considered. Proton and neutron wave functions were produced by a unique mean-field potential, in an ls coupling scheme. The nucleon-nucleon interactions employed contain only central terms, and the correlations are scalar functions. This simplified problem was used to test the theoretical and numerical feasibility of the approach. Results for binding energies of $^{16}$O and $^{40}$Ca model nuclei were reported in [27], while the corresponding momentum distributions were presented in a follow-up article [28].

A more realistic description of doubly-closed-shell nuclei was given in Ref. 29. Protons and neutrons were treated separately, and the single-particle wave functions were written in a jj coupling scheme. The RFHNC equations required a non-trivial reformulation. Binding energies, matter densities, and momentum distributions have been calculated for various doubly-magic nuclei up $^{208}$Pb. Also in this case, however, simple central interactions and scalar correlations were assumed. Within the same framework, we described hypernuclei containing a single Λ particle by considering the hyperon as an impurity in the nucleonic system [30].

In the next step, the RFHNC equations were extended to handle operator-dependent correlations that do not commute with the hamiltonian, nor among themselves. The recourse for implementation was the single-operator-chain (SOC) approximation. Because of the technical difficulties involved, the RFHNC/SOC equations have first been formulated to deal with spin- and isospin-saturated nuclei, and the single-particle wave functions described in the ls coupling scheme. Again, only $^{16}$O and $^{40}$Ca nuclei could be treated. The results of these calculations were presented in Ref. 31. In this phase of the project, Adelchi’s great expertise in FHNC nuclear-matter calculations was exploited to the utmost. The calculations of Ref. 31 were performed with two-nucleon interactions only. The results of fully realistic calculations, in which two-nucleon interactions of the Argonne-Urbana family are supplemented with appropriate three-body forces, have been described in Refs. 32, 33.

A formulation of the RFHNC/SOC equations general enough to deal with protons and neutrons separately in the more realistic jj coupling scheme was finally achieved. In Ref. 34, binding energies and density distributions have been reported for the $^{12}$C, $^{16}$O, $^{40}$Ca, $^{48}$Ca, and $^{208}$Pb nuclei. To the best of our knowledge, these are the first calculations of medium-heavy nuclei carried out with fully realistic interactions containing both two- and three-body forces.

We feel now as if we have reached the top of the mountain. The hardest work is done, and we have before us an expansive vista of promising applications of the RFHNC/SOC computational scheme. The absence of Adelchi weighs heavily on this new phase of our project. His talent, his leadership, his experience and, not least, his subtle sense of humor, would have given a different shape to our future work.

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Ground state of medium-heavy doubly-closed shell nuclei in correlated basis function theory

One Body Density Matrix, Natural Orbits and Quasi Hole States in Correlated ground state of

Ground state of N=Z doubly closed shell nuclei in CBF theory

Inclusive cross section ratios at

A correlation model for lambda-hypernuclei

Model calculations of doubly closed shell nuclei in CBF theory (I)

Higher-order effects in inclusive electron-nucleus scattering

Scattering of GeV electrons by nuclear matter

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Microscopic calculation of the longitudinal response of nuclear matter

S Energy and structure of dilute hard- and soft-sphere gases

Bose-Einstein condensates in the large gas parameter regime

Beyond the Gross-Pitaevskii equation approximation: Local density vs. correlated basis approach for trapped bosons

Effective mass of one

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Deep inelastic response of liquid Helium

Model independent bounds for the potential and kinetic energy of liquid ^4He at zero temperature

Deep inelastic response of liquid Helium

Microscopic calculation of the excitation spectrum of one ^3He impurity in liquid ^4He

A microscopic approach to the response of ^3He-^4He mixtures

Model calculations of doubly closed shell nuclei in CBF theory III. j-j coupling and isospin dependence

Model calculations of doubly closed shell nuclei in CBF theory (II). One body density matrix and momentum distribution

Model calculations of doubly closed shell nuclei in CBF theory (I). One body density matrix and momentum distribution

A correlation model for lambda-hypernuclei

Ground state of N=Z doubly closed shell nuclei in CBF theory

Correlated ground state of ^16O and ^40Ca

One Body Density Matrix, Natural Orbits and Quasi Hole States in ^16O and ^40Ca

Ground state of medium-heavy doubly-closed shell nuclei in correlated basis function theory