Integrability in AdS/CFT correspondence

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AdS/CFT duality predicts a weak-strong coupling relation between type IIB superstring moving on the background $AdS_5 \times S^5$ and four dimensional superconformally invariant $\mathcal{N} = 4$ super Yang-Mills (SYM) theory. The two sides of the correspondence are quantum integrable. In particular, string state energies are in 1-1 correspondence with anomalous dimensions of composite operators in the gauge theory. The integrable structure is well understood at least in the non-trivial 't Hooft planar limit of the gauge theory which corresponds to free quantum superstring. The factorized S-matrix and its Bethe equations are known in the perturbative regime and provide multi-loop anomalous dimensions of several classes of operators completely bypassing the evaluation of huge numbers of Feynman diagrams.

This very nice picture has a very interested connection with QCD which shares with $\mathcal{N} = 4$ SYM the pure gauge gluon sector. Indeed, anomalous dimensions of twist-2 operators are essentially identified with splitting functions appearing in the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) description of deep inelastic scattering (DIS). This means that integrability offers the unique possibility of studying physical properties of splitting functions at prohibitively large loop order.

Indeed, it is well known that splitting function, in usual Bjorken parametrization $P = P(x)$ have a very rich physical structure. The quasi-elastic $x \to 1$ limit is governed by soft gluon emission which is known to be almost-classical in the Low-Burnett-Kroll sense and universal. The leading contribution to $P(x)$ in this limit is related to the universal cusp anomalous dimension of Wilson loops. The opposite diffractive limit $x \to 0$ is instead described by the Belitsky-Fadin-Kuraev-Lipatov equation which predicts the leading singularities in the analytically continued Regge amplitudes. In the DIS context, this means the Pomeron trajectory. Also, and recently, $P(x)$ is also known to obey a very deep physical constraint which we dub generalized Gribov-Lipatov reciprocity. It stems from the simple observation that the DIS process and electron-positron annihilation into hadrons are crossed processes. This crossing symmetry can be enforced at the level of the two radiation cascades leading to a reformulation of a reciprocity respecting DGLAP evolution. When this is done, the splitting functions display a very interesting hierarchy of Moch-Vogt-Vermaseren-like relations in the subleading terms as $x \to 1$.

All these properties can be tested in the simpler context of $\mathcal{N} = 4$ SYM where one consider twist L operators in the $sl(2)$ closed subsector of the full $psu(2, 2|4)$ theory. These have the general form

$$\mathcal{O}_{n_1 \cdots n_L} = \mathrm{Tr}\,\{D^{n_1}\varphi \cdots D^{n_L}\varphi\}\,,$$

where φ is one of the scalar fields and D is a light-cone projected covariant derivative.

In the last year, we have obtained several important results for these operators and related ones. In particular our main results have been

1. We have fully proved reciprocity of twist-2 fields at four loops [2].
2. Reciprocity holds at higher twist and we have computed the five-loop anomalous dimension for twist-3 fields proving its reciprocity symmetry among other properties [1].
3. Similar analysis have been devoted to larger higher rank sectors to cover twist-operators which at one-loop involve different elementary fields. This is particularly interesting for gauge operators [10,4].
4. Various technical problems related to the analysis of the quasi-elastic limit are solved in [11,5].
5. It is possible to devise sum-rules for the anomalous dimensions of twist-operators. This is interesting in connection to the problem of duality since the identification of string duals to twist-operators is completely solved only in the case of the ground state trajectory dual to multiply folded string semiclassical states[9].

All these problems concern the weakly coupled gauge theory side of the AdS/CFT correspondence. Of course, integrability says a lot about the flow to the strongly coupled regime where we have to compute corrections to semiclassical string states. In particular, it is very interesting to study the large spin corrections to the folded string which should provide a test of reciprocity at strong coupling [3,6].

Finally, in the strict thermodynamical limit, many simplifications occur in the strong coupling analysis since Bethe equations can be taken in a continuum limit and turned in more tractable integral equations. This is discussed in [8,7,12].

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