Renormalizable SU(5) $\otimes$ A_4 SUSY GUT

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The experimental discovery of flavor oscillations of neutrinos with the consequence that their mass is different from zero is certainly a clear indication that there is New Physics beyond the content of the Standard Model. One of the most attractive and beautiful scenario in which to try to set this information is represented by the Grand Unification Theories (GUT); these last ones, in fact, are able to realize the possibility to describe in a high energy limit ($\sim 10^{16}$ GeV) the unification of three of the four fundamental forces, the electromagnetic, the weak and the strong one, into a single gauge interaction, as suggested, without succeeding, from the same Standard Model as a consequence of the gauge coupling constants running. The first step to do in order to analyze a context of grand unification is, therefore, to choose a gauge group in which to embed the one of the Standard Model $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$; several anomalies free choices can be made [1], going from the minimal one, represented by SU(5), till the so called "exceptional" ones, represented by E_6 and E_8. As a further feature it's relatively simple to take into account also the introduction of supersymmetry with R-parity into this kind of context, using the fact that the gauge structure of the entire theory remains untouched [2]. Inside a unification theory, moreover, it is possible also to try to find an answer to some important and unsolved questions in flavor physics:

- the low energy data of the quark mixing matrix U_{CKM} that are well described by the Wolfenstein parametrization [3],

$$U_{CKM} \equiv U_U^\dagger U_D \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (1)$$

as well as the hierarchy for the quark masses

$$\begin{aligned} M_U^{diag} &\simeq & m_t(\lambda^8, \lambda^4, 1)\,, \\ M_D^{diag} &\simeq & m_b(\lambda^4, \lambda^2, 1), \end{aligned} \quad (2)$$

where $U_{U,D}$ are the left mixing matrix in the quark sector and $\lambda \approx 0.22$ is the Cabibbo angle;

- the difference between the CKM quark mixing matrix, described at first order by the Cabibbo rotation, and the lepton mixing matrix, well described through the *tri-bimaximal* matrix [4]

$$U_{TBM} \equiv U_E^\dagger U_\nu \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (3)$$

with a zero $(1,3)$ entry.

The introduction of a discrete flavor symmetry, i.e. a symmetry that acts on different families, and its breaking in the low energy limit can be able to reproduce if not all at least some of these phenomenological features as an example the choice of the discrete group A_4, the symmetry group of the tetrahedron [5], is motivated from the possibility to obtain (3) in a simple way just assigning a flavor symmetry charge to the Standard Model fields and introducing extra Higgs multiplet(s) in order to have a Majorana mass term for the neutrinos. Despite several attempts, however, seems to be hard to consider the combination of a flavor symmetry as A_4 within a gauge GUT symmetry, without losing important prerogatives as renormalizability or without turning to a simple and unwonted fit of the experimental data.

In order to investigate more deeply this aspect of the theory, we have choose to work into the simplest context of SUSY SU(5) unification, trying to add the A_4 flavor symmetry and to preserve its nice features without forsake the request of renormalizability. In the minimal $SU(5)$ scenario, the field content is

$$\begin{aligned} &\text{Standard Matter}: \qquad\qquad\qquad\qquad (4) \\ &\overline{\mathbf{5}}_{T,i} = \left(\widehat{D}_i^c, \widehat{E}_i, \widehat{N}_i\right), \mathbf{10}_{T,i} = \left(\widehat{U}_i^c, \widehat{Q}_i, \widehat{E}_i^c\right); \\ &\text{Higgs Sector}: \quad \mathbf{5}_H, \overline{\mathbf{5}}_H, \mathbf{45}_H, \overline{\mathbf{45}}_H, \mathbf{24}_H\,. \end{aligned}$$

where i is the family flavor index and where $\mathbf{5}_H$, $\overline{\mathbf{5}}_H$ and $\mathbf{24}_H$ are necessary in order to break the SU(5) gauge symmetry into the Standard Model one and successively into the residual $\mathrm{SU}(3)_C \otimes \mathrm{U}(1)_{em}$, while $\mathbf{45}_H$ and $\overline{\mathbf{45}}_H$ are necessary in order to avoid the wrong prediction $M_D^T = M_E$ in the Yukawa sector; with this content it's straightforward to write down the correct supersymmetric Lagrangian with exact R-parity symmetry.

A more realistic supersymmetric GUT based on SU(5) that we have considered [6] is the so called *Supersymmetric Adjoint* one [7], in which the mass matrix of the neutrinos is obtained with the hybrid type I + type III seesaw mechanism, that is with the possibility that the right handed neutrinos could form both a $SU(2)_L$ singlet (type I) and a $SU(2)_L$ triplet (type III); these contributions will be integrated out of the theory in order to have an effective Majorana mass operator for the light left handed active neutrinos, taking advantage of the fact that the adjoint representation of SU(5), or $\mathbf{24}_T$, contains simultaneously these two terms in its decomposition according to the Standard Model gauge subgroup. In this situation the field content of the theory is obtained by adding

$$\text{Exotic Matter} : \mathbf{24}_T. \tag{5}$$

The next step is to assign an A_4 charge to the previous listed supermultiplets, considering all the way in which this operation can be done in such a way to promote A_4 as a symmetry of the entire Lagrangian. We have found 5 allowed possibilities, but just one of these, unfortunately, leads to a phenomenologically interesting situation, that is the case in which all the supermultiplets of the theory transform under A_4 according to the irreducible three dimensional representation except for $\mathbf{24}_H$ that is a A_4 singlet while in the Higgs sector the supermultiplets have to acquire a vev into the following directions (into the flavor space):

$$\langle \mathbf{5}_H \rangle = v_5(1,0,0), \quad \langle \overline{\mathbf{5}}_H \rangle = v_{\overline{5}}(1,0,0); \tag{6}$$

$$\langle \overline{\mathbf{45}}_H \rangle = v_{\overline{45}}(1,1,1), \quad \langle \mathbf{45}_H \rangle = (v_{45}, \delta v_{45}, \delta v_{45}); \tag{7}$$

the presence of the A_4 symmetry is able to give to the mass matrices in the charged leptonic sector the nice form:

$$M = \begin{pmatrix} 0 & \gamma_1 & \gamma_2 \\ \gamma_2 & 0 & \Gamma_1 \\ \gamma_1 & \Gamma_2 & 0 \end{pmatrix}, \tag{8}$$

where a kind of hierarchy between the first and the other two families could be reproduced after a block diagonalization assuming:

$$\Gamma_i \propto v_5, v_{\overline{5}} \gg \gamma_i \propto v_{45}, v_{\overline{45}}; \tag{9}$$

we are in fact in this case able to fit the data of the masses for (u,c,t), (d,s,b) quarks and for (e,μ,τ) as well as the off diagonal corrections for the CKM-matrix, defined as functions of the same parameters γ_i and Γ_i in the mass matrices (8) as a consequences of the rotations in the bi-unitary diagonalization procedure. Going into the neutrino sector, the Majorana mass matrix for the left handed neutrinos comes from, as previously explained, two simultaneous contributions of type I and type III seesaw mechanism, leading to the general form:

$$M_\nu = \left(M_\nu^I\right)^T \frac{1}{M_1} M_\nu^I + \left(M_\nu^{III}\right)^T \frac{1}{M_3} M_\nu^{III}, \tag{10}$$

where M_ν^I and M_ν^{III} are the mass matrices from the Dirac interactions between the left handed neutrinos and the heavy right handed ones while M_3 and M_1 are the Majorana masses for the same right handed triplet and for the singlet. In this situation the action of a flavor symmetry can just give a structure inside the matrices M_ν^I and M_ν^{III}, because they directly come from the Lagrangian, but this last feature is completely lost when these matrices are combined in (10). As a consequence in this hybrid context the A_4 symmetry loses its interest as a good symmetry for the tri-bimaximal mixing. Even if with this important conclusion in mind, we are able to obtain, just from a brutal fit, the predicted mixing in (3) from (10).

In this work we have considered the possibility to enlarge the SU(5) gauge unification symmetry with the inclusion of the discrete flavor symmetry of the A_4 group. We are able to reproduce the exact path that leads to a correct description of the mixing in the leptonic sector without losing a fundamental propriety of the unification theory as the renormalizability in the context in which the neutrino mass matrix is generated through the mediation of an extra matter triplet contained into the $\mathbf{24}_T$ multiplet of SU(5). Even if the GUT scale is very close to the Planck scale, in fact, we think that renormalizability have to be a fundamental characteristic of the unification theory, in order to avoid the presence of high-energy operators. However in this model the nice feature of the A_4 flavor symmetry is not conserved and the mixing can be reproduced only with a fit. Other renormalizable models with the same flavor symmetry and gauge group are under investigations.

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