# Relativistic bound states

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The interest in this subject stems from the discovery of a mass for the neutrino. Even if the neutrino interacts only through the weak interactions, one must wonder if say a neutrino-lepton bound state exists.

The first question would seem to be *are the weak interactions an attractive force*? Actually, since we are considering the bound states of two fermions, it can be shown that the scalar and tensor terms have different signs for the potential. So one of them must be a candidate bound state. In non relativistic potential theory there is a strange effect. In one dimension, a well potential will yield a bound state if it is sufficiently deep and broad. In three dimensions there is always a bound state. This is connected to the fact that the radius variable is non negative and this implies an infinite potential wall at the origin. Considering the Schrödinger equation it can be shown that for weak Yukawa potentials bound states do not arise. However, in previous works [1,2], Stefano De Leo and I have shown that there is a relativistic effect (Yukawa amplification) which suggests that such a bound states might indeed exist. These states would be highly relativistic and thus beyond the simple treatment of potential bound states.

The first relativistic corrections to the Schroedinger equation contains the Darwin term:

$$\nabla^2 V(r)$$

For a Coulomb potential this gives the essential delta function term that makes bound state levels only j-dependent. An automatic result for the Dirac equation. But for a Yukawa potential with an exchanged particle mass of M, there is an additional contribution proportional to $(M/m)^2$ times the original Yukawa (where m is the reduced particle mass).

We have tried to treat this directly with the Dirac equation without the need of approximations but as the mass of the exchanged particle increases, the bound state drops into the Klein energy zone where only oscillatory solutions exist everywhere. This obfuscates the tracking of bound states. This is probably a limitation of the single particle equation itself.

This has lead to the question of proving or disproving the existence of a bound state in field theory. This is surprisingly hard to do. The best suggestion is the comparison of tree and box diagrams. Bound states exist when these become comparable in strength. This works for low mass exchanges but for weak interactions the analysis must be extended to high mass exchanges (those of the intermediate vector bosons). We have tested this for some simple models such as interacting bosons with boson exchanges (ref.1) and for interacting fermions (always with boson exchanges). In both these cases the existence of bound states has been excluded. However neither is a gauge theory. The difficulty of applying these techniques mediately to weak interacting fermions is that the calculations must be renormalized and both neutral and charged currents need to be included. This study is ongoing.

## REFERENCES

1. S. De Leo and P. Rotelli, Phys. Rev. D, 78 (2008) 025006
2. S. De Leo and P. Rotelli, Eur. Phys. Jour. C. 53 (2008) 659