Nonlinear Physics

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The main objectives of this research activity are:

i) the study of new algebraic and analytic aspects of models integrable or solvable in some sense, both continuous and discrete, with special attention to multidimensional systems, and their physical applications, f.i. in field theory, plasma physics, nonlinear optics, multidimensional ferromagnets and surface and interface dynamics;

ii) the application of techniques of the theory of integrable and solvable systems to several interesting problems in mathematics (differential geometry, discrete geometry) and in quantum information theory and the extension of these techniques to non integrable physical systems, f.i. in topological quantum field theory, string theory, Einstein equations.

The scientific activity is articulated along different research lines (to the extent they can be separated, there being of course much synergy among them). An outline of the research program in each of these lines, including the identification of the researchers (both internal and external) involved, is set out below. A list of recent publications and scientific activities by group members can be found, instead, at `http://www.fisica.unisalento.it/~landolfi/NLPA.pdf`.

Extended Resolvent and applications (Boiti, Pempinelli, Prinari with A Pogrebkov (Moscow))

In several years of cooperation a new mathematical object, called Extended Resolvent, has been developed. This object generalizes the classical resolvent of differential operators and it can be used to study the nonlinear integrable evolution equations, as the Kadomtsev-Petviashvili equations (KPI and KPII), specifically in the physically interesting case of wave solutions. The case of N solitons on an arbitrary background has been completely solved for the first time for KPI. During 2008, using the Extend Resolvent approach, we studied the spectral properties of the heat operator which is the linear spectral problem associated to KPII. We introduced twisting transformations of the heat operator and used them, at the same time, to superimpose à la Darboux N solitons to a generic smooth, decaying at infinity, potential and to generate the corresponding Jost solutions. These twisting operators are also used to study the existence of the related Extended Resolvent. Existence and uniqueness of the Extended Resolvent is considered in detail by means of the special examples. In particular, we showed that here there exist annihilators of the extended heat operator that are exponentially decaying functions of spatial variables. Existence of these annihilators is the new and unexpected feature of such potentials, that has no analog either in the one-dimensional, or in a one-soliton two-dimensional cases.

Inverse Scattering, its extensions and applications (Prinari with M Ablowitz and S Chakravarty (Colorado), G Biondini (Buffalo), A Trubatch (Montclair University), M Lo Schiavo (Roma), K Maruno (ITPA))

We have studied the strong coupling limit of nonlinear Schrodinger systems by using Hirotas bilinear method and the Pfaffian technique. We have written an invited contribution to a peer reviewed encyclopedia on continuous and discrete Nonlinear Schrodinger systems and their applications. We also have recently started investigating problems of population dynamics and quality analysis using generalized kinetic methods and dynamical systems approaches. More precisely, we proposed a tool to analyze and predict the quality evolution of a social structure with respect to time when the external events are known. The approach is based on the balanced evolution of statistical variables at a microscopic level and a set of evolution equations that describe the dynamics of the probability density function of each population over these microscopic variables. The density functions are then used to construct the phenomenological, macroscopic picture.

D-bar-dressing method, its extensions and applications (Konopelchenko with L Martinez Alonso (Madrid), E Medina (Cadiz) and L Bogdanov (Moscow), W K Schief (Berlin), G Ortenzi (Milano))

Novel pure algebraic approach to the theory of deformations of associative algebras has been developed. It contains theories of coisotropic, quantum and discrete deformations as particular realizations. It is shown that such deformations are described by dispersionless, dispersive, discrete integrable systems like the oriented associativity equation, WDVV equation, Kadomtsev-Petviashivili hierarchy, its discrete version, Hirota-Miwa equation, Darboux system and third order ordinary differential equations with the Painleve property from the Chazy-Bureau list. These results demonstrate deep algebraic roots of the integrable systems.

The quasiclassical generalized Weierstrass representation for corrugated surfaces has been found and studied. It was shown that the deformations of such surfaces is governed by the dispersionless modified Veselov-Novikov hierarchy.

Symmetries of Nonlinear Field Models (Martina with P Horvathy (Tours), D Levi (Rome III), M Grundland and P Winternitz (Montreal), A Protogenov (Nizhny-Novgorod))

We have performed studies on the symmetry group structure of the models, the search of the integrability of special reductions appearing both in the HEP Skyrme-Faddeeev model and in CMP multi-parameters gauged Landau-Ginzburg theories (mainly with Protogenov), which admit knotted vortex-like structures indexed by a topological index. Special approximated solutions are discussed.

Entanglement and Quantum Computation (Soliani, Martina and Ruggeri)

The quantum entanglement quantification and measurement has been considered for bipartite systems which have infinite dimensional Hilbert spaces. A toy model of oscillators has been introduced and the concept of entanglement gap has been applied, remaking the experimental conditions under which measurements of energy on a sub-system can be witness of entanglement for the whole system.

Generalized observables and phase operators in Quantum Mechanics (Landolfi and Gianfreda with M.G.A. Paris (Milan))

The research activity deals with the problem of defining phase/angle operators for quantum systems. Spectral properties of a formal Weyl-ordered expansion for the action operator associated with systems described by time-depending quadratic Hamiltonians are currently under study. Possible results are expected to be useful in describing the loss of coherence during evolution of systems of interest in Quantum Cosmology and Quantum Optics. Besides, generalized measurements schemes for the feasible phase of some non-normal multi-photon operators are under development.

Chaotic systems and applications (Renna and D'Innocenzo)

Starting from the equation, which describes the motion of a linear oscillator subjected to an external position-dependent force, we have obtained a two-dimensional map that in the limit of zero dissipation coincides with the Poincar map of a kicked oscillator. The behavior of the oscillator was investigated numerically as a function of the strength of external odd perturbations and of the friction parameter. The map can be continued to the limit of very strong dissipation, where it turns into a one-dimensional map. Also we have studied about a statistical model for climate change detection, based on a Bayesian approach.

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