Estimating the physical parameters of the Sgr A^* black hole

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J. A. Wheeler was one of the greatest scientists of the twentieth century because of his many contributions to nuclear physics, theory of quantum measurement, general relativity and relativistic astrophysics and to his memory has been dedicated the Second Italian-Pakistani Joint Workshop on Relativistic Astrophysics organized in 2009 in Pescara [1]. The last scientific paper by Wheeler was on retro-MACHOs (we prefer to call them retro-lensing or relativistic images), written in collaboration with D. Holz [2] (see also [3]). The authors took seriously something that was well known to be possible around a black hole (BH) in principle. They considered our Sun as the source of light rays that, if the impact parameter is right, may go around the BH and reach the observer forming a ring around the BH. Light rays with a slightly smaller impact parameter may go twice around the BH and then escape to the observer forming a slightly smaller ring, and so on. So, in the case of perfect alignment of source, lens and observer, an infinite series of rings should appear. Therefore, they suggested that a survey of the sky be made to look for these rings as a way to search for isolated BHs around our solar system.

In general, a complication of the effect is due to the Earth rotation about the Sun so that the retro-lensing image magnitude changes with time in a periodic way. The main problem of an eventual survey to search for such relativistic events is that our Sun is not a very bright source and even using the HST, only a BH heavier than 10 $M_\odot$ within 0.01 pc might be revealed in this way. Although it may be expected that sooner or later a survey of the sky to search for such rings will take place, it is for the moment much better to consider systems composed of massive BHs and luminous stars and look, therefore, towards well known BH candidates. This is exactly what has been suggested in [4] soon after the Holz and Wheeler paper appeared, that is the supermassive black hole at the galactic center (Sgr A^*) is the most interesting retro-lens to look for.

A classical method for estimating the physical parameters (in particular mass and angular momentum) of BHs and in particular of the candidate supermassive black hole at Sgr A^* is to look for the periastron or apoastron shift of the stars orbiting around it (the so-called S-stars). However, the amount of the apoastron shift strongly depends also on the distribution of both the stars and the dark matter around the BH making it practically impossible to estimate the BH parameters, due to the large number of model parameters involved. A more direct way to estimate the central BH parameters, including in principle also its electric charge) is to measure the shape [5,6] and also the spectrum of the retro-lensing images of the brightest and innermost stars in the K-band (or also the shape of the BH shadow in the radio band). In particular, as regards the photon spectral change if it goes around a spinning BH, the photon that go around very close to the BH enters and then exits the region where frame dragging is significant. This speeds up the BH by a minuscule amount if the photon is co-rotating with it (and slows it down when it is counter-rotating), as some angular momentum and energy are imparted to it while it carries the photon along. Now the photon will have somewhat more energy than when it was emitted by the source. The longer it is “carried along” the more energy is gained. The closer it gets to the hole the more the effect is. This is simply the analogue for photons, of the “gravitational slingshot” used to accelerate spacecraft, that gives us an estimate of the energy gained by the photon. One can obtain the photon frequency change by invoking the formula: $\delta\nu/\nu = \delta t/t$, where δt is the time delay due to the rotation of the hole and t is the time spent in the strong field of the hole. If the light goes round the hole and is reflected, the time taken is the distance traveled, which is half way round the hole (at light speed), which is πb. The time delay is due to the cross term in the metric, responsible for frame dragging, $2ma/b$ and acts over an angle 2π. Thus, the total effect for the spectral shift for

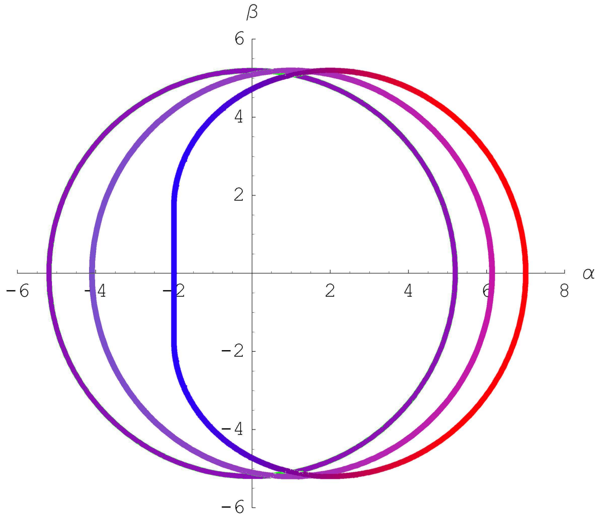

Figure 1. The precise shape of the retro-lensing image in the case of perfect alignment, with the spin perpendicular to the line of sight, is shown. Here, α and β are the celestial coordinates of the image as seen by an observer at infinity. The circular image corresponds to the retro-lensing image for a Schwarzschild black hole. The second image is that expected for a Kerr black hole with rotation parameter $a = 0.5$. The third one corresponds to a maximally rotating Kerr black hole with $a = 1$. The coloring of the rings is given to indicate the frequency shift due to the spin effect (not giving the precise color shifts). Here the black hole is assumed to rotate counterclockwise as seen from above. The unit of length along the coordinate axes α and β is M, so that the image in the Schwarzschild case is at a distance of $3\sqrt{3}/2$ Schwarzschild radii from the black hole center.

the $n^{\rm th}$ retro-lensed ring is

$$\left(\frac{\delta\nu}{\nu}\right)_n = 4\frac{ma}{b^2}(2n-1)\ . \qquad (1)$$

Indeed, $4ma/b^2$ is the effect that would be expected for the primary retro-lensing image. The secondary one would have an additional time delay of going once around, i.e. it would be 3 times this value and the next 5 times, so that for the $n^{\rm th}$ ring it would be $(2n-1)$ times this value. The shape of the retro-lensing image in the perfect alignment case is shown in Fig. 1. The circular shape corresponds to a Schwarzschild black hole, the second image to a Kerr black hole with $a = 0.5$, the third one to a maximally rotating Kerr black hole ($a = 1$). Here, the black hole is assumed to rotate counterclockwise as seen from above. As is clear from the discussion above, in the Schwarzschild case the retro-lensing image has everywhere the same color, since no spectral shift effects are present in this case. For a Kerr black hole, instead, the light rays co-rotating with the black hole form a closer image which is also blue-shifted with respect to that formed by counter-rotating photons. The maximal effect is obtained for a Kerr black hole with $a = 1$.

By combining these measurements, also taking into account the proper motion of the supermassive BH, it will be possible in the near future to estimate both the BH parameters and those of the stellar and dark matter distributions around it [7–9].

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