The Kolmogorov stochasticity parameter: a new tool to analyze CMB maps

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Since the discovery of the anisotropy of the cosmic microwave background (CMB) radiation by COBE, the pixelized sky maps of CMB temperature were among the basic sources of cosmological information [7]. Later experiments, including another full sky coverage by Wilkinson Microwave Anisotropy Probe (WMAP) [4], produced higher sensitivity and resolution maps, enabling the determination of the values of a set of cosmological parameters. In addition, CMB polarization measurements (see [6]) represent a very important channel of information, and higher accuracy data are expected in forthcoming experiments. Here we advance another concept of a digital sky, namely, a map of the degree of CMB temperature randomness using the Kolmogorov stochasticity parameter (KSP).

The stochastic parameter and the statistic introduced by Kolmogorov [5] lead to measurement of the objective randomness degree of finite sequences resulting from dynamical systems or number theory [1]. In particular, let $\{X_1, X_2, \ldots, X_n\}$ be n independent values of the same real-valued random variable X ordered in an increasing manner $X_1 \leq X_2 \leq \cdots \leq X_n$ and let $F(x) = P\{X \leq x\}$ be a cumulative distribution function (CDF) of X. Their *empirical distribution function* $F_n(x)$ is defined by the relations

$$F_n(x) = \begin{cases} 0 & x < X_1 \\ k/n & X_k \leq x < X_{k+1}, \quad k = 1, \ldots, n-1 \\ 1 & X_n \leq x. \end{cases}$$

Kolmogorov's stochasticity parameter λ_n has the following form ([5], [1])

$$\lambda_n = \sqrt{n} \sup_x |F_n(x) - F(x)|. \tag{1}$$

Kolmogorov proved in [5] that for any continuous CDF F, $\lim_{n\to\infty} P\{\lambda_n \leq \lambda\} = \Phi(\lambda)$, where $\Phi(0) = 0$,

$$\Phi(\lambda) = \sum_{k=-\infty}^{+\infty} (-1)^k e^{-2k^2\lambda^2}, \lambda > 0, \tag{2}$$

the convergence is uniform, and Φ (the Kolmogorov's distribution) is independent on F.

We apply KSP to measure the objective stochasticity degree of WMAP datasets. The creation of Kolmogorov's map (K-map) (Fig. 1) estimates the stochasticity parameter for ordered sequences of the CMB temperature values assigned to a set of pixels in a given region of the sky, as it was first performed in [3]. It does not seems trivial that the resulting digitized sky map would duplicate or, on the contrary, differ from the features in the conventional temperature maps. The first remarkable result in the obtained K-map is, indeed, the clearly distinguished Galactic disk (see Fig. 1), with high and nearly uniform Kolmogorov's parameter, thus outlining it upon the CMB contribution in the rest of the sky. The second remarkable results is related to the efficiency to find point sources in the CMB maps. Indeed, the Kolmogorov's parameter quantifies, for a numerical series, the degree of randomness of it, given an underlying probability distribution function. This degree of randomness can be related to possible inhomogeneities of the Universe and, in particular, it could help to disentangle the cosmological signal from a non cosmological one as the emission of point sources.

To this aim, we constructed and analyzed Kolmogorov's map for WMAPs 7-year temperature data and obtained a list of 12 regions of a degree scale with anomalously high value of Kolmogorov's function (Fig. 2). Among those regions, 6 coincide with sources given in the catalog of the WMAPs point sources, where they are identified as radio galaxies, while all 12 regions coincide with gamma-ray sources discovered by Fermi satellites Large Area Telescope (LAT) [2]; 4 among these regions in both catalogs are overlapping mutually. Indeed, the release of Fermi/LAT 1FGL catalog confirmed the prediction that all 12 CMB structures are identified in that catalog, thus showing the predictive power of the approach.

The future increase in the angular resolution of the CMB temperature maps will enable to apply this method to smaller scales, i.e. to obtain the averaged Kolmogorov function for large enough number of pixels, and hence to increase its source detecting power. At the available angular resolution of WMAP the method enables to identify at high confidence level only a sample of 12 regions, which is certainly smaller than the number of sources identified by WMAPs mask. However, even at present resolution the method identifies sources not noticed by WMAPs procedure, which appear to be Fermi/LAT gamma sources [2]. Most of those sources appear to be blazar/quasars and active galaxy, i.e. multi-wavelength and superluminous objects, showing a new method for their identification based on totally different idea, the statistic of the signal. The absence of some of the detected sources in the initial 0FGL catalog and their appearance in the next, 1FGL catalog, shows the predictive power of the approach. The Kolmogorov parameter thus enables to detect point sources in the CMB maps not noticed by other criteria. The applicability of the method will increase at higher resolution maps as those expected from the PLANCK mission.

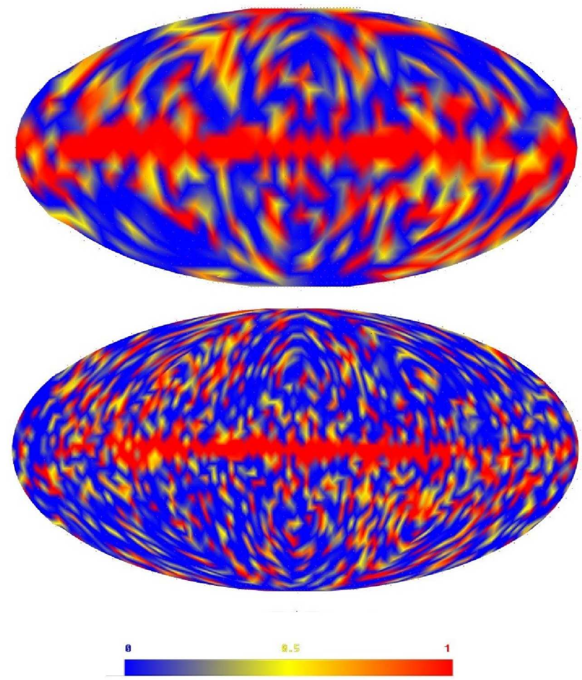

Figure 1. Kolmogorov maps, i.e. the degree of randomness in CMB sky. WMAP's 5-year W-band, 94 GHz data are used; upper map is for Nside = 8, the lower for Nside = 16. The Galactic disk is clearly distinguished.

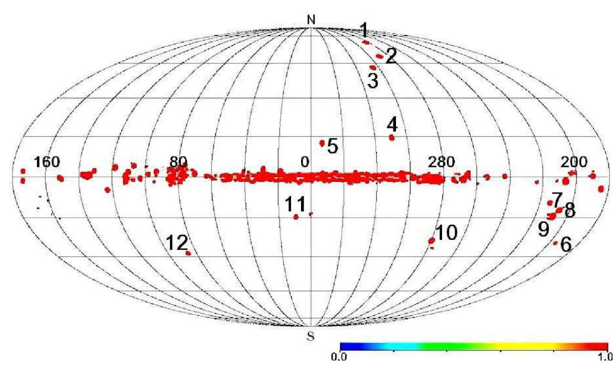

Figure 2. The location of the 12 high Φ regions in the sky, i.e. those outside the Galactic disk with $|b| > 10°$.

REFERENCES

1. V. Arnold. OPEN PROBLEM: Orbits' statistics in chaotic dynamical systems. *Nonlinearity*, 21, 2008.
2. The Fermi-LAT Collaboration. Fermi Large Area Telescope First Source Catalog. *Astrophys. J. Suppl.*, 188:405–436, 2010.
3. V. G. Gurzadyan and A. A. Kocharyan. Kolmogorov stochasticity parameter measuring the randomness in the cosmic microwave background. *Astron. Astrophys. Lett.*, 492:L33–L34, 2008.
4. G. Hinshaw et al. Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations:Data Processing, Sky Maps, & Basic Results. *Astrophys. J. Suppl.*, 180:225–245, 2009.
5. A. N. Kolmogorov. Sulla determinazione empirica di una legge di distribuzione. *G.Ist.Ital.Attuari*, 4:83–91, 1933.
6. John Kovac et al. Detection of polarization in the cosmic microwave background using DASI. *Nature*, 420:772–787, 2002.
7. George F. Smoot et al. Structure in the COBE differential microwave radiometer first year maps. *Astrophys. J.*, 396:L1–L5, 1992.