Self-consistent CRPA formalism with finite-range interaction

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Self-consistent Random Phase Approximation (RPA) approaches which consider the continuum have been proposed already in the second half of the '70s, but they are applicable only to zero-range interactions. There are however various drawbacks in the use of these forces, many of them discussed already in Ref. [1] where the D1 parameterization of the finite-range Gogny interaction was proposed. Other drawback are related to the fact that RPA calculations with zero-range interactions produce more collectivity with increasing value of the momentum transfer than those done with finite-range interactions [2]. In addition, finite-range interactions provide a better description of unnatural parity excitations [3–5]. Finally, finite-range interactions are more directly comparable with microscopic nucleon-nucleon force. In the literature there are few examples of CRPA calculations done with finite-range interactions [6,7], and, to the best of our knowledge, only a single case of self-consistent Continuum RPA (CRPA) calculation [8].

We present a technique which treats, without approximations, the continuum part of the excitation spectrum in RPA calculations with finite-range interactions [9]. The CRPA secular equations whose solution provides the values of the RPA amplitudes X and Y can be written as

$$
\begin{aligned}
&(\epsilon_p - \epsilon_h - \omega)\, X^{\nu}_{ph}(\epsilon_p) \; + \\
&\sum_{p'h'} \sum\!\!\!\!\!\!\int_{\epsilon_{p'}} \left[v^{J}_{ph,p'h'}(\epsilon_p,\epsilon_{p'})\, X^{\nu}_{p'h'}(\epsilon_{p'}) \right. \\
&\left. + u^{J}_{ph,p'h'}(\epsilon_p,\epsilon_{p'})\, Y^{\nu}_{p'h'}(\epsilon_{p'}) \right] \;=\; 0\,, \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
&(\epsilon_p - \epsilon_h + \omega)\, Y^{\nu}_{ph}(\epsilon_p) \; + \\
&\sum_{p'h'} \sum\!\!\!\!\!\!\int_{\epsilon_{p'}} \left[v^{J*}_{ph,p'h'}(\epsilon_p,\epsilon_{p'})\, Y^{\nu}_{p'h'}(\epsilon_p) \right. \\
&\left. + u^{J*}_{ph,p'h'}(\epsilon_p,\epsilon_{p'})\, X^{\nu}_{p'h'}(\epsilon_{p'}) \right] \;=\; 0\,. \qquad (2)
\end{aligned}
$$

In the above equations v and u contain the direct and exchange interaction matrix elements, ϵ_α is the single particle energy of particle or hole α and ω the excited state energy. Our method of solving the CRPA equations consists in reformulating the secular equations (1) and (2) with new unknown functions which do not have an explicit dependence on the continuous particle energy ϵ_p. The new unknowns are the channel functions f and g defined as:

$$
f^{\nu}_{ph}(r) \;=\; \sum\!\!\!\!\!\!\int_{\epsilon_p} X^{\nu}_{ph}(\epsilon_p)\, R_p(r,\epsilon_p)\,, \qquad (3)
$$

and

$$
g^{\nu}_{ph}(r) \;=\; \sum\!\!\!\!\!\!\int_{\epsilon_p} Y^{\nu *}_{ph}(\epsilon_p)\, R_p(r,\epsilon_p) \qquad (4)
$$

where R is the radial part of the single particle wave function obtained with Hartree Fock (HF) calculations. In this way we write a new set of CRPA secular equations with these unknowns:

$$
\begin{aligned}
&\mathcal{H}\,[f_{ph}(r)] - (\epsilon_h + \omega) f_{ph}(r) = \\
&-\mathcal{F}^{J}_{ph}(r) + \sum_{\epsilon_i<\epsilon_{\rm F}} \delta_{ip} R_i(r) \int {\rm d}r_1 r_1^2 R^*_i(r_1) \mathcal{F}^{J}_{ph}(r_1) \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{H}\,[f_{ph}(r)] - (\epsilon_h - \omega) g_{ph}(r) = \\
&-\mathcal{G}^{J}_{ph}(r) + \sum_{\epsilon_i<\epsilon_{\rm F}} \delta_{ip} R_i(r) \int {\rm d}r_1 r_1^2 R^*_i(r_1)\, \mathcal{G}^{J}_{ph}(r_1) \qquad (6)
\end{aligned}
$$

where we have defined

$$
\begin{aligned}
\mathcal{F}^{J}_{ph}(r) \;=\; &\sum_{p'h'} \int {\rm d}r_2\, r_2^2 \\
\Bigg\{ &R^*_{h'}(r_2) \Big[V^{J,{\rm dir}}_{ph,p'h'}(r,r_2)\, R_h(r)\, f_{p'h'}(r_2) \\
&- V^{J,{\rm exc}}_{ph,p'h'}(r,r_2)\, f_{p'h'}(r)\, R_h(r_2) \Big] \\
+\, &g^*_{p'h'}(r_2) \Big[U^{J,{\rm dir}}_{ph,p'h'}(r,r_2)\, R_h(r)\, R_{h'}(r_2) \\
&- U^{J,{\rm exc}}_{ph,p'h'}(r,r_2)\, R_{h'}(r)\, R_h(r_2) \Big] \Bigg\}\,, \qquad (7)
\end{aligned}
$$

and $\mathcal{G}^J_{ph}$ is obtained from the above equation by interchanging the f and g channel functions. In this equation U and V contain the interaction matrix element.

With this procedure, we have changed a set of algebraic equations with unknowns depending on the continuous variable ϵ_p into a set of integro-differential equations with unknowns depending on the distance from the center of coordinates. The solution of this problem requires to impose the proper boundary conditions. We call open those channels where the particle is in the continuum, i. e. it has positive energy, and closed the other ones. After fixing the angular momentum J and the parity Π of the excited state, for each value of the excitation energy ω, we solve Eqs. (5) and (6) a number of times equal to the number of the open channels. Every time we impose a different boundary condition, i.e. that the particle is emitted only in a specific channel, which we call elastic channel and label as p_0h_0. For an open ph channel, we impose that the outgoing asymptotic behaviour of the channel function $f^{p_0h_0}_{ph}$ is

$$f^{p_0h_0}_{ph}(r\to\infty)\;\to\;R_{p_0}(r,\epsilon_p)\,\delta_{p,p_0}\,\delta_{h,h_0}\;+\;\lambda\,H^-_p(\epsilon_h+\omega,r)\,, \qquad (8)$$

where λ is a complex normalization constant and $H^-_p(\epsilon_h+\omega,r)$ is an ingoing Coulomb function or a Hankel function in case of a proton or neutron channel, respectively.

In the case of a closed channel, the asymptotic behaviour is given by a decreasing exponential function

$$f^{p_0h_0}_{ph}(r\to\infty)\to\frac{1}{r}\exp\left[-r\left(\frac{2m|\epsilon_h+\omega|}{\hbar^2}\right)^{\frac{1}{2}}\right], \qquad (9)$$

as in the case of the channel functions $g^{p_0h_0}_{ph}$,

$$g^{p_0h_0}_{ph}(r\to\infty)\to\frac{1}{r}\exp\left[-r\left(\frac{2m|\epsilon_h-\omega|}{\hbar^2}\right)^{\frac{1}{2}}\right]. \qquad (10)$$

We solve the CRPA secular equations (5) and (6) by using a procedure similar to that presented in Ref. [7]. The channel functions f and g are expanded on a basis of sturmian functions Φ^μ_p which obey the required boundary conditions (8)-(10). Each sturmian funciton is characterized by the particle energy, the index p, and the number of nodes μ appearing at distances smaller than a cut-off radius a. The expression of one of the new CRPA secular equation is:

$$\sum_\mu\sum_{p'h'}\Bigg\{\Bigg[\delta_{pp'}\delta_{hh'}\Big(\delta_{\mu\nu}-\langle(\Phi^{\nu+}_p)^*|\mathcal{U}|\Phi^{\mu+}_p\rangle+\langle(\Phi^{\nu+}_p)^*I|\mathcal{W}|I\Phi^{\mu+}_p\rangle$$
$$+\sum_{\epsilon_i<\epsilon_F}\delta_{ip}(\epsilon_i-\epsilon_h-\omega)\langle(\Phi^{\nu+}_p)^*|R_i\rangle\langle(R_i)^*|\Phi^{\mu+}_p\rangle\Big)$$
$$-\Big(\langle(\widetilde{\Phi}^{\nu+}_p)^*R_{h'}|V^{J,\mathrm{dir}}_{ph,p'h'}|R_h\widetilde{\Phi}^{\mu+}_{p'}\rangle$$
$$-\langle(\widetilde{\Phi}^{\nu+}_p)^*R_{h'}|V^{J,\mathrm{exc}}_{ph,p'h'}|\widetilde{\Phi}^{\mu+}_{p'}R_h\rangle\Big)\Bigg]c^{\mu+}_{p'h'}$$
$$-\Big(\langle(\widetilde{\Phi}^{\nu+}_p)^*\widetilde{\Phi}^{\mu-}_{p'}|U^{J,\mathrm{dir}}_{ph,p'h'}|R_hR_{h'}\rangle$$
$$-\langle(\widetilde{\Phi}^{\nu+}_p)^*\widetilde{\Phi}^{\mu-}_{p'}|U^{J,\mathrm{exc}}_{ph,p'h'}|R_{h'}R_h\rangle\Big)(c^{\mu-}_{p'h'})^*\Bigg\}=$$
$$=\langle(\widetilde{\Phi}^{\nu+}_p)^*R_{h_0}|V^{J,\mathrm{dir}}_{ph,p_0h_0}|R_hR_{p_0}(\epsilon_{p_0})\rangle$$
$$-\langle(\widetilde{\Phi}^{\nu+}_p)^*R_{h_0}|V^{J,\mathrm{exc}}_{ph,p_0h_0}|R_{p_0}(\epsilon_{p_0})R_h\rangle, \qquad (11)$$

where the unknowns are the sturmian function expansion coefficientsc^μ_{ph}. In the above expressions, with the bra and ket integration convention we indicate integrations on radial variables only. The number of these integrations is given by the number of the functions inserted between the bra and ket symbols. For this reason we have inserted the symbol I indicating the identity function. The expression of the second CRPA equation for g is symmetric to Eq. (11).

Summarizing, we have transformed the CRPA secular equations (1) and (2) into a set of algebraical equations whose unknowns are the expansion coefficients c^μ_{ph}. These equations have a solution for each value of the excitation energy ω above the nucleon emission threshold and can be solved by using traditional diagonalization techniques and used with finite range interactions.

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