Subleading three-nucleon contact interaction

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One of the central tasks in nuclear physics is the determination of the interaction among nucleons. Traditionally, the two-nucleon ($2N$) forces have been modeled phenomenologically. By now, these models reach a remarkable degree of accuracy, providing an excellent fit to the very large body of experimental data. More recently, a new class of potentials, so called chiral potentials, have been developed, which establish a solid link between the nuclear interaction and the underlying strong interaction theory. They are derived in the framework of an effective field theory, incorporating all symmetry properties of QCD.

The effective theory is formulated in terms of nucleons and pions, whose interactions are constrained by the (approximate and spontaneously broken) chiral symmetry of QCD. The effect of heavier degrees of freedom is encoded in the value of (an infinite set of) coupling constants, the low-energy constants (LECs). The theory is organized as a systematic low-energy expansion, in powers of small momenta or quark masses (or rather, pion masses), so that at each order of the chiral expansion, only a finite number of LECs is involved. The occurrence of new LECs, as one proceeds in the expansion, limits the predictive power of the theory, which is lower and lower as the scale of "new physics" (mass of particles not explicitly included in the theory) becomes closer and closer. Nevertheless, it can be said that the effective theory approach represents a real progress, compared to traditional phenomenological potentials, as it is systematic and susceptible in principle of refinement. It allows, for instance, to justify the hierarchy of nuclear forces: n-nucleon forces are twice suppressed with respect to the $(n-1)$-nucleon forces.

Parallel to these developments there has been tremendous progress in the numerical techniques for the determination of bound and scattering states of few-nucleon systems. The latter then constitute a privileged theoretical laboratory to test different models of the nuclear interaction. Presently the low-energy expansion for the $2N$ potential has been carried up to the next-to-next-to-next-to leading order (N3LO), yielding a χ^2 per datum very close to one.

The status of the three-nucleon ($3N$) interaction is not as satisfactory. While its presence is necessary if one wants to reproduce the 3- and 4-nucleon binding energies, all currently available $3N$ potentials fail to simultaneously describe also the neutron-deuteron doublet scattering length [1]. As a matter of fact, several low-energy data in the nucleon-deuteron system remain still unexplained (e.g. the so-called A_y puzzle). This is not really surprising, since the adopted $3N$ interaction models only contain a few free parameters, contrary to what happens for the $2N$ interaction, which is parametrized by more than 20 adjustable parameters. For example, the chiral $3N$ force at N2LO only contains 2 adjustable LECs, and its extension to N3LO does not involve any new LEC.

In this context, starting with the observation that the discrepancies between theory and experiment in the $3N$ system arise at very low energy, we propose to refine that part of the interaction which stems from purely nucleonic vertices. Indeed, at very low energies, even the pions can be integrated out of the theory (their effect being subsumed in the values of the LECs), giving rise to the "pionless" effective theory, and the interaction among nucleons is due to contact vertices. It should be noticed that purely contact vertices are also part of the "pionful" version of the effective theory: a matching procedure could in principle be used to find the relationship between the values of the LECs in both theories. A $3N$ contact term is already part of the N2LO chiral ("pionful") $3N$ interaction. Invariance under parity dictates that subleading $3N$ contact terms contain two (covariant) derivatives of nucleon fields; they would therefore contribute, in the pionful version of the effective theory, at N4LO: they are not considered in any of the currently available models. Our task is twofold: first, classify all possible terms of this kind, second, fit the accompanying LECs to $3N$ data, in association with a given $2N$ potential model. We report here our progress in the first task, sticking to isospin conserving structures.

Contact terms are not much constrained by chiral symmetry, but they have to respect the discrete symmetries of the underlying theory, namely (besides parity) charge conjugation, or better to say, in a non-relativistic theory, time-reversal invariance. Using translational invariance (or momentum conservation) the only possible space-structures can be taken to be of the form

$$\begin{array}{l} X^+_{A,ij} = (N^\dagger \overleftrightarrow{\nabla}_i N)(N^\dagger \overleftrightarrow{\nabla}_j N)(N^\dagger N) \\ X^+_{B,ij} = \nabla_i (N^\dagger N) \nabla_j (N^\dagger N)(N^\dagger N) \\ X^-_{C,ij} = i\nabla_i (N^\dagger N)(N^\dagger \overleftrightarrow{\nabla}_j N)(N^\dagger N) \\ X^+_{D,ij} = (N^\dagger \overleftrightarrow{\nabla}_i \overleftrightarrow{\nabla}_j N)(N^\dagger N)(N^\dagger N), \end{array} \quad (1)$$

where the i is required by the hermiticity condition and (hereafter) the superscripts denote the time-reversal properties. The relevant isospin-conserving structures are

$$\begin{array}{l} T^+ = \mathbf{1}, \quad \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \quad \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3, \quad \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3, \\ T^- = \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3, \end{array} \quad (2)$$

where the subscripts of the Pauli matrices refer to the nucleon bilinears they belong to. Even (odd) combinations of $X \otimes T$ structures under time-reversal have to be associated with spin structures containing even (odd) numbers of σ matrices. Finally, the spin-space indices have to be contracted with Kronecker δ's or Levi-Civita tensors ϵ's. In this way, a list of 146 operators is obtained, evidently too large to be practicable. However, there are further constraints that have to be imposed on these operators: symmetry properties under permutations, encoded in the anticommuting nature of the nucleon field N, and the requirements of Poincaré covariance. The former are conveniently expressed as Fierz-like relations, i.e. examining the effect of exchanging spin (or isospin) indices,

$$\begin{array}{l} (\mathbf{1})[\mathbf{1}] = \frac{1}{2}(\mathbf{1})[\mathbf{1}) + \frac{1}{2}(\boldsymbol{\sigma}] \cdot [\boldsymbol{\sigma}) \\ (\sigma^i)[\mathbf{1}] = \frac{1}{2}(\sigma^i][\mathbf{1}) + \frac{1}{2}(\mathbf{1})[\sigma^i) - \frac{i}{2}\epsilon^{ijk}(\sigma^j][\sigma^k) \\ (\sigma^i)[\sigma^j] = \frac{1}{2}\left\{\delta^{ij}(\mathbf{1})[\mathbf{1}) - \delta^{ij}(\boldsymbol{\sigma}] \cdot [\boldsymbol{\sigma}) + (\sigma^i][\sigma^j)\right. \\ \left. +(\sigma^j][\sigma^i) + i\epsilon^{ijk}(\sigma^k][\mathbf{1}) - i\epsilon^{ijk}(\mathbf{1})[\sigma^k)\right\}, \end{array} \quad (3)$$

where $\mathbf{1}$ is the identity operator in the one-particle spin space, and (,) and [,] denote spin (isospin) indices of the enclosed operator. Indices rearrangements also involve the field derivatives: for instance, under permutation of nucleons 1-2,

$$\begin{array}{l} \overleftrightarrow{\nabla}_1 \to \frac{1}{2}(\overrightarrow{\nabla}_2 + \overleftrightarrow{\nabla}_2 - \overrightarrow{\nabla}_1 + \overleftrightarrow{\nabla}_1), \\ \overrightarrow{\nabla}_1 \to \frac{1}{2}(\overrightarrow{\nabla}_2 + \overleftrightarrow{\nabla}_2 + \overrightarrow{\nabla}_1 - \overleftrightarrow{\nabla}_1). \end{array} \quad (4)$$

Simultaneous Fierz rearrangements of spin and isospin indices of nucleon fields 1–2, 1–3 and 2–3 allow then to derive a set of 3×146 linear relations among the operators. It turns out that 132 of these relations are linearly independent, leaving a set of 14 two-derivative $3N$ contact operators.

Relativity constraints can be imposed order by order in the low-energy expansion [2]. They yield four additional independent relations, so that the final number of operators is reduced to 10 [3]. A $3N$ potential in momentum space can then be derived, involving 10 unknown LECs. With a cutoff depending only on momentum transfers, this potential can be given a local form in coordinate space,

$$\begin{array}{l} V = (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \\ +E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\ +(E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\ +(E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k)(\mathbf{L} \cdot \boldsymbol{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\ +(E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \end{array}$$

where the E_i are LECs, S_{ij} and $(\mathbf{L} \cdot \boldsymbol{S})_{ij}$ are respectively the tensor and spin-orbit operators for particles i and j, and the function $Z(r)$ is the Fourier transform of the cutoff function $F(\mathbf{k}^2; \Lambda)$,

$$Z_0(r; \Lambda) = \int \frac{d\mathbf{p}}{(2\pi)^3} \mathrm{e}^{i\mathbf{p}\cdot\mathbf{r}} \; F(\mathbf{p}^2; \Lambda) \quad (5)$$

In the derivation of the $3N$ potential, a choice of basis has been made such that most terms in the potential can be viewed as an ordinary interaction of particles ij with a further dependence on the coordinate of the third particle. In particular, the terms proportional to E_7 and E_8 are of spin-orbit type, and, as already suggested in the literature, could be useful to solve the A_y puzzle.

It should also be remembered that, in few-nucleon calculations, also (no-derivative) four-nucleon ($4N$) contact terms contribute at the same level of accuracy in the low-energy expansion. By imposing all constraints from discrete symmetries and from Fierz rearrangements, it turns out that only one such operator exists [3]. This allows to decouple, to a certain extent, the $3N$ sector from the $4N$ sector, if one is willing to fix all the LECs from the data. For instance, one can in principle adjust the subleading $3N$ potential in the $A = 3$ systems without worrying much about the consequences for the α particle binding energy: the latter could always be reproduced by adjusting the $4N$ contact term.

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