Relativistic covariance of the 2-nucleon contact interactions in the even- and odd-parity sectors

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Contact interactions are a crucial ingredient of the effective field theory (EFT) description of the nuclear forces. Being associated with unknown low-energy constants (LECs) it is important to identify a minimal and complete set of such operators: using an overcomplete set, containing redundant operators, would artificially increase the number of LECs, which could never be determined from observables. In Ref. [1] we concentrate on the constraints from relativistic covariance at order p (parity-violating) and p^2 (parity-conserving) of the low-energy expansion. We construct the most general hermitian Lagrangian density allowed by invariance under transformations of the Lorentz group and by the discrete symmetries of the strong interaction and perform a non-relativistic reduction thereof. Up to the considered orders, relativistic covariance is maintained in the latter process.

The relativistic Lagrangian is written in terms of fermion fields containing both positive and negative-energy components. (However, to the extent that we are interested in the Lagrangians up to $O(p^2)$, we can forget the negative energy components.) It consists of products of fermion bilinears with space-time structures

$$(\bar\psi \overleftrightarrow{\partial}_{\mu_1} ... \overleftrightarrow{\partial}_{\mu_i} \Gamma_1 \psi)\partial_{\lambda_1}...\partial_{\lambda_k}(\bar\psi \overleftrightarrow{\partial}_{\nu_1} ... \overleftrightarrow{\partial}_{\nu_k} \Gamma_2 \psi), \tag{1}$$

where $\overleftrightarrow{\partial} = \underrightarrow{\partial} - \underleftarrow{\partial}$ and $\Gamma_{1,2}$ are generic elements of the Clifford algebra, that can be expanded in the usual basis $\mathbf{1}$, γ_5, γ_μ, $\gamma_\mu\gamma_5$ and $\sigma_{\mu\nu}$, including the metric and Levi-Civita tensors, $\epsilon^{\mu\nu\rho\sigma}$ (with the convention $\epsilon^{0123} = -1$). The Lorentz indices on the partial derivatives are contracted among themselves and/or with those in the $\Gamma_{1,2}$. In the parity-conserving sector, in order to have flavour singlets, the flavour structure of the two bilinears must be either $1 \otimes 1$ or $\tau^a \otimes \tau^a$. However, the latter needs not be considered, as it can be eliminated by Fierz rearrangement. The flavour structure is more complex in the parity-violating sector. In this case the effective Lagrangian includes the electroweak current-current interaction which gives rise to $\Delta I = 0, 1, 2$ operators, I denoting the isospin.

Notice that the derivatives acting on the entire fermion bilinear are suppressed in the chiral counting, $\partial \sim O(p)$, while $\overleftrightarrow{\partial} \sim O(1)$ due to the presence of the heavy fermion mass scale. Therefore, at each chiral order, only a finite number of ∂ appears, while it is possible to have any number of $\overleftrightarrow{\partial}$. The situation is not so troubling though: for instance the contracted product $\overleftrightarrow{\partial}_\mu \overleftrightarrow{\partial}^\mu$ yields a squared mass term (without derivatives) plus a $\partial_\mu\partial^\mu$ operator acting on the whole bilinear, which is suppressed by $O(p^2)$. In general, no two Lorentz indices inside a fermion bilinear can be contracted with one another, except for the Levi-Civita tensors and for the (suppressed) ∂^2 acting on the whole bilinear. This is due to the fact that we can always make use of the fermion field equations of motion to eliminate terms with $\not\partial\psi$ in favor of terms without derivatives.

The Lagrangian should be hermitian and invariant under $\mathcal{C}$ (charge conjugation) and $\mathcal{P}$ (parity). While the hermiticity condition does not impose any constraint, since one can always multiply the single bilinears by appropriate factors of i, the $\mathcal{C}$ and $\mathcal{P}$ symmetry must be enforced.

Following these criteria a complete but non-minimal set consisting of 54 $\mathcal{P}$- and $\mathcal{C}$-conserving operators, denoted as $\widetilde{O}_i$, is obtained.

The non-relativistic reduction of the $\widetilde{O}_i$ up to terms of order Q^2 is obtained starting from the relativistic field

$$\psi^{(+)}(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{m}{E_\mathbf{p}} b_s(\mathbf{p}) u^{(s)}(\mathbf{p}) \mathrm{e}^{-ip\cdot x}, \tag{2}$$

with normalizations,

$$\begin{array}{l} \{b_s(\mathbf{p}), b^\dagger_{s'}(\mathbf{k})\} = \frac{E_\mathbf{p}}{m}\delta_{ss'}(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{k}), \\ \bar u^{(s)}(\mathbf{k}) u^{(s')}(\mathbf{k}) = \delta_{ss'}, \end{array} \tag{3}$$

to the non-relativistic one,

$$N(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \phi^{(s)} \tilde b_s(\mathbf{p}) \mathrm{e}^{-ip\cdot x}, \tag{4}$$

with $\phi^{(s)}$ a two-component spin doublet, and the

O_S	$(N^\dagger N)(N^\dagger N)$
O_T	$(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \boldsymbol{\sigma} N)$
O_1	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)^2 + \text{h.c.}$
O_2	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}} N)$
O_3	$(N^\dagger N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{h.c.}$
O_4	$i\,(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \times \boldsymbol{\sigma} N) + \text{h.c.}$
O_5	$i\,(N^\dagger N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} \times \overrightarrow{\boldsymbol{\nabla}} N)$
O_6	$i\,(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \times \overrightarrow{\boldsymbol{\nabla}} N)$
O_7	$(N^\dagger \boldsymbol{\sigma}\cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma}\cdot \overrightarrow{\boldsymbol{\nabla}} N) + \text{h.c.}$
O_8	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \sigma^k \overrightarrow{\nabla^j} N) + \text{h.c.}$
O_9	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \sigma^j \overrightarrow{\nabla^k} N) + \text{h.c.}$
O_{10}	$(N^\dagger \boldsymbol{\sigma}\cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} N)$
O_{11}	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \overleftarrow{\nabla^j} \sigma^k N)$
O_{12}	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \overleftarrow{\nabla^k} \sigma^j N)$
O_{13}	$(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} \overrightarrow{\nabla^j} N)(N^\dagger \sigma^j N) + \text{h.c.}$
O_{14}	$2\,(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \sigma^j \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \sigma^j N)$

Table 1
Operators entering the LO (Q^0) and N^{2}LO (Q^2) contact interactions. The left (right) arrow on ∇ indicates that the gradient acts on the left (right) field. Normal-ordering of the field operator products is understood. In fact this list is redundant, as the relations $O_7 + 2O_{10} = O_8 + 2O_{11}$ and $O_4 + O_5 - O_6 = 0$ may be shown to hold.

operators $\tilde{b}_s(\mathbf{p}) \equiv \sqrt{m/E_\mathbf{p}}\, b_s(\mathbf{p})$, according to

$$\psi^{(+)}(x) = \begin{pmatrix} 1 + \frac{\nabla^2}{8m^2} \\ -i\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}}{2m} \end{pmatrix} N(x) + o(p^2). \quad (5)$$

Partial integrations and use of the fields' equations of motion to eliminate time derivatives allow to express them as linear combinations of the operator basis O_i, defined in Table 1.

As a result, a complete basis of operators can be defined as

$$\begin{array}{l} O_S + (O_1 + O_3 + O_5 + O_6)/(4m^2) \\ O_T - (O_5 + O_6 - O_7 + O_8 + 2\,O_{12} + O_{14})/(4m^2) \\ O_1 + 2\,O_2 \\ 2\,O_2 + O_3 \\ O_9 + 2\,O_{12} \\ O_9 + O_{14} \\ O_5 - O_6 \\ O_7 + 2\,O_{10} \\ O_7 + O_8 + 2\,O_{13} \end{array} \quad (6)$$

consisting of 2 leading (of order Q^0) and 7 subleading (Q^2) operators.

For the PV Lagrangian we consider hermitean, $\mathcal{P}$ odd, and $\mathcal{CP}$ even products of a pair of bilinears. In this case, isoscalar, isovector and isotensor operators have to be constructed, however, so that the total electric charge be conserved (that is, the terms must commute with the third component of the isospin operator, T_z). Let us denote with τ_0 the 2×2 identity matrix and $\boldsymbol{\tau}$ the Pauli matrices acting on the flavour degrees of freedom. Then the most general product of fermion bilinears can be written as

$$\widetilde{O}^k_{AB} = \sum_{a,b=0}^{4} F^k_{ab} (\overline{\psi}\,\tau_a \Gamma_1\,\psi)\,(\overline{\psi}\,\tau_b \Gamma_2\,\psi)\ , \quad (7)$$

where Γ_1 and Γ_2 stand for an element of the Clifford algebra plus a generic combination of four-gradients (both $\overleftrightarrow{\partial}^\lambda$ and/or ∂^λ). There are six possible choices for the coefficients F^k_{ab}, as detailed in Table 2.

k	F^k_{ab}	$\mathcal{C}$
1	$\delta_{a,0}\delta_{b,0}$	+
2	$\delta_{a,b} - \delta_{a,0}\delta_{b,0}$	+
3	$\delta_{a,3}\delta_{b,0} + \delta_{a,0}\delta_{b,3}$	+
4	$\delta_{a,3}\delta_{b,0} - \delta_{a,0}\delta_{b,3}$	+
5	$\delta_{a,1}\delta_{b,1} + \delta_{a,2}\delta_{b,2} - 2\delta_{a,3}\delta_{b,3}$	+
6	$i\,[\delta_{a,1}\delta_{b,2} - \delta_{a,2}\delta_{b,1}]$	−

Table 2
Possible choices for the coefficients F^k_{ab}. In the last column, the transformation properties of the operators under charge conjugation ($\mathcal{C}$) are reported.

Note that for $k = 1, 2$ the resulting operator is isoscalar, for $k = 3, 4, 6$ isovector, and for $k = 5$ isotensor. In the last column, the transformation properties of the six operators under charge conjugation ($\mathcal{C}$) are reported (clearly all of them are even under parity and hermitean conjugation). Since the products of bilinears considered now should be odd under parity, they must be odd also under charge conjugation. Following the criteria laid out in Ref. [2], a complete but non-minimal set consisting of 58 $\mathcal{P}$-odd and $\mathcal{C}$-odd operators contributing to the order $\mathcal{O}(Q)$, denoted as $\widetilde{O}^{PV}_i$.

The non-relativistic reductions of the 58 operators can be performed using Eq. (5). At $\mathcal{O}(Q)$ they are expressed as linear combinations of 10 non-relativistic operators. Using Fierz relations for the Pauli matrices it is possible to show that there are only five $\mathcal{O}(q)$ PV contact terms, as already found in Ref. [3].

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