The breakup rate of small solid aggregates in turbulent suspensions

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In many different applications (see e.g. [1]), it happens that small solid aggregates such as clusters or flocs are suspended in turbulent flows. Hydrodynamic stresses due to the turbulent nature of the flow act to prevent the formation of very large aggregates, and contribute to disperse them. But, turbulence is crucial for the reverse process also, i.e. transformation of small particles of colloidal size into aggregates of few micrometres to millimetres. Indeed, turbulence not only creates hydrodynamic stress that can cause restructuring and breakup of aggregates [1],but also increases the rate of collision among particles, by inducing high velocity differences and preferential concentration within the particle field [2,3].

Recently, we have investigated the breakup of small solid aggregates in a homogeneus and isotropic 3D fully developed turbulent flow, at Reynolds number based on Taylor microscale $Re_\lambda \sim 400$. Details of the numerical simulations and database can be found in [2].

As a starting point for our study, we focused on the modelling of the aggregates breakup rate in a very simple set-up. We consider a dilute suspension of very small aggregates - much smaller then the Kolmogorov scale of the flow-, so in the range of 25 to 100 μm for typical turbulent flows, and with negligible inertia. Further, we assume that aggregates concentration is such that they do not modify the flow. Hence, their evolution is identical to that of passive point-like particles. Moreover, the aggregates are brittle and breakup occurs instantaneously once being subject to a hydrodynamic stress that exceeds a critical value. For small and inertialess aggregates, the hydrodynamic stress exerted by the flow is $\sim\mu(\varepsilon/\nu)^{1/2}$, where μ and ν are the dynamic and kinematic viscosity, respectively and $\varepsilon = \nu \sum_{i,j=1}^{3} (\partial u_i/\partial x_j)^2$ is the local energy dissipation per unit mass, and $\partial u_i/\partial x_j$ are the components of the velocity gradient tensor. Finally, as customary, we assume the existence of a constituent power-law relation relating the mass of the aggregates and the critical value of the turbulent stress at which they break, i.e. $\varepsilon_{\rm cr}$. This states that larger aggregates break at a lower stress than smaller ones: $\varepsilon_{\rm cr}(\xi) = \langle\varepsilon\rangle(\xi/\xi_s)^{-1/q}$ where the exponent q is related to the aggregate structure, ξ is the mass of an aggregate, and ξ_s is the characteristic aggregate mass. In this set-up, the key role is played by the turbulent velocity gradients across the aggregate ε, which are known to possess strongly non-Gaussian, intermittent statistics [4].

We propose a first-principle way to compute the fragmentation rate is through *exit-time* statistics [5], which is equivalent to measuring the distribution of the time necessary to observe the first occurrence, along the trajectory, of a local hydrodynamic stress ε strong enough to break the aggregate itself $\varepsilon > \varepsilon_{\rm cr}$. Hence, the fragmentation rate $f_{\varepsilon_{\rm cr}}$ is nothing but the inverse of the first moment of the exit-time distribution:

$$f_{\varepsilon_{\rm cr}} = \left[\int_0^\infty d\tau\ \tau \mathcal{P}_{\varepsilon_{\rm cr}}(\tau)\right]^{-1} = \frac{1}{\langle\tau(\varepsilon_{\rm cr})\rangle_{ex}}\,, \qquad (1)$$

where $\varepsilon_{\rm cr}(\xi)$ is the critical energy dissipation (or critical stress) needed to break an aggregate of *mass* ξ, and $\mathcal{P}_{\varepsilon_{\rm cr}}(\tau)$ is the distribution function of the exit-times τ for a fixed critical threshold $\varepsilon_{\rm cr}$.

A way to estimate the fragmentation rate was proposed some years ago by Loginov [6]. This is

$$f^E_{\varepsilon_{\rm cr}} = \frac{1}{\langle T(\varepsilon_{\rm cr})\rangle} = \frac{\int_0^\infty d\dot\varepsilon\ \dot\varepsilon p_2(\varepsilon_{\rm cr},\dot\varepsilon)}{\int_0^{\varepsilon_{\rm cr}} d\varepsilon\ p(\varepsilon)}\,. \qquad (2)$$

Here the numerator is the Rice formula (see [7]) giving the mean number of crossings of $\varepsilon_{\rm cr}$ in terms of the joint probability of dissipation and its time derivative $p_2(\varepsilon,\dot\varepsilon)$; the denominator is the measure of the total time spent in the region with $\varepsilon < \varepsilon_{\rm cr}$. Notice that the integration in the numerator goes only on positive values in order to

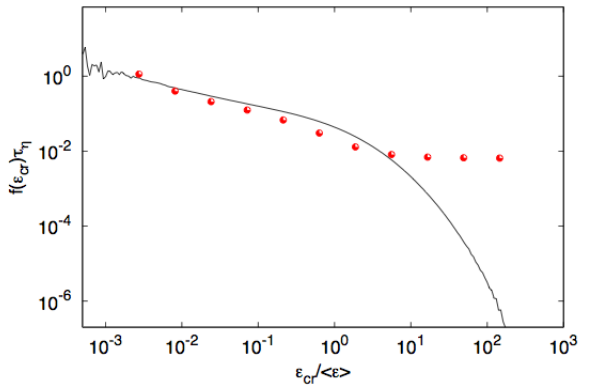

Figure 1. Plot of the breakup rate function $f_{\varepsilon_{\rm cr}}$ measured in our simulations following eq. (1) (red dots), is compared the to the approximate Loginov expression (2), (continuous line). The fragmentation rate is made adimensional by multiplying it by the Kolmogorov time scale of the flow τ_η. Data are obtained from a Direct Numerical Simulation of homogeneous and isotropic turbulence at $Re_\lambda = 400$, seeded with millions of tracer particles.

consider only up-crossing of the threshold $\varepsilon_{\rm cr}$ [7]. A clear *experimental* advantage of Eq. (2) is that it is *quasi-Eulerian*: it does not require to follow trajectories, since it depends only the spatial distribution of dissipation and of its first time derivative in the flow. Let us remark that expressions (1) and (2) are not strictly equivalent [5].

In Figure 1, the breakup rate function $f_{\varepsilon_{\rm cr}}$ obtained according to equation (1) is compared with the estimation proposed by Loginov [6]. The agreement of the two curves is very good for small values of the critical energy dissipations - corresponding to aggregates of large mass. The reasons for which it does not properly work in the limit of large critical energy dissipations - corresponding to small masses-, is due to statistical convergence: indeed to break up a small mass a high stress is needed, and hence very long trajectories are needed.

To conclude, we have introduced a first-principle definition of the fragmentation rate of small solid aggregates in turbulent flows, and we have shown that the Loginov expression represents a valid approximation when Lagrangian measurements of aggregates are not available. If a model for aggregation is available, the mean-field Smoluchowski equation for the the time evolution of an ensemble of aggregates undergoing breakup and coalescence can be solved (see [5]).

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