## Time fluctuations of vegetation patterns and desertification transitions

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The identification of early warning signals of regime shifts in ecosystems is a crucial issue since these shifts can imply severe losses of ecological and economic resources. In particular, environmental stresses induced either by an excess of anthropic load or by an increased frequency of meteorological extreme events, can give rise in arid or semi-arid ecosystems to desertification transitions. Many recent studies highlighted the interest in vegetation patchiness analyses as a tool to provide indicators of desertification risk. Recently, Kefi et al. [1] investigated the influence of external stress on the spatial organization of the vegetation, by combining modelling and field data from three Mediterranean arid grazed ecosystems in Spain, Greece and Morocco. The model, based on a stochastic cellular automaton (CA), was accounting for several ecological mechanisms. The ecological landscape was then determined by a certain number of parameters. In particular, we mention the mortality parameter, m, associated with the external stress (grazing pressure) and the facilitation parameter, f, controlling the strength of local cooperative interactions among plants. The analysis of both field data and numerical simulations showed [1] that, far from transition, vegetation patch-size distribution,  $\Pi(s)$ , follows a power law, where s is the size of the vegetation patches, i.e. the cluster size. Moreover, Kefi et al. found that at increasing m or at decreasing  $f,\,\Pi(s)$  significantly deviates from a power law, showing an exponential cutoff. Then, Kefi et al. proposed that these deviations from a power-law patch-size distribution may be warning signals for the onset of desertification processes [1]. An explanation of the origin of this Pareto-like behavior of the spatial distributions in ecosystems has been advanced by Manor and Shnerb [2] by a map of the Kefi model to a Markov birth-death stochastic process. However, such studies and many others in the literature [3,4] devoted to desertification risk, mainly focus on the analysis of the spatial fluctuations of the vegetation pattern, while only few authors investigated time fluctuations.

We have investigated the time fluctuations

properties of some global and local quantities associated with the steady states of the CA model of Kefi et al. Precisely, by making use of numerical simulations, we have considered the behavior as a function of m and N (system size) of the total living cell density (total vegetation mass fraction)  $\rho^+$ , of the dead cell density (fraction of the system surface without vegetation)  $\rho^0$  and of the degraded cell density (fraction of the system) surface with compromised soil quality)  $\rho^{-}$ , where  $\rho^+ + \rho^0 + \rho^- = 1$ . Furthermore we have considered the behavior of the size of the biggest living cell cluster,  $S_B$ . In particular, we have analyzed the distributions of the time fluctuations of these quantities, by calculating the probability density function (PDF) and the first three moments of the distributions as a function of m and N.

We have found that the system undergoes a critical transition for a given value  $m_c$  of the mortality parameter, associated with the desertification transition, as proved by the scaling behavior of both the average  $\langle \rho^+ \rangle$  and the variance  $\langle (\delta \rho^+)^2 \rangle$  of the living cell density as a function of the difference  $m - m_c$ . Indeed,  $\langle \rho^+ \rangle \to 0$ and  $\langle (\delta \rho^+)^2 \rangle \to \infty$  for  $m \to m_c$ . Consistently with this statement, the time fluctuations of the biggest cluster size  $S_B$  at  $m \approx m_c$  strongly deviate from Gaussianity, by displaying a positive skewness persistent at increasing N. Precisely, the fluctuations of  $S_B$  follow the universal Bramwell-Holdsworth-Pinton (BHP) distribution [6,7] independently of the system size [5].

Moreover, we have found that the variance of  $S_B$  exhibits a sharp peak at a given value  $m^*$  well below to  $m_c$ . The value of  $m^*$  is the same of that reported by Kefi et al. as corresponding to the emergence of the exponential cut-off in the power-law patch-size distribution and indicated by these authors as a early warning signal for the onset of desertification. We explain this peak in the  $S_B$  variance at  $m^*$  in terms of percolation threshold for the living cells. However, we have also found that a strong non-Gaussianity of the  $S_B$  fluctuations, now with negative skewness, occurs at  $m < m^*$ , characterized in this case by a weak dependence on N (see Fig. 1). So we pro-



Figure 1. Normalized PDF of the time fluctuations of the biggest cluster sizes calculated for a value  $m^* = 0.09$  of the mortality parameter and different system size: N = 100 (black squares) and N = 150 (red circles). The solid curve corresponds to the BHP distribution [6].

pose that this change of skewness, from negative to positive values, and in particular this strong non-Gaussianity at  $m < m^*$  provides a "very" early warning of the onset of desertification transition.

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