Analysing quality with Generalized Kinetic Methods

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Interest in mathematical models that describe and analyze living elements is increasing. In particular, the field known in the literature as Generalized Kinetic Theory is proving to be sufficiently general and conveniently adaptable to perform the difficult task of treating advanced and complex systems such as those that refer to human individuals.

Generalized kinetic models represent a fruitful predictive and descriptive tool in the area of the social sciences. These models transfer the methodology developed for systems of a great number of interacting particles (typically in the field of kinetic theory of classical particles) to various other fields of research.

In Ref. [1], we proposed to use generalized kinetic theory to obtain the (statistical) description of the time evolution of a global variable - "atmosphere" - related to the quality of a complex system such as a medical ward. The work was originally motivated by the analysis of experimental data collected for almost 10 years in an acute psychiatric in-patient care unit (SPDC, located in San Pietro Vernotico, Brindisi - Italy) under the direct responsibility of one of the authors (A.V. Serio, MD), in the framework of a regional research project for monitoring and improving the quality psychiatric wards. The ward has been considered as a closed system containing two populations: patients and medical and nursing staff (staff, for shortness). The dynamics, and the mutual relations among the individuals, have been modelled as strictly dependent on the occurring (or perceived as occurring) atmosphere, and on the effects produced on each individual by the performance/behavior of all the others. The experimental data collected over the years are both quantitative and qualitative and include: 1. monitoring a "macroscopic" variable called Ward Atmosphere (or also "therapeutic atmosphere"); 2. presence of medical and nursing staff (on three shifts per day); 3. critical or sentinel events (such as episodes of aggressiveness or violence, accidents, restraints, escapes, etc); 4. internal and external events both of positive and negative nature (such as visits by mental health community teams or relatives, social activities, leaves; uneasy admissions, such as patients on involuntary admission or at their first hospitalization); 5. ordinary flux data (daily admissions/discharges of patients).

The approach we proposed belongs to the so-called Generalized Kinetic Theory and in our statistical picture (of the "Boltzmann" kind) the underlying structure is referred by a microscopic state variable [denoted by activity of the actors, and taking values in some bounded intervals], subject to actions both of internal and of external nature. The mathematical framework is developed for a general setting, but in the specific case considered here, there are two populations (P_1, patients, composed by $\mathcal{N}_1(t)$ individuals, and P_2, operators, composed by $\mathcal{N}_2(t)$ individuals).

Individuals of the same population are identical, and only addressed to by a state variable denoting their activity, which is assumed to be a scalar random variable over some (hidden) measure space of elementary events, and realized by distinct variables, one per each of the system populations: u_1, u_2, with $u_i \in I_i = [0,1]$ for $i = 1,2$. What in fact is of interest are not the mentioned variables (whichever their character be, continuous or discrete), rather the random processes represented by the family of such state variables over the time interval $[0,T]$. That is:

$$u_1, u_2 : t \in [0,T] \to u_i(t) \in I_i \qquad i = 1,2$$

Note that if referred to actors of different populations, the activity has different meanings. In the particular case we are interested in, the state variable u_1 measures the *psychotic behavior* of the patients while u_2 measures the *stress* of the operators. The corresponding probability density functions

$$f_i : (t,u) \in [0,T] \times I_i \to f_i(t,x) \in [0,\infty[$$

that refer on how the individuals of each population $i = 1,2$ are distributed with respect to their state variable, are the objects of our study.

The main assumption in that each individual of the system is subject to actions of external and of internal nature. Actions of internal nature, or interactions, are further subdivided into actions of social character and actions of direct character; the former ones due to means computed over specific actors' ensembles, the latter to direct relations among single actors. Actions of external nature act by means of a term with the structure of a field. Interactions among the actors are identified, as usual in kinetic theories, by means of convenient encounter frequencies and change of state probabilities.

The dynamics is then ruled by a set of nonlinear (integro-differential) evolution equations. More specifically, the total variation rate of density f_1, f_2 equals the balance of gain & loss terms due to the interactions among the individuals, i.e.:

$$\partial_t f_i + \partial_u \Phi_i[\mathbf{f}] = \mathcal{G}_i[\mathbf{f}] - \mathcal{L}_i[\mathbf{f}]\,, \qquad i = 1, 2.$$

Note that the implicit dependence on the densities $\mathbf{f} := (f_1, f_2)$ is recalled by the square brackets.

Unlike other generalized kinetic models, here no change of populations are (obviously) allowed.

Note that gain & loss terms depend on internal actions (direct interactions among single actors); the convective term on functions $f_i(t, u)$ has the structure of a (local) net flow $\partial_u \Phi_i$ where $\Phi_i[\mathbf{f}](t, u) = \mathcal{K}_i[\mathbf{f}](t, u) f_i(t, u)$, with drift velocity $\mathcal{K}_i[\mathbf{f}]$ accounting for the expected speed of (mass) change due to external events, to actions of global character to be ascribed to entire ensembles of actors, and to critical events.

The interactions can modify the state of a test actor with a rate and a probability specified by convenient functions. In particular, in our model the interaction terms will be described by certain functions $\eta_{i,j}(t, x, y)$, denoting the rate of events such that a test individual, of population P_i in the state x, encounters an individual of population P_j in the state y and $\psi_{i,j}[\mathbf{f}](t, x, y; x')$, denoting the probability density about the outgoing state x' of the test individual of population P_i in a state x after an event wherein he encountered an individual of population P_j in the state y. Probabilities are conveniently assumed to be normalized Gaussians of appropriate means $\mu = \mu_{i,j}(x, y)$ and variations $\sigma = \sigma_{i,j}(x, y)$ that are the only functions left to be modelled.

Macroscopic quantities are actually "measurable". For example, the mean activity of population P_i:

$$U_i(t) = \mathcal{N}_i(t) \int_{I_i} u f_i(t, u) du\,, \qquad i = 1, 2$$

and convenient weights may be properly defined such that

$$U(t) = \alpha_1 U_1(t) + \alpha_2 U_2(t)$$

eventually provides the global ward atmosphere (or expected quality) of the service at time t.

We have also developed an (inequivalent) picture developed with the assumption that the actors' state variables, rather than being continuous real valued random variables, are instead discrete real valued ones. This approach is under many respects more suitable to describe the specific case, where quality has not been measured as a real variable [its values have in fact been attributed according to a phenomenological measure scale.]

Regardless of the model (continuous or discrete) being used, our aim is twofold:

- descriptive: given an initial value for f_i, compatible with the atmosphere for some "past" time $t = 0$, solve the system of evolution equations and fit the parameters by comparing with the available experimental data.

- predictive: use the model obtained in this way to predict the time evolution and the effects that possible readjustments of the structure may produce on it.

In particular, our analysis aims at singling out and characterizing the physical variables that control the dynamics, and at predicting the effect that possible readjustments of the structure may produce on it.

We are currently working [2] on the discrete analog of the model proposed in [1], where the statistical description of patients and staff populations has been assumed to be correctly acquired on discretizing their activities into two sets of five real values; correspondingly, two sets of five probabilities refer about the (percentage) number of actors that at any instant of time are expected to be found in five discrete states, corresponding to a color code used to measure the atmosphere. Hence, the mathematical model consists of a system of 10 coupled, nonlinear, nonlocal ordinary differential equations for the probability variables. The equations depend on a set of physical parameters that describe, at the "mesoscopic" scale, the nature and frequency of the direct interactions among the actors, as well as the effect of external positive or negative events, work-load, and mean field terms (which specify how the individuals are affected by the overall ward state). Aim of the analysis is to make use of the history data set, and single out and characterize those physical parameters that control the dynamics, to ultimately predict, on a short-time scale, the possible outbreak of a crisis, and, on a long-time scale, the effects that planned or un-planned readjustments of the structure may produce on it.

A sophisticated numerical (fortran) code has been implemented to solve the initial value problem for the system of ODEs, both when the input/initial value data are read from the historic series, and when they are randomly simulated via Monte Carlo processes. Analyzing a large number of numerical simulations, we have been able to identify those parameters whose variations the model is most sensitive to, and choose their optimal values.

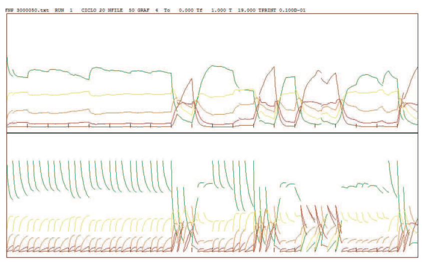

Figure 1. Numerical simulations showing the probability density functions for patients (top) and staff (bottom) relative to the case period May 27, 2001 – June 15, 2001. The simulation shows critical regimes (the appearance of red/brown peaks), which indeed have a direct correspondence with what has been observed in the hospital in the period of time under consideration.

In order to compare the predictions of model with the experimental data, it is necessary to convert the output of the numerical simulations, i.e., the two probability density functions for patients and staff, into a single, sequence of integer values in $\{2, 4, 6, 8, 10\}$ that represent the estimate for the ward atmosphere on each shift for the given period.

From the output of the numerical simulations, estimated values for the atmosphere have been obtained based on ad hoc algorithms. The comparison for two periods is shown in Fig. 2.

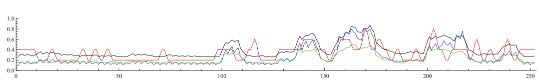

Figure 2. Comparison among historical and simulated atmospheres for period May 6 - Jun 1, 2001, with history (red), U (green), S (blue), R (black).

In the future, the model will be used as a predictive tool to help understanding which parameters/events most deeply influence the quality of the structure, and which adjustments, on different time scales, may help improving the ward performance.

REFERENCES

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