

$$\vec{p}_a^2 = \vec{p}_b^2 + p_b^2 + 2|\vec{p}_b| |p_b|$$

4)

$$|\vec{p}_a| = |\vec{p}_b| + |p_b| \quad M|\vec{v}_a| = M|\vec{v}_b| + m|v_b|$$

5)

$$|v_b| = \frac{M(|v_a| - |\vec{v}_b|)}{m}$$

6)

Dall'energia cinetica 1)

$$M|\vec{v}_a|^2 = M|\vec{v}_b|^2 + m \frac{M^2}{m^2} (|\vec{v}_a|^2 + |\vec{v}_b|^2 - 2|\vec{v}_a||\vec{v}_b|)$$

$$\left(M + \frac{M^2}{m}\right) |\vec{v}_b|^2 - 2 \frac{M^2}{m} |\vec{v}_a||\vec{v}_b| - \left(M - \frac{M^2}{m}\right) |\vec{v}_a|^2 = 0$$

$$|\vec{v}_b| = \frac{1}{2 \left(M + \frac{M^2}{m}\right)} \left[ +2 \frac{M^2}{m} |\vec{v}_a| \pm \left( 4 \frac{M^4}{m^2} |\vec{v}_a|^2 + 4 \left(M + \frac{M^2}{m}\right) \left(M - \frac{M^2}{m}\right) |\vec{v}_a|^2 \right)^{1/2} \right]$$

$$= \frac{1}{2} \frac{m}{mM + M^2} \left[ +2 \frac{M^2}{m} |\vec{v}_a| \pm |\vec{v}_a| 2 \left( \frac{M^2}{m^2} + M^2 - \frac{M^4}{m^2} \right)^{1/2} \right]$$

$$= \frac{1}{2} \frac{m}{mM + M^2} \left[ \frac{+M^2}{m} \pm M \right] |\vec{v}_a|$$

con segno +  $|\vec{v}_a| = |\vec{v}_b|$   
 con segno - ottengo

$$= \frac{m}{mM + M^2} \frac{-Mm + M^2}{m} |\vec{v}_a| = \frac{M-m}{M+m} |\vec{v}_a| = |\vec{v}_b|$$

7)

