Massima energia trasferita

$$w_{max} = E_a - E_b = \frac{1}{2} M \left(\vec{V}_a^2 - \vec{V}_b^2 \right) = \frac{1}{2} M |\vec{V}_a|^2 \left(1 - \left(\frac{M-m}{M+m} \right)^2 \right)$$

$$= E \left[\frac{M^2 + m^2 + 2Mm - M^2 - m^2 + 2Mm}{(M+m)^2} \right] = E \frac{4mM}{(M+m)^2}$$ 8

Per $m = M$ $w_{max} = E$ tutta l'energia trasferita in un urto

Per $M = m_\mu = 207 m_e$

$$\frac{w_{Max}}{E} = \frac{4 m_e (207 m_e)}{(208 m_e)^2} = 0.0192$$ 9

Espressione relativistica

$$w_{max}^{rel} = \frac{2\gamma^2 m v^2}{1 + 2\gamma \frac{m}{M} + \frac{m^2}{M^2}}$$ 10

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c}$$

$$E^2 = M^2 c^4 + p^2 c^2 \qquad E^2 - M^2 c^4 = p^2 c^2 = M^2 v^2 c^2$$

$$E = E_{kin} + Mc^2 \qquad v^2 c^2 = \frac{E^2}{M^2} - c^4$$

$$\beta^2 = \left(\frac{v}{c} \right)^2 = \frac{E^2}{M^2 c^4} - 1 = \frac{(E_{kin} + Mc^2)^2}{M^2 c^4} - 1$$ 11